Spontaneous emission near the edge of a photonic band gap

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We study spontaneous emission near the edge of a photonic band gap. Instead of a simple exponential decay in the vacuum, spontaneous emission displays an oscillatory behavior. A single photon-atom bound dressed state exhibits a fractional steady-state atomic population on the excited state. For a three-level atom we evaluate the spectral splitting and subnatural linewidth of spontaneous emission. In the presence of $N-1$ unexcited atoms we show that the collective time scale factor is equal to N^ϕ, where $\phi = \frac{2}{3}$ for an isotropic band gap and $\phi = 1$ or 2 for anisotropic two-dimensional or three-dimensional band edges, respectively.

I. INTRODUCTION

Spontaneous emission is a fundamental concept in atomic physics. A new generation of experiments reveals that spontaneous radiation from excited atoms can be greatly suppressed or enhanced by placing the atoms between mirrors or in cavities [1,2]. The spectrum of spontaneous emission can display vacuum Rabi splitting in a resonant high-Q cavity [3].

This modification of spontaneous emission arises from the fact that a dielectric cavity acts as a local resonance mode for electromagnetic wave propagation. There is a long lifetime for radiation injected into the cavity and a corresponding scattering resonance for radiation outside of the cavity. The extent of isolation of modes inside the resonator from modes outside is measured by the quality factor of the cavity.

A perfect isolation of electromagnetic modes is possible if a localized state of light can be formed. It has been suggested that such states may arise in strongly scattering dielectric microstructures [4]. In a periodic dielectric array, this can be facilitated by a synergetic interplay between the microcavity resonances of individual dielectric particles and the Bragg scattering resonance of the array. This leads to the formation of a photonic band gap (PBG). It was suggested that this would be accompanied by the inhibition of spontaneous emission [5] as well as the formation of strongly localized states of light [6]. The existence of photonic band gaps has been demonstrated both computationally [7] and experimentally [8]. Furthermore, when an atom with a resonant transition within the frequency gap is placed in the photonic band-gap material, it has been predicted that the excited atom forms a photon-atom bound state [9,10], the optical analog of an electron-impurity level bound state in the gap of a semiconductor.

In this paper, we derive the dynamical and spectral properties of spontaneous emission near the edge of a PBG. Instead of a simple exponential decay as it is in the vacuum, spontaneous emission displays an oscillatory behavior. A photon-atom bound dressed state occurs even when the atomic resonant frequency lies outside

(near) the gap. This bound dressed state leads to a fractional steady-state of the single-atom population in the excited state. For a three-level atom, this fractionalized state can be probed experimentally. In particular, we derive the spectral splitting and subnatural linewidth of spontaneous emission into the third level. We also investigate spontaneous emission of an excited two-level atom in the presence of $N-1$ unexcited atoms. In particular, we find that the collective time scale factor for emission is proportional to N^ϕ, where $\phi = \frac{2}{3}$ for an isotropic band gap and $\phi = 1$ or 2 for anisotropic two- and three-dimensional (3D) band edges. This feature is distinct from both cavity QED [11,12] and the free space cases [13], where the collective scale factors are equal to $N^{1/2}$ and N, respectively.

II. SINGLE-ATOM SPONTANEOUS EMISSION

We begin by investigating a two-level atom coupled to the radiation field in a three-dimensional periodic dielectric [9]. The atom has excited state $|2\rangle$, ground state $|1\rangle$, and resonant transition frequency ω_{21}.

The Hamiltonian of the system in the interaction picture takes the form

$$H = \sum_\lambda \hbar \Delta_\lambda a_\lambda^\dagger a_\lambda + i\hbar \sum_\lambda g_\lambda (a_\lambda^\dagger \sigma_{12} - \sigma_{21} a_\lambda) , \quad (2.1)$$

where $\sigma_{ij} = |i\rangle\langle j|$ $(i,j = 1,2)$ are the atomic operators; a_λ and $a_\lambda^\dagger$ are the radiation field annihilation and creation operators; $\Delta_\lambda = \omega_\lambda - \omega_{21}$ is a detuning of the radiation mode frequency ω_λ from the atomic resonant frequency ω_{21} and g_λ is the atomic field coupling constant

$$g_\lambda = \frac{\omega_{21} d_{21}}{\hbar} \left[\frac{\hbar}{2\epsilon_0 \omega_\lambda V} \right]^{1/2} e_\lambda \cdot u_d . \quad (2.2)$$

Here d_{21} and u_d are the absolute value and unit vector of the atomic dipole moment, V is the sample volume, $e_\lambda \equiv e_{k,\sigma}$ are the two transverse (polarization) unit vectors, and ϵ_0 is the Coulomb constant.

Assume the atom is initially on the excited state $|2\rangle$ and the field is in the vacuum state. The wave function of the system then has the form

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$$|\psi(t)\rangle = b_2(t)|2,\{0\}\rangle + \sum_\lambda b_{1,\lambda}(t)|1,\{\lambda\}\rangle e^{-i\Delta_\lambda t} . \tag{2.3}$$

The state vector $|2,\{0\}\rangle$ describes the atom in its excited state $|2\rangle$ and no photons present, whereas the state vector $|1,\{\lambda\}\rangle$ describes the atom in its ground state $|1\rangle$ and a single photon in mode $\{\lambda\}$.

The time-dependent Schrödinger equation projected on the one-photon sector of the Hilbert space takes the form

$$\frac{d}{dt}b_2(t) = -\sum_\lambda g_\lambda b_{1,\lambda}(t)e^{-i\Delta_\lambda t} , \tag{2.4}$$

$$\frac{d}{dt}b_{1,\lambda}(t) = g_\lambda b_2(t)e^{i\Delta_\lambda t} . \tag{2.5}$$

The formal solution of Eq. (2.5) is

$$b_{1,\lambda}(t) = g_\lambda \int_0^t b_2(t')e^{i\Delta_\lambda t'}dt' . \tag{2.6}$$

Substituting Eq. (2.6) into Eq. (2.4) we have

$$\frac{d}{dt}b_2(t) = -\sum_\lambda g_\lambda^2 \int_0^t b_2(t')e^{-i\Delta_\lambda(t-t')}dt' . \tag{2.7}$$

The Laplace transform

$$\tilde{b}_2(s) = \int_0^\infty e^{-st}b_2(t)dt , \tag{2.8}$$

can be found from Eq. (2.7) as

$$\tilde{b}_2(s) = \left[s + \sum_\lambda g_\lambda^2 \frac{1}{s+i(\omega_\lambda - \omega_a)} \right]^{-1} . \tag{2.9}$$

Converting the mode sum over transverse plane waves into an integral and performing the angular integral we obtain

$$\tilde{b}_2(s) = \left[s + \frac{\omega_{21}^2 d_{21}^2}{6\pi^2\epsilon_0\hbar} \int_0^\Lambda \frac{k^2 dk}{\omega_k[s+i(\omega_k - \omega_{21})]} \right]^{-1} . \tag{2.10}$$

Here, Λ is the cutoff in the photon wave vector [14] since photons of energy higher than the electron rest mass mc^2 probe the relativistic structure of the electron wave packet.

For a broadband of the density of states, such as in vacuum, one can use the Wigner-Weisskopf approximation [15]. That is, only the pole contribution of $s(s \to 0^+)$ in the integral of Eq. (2.10) is retained:

$$\lim_{s\to 0^+} \frac{1}{s+i(\omega_k - \omega_a)} = -i\mathrm{P}\frac{1}{\omega - \omega_a} + \pi\delta(\omega_k - \omega_a) . \tag{2.11}$$

Substituting Eq. (2.11) into Eq. (2.10) yields

$$\tilde{b}_2(s) = [s + i\delta_{21} + \tfrac{1}{2}\gamma_{21}]^{-1} , \tag{2.12}$$

where δ_{21} and γ_{21} are the usual Lamb shift and spontaneous emission rate, respectively [15]. Clearly, the spontaneous emission decay is purely exponential.

The Wigner-Weisskopf perturbation theory, however, is inadequate when the density of electromagnetic modes changes rapidly in the vicinity of the atomic transition

frequency ω_{21}. In this case, we must perform an exact integration in Eq. (2.10). For the purpose of discussion we consider a simple model Hamiltonian for electromagnetic waves in a three-dimensional periodic dielectric. The photon dispersion relation ω_k is chosen to be isotropic and satisfies the transcendental equation

$$4n\cos(kL) = (1+n)^2\cos[(2na+b)(\omega_k/c)]$$
$$-(1-n)^2\cos[(2na-b)(\omega_k/c)] . \tag{2.13}$$

This is obtained from the exact solution of the scalar wave equation with dielectric constant [9,16]

$$\epsilon(x) = \sum_{m=-\infty}^\infty u(x-mL) , \tag{2.14}$$

where

$$u(x) = \begin{cases} n^2 - 1 & \text{if } |x| < a \\ 0 & \text{otherwise.} \end{cases} \tag{2.15}$$

Here n is the refractive index of the scatterer, a is its radius, and $2a+b=L$ is the lattice constant. For the special case $2na=b$, ω_k can be found analytically from Eq. (2.13) as

$$\omega_k = \frac{c}{4na}\arccos\left[\frac{4n\cos(kL)+(1-n)^2}{(1+n)^2}\right] . \tag{2.16}$$

By symmetrizing ω_k given in Eq. (2.16) to all directions in k space, we produce photonic band gaps at the spheres $|\mathbf{k}| = m\pi/L$ with $m = 1,2,3\ldots$. Near the band-gap edges the density of states becomes singular [9], the atom-field interaction becomes strong, and we can expect dynamical features of spontaneous emission decay. For $k \cong k_0 \equiv \pi/L$ the dispersion relation near the band edge ω_c can be approximated by [9]

$$\omega_k = \omega_c + A(k-k_0)^2 , \tag{2.17}$$

where $A \cong \omega_c/k_0^2$.
Using Eq. (2.17), we evaluate Eq. (2.10) as

$$\tilde{b}_2(s) = \frac{(s-i\delta)^{1/2}}{s(s-i\delta)^{1/2} - (i\beta)^{3/2}} . \tag{2.18}$$

Here

$$\beta^{3/2} = \frac{\omega_{21}^{7/2}d^2}{6\pi\epsilon_0\hbar c^3} , \tag{2.19}$$

and $\delta = \omega_{21} - \omega_c$. The amplitude $b_2(t)$ is given by the inverse Laplace transform

$$b_2(t) = \frac{1}{2\pi i}\int_{\epsilon-i\infty}^{\epsilon+i\infty} e^{st}\tilde{b}_2(s)ds , \tag{2.20}$$

where the real number ϵ is chosen so that $s = \epsilon$ lies to the right of all the singularities (poles and branch point) of function $\tilde{b}_2(s)$. The inverse Laplace transform of (2.18) yields

$$b_2(t) = 2a_1 x_1 e^{\beta x_1^2 t + i\delta t} + a_2(x_2 + y_2)e^{\beta x_2^2 t + i\delta t}$$

$$- \sum_{j=1}^{3} a_j y_j [1 - \Phi(\sqrt{\beta x_j^2 t})]e^{\beta x_j^2 t + i\delta t}, \qquad (2.21)$$

where

$$x_1 = (A_+ + A_-)e^{i(\pi/4)}, \qquad (2.22)$$

$$x_2 = (A_+ e^{-i(\pi/6)} - A_- e^{i(\pi/6)})e^{-i(\pi/4)}, \qquad (2.23)$$

$$x_3 = (A_+ e^{i(\pi/6)} - A_- e^{-i(\pi/6)})e^{i(3\pi/4)}, \qquad (2.24)$$

$$A\pm = \left[\frac{1}{2} \pm \frac{1}{2}\left[1 + \frac{4}{27}\frac{\delta^3}{\beta^3}\right]^{1/2}\right]^{1/3} \qquad (2.25)$$

$$a_j = \frac{x_j}{(x_j - x_i)(x_j - x_k)} \quad (j \neq i \neq k; j, i, k = 1, 2, 3), \qquad (2.26)$$

$$y_j = \sqrt{x_j^2} \quad (j = 1, 2, 3), \qquad (2.27)$$

and $\Phi(x)$ is the error function, whose series and asymptotic representations are given in Ref. [17]. For $\delta = 0$ one can find from Eq. (2.22) that $\beta x_1^2 = i\beta$. That is, the value β given in Eq. (2.19) can be considered as a resonant frequency splitting, an analog of the vacuum Rabi splitting in cavity quantum electrodynamics [3]. For large βt, the terms of higher order than $(\beta t)^{-3/2}$ can be ignored, and Eq. (2.21) reduces to

$$b_2(t) \cong 2a_1 x_1 e^{\beta x_1^2 t + i\delta t} + a_2(x_2 + y_2)e^{\beta x_2^2 t + i\delta t}$$

$$+ \frac{1}{2\sqrt{\pi}}\left[\sum_{j=1}^{3}\frac{a_j}{x_j^2}\right]\frac{e^{i\delta t}}{(\beta t)^{3/2}}. \qquad (2.28)$$

Clearly from Eqs. (2.21) and (2.28) the atomic level splits into dressed states caused by the strong interaction between the atom and its own radiation. Using Eqs. (2.22)–(2.25), it follows that $y_2 = x_2$ if

$$\delta > \frac{-3\beta(2 + \sqrt{3})}{[1 + (2 + \sqrt{3})^3]^{2/3}}. \qquad (2.29)$$

It follows that these dressed states occur at frequencies $\omega_c - \beta\,\mathrm{Im}(x_1^2)$ and $\omega_c - \beta\,\mathrm{Im}(x_2^2)$. In the case when ω_{21} is far inside the gap so that the inequality (2.29) is not satisfied we have $y_2 = -x_2$ and the second term in Eqs (2.21) and (2.28) vanishes, i.e., there is no true atomic level splitting. Clearly from Eq. (2.28), the branch point contribution yields the "quasidressed" (not truly exponential) state at the band-edge frequency ω_c. The interference between the dressed states and "quasidressed" state leads to oscillatory behavior of spontaneous emission decay. This is depicted in Fig. 1(a). This phenomenon is quite distinct from a simple exponential decay which occurs in the vacuum.

Using Eqs. (2.22)–(2.25) we can also show that $x_1^2 = i|x_1|^2$. That is, the corresponding dressed state at the frequency $\omega_c - \beta|x_1|^2$ is the photon-atom bound dressed state with no decay. A photon which is emitted by the atom in this dressed state will exhibit tunneling on

a length scale given by the localization length before being Bragg reflected back to the emitting atom. The photon-atom bound state *inside* the band gap has been predicted in Ref. [9]. We emphasize here that the photon-atom bound dressed state is present even when the resonant atomic frequency ω_{21} lies *outside* the band gap. This bound dressed state leads to the fractionalized steady-state atomic population in the excited state $P_s = \lim_{t \to \infty}|b_2(t)|^2 = 4|a_1 x_1|^2$ [Fig. 1(b)]. Clearly, the nonzero steady-state atomic population in the excited state is present even when the resonant atomic frequency ω_{21} lies *outside* the band gap, where the density of states is not equal zero. The atomic level splitting, oscillatory behavior, and fractionalized steady-state atomic population in the excited state are all direct consequences of strong interaction between the atom and its own localized radiation when the atomic resonant frequency lies near
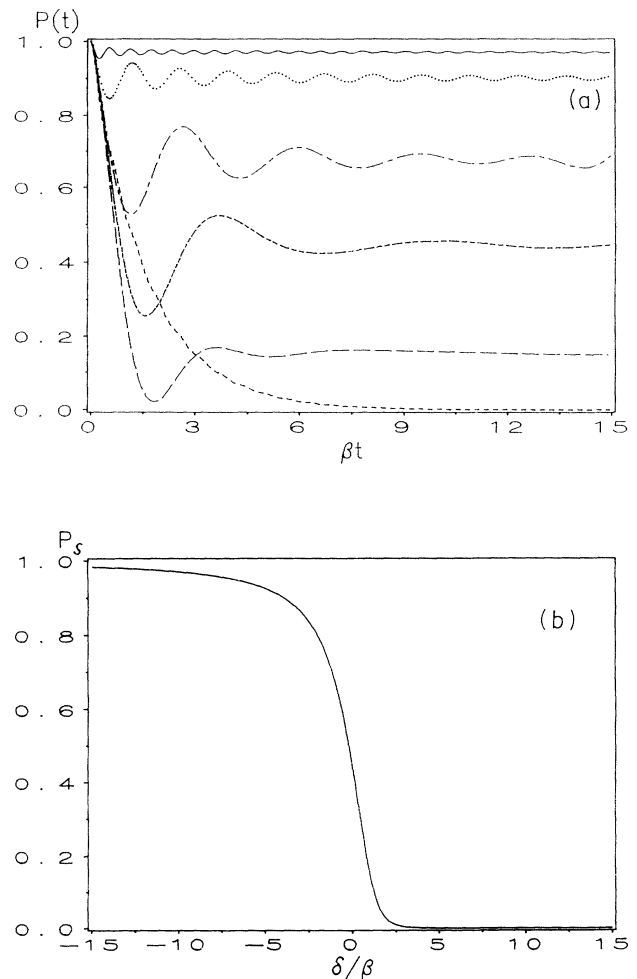

FIG. 1. (a) Atomic population on the excited state, $P(t) = |b_2(t)|^2$ as a function of βt and for various values of detuning from photonic band edge $\delta = -10\beta$ (solid curve), $\delta = -4\beta$ (dotted curve), $\delta = -\beta$ (long-short-dashed curve), $\delta = 0$ (long-short-short-dashed curve), $\delta = \beta$ (long-dashed curve), and $\delta = 10\beta$ (short-dashed curve). (b) Steady-state atomic population $P_s = \lim_{t \to \infty}|b_2(t)|^2 = 4|a_1 x_1|^2$ as a function of δ/β.

the edge of a perfect photonic band gap. These properties are strongly dependent on the detuning of the resonant atomic frequency ω_{21} from the band-edge frequency ω_c. Physically, the atom exchanges energy back and forth with its own radiation, backscattered after tunneling a localized distance. This in turn is a result of the vacuum Rabi splitting of the atomic level by the photonic band edge. One level of the doublet is a localized state within the photonic band gap, whereas the other level is a resonance in the extended state continuum. The frequency of oscillations is directly determined by the magnitude of the atomic level splitting. It is distinct from the well-known Jaynes-Cummings oscillations which arise from the interaction of the atom with an isolated cavity or dielectric mode [18,19]. In our model, no defect mode is present.

The excited-state population density can be measured via absorption of a probe beam at different decay times [20]. Alternatively, the nature of the fractionalized excited state can be probed by spontaneous emission from the excited state into the third level $|3\rangle$ of a λ configuration (Fig. 2). Assume that the transition frequency ω_{23} lies far from the gap, so that we can use the Wigner-Weisskopf approximation (2.11) for spontaneous emission $|2\rangle \rightarrow |3\rangle$. The Laplace transform $\tilde{b}_2(s)$, Eq. (2.8), can be found in this case as

$$\tilde{b}_2(s) = \frac{(s-i\delta)^{1/2}}{s(s-i\delta)^{1/2}+(i\delta_{23}+\frac{1}{2}\gamma_{23})(s-i\delta)^{1/2}-(i\beta)^{3/2}},$$

(2.30)

where δ_{23} and γ_{23} are the Lamb shift and spontaneous emission decay of the transition $|2\rangle \rightarrow |3\rangle$. The inverse Laplace transform of Eq. (2.30) gives exactly the same form of $b_2(t)$ in Eq. (2.21) except δ in Eq. (2.25) must be replaced by $\delta + \delta_{23} - (i/2)\gamma_{23}$. The influence of the third level $|3\rangle$ on the dynamics of spontaneous emission is shown in the Fig. 3. Clearly, oscillations in spontaneous emission decay occur even for relatively large values of γ_{23}.

The spectrum of spontaneous emission $|2\rangle \rightarrow |3\rangle$ also exhibits interesting properties. This spectrum is given by

$$S(\omega_\lambda) \sim |\tilde{b}_2[-i(\omega_\lambda - \omega_{23})]|^2 ,$$
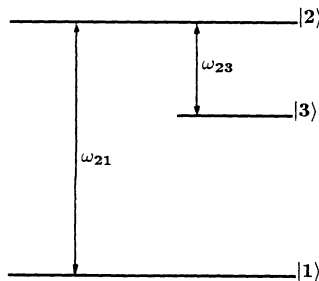
(2.31)

FIG. 2. Three-level atom of a λ configuration. The transition frequency ω_{21} lies near the band edge while ω_{23} is assumed to be far from the gap.
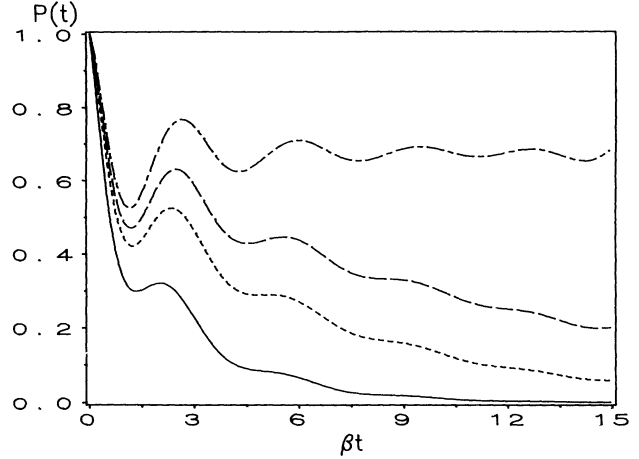
FIG. 3. Atomic population on the excited level $|2\rangle$ of a three-level atom as a function of βt for $\delta = -\beta$, Lamb shift $\delta_{23} = 0$ and for various values of the spontaneous emission decay rate $\gamma_{23} = 0$ (short-long-dashed curve), $\gamma_{23} = 0.1\beta$ (long-dashed curve), $\gamma_{23} = 0.2\beta$ (short-dashed curve), and $\gamma_{23} = 0.5\beta$ (solid curve).

where $\tilde{b}_2(s)$ is given in Eq. (2.30). In Fig. 4 we plot spectrum $S(\omega_\lambda)$ for different values of $\delta = \omega_{21} - \omega_c$. Clearly, the spectrum $S(\omega_\lambda)$ splits into a doublet with peaks at $\omega_{23} - \beta \operatorname{Im}(x_1^2) - \delta$ and $\omega_{23} - \beta \operatorname{Im}(x_2^2) - \delta$. This splitting is analogous to the Autler-Townes splitting [21]. In our case, however, there is no external field and splitting is caused entirely by strong interaction between the atom and its own radiation field. The linewidth of the left sideband can be much smaller than γ_{21} (solid curve in Fig. 4), the natural linewidth of the spontaneous transition $|2\rangle \rightarrow |3\rangle$.
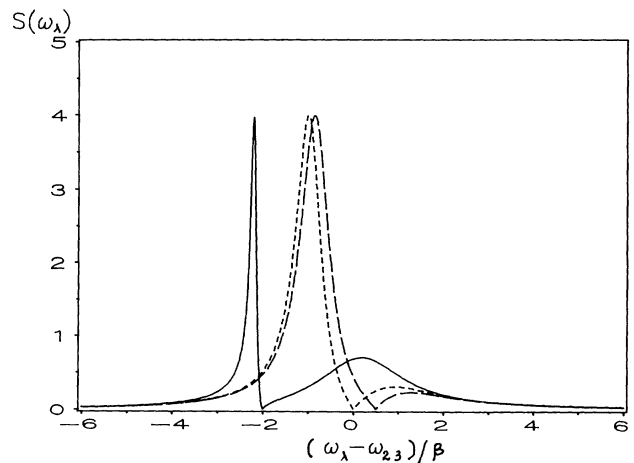
FIG. 4. Autler-Townes spectrum $S(\omega_\lambda)$ of spontaneous emission $|2\rangle \rightarrow |3\rangle$ (in a system of the resonant frequency splitting $\beta = 1$) for $\delta_{23} = 0$, $\gamma_{23} = 1$, and for various values of detuning from the band edge $\delta = -0.5$ (long-dashed curve), $\delta = 0$ (short-dashed curve) and $\delta = 2$ (solid curve).

III. INFLUENCE OF $N-1$ UNEXCITED ATOMS

In this section we investigate spontaneous emission of an excited two-level atom in a presence of $N-1$ unexcited atoms in a PBG. We limit our studies to the Dicke model [22] in a perfect photonic band gap. The Hamiltonian (2.1) for the multiatom case is given by

$$H = \sum_\lambda \hbar\Delta_\lambda a_\lambda^\dagger a_\lambda + i\hbar\sum_\lambda g_\lambda(a_\lambda^\dagger J_{12} - J_{21}a_\lambda) , \tag{3.1}$$

where

$$J_{ij} = \sum_{k=1}^{N} \sigma_{ij}^{(k)} \quad (i,j=1,2) . \tag{3.2}$$

Assume that the atomic system is initially in the symmetrical superradiant state [22] $|J, M = 1-J\rangle$ with only one atom being in the excited state and the field is in the vacuum state. Here $|J, M\rangle$ states are the normalized eigenstates of operators $J_3 = \frac{1}{2}(J_{22} - J_{11})$ and $J^2 = \frac{1}{2}(J_{21}J_{12} + J_{12}J_{21}) + J_3^2$. The wave function of the system than has the form

$$|\psi(t)\rangle_N = b_{2N}|J, M = 1-J, \{0\}\rangle$$
$$+ \sum_\lambda b_{1N,\lambda}(t)|J, M = -J, \{\lambda\}\rangle e^{-i\Delta_\lambda t} . \tag{3.3}$$

The state vector $|J, M = 1-J, \{0\}\rangle$ describes the atomic system in the state $|J, M = 1-J\rangle$ and no photons present, whereas the state vector $|J, M = 1-J, \{\lambda\}\rangle$ describes that all atoms are in their ground state and a single photon in mode $\{\lambda\}$.

The time-dependent Schrödinger equation projected on the one-photon sector of the Hilbert space takes the form

$$\frac{d}{dt}b_{2N}(t) = -\sqrt{N}\sum_\lambda g_\lambda b_{1N,\lambda}(t)e^{-i\Delta_\lambda t} , \tag{3.4}$$

$$\frac{d}{dt}b_{1N,\lambda}(t) = \sqrt{N}g_\lambda b_{2N}(t)e^{i\Delta_\lambda t} . \tag{3.5}$$

Equations (3.4) and (3.5) have the same forms as Eqs. (2.4) and (2.5) except the factor $\sqrt{N}$. The Laplace transform $\tilde{b}_{2N}(s)$ can be found from Eqs. (3.4) and (3.5) as

$$\tilde{b}_{2N}(s) = \left[s + N\frac{\omega_{21}^2 d_{21}^2}{6\pi^2\epsilon_0\hbar}\int_0^\Lambda \frac{k^2 dk}{\omega_k[s+i(\omega_k-\omega_{21})]}\right]^{-1} . \tag{3.6}$$

In free space with a continuous broadband density of state we can use the Wigner-Weisskopf approximation (2.11). The atomic population in the excited state is given by

$$|b_{2N}(t)|^2 = e^{-N\gamma_{21}t} . \tag{3.7}$$

That is, the collective time scale factor is proportional to N.

In the case when the atomic resonant frequency lies near the edge of a PBG, we used the dispersion relation (2.17) and the Laplace transform $\tilde{b}_{2N}$ is found as

$$\tilde{b}_{2N}(s) = \frac{(s-i\delta)^{1/2}}{s(s-i\delta)^{1/2} - (i\beta_N)^{3/2}} , \tag{3.8}$$

where

$$\beta_N^{3/2} = N\beta^{3/2} . \tag{3.9}$$

The inverse Laplace transform of (3.8) yields

$$b_{2N}(t) = 2a_1 x_1 e^{\beta N^{2/3}x_1^2 t + i\delta t}$$
$$+ a_2(x_2 + y_2)e^{\beta N^{2/3}x_2^2 t + i\delta t}$$
$$- \sum_{j=1}^3 a_j y_j [1 - \Phi(\sqrt{\beta N^{2/3}x_j^2 t})]$$
$$\times e^{\beta N^{2/3}x_j^2 t + i\delta t} , \tag{3.10}$$

where the qualities x_j, y_j, and a_j $(j=1,2,3)$ are given in Eqs. (2.22)–(2.27) except that β in Eq. (2.25) is replaced by $N^{2/3}\beta$. Clearly $b_{2N}(t)$ has the same form as $b_2(t)$ except with the factor β of a single atom case replaced by $N^{2/3}\beta$. This means that in the presence of $N-1$ unexcited atoms, the resonant frequency splitting increases by the factor of $N^{2/3}$. For $\delta=0$ one can find from Eq. (2.33) that $\beta N^{2/3}x_2^2 = -\frac{1}{2}(i\sqrt{3}+1)\beta N^{2/3}$. Clearly the rate of spontaneous emission is also increased by a factor of $N^{2/3}$. In the case when atoms are identical but distinguishable, an effect similar to "radiation trapping" predicted for the case when atoms are in a cavity [11] may occur. That is, the system will not emit its energy as the number of atoms N becomes very large, and the energy is trapped in the single atom.

IV. COLLECTIVE SCALE FACTOR IN ANISOTROPIC BAND GAPS

The collective time scale factor $N^{2/3}$ was found in previous section using the isotropic model of a PBG described by Eq. (2.16). The exponent of N, however depends sensitively on the dimension of the phase space occupied by band-edge photons of vanishing group velocity and the resulting band-edge singularity in the overall photon density of states. In an isotropic band edge, we have overestimated this phase space using the entire sphere $|\mathbf{k}| = \pi/L$. For a real dielectric crystal in three dimensions with an allowed point-group symmetry, the band edge is associated with a specific point $\mathbf{k} = \mathbf{k}_0$ rather than the entire sphere $|\mathbf{k}| = |\mathbf{k}_0|$ [6]

$$\omega_\mathbf{k} \cong \omega_c + A(\mathbf{k} - \mathbf{k}_0)^2 . \tag{4.1}$$

The dispersion relation (4.1) leads to a photonic density of states $\rho(\omega)$ at a band edge ω_c which behaves as $(\omega - \omega_c)^{(d/2)-1}$ for $\omega \geq \omega_c$. Here d is the band-edge dimension [9]. In this section we discuss briefly the influence of the anisotropic dispersion relation on the collective scale factor. The Laplace transform $\tilde{b}_{2N}(s)$ in this case can be found as

$$\tilde{b}_{2N}(s) = \left[s + \frac{N\alpha}{(2\pi)^d}\int \frac{d^d\mathbf{k}}{\omega_\mathbf{k}[s+i(\omega_\mathbf{k}-\omega_{21})]}\right]^{-1} , \tag{4.2}$$

where

$$\alpha \cong \frac{\omega_{21}^2 d_{21}^2}{2\epsilon_0\hbar} , \tag{4.3}$$

and $d=1,2,3$ is the band-edge dimension. Using the dispersion relation (4.1) and changing variables of integration to $q=k-k_0$ we can write Eq (4.2) in the form

$$\tilde{b}_{2N}(s) = \left[s - \frac{iN\alpha}{(2\pi)^d} \int \frac{d^d q}{(\omega_c + Aq^2)[Aq^2 - is]} \right]^{-1}. \quad (4.4)$$

For simplicity we assume $\omega_c = \omega_{21}$, i.e., the atomic resonance frequency lies at the edge of a PBG. The integral in Eq. (4.4) can be evaluated by contour integration and $\tilde{b}_{2N}(s)$ is given by

$$\tilde{b}_{2N}(s) = \begin{cases} \left[s - i^{3/2} \dfrac{N\alpha_1}{\sqrt{s}} \right]^{-1} & \text{if } d=1 \\[2ex] \left[s - iN\alpha_2 \ln\left[\dfrac{i\omega_c}{s} \right] \right]^{-1} & \text{if } d=2 \\[2ex] [s - iN\alpha_3 + \sqrt{i}N\alpha_4\sqrt{s}]^{-1} & \text{if } d=3, \end{cases} \quad (4.5)$$

where

$$\alpha_1 = \frac{\alpha}{2\sqrt{A}\,\omega_c}, \quad (4.6)$$

$$\alpha_2 = \frac{\alpha}{4\pi A \omega_c}, \quad (4.7)$$

$$\alpha_3 = \frac{\alpha}{4\pi A^{3/2}\sqrt{\omega_c}}, \quad (4.8)$$

$$\alpha_4 = \frac{\alpha}{4\pi A^{3/2}\omega_c}. \quad (4.9)$$

Dynamical properties of spontaneous emission can be studied in detail from the inverse Laplace transform of $\tilde{b}_{2N}(s)$ given in Eq. (4.5). The most important distinction of anisotropic band edges from isotropic bandgaps is in the collective scale factor, which we can evaluate directly from Eq. (4.5). For 1D band edges, $\tilde{b}_{2N}(s)$ has a pole

when $s^{3/2} \sim N$, that is the collective scale factor is equal $N^{2/3}$, the same as for the isotropic band gap. For 2D band edges, $\tilde{b}_{2N}(s)$ has a pole $s \sim N$ with a weak logarithmic correction, i.e., the collective scale factor is approximately equal to N as for free space. In the 3D band-edge case, $\tilde{b}_{2N}(s)$ has a pole $s \sim N^2$. As a result the collective scale factor becomes N^2. The last case may lead to the intensity of superradiance being proportional to N^3, that is much more intense than Dicke superradiance. This problem will be discussed in detail elsewhere.

V. CONCLUSIONS

In summary, we have derived a number of features of spontaneous emission from two and three-level atoms in which one transition frequency lies near the edge of a PBG. These include temporal oscillations, fractionalized steady-state atomic population on the excited state, spectral splitting and subnatural linewidth. The influence of N-1 unexcited atoms has also been considered. In particular the collective time scale factor is equal to N^ϕ, where $\phi = \frac{2}{3}$ for an isotropic bandgap and $\phi = 1$ or 2 for anisotropic 2D or 3D band edges, respectively. These are all direct consequences of photon localization as embodied in the photon-atom bound state. One possible experimental realization of these effects may arise from laser-cooled atoms in the void regions of a photonic band-gap material. Another possibility is the use of an organic impurity molecule such as pentacene. It has been shown that such molecules exhibit very narrow linewidths when placed in appropriate solid hosts [23].

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