Spontaneous excitation of an accelerated atom: The contributions of vacuum fluctuations and radiation reaction

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We consider an atom in interaction with a massless scalar quantum field. We discuss the structure of the rate of variation of the atomic energy for an arbitrary stationary motion of the atom through the quantum vacuum. Our main intention is to identify and to analyze quantitatively the distinct contributions of vacuum fluctuations and radiation reaction to the spontaneous excitation of a uniformly accelerated atom in its ground state. This gives an understanding of the role of the different physical processes underlying the Unruh effect. The atom's evolution into equilibrium and the Einstein coefficients for spontaneous excitation and spontaneous emission are calculated.

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I. INTRODUCTION

Spontaneous emission is one of the most prominent effects in the interaction of atoms with radiation. Two heuristic pictures have been put forward to explain why an atom in an excited state loses energy and radiates. The first one is inspired by classical electrodynamics: It is well known that, classically, accelerated electrons in atoms radiate. The radiation field reacts back on the atom, causing a loss of atomic energy. This is called *radiation reaction*. However, the fact that the rate of change of an atom's internal energy is always negative leads to the instability of classical atoms with unacceptable consequences.

In a semiclassical theory, on the other hand, where quantum-mechanical atoms interact with a classical radiation field, only stimulated emission and absorption are predicted; spontaneous emission is not present. According to such a theory, excited atoms in the vacuum do not radiate. This has lead to the idea that spontaneous emission is connected with the quantum fluctuations of the radiation field. In particular, spontaneous emission has been interpreted as stimulated emission induced by *vacuum fluctuations* [1]. When making the argument quantitative, however, the following question arises: since stimulated emission and absorption have equal Einstein B coefficients, why do vacuum fluctuations not induce "spontaneous absorption" [2]?

Quantum field theoretical investigations of the roles of vacuum fluctuations and radiation reaction in spontaneous emission have been carried out since 1973 [3–7]. A Heisenberg picture approach has always been used, since it allows an easy comparison of quantum-mechanical and classical concepts. In these studies, the notion of vacuum fluctuations was connected with the free solutions of the Heisenberg equations for the quantum field, i.e., the field that is present even in the vacuum. Radiation reaction was incorporated via the source field, which is the part of the field caused by the presence of the atom itself. Surprisingly, it turned out that seemingly the contributions of vacuum fluctuations and radiation reaction can to a large extent be chosen arbitrarily, depending on the ordering of commuting atom and field variables.

Dalibard, Dupont-Roc, and Cohen-Tannoudji (DDC) argued in [8] and [9] that there exists a preferred operator ordering: only if one chooses a symmetric ordering are the distinct contributions of vacuum fluctuations and radiation reaction to the rate of change of an atomic observable separately Hermitian and able to possess an independent physical meaning. Using this prescription, one finds that for an atom in an excited state, vacuum fluctuations and radiation reaction contribute equally to the rate of change of the atomic excitation energy. For an atom in the ground state, on the other hand, the two contributions cancel precisely. There is a balance between vacuum fluctuations and radiation reaction that prevents transitions from the ground state and ensures its stability. In the same way, the formalism of DDC can be used to study separately the effects of the two physical mechanisms in a variety of situations. For example, there have been investigations of the radiative properties of atoms near a conducting plane [10,11] or of atomic level shifts in cavities [12]. All these considerations refer to an atom at rest.

In this paper, we want to study the modified effects of vacuum fluctuations and radiation reaction for an *accelerated* atom coupled to a quantum field in free space. It is well known that a uniformly accelerated atom in its ground state is spontaneously excited even in the vacuum [13] (cf. also [14]). This process, which is called the *Unruh effect*, is connected with the emission of a particle [15,16]. The intimate relation between spontaneous emission and the Unruh effect has been noted previously [17].

To understand the physical mechanisms responsible for the spontaneous excitation of an accelerated atom, it appears promising to apply the methods developed in the

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theory of spontaneous emission for inertial atoms to the case of accelerated atoms. We will identify the contributions of vacuum fluctuations and radiation reaction to the rate of change of the atomic Hamiltonian in the accelerated case. As we will see, the effect of vacuum fluctuations on the atom is changed by the acceleration whereas the contribution of radiation reaction is completely unaltered. This leads to the following picture: for an atom in an excited state, there will be spontaneous emission with a modified transition rate. For an atom in the ground state, however, the balance between vacuum fluctuations and radiation reaction is no longer perfect. Due to the modified vacuum fluctuation contribution, transitions to an excited state become possible even in the vacuum: this is the Unruh effect. In working out the respective details, we substantiate in a quantitative way the intuitive picture described by Sciama, Candelas, and Deutsch [18] generalizing the discussion in Ref. [19]. We mention that as far as the Unruh effect is concerned, an alternative discussion based on a classical notion of vacuum fluctuations has been given in [20].

The paper is organized as follows. In Sec. II we introduce the model of an atom coupled to the radiation field. For simplicity, we chose a two-level atom and a scalar quantum field. In Sec. III the Heisenberg equations of motion are derived and formally solved. In Sec. IV we identify the contributions of vacuum fluctuations and radiation reaction to the rate of change of an arbitrary atomic observable. We generalize the formalism of DDC [9] to an atom in arbitrary stationary motion. The special case of the evolution of the atom's excitation energy is considered. It is applied in Sec. V to the spontaneous emission from an inertial atom in vacuum and in a heat bath. In Sec. VI we treat a uniformly accelerated atom and discuss the physical reasons for its spontaneous excitation. Finally, in Sec. VII we derive the evolution of the atom's energy and the Einstein coefficients for spontaneous emission and the Unruh effect.

II. INTERACTION OF A TWO-LEVEL ATOM AND A SCALAR QUANTUM FIELD

We want to study the interaction of a two-level atom and a real scalar massless quantum field as a simplified model of quantum electrodynamics. $x \leftrightarrow (t, \vec{x})$ are the Minkowski coordinates referring to an inertial reference frame. We consider an atom on a stationary trajectory $x(\tau) = (t(\tau), \vec{x}(\tau))$, where τ denotes its proper time. Throughout the paper, the time evolution of the coupled system will be described with respect to τ. The stationary trajectory guarantees that the undisturbed atom has stationary states which are called $|-\rangle$ and $|+\rangle$, with energies $-\frac{1}{2}\omega_0$ and $+\frac{1}{2}\omega_0$ and a level spacing ω_0. The Hamiltonian that governs the time evolution of the atom with respect to τ can then be written in Dicke's [21] notation

$$H_A(\tau) = \omega_0 R_3(\tau), \tag{1}$$

where $R_3 = \frac{1}{2}|+\rangle\langle+| - \frac{1}{2}|-\rangle\langle-|$ and $\hbar = c = 1$.

The free Hamiltonian of the quantum field which generates the time evolution with regard to t is given by

$$\tilde{H}_F(t) = \int d^3k\, \omega_{\vec{k}}\, a^\dagger_{\vec{k}} a_{\vec{k}}. \tag{2}$$

$a^\dagger_{\vec{k}}$ and $a_{\vec{k}}$ are the creation and annihilation operators for a "photon" with momentum $\vec{k}$. $\tilde{H}_F(t)$ governs the time evolution of the field in t, the inertial time of the laboratory system. We change to the new time variable τ. Heisenberg's equations of motion show that the Hamiltonian with respect to τ is given by

$$H_F(\tau) = \int d^3k\, \omega_{\vec{k}}\, a^\dagger_{\vec{k}} a_{\vec{k}} \frac{dt}{d\tau}. \tag{3}$$

The decomposition of the field operator in terms of creation and annihilation operators reads

$$\phi(t, \vec{x}) = \int d^3k\, g_{\vec{k}} \left[a_{\vec{k}}(t) e^{i\vec{k}\vec{x}} + a^\dagger_{\vec{k}}(t) e^{-i\vec{k}\cdot\vec{x}} \right] \tag{4}$$

where $g_{\vec{k}} = [2\omega_{\vec{k}}(2\pi)^3]^{-\frac{1}{2}}$.

We couple the atom and the field by the scalar counterpart of the electric dipole interaction

$$H_I(\tau) = \mu R_2(\tau)\phi(x(\tau)). \tag{5}$$

The coupling is effective only on the trajectory $x(\tau)$ of the atom. μ is a coupling constant which we assume to be small. R_2 is a matrix that connects only different atomic states: $R_2 = \frac{1}{2}i(R_- - R_+)$, where $R_+ = |+\rangle\langle-|$ and $R_- = |-\rangle\langle+|$ are the atomic raising and lowering operators. The operators R_3 and $R_\pm$ obey angular momentum algebra: $[R_3, R_\pm] = \pm R_\pm$, $[R_+, R_-] = 2R_3$.

The Heisenberg equations of motion for the dynamical variables of the atom and the field can be derived from the Hamiltonian $H = H_A + H_F + H_I$:

$$\frac{d}{d\tau} R_\pm(\tau) = \pm i\omega_0 R_\pm(\tau) + i\mu\phi(x(\tau))[R_2(\tau), R_\pm(\tau)], \tag{6}$$

$$\frac{d}{d\tau} R_3(\tau) = i\mu\phi(x(\tau))[R_2(\tau), R_3(\tau)], \tag{7}$$

$$\frac{d}{dt} a_{\vec{k}}(t(\tau)) = -i\omega_{\vec{k}} a_{\vec{k}}(t(\tau)) + i\mu R_2(\tau)[\phi(x(\tau)), a_{\vec{k}}(t(\tau))]\frac{d\tau}{dt}. \tag{8}$$

In Eq. (8), t must be considered as a function of τ. We prefer to leave the commutators occurring in (6)–(8) unevaluated because this will simplify the physical picture in the later sections.

The solutions of the equations of motion can be split into two parts: (i) the *free part*, which is present even in the absence of the coupling, and (ii) the *source part*, which is caused by the interaction between atom and field and contains the coupling constant μ:

$$R_\pm(\tau) = R^f_\pm(\tau) + R^s_\pm(\tau),$$
$$R_3(\tau) = R^f_3(\tau) + R^s_3(\tau),$$
$$a_{\vec{k}}((\tau)) = a^f_{\vec{k}}(t(\tau)) + a^s_{\vec{k}}(t(\tau)).$$

Formal integration of the equations of motion yields

$$R^f_\pm(\tau) = R^f_\pm(\tau_0)e^{\pm i\omega_0(\tau-\tau_0)},$$
$$R^s_\pm(\tau) = i\mu \int_{\tau_0}^{\tau} d\tau'\, \phi^f(x(\tau'))[R^f_2(\tau'), R^f_\pm(\tau)], \tag{9}$$

$$R^f_3(\tau) = R^f_3(\tau_0),$$
$$R^s_3(\tau) = i\mu \int_{\tau_0}^{\tau} d\tau'\, \phi^f(x(\tau'))[R^f_2(\tau'), R^f_3(\tau)], \tag{10}$$

$$a^f_{\vec{k}}(t(\tau)) = a^f_{\vec{k}}(t(\tau_0))e^{-i\omega_{\vec{k}}[t(\tau)-t(\tau_0)]},$$
$$a^s_{\vec{k}}(t(\tau)) = i\mu \int_{\tau_0}^{\tau} d\tau'\, R^f_2(\tau')[\phi^f(x(\tau')), a^f_{\vec{k}}(t(\tau))]. \tag{11}$$

In the source parts of the solutions, all operators on the right-hand side have been replaced by their free parts, which is correct to first order in μ. From (11), we can construct the free and source part of the quantum field ϕ:

$$\phi^f(t(\tau), \vec{x}(\tau)) = \int d^3k\, g_{\vec{k}}[a_{\vec{k}}(0)e^{i\vec{k}\vec{x}(\tau)-i\omega_{\vec{k}}t(\tau)} + a^\dagger_{\vec{k}}(0)e^{-i\vec{k}\vec{x}(\tau)+i\omega_{\vec{k}}t(\tau)}],$$
$$\phi^s(t(\tau), \vec{x}(\tau)) = i\mu \int_{\tau_0}^{\tau} d\tau'\, R^f_2(\tau')[\phi^f(x(\tau')), \phi^f(x(\tau))]. \tag{12}$$

III. VACUUM FLUCTUATIONS AND RADIATION REACTION

We assume that the initial state of the field is the vacuum $|0\rangle$, while the atom is prepared in the state $|a\rangle$, which may be $|+\rangle$ or $|-\rangle$. In principle, the time evolution of the mean value of any atomic observable G could be calculated as the solution of a coupled set of Heisenberg equations analogous to (6)–(8). Our aim, however, is to identify and separate on the basis of (12) in the rate of change of $G(\tau)$ the contributions that are caused by two distinct physical mechanisms: (i) the change in G produced by the fluctuations of the quantum field which are present even in the vacuum—this part is related to the *free* part of the field and is called the contribution of the *vacuum fluctuations* to $\frac{dG}{d\tau}$—and (ii) the change in G due to the interaction with that part of the field which is caused by the atom itself. This is the contribution of *radiation reaction* to $\frac{dG}{d\tau}$ and is connected with the source part of the field.

The task of identifying the contributions of vacuum fluctuations and radiation reaction in the dynamics of a small system coupled to a reservoir has been considered by Dalibard, Dupont-Roc, and Cohen-Tannoudji [8,9]. We will apply their formalism to the problem of a two-level atom coupled to the radiation field and generalize it to arbitrary stationary trajectories $x(\tau)$ of the atom.

The Heisenberg equations of motion for an arbitrary atomic observable $G(\tau)$ is given by

$$\frac{d}{d\tau}G(\tau) = i[H_A(\tau), G(\tau)] + i[H_I(\tau), G(\tau)]. \tag{13}$$

We are interested only in the part of $\frac{dG}{d\tau}$ due to the interaction with the field:

$$\left(\frac{dG(\tau)}{d\tau}\right)_{\text{coupling}} = i\mu\phi(x(\tau))[R_2(\tau), G(\tau)]. \tag{14}$$

In order to identify the contributions of vacuum fluctuations and radiation reaction, we must investigate in (14) the effects of ϕ^f and ϕ^s separately.

At this point, however, an operator ordering problem arises. The feature that all atomic observables commute with ϕ is preserved in time because of the unitary evolution. This is not true for ϕ^f and ϕ^s separately. The reason for this is that the source part of ϕ picks up contributions of atomic observables during its time evolution and vice versa [cf. (9)–(11)]. Because (14) contains products of atomic and field operators, we must therefore choose an operator ordering in (14) when discussing the effects of ϕ^f and ϕ^s separately:

$$\left(\frac{dG(\tau)}{d\tau}\right)_{\text{coupling}} = i\mu(\lambda\phi(x(\tau))[R_2(\tau), G(\tau)] + (1-\lambda)[R_2(\tau), G(\tau)]\phi(x(\tau))), \tag{15}$$

with an arbitrary real λ. Different operator orderings will lead to the same final results for physical quantities on the left-hand side of (15), but will yield different interpretations concerning the roles played by vacuum fluctuations and radiation reaction [3–7]. However, DDC noticed that there exists a preferred ordering prescription: they showed that only if a symmetric ordering ($\lambda = \frac{1}{2}$) of atomic and field variables is adopted, $(dG/d\tau)_{VF}$ and $(dG/d\tau)_{RR}$ are both Hermitian. They argued that only under this condition, the effects of vacuum fluctuations and radiation reaction can possess an independent physical meaning.

Adopting the symmetric ordering prescription in (14), we can identify the contribution of the *vacuum fluctuations* to $\frac{dG}{d\tau}$,

$$\left(\frac{dG(\tau)}{d\tau}\right)_{VF} = \frac{1}{2}i\mu(\phi^f(x(\tau))[R_2(\tau), G(\tau)] + [R_2(\tau), G(\tau)]\phi^f(x(\tau))), \tag{16}$$

which goes back to ϕ^f, and the contribution of *radiation reaction*,

$$\left(\frac{dG(\tau)}{d\tau}\right)_{RR} = \frac{1}{2}i\mu(\phi^s(x(\tau))[R_2(\tau), G(\tau)] + [R_2(\tau), G(\tau)]\phi^s(x(\tau))), \tag{17}$$

which goes back to the source part ϕ^s of the field. We note that if the initial state of the field is not the vacuum but some large reservoir of ϕ particles (photons), the expression (16) represents the reservoir fluctuations $(dG/d\tau)_{RF}$. The vacuum can be regarded as a particular reservoir.

IV. RATE OF VARIATION OF THE ATOMIC ENERGY IN VACUUM FOR ARBITRARY STATIONARY MOTION

We are now prepared to identify the contributions of vacuum fluctuations and radiation reaction in the evolution of the atom's excitation energy, which is given by the expectation value of $H_A = \omega_0 R_3(\tau)$. The free part of the atomic Hamiltonian is constant in time so that the rate of change of H_A consists only of the two contributions obtained from (16) and (17):

$$\left(\frac{dH_A(\tau)}{d\tau}\right)_{VF} = \frac{1}{2} i\omega_0\mu(\phi^f(x(\tau))[R_2(\tau), R_3(\tau)] + [R_2(\tau), R_3(\tau)]\phi^f(x(\tau))), \quad (18)$$

$$\left(\frac{dH_A(\tau)}{d\tau}\right)_{RR} = \frac{1}{2} i\omega_0\mu(\phi^s(x(\tau))[R_2(\tau), R_3(\tau)] + [R_2(\tau), R_3(\tau)]\phi^s(x(\tau))). \quad (19)$$

We can separate $R_2(\tau)$ and $R_3(\tau)$ into their free part (zeroth order in μ) and source part (first order in μ); cf. (9) and (10). In a perturbative treatment, we take into account only terms up to order μ^2. We will also express all terms on the right-hand side of (18) and (19) in terms of the free parts R_2^f, R_3^f, and ϕ^f. This will allow us to describe the atom's evolution with respect to simple statistical functions of the atom and the field. Therefore we use, for the corresponding source parts, the expressions (9)–(11) from the solutions of the Heisenberg equations above. We obtain, up to order μ^2,

$$\begin{aligned}\left(\frac{dH_A(\tau)}{d\tau}\right)_{VF} = {} & \frac{1}{2} i\omega_0\mu(\phi^f(x(\tau))[R_2^f(\tau), R_3^f(\tau)] \\ & + [R_2^f(\tau), R_3^f(\tau)]\phi^f(x(\tau))) \\ & - \frac{1}{2}\omega_0\mu^2 \int_{\tau_0}^{\tau} d\tau' \, \{\phi^f(x(\tau)), \phi^f(x(\tau'))\} \\ & \times [R_2^f(\tau'), [R_2^f(\tau), R_3^f(\tau)]], \end{aligned} \quad (20)$$

$$\begin{aligned}\left(\frac{dH_A(\tau)}{d\tau}\right)_{RR} = {} & \frac{1}{2}\omega_0\mu^2 \int_{\tau_0}^{\tau} d\tau' \, [\phi^f(x(\tau)), \phi^f(x(\tau'))] \\ & \times (R_2^f(\tau')[R_2^f(\tau), R_3^f(\tau)] \\ & + [R_2^f(\tau), R_3^f(\tau)]R_2^f(\tau')), \end{aligned} \quad (21)$$

where curly brackets denote the anticommutator and we have used the fact that free atom and field variables commute.

We are interested only in atomic observables. Consequently, we perform an averaging over the field degrees of freedom by taking the vacuum expectation value of (20) and (21). The right-hand sides of (20) and (21) contain only free operators so that only ϕ^f is affected. Since $\langle 0|\phi^f|0\rangle = 0$, the first line of (20) does not contribute and we obtain

$$\begin{aligned}\langle 0|\frac{dH_A(\tau)}{d\tau}|0\rangle_{VF} = {} & -\omega_0\mu^2 \int_{\tau_0}^{\tau} d\tau' \, C^F(x(\tau), x(\tau')) \\ & \times [R_2^f(\tau'), [R_2^f(\tau), R_3^f(\tau)]], \end{aligned} \quad (22)$$

$$\begin{aligned}\langle 0|\frac{dH_A(\tau)}{d\tau}|0\rangle_{RR} = {} & \omega_0\mu^2 \int_{\tau_0}^{\tau} d\tau' \, \chi^F(x(\tau), x(\tau')) \\ & \times \left\{R_2^f(\tau'), [R_2^f(\tau), R_3^f(\tau)]\right\}. \end{aligned} \quad (23)$$

The statistical functions C^F and χ^F of the field are well known from linear response theory [22]. The *symmetric correlation function* of the field is given by

$$C^F(x(\tau), x(\tau')) = \frac{1}{2}\langle 0|\{\phi^f(x(\tau)), \phi^f(x(\tau'))\}|0\rangle. \quad (24)$$

It is sometimes also called Hadamard's elementary function and describes the fluctuations of the quantum field in the vacuum state. The *linear susceptibility* of the field is defined as

$$\chi^F(x(\tau), x(\tau')) = \frac{1}{2}\langle 0|[\phi^f(x(\tau)), \phi^f(x(\tau'))]|0\rangle \quad (25)$$

and is also known as a Pauli-Jordan or Schwinger function. According to (23), it describes the linear response of the averaged rate $\frac{dH_A(\tau)}{d\tau}$ on fluctuations of the atom. Note that the statistical functions of the field have to be evaluated along the trajectory of the atom.

We do not intend to deal with the operator dynamics of Eqs. (22) and (23). Instead we are interested in the evolution of expectation values of atomic observables. Accordingly, we take the expectation value of Eqs. (22) and (23) in the atom's state $|a\rangle$. We can replace with (7) to order μ^2 the commutator $\omega_0[R_2^f(\tau), R_3^f(\tau)]$ with $i\frac{d}{d\tau}R_2^f(\tau)$ and obtain as a central result

$$\left\langle \frac{dH_A(\tau)}{d\tau}\right\rangle_{VF} = 2i\mu^2 \int_{\tau_0}^{\tau} d\tau' \, C^F(x(\tau), x(\tau'))\frac{d}{d\tau}\chi^A(\tau, \tau'), \quad (26)$$

$$\left\langle \frac{dH_A(\tau)}{d\tau}\right\rangle_{RR} = 2i\mu^2 \int_{\tau_0}^{\tau} d\tau' \, \chi^F(x(\tau), x(\tau'))\frac{d}{d\tau}C^A(\tau, \tau'), \quad (27)$$

where $\langle \cdots \rangle = \langle 0, a| \cdots |0, a\rangle$. The rate of change of the atom's excitation energy is expressed entirely in terms of statistical functions. The statistical functions of the atom are defined analogously to (24) and (25):

$$C^A(\tau, \tau') = \frac{1}{2}\langle a|\{R_2^f(\tau), R_2^f(\tau')\}|a\rangle, \quad (28)$$

$$\chi^A(\tau, \tau') = \frac{1}{2}\langle a|[R_2^f(\tau), R_2^f(\tau')]|a\rangle. \quad (29)$$

C^A is called the *symmetric correlation function* of the atom in the state $|a\rangle$, χ^A its *linear susceptibility.* It will be important that C^A and χ^A do not depend on the trajectory of the atom, but characterize only the atom itself.

The physical picture implied by Eqs. (26) and (27) can be expressed in the following way [8]: (i) the field fluctuates and acts on the atom, which is polarized (contribution of the vacuum fluctuations), and (ii) the atom fluctuates and perturbs the field, which in turn reacts back on the atom (contribution of radiation reaction). The intimate relation between dissipative and fluctuative processes provides an example for the fluctuation-dissipation theorem [23,2]. Because of the symmetric operator ordering chosen in Sec. III, only the commutator appears in χ^F of (24). Accordingly, the radiation reaction contribution (27) does not depend on the state of the field. This is plausible since radiation reaction is connected only with the part of the field radiated by the atom and justifies again the choice of the symmetric ordering.

Finally, we want to give the explicit forms of the statistical functions of the atom and the field, which will be useful in the following sections. We obtain, for the statistical functions of the atom,

$$C^A(\tau,\tau') = \frac{1}{2}\sum_b |\langle a|R_2^f(0)|b\rangle|^2 \times (e^{i\omega_{ab}(\tau-\tau')} + e^{-i\omega_{ab}(\tau-\tau')}), \tag{30}$$

$$\chi^A(\tau,\tau') = \frac{1}{2}\sum_b |\langle a|R_2^f(0)|b\rangle|^2 \times (e^{i\omega_{ab}(\tau-\tau')} - e^{-i\omega_{ab}(\tau-\tau')}), \tag{31}$$

where $\omega_{ab} = \omega_a - \omega_b$ and the sum extends over a complete set of atomic states.

The statistical functions of the field are well known from special relativistic quantum field theory [24] with reference to Minkowski coordinates t and $\vec{x}$. They can be written as functions of τ

$$\begin{aligned} C^F(x(\tau),x(\tau')) &= \frac{1}{8\pi^2}\frac{1}{|\vec{x}|}\left(\frac{\mathrm{P}}{\Delta t + |\Delta\vec{x}|} - \frac{\mathrm{P}}{\Delta t - |\Delta\vec{x}|}\right) \\ &= -\frac{1}{8\pi^2}\left(\frac{1}{(\Delta t + i\epsilon)^2 - |\Delta\vec{x}|^2} + \frac{1}{(\Delta t - i\epsilon)^2 - |\Delta\vec{x}|^2}\right), \end{aligned} \tag{32}$$

$$\chi^F(x(\tau),x(\tau')) = \frac{i}{8\pi}\frac{1}{|\Delta\vec{x}|}[\delta(\Delta t + |\Delta\vec{x}|) - \delta(\Delta t - |\Delta\vec{x}|)], \tag{33}$$

where $\Delta t = t(\tau) - t(\tau')$, $\Delta\vec{x} = \vec{x}(\tau) - \vec{x}(\tau')$, and P denotes the principal value.

V. SPONTANEOUS EMISSION FROM A UNIFORMLY MOVING ATOM

In this section, we apply the previously developed formalism to the well known problem of the spontaneous emission from an inertially moving atom. This will provide a basis for the discussion of the role of vacuum fluctuations and radiation reaction in the more general case of accelerated atoms in the following section. We consider an atom with constant velocity $\vec{v}$ on the trajectory

$$t(\tau) = \gamma\tau, \quad \vec{x}(\tau) = \vec{x}_0 + \vec{v}\gamma\tau, \tag{34}$$

where $\gamma = (1 - v^2)^{-\frac{1}{2}}$. The statistical functions of the field are easily calculated from their general forms (32) and (33):

$$C^F(x(\tau),x(\tau')) = -\frac{1}{8\pi^2}\left(\frac{1}{(\tau - \tau' + i\epsilon)^2} + \frac{1}{(\tau - \tau' - i\epsilon)^2}\right), \tag{35}$$

$$\chi^F(x(\tau),x(\tau')) = -\frac{i}{4\pi(\tau - \tau')}\,\delta(\tau - \tau'). \tag{36}$$

The contributions (26) and (27) to spontaneous emission can now be evaluated using the statistical functions of the atom, which are given by (30) and (31). With a substitution $u = \tau - \tau'$ we get

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{VF} = \frac{\mu^2}{8\pi^2}\sum_b \omega_{ab}|\langle a|R_2^f(0)|b\rangle|^2 \times \int_{-\infty}^{+\infty} du \left(\frac{1}{(u+i\epsilon)^2} + \frac{1}{(u-i\epsilon)^2}\right) \times e^{i\omega_{ab}u}, \tag{37}$$

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{RR} = -i\frac{\mu^2}{4\pi}\sum_b \omega_{ab}|\langle a|R_2^f(0)|b\rangle|^2 \times \int_{-\infty}^{+\infty} du\,\delta'(u)e^{i\omega_{ab}u}, \tag{38}$$

where we have extended the range of integration to infinity for sufficiently long times $\tau - \tau_0$. We have also used the identity $\delta(u) = -u\delta'(u)$.

After the evaluation of the integrals we obtain for the contribution of the vacuum fluctuations to the rate of change of atomic excitation energy

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{VF} = -\frac{\mu^2}{2\pi}\left(\sum_{\omega_a > \omega_b} \frac{1}{2}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2 - \sum_{\omega_a < \omega_b} \frac{1}{2}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\right). \tag{39}$$

This result possesses an interesting interpretation [8]. Consider first the case when the atom is initially in the excited state ($|a\rangle = |+\rangle$). Then only the first term ($\omega_a > \omega_b$) contributes. The vacuum fluctuations lead to a deexcitation of the atom in the excited state: $\langle dH_A(\tau)/d\tau\rangle_{VF} < 0$. This conforms with the old heuristic picture of spontaneous emission as stimulated emission induced by vacuum fluctuations [1]. If, on the other hand, the atom is initially in the ground state ($|a\rangle = |-\rangle$), there is only a contribution from the second term ($\omega_a < \omega_b$). We see that vacuum fluctuations tend to excite an atom in the ground state: $\langle dH_A(\tau)/d\tau\rangle_{VF} > 0$. Note that, if only the effects of vacuum fluctuations are taken into account, both spontaneous excitation and deexcitation occur with equal frequency. Although spontaneous excitation does not occur for inertial atoms, this result should not come as a surprise if we take the heuristic picture seriously: since stimulated excitation and deexcitation have equal Einstein B coefficients, vacuum fluctuations should stimulate atomic excitation as well as deexcitation [2].

The contribution of radiation reaction to the change in the atom's energy becomes

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{RR} = -\frac{\mu^2}{2\pi}\left(\sum_{\omega_a > \omega_b} \frac{1}{2}\omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 + \sum_{\omega_a < \omega_b} \frac{1}{2}\omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \right). \quad (40)$$

The effect of radiation reaction always leads to a loss of atomic energy, $\langle dH_A(\tau)/d\tau\rangle_{RR} < 0$, independent of whether the atom is initially in the ground or excited state. This can be compared with radiation reaction in the classical theory, which has the same effect and results in the instability of classical atoms.

The total rate of change of the atomic excitation energy is obtained by adding the contributions of vacuum fluctuations and radiation reaction:

$$\left\langle \frac{dH_A}{d\tau} \right\rangle_{\rm tot} = \left\langle \frac{dH_A}{d\tau} \right\rangle_{VF} + \left\langle \frac{dH_A}{d\tau} \right\rangle_{RR}. \quad (41)$$

We observe that the effects of both contributions for an atom in the ground state ($\omega_a < \omega_b$) have equal magnitudes but opposite sign so that they exactly cancel:

$$\left\langle \frac{dH_A}{d\tau} \right\rangle_{\rm tot} = -\frac{\mu^2}{2\pi} \sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2. \quad (42)$$

In the ground state of an atom there is a balance between vacuum fluctuations and radiation reaction which establishes that no spontaneous excitation to higher levels is possible for inertial atoms [8,19], only transitions to lower-lying levels (spontaneous emission) occur. This balance also ensures the stability of the ground state: an inertial atom in its ground state does not radiate. Equation (42) shows, on the other hand, that for an atom in the excited state, the effects of vacuum fluctuations and radiation reaction add with equal contributions to the familiar phenomenon of spontaneous emission.

To calculate the Einstein A coefficient for the spontaneous emission of inertially moving atoms, we simplify Eq. (42) by noting that

$$\sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \pm \sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 = \begin{cases} \frac{1}{4}\omega_0^2 \\ -\frac{1}{2}\omega_0^2\langle a|R_3^f(0)|a\rangle. \end{cases} \quad (43)$$

In order μ^2, we can replace $\omega_0\langle R_3^f\rangle$ by $\langle H_A\rangle$ and obtain a differential equation for the atomic excitation energy:

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\rm tot} = -\frac{\mu^2}{8\pi}\omega_0\left(\frac{1}{2}\omega_0 + \langle H_A(\tau)\rangle\right). \quad (44)$$

Note that no factors of γ appear in (44) because we took care to express all observables in the rest frame of the atom. The solution of (44) is

$$\langle H_A(\tau)\rangle = -\frac{1}{2}\omega_0 + \left(\langle H_A(0)\rangle + \frac{1}{2}\omega_0\right)e^{-A\tau}, \quad (45)$$

the familiar exponential decay to the atomic ground state $\langle H_A\rangle = -\frac{1}{2}\omega_0$. The spontaneous emission rate is given by the Einstein A coefficient of the scalar theory:

$$A = \frac{\mu^2}{8\pi}\omega_0. \quad (46)$$

How would the above results have been modified if the initial state of the field were not the vacuum, but a bath of thermal radiation, described by a density matrix $\rho = \exp(-\beta H)$ with $\beta = 1/kT$? The formalism can be easily generalized to that case. The vacuum expectation value in (22) and (23) has simply to be replaced by a reservoir average. The statistical functions of the field (24) and (25) are then replaced by thermal Green's functions

$$C_\beta^F(x(\tau), x(\tau')) = \frac{1}{2}\mathrm{Tr}(\rho\{\phi(x(\tau)), \phi(x(\tau'))\}), \quad (47)$$

$$\chi_\beta^F(x(\tau), x(\tau')) = \frac{1}{2}\mathrm{Tr}(\rho[\phi(x(\tau)), \phi(x(\tau'))]), \quad (48)$$

where the trace extends over the field degrees of freedom. The contributions of reservoir fluctuations and radiation reaction to the rate of change of the atom's excitation energy can be found according to (26) and (27) with the formulas (47) and (48) for the finite temperature statistical functions of the field.

The thermal Green's functions (47) and (48) can be expressed in terms of the statistical functions for the vacuum. As mentioned above, the linear susceptibility of the field does not depend on the state of the field and is therefore equal to that of the vacuum:

$$\chi_\beta^F(x(\tau), x(\tau')) = \chi^F(x(\tau), x(\tau')). \quad (49)$$

Hence we have to deal only with the symmetric correlation function C_β^F of the field. It can be generally shown that C_β^F is connected to the vacuum symmetric correlation function by [cf. [25], Eq. (2.111)]

$$C_\beta^F(t(\tau), \vec{x}(\tau), t(\tau'), \vec{x}(\tau')) = \sum_{k=-\infty}^{+\infty} C^F(t(\tau) + ik\beta, \vec{x}(\tau), t(\tau'), \vec{x}(\tau')). \quad (50)$$

This function appears in the expression (26) for the contribution of reservoir fluctuations to the change of the atomic energy. The terms $k \neq 0$ in (50) represent the influence of the thermal heat bath.

Let us consider an atom at rest in a thermal bath of radiation. As we have seen, the contribution of radiation reaction remains unchanged and is given by (40). To determine the contribution of reservoir fluctuations, we have to calculate the finite temperature symmetric correlation function. If we evaluate (50) for an atom at rest in the thermal bath [trajectory (34) with $\vec{v}=0$, $\gamma=1$], we obtain

$$C^F_\beta(x(\tau),x(\tau')) = -\frac{1}{8\pi^2}\sum_{k=-\infty}^{+\infty}\left(\frac{1}{(\tau-\tau'+ik\beta+i\epsilon)^2} + \frac{1}{(\tau-\tau'+ik\beta-i\epsilon)^2}\right). \quad (51)$$

We will not discuss here the evaluation of (26) with the thermal Green's function (51). This will be postponed to below, where a comparison with the case of a uniformly accelerated atom can be made.

VI. UNIFORMLY ACCELERATED ATOM

Let us now generalize the preceding discussion to the case of a uniformly accelerating atom. We go back to the results of Sec. IV and specify the atom trajectory by

$$t(\tau) = \frac{1}{a}\sinh a\tau,\quad z(\tau) = \frac{1}{a}\cosh a\tau,\quad x(\tau)=y(\tau)=0, \quad (52)$$

where a is the proper acceleration. The statistical functions of the field for the trajectory (52) can be evaluated from their general forms (32) and (33). After some algebra, we obtain

$$\begin{aligned} C^F(x(\tau),x(\tau')) &= -\frac{a^2}{32\pi^2}\left(\frac{1}{\sinh^2[\frac{a}{2}(\tau-\tau')+ia\epsilon]} + \frac{1}{\sinh^2[\frac{a}{2}(\tau-\tau')-ia\epsilon]}\right) \\ &= -\frac{1}{8\pi^2}\sum_{k=-\infty}^{\infty}\left(\frac{1}{(\tau-\tau'+\frac{2\pi i}{a}k+2i\epsilon)^2} + \frac{1}{(\tau-\tau'+\frac{2\pi i}{a}k-2i\epsilon)^2}\right), \end{aligned} \quad (53)$$

$$\chi^F(x(\tau),x(\tau')) = -\frac{i}{8\pi}\frac{a}{\sinh\frac{a}{2}(\tau-\tau')}\,\delta(\tau-\tau'). \quad (54)$$

To obtain the second line, Eq. (4.3.92) from Ref. [26] has been used. The stationarity of the motion of the atom is reflected by the fact that only the time difference $\tau-\tau'$ appears. Comparison of (53) with (51) and of (54) with (49) and (36) shows complete agreement of the statistical functions C^F and χ^F of the field for a trajectory with a =const through the vacuum on one hand and an inertial trajectory which is at rest with respect to a thermal bath with temperature $T=a/2\pi$ on the other. This well-known fact has consequences for the separate discussion of the influence of vacuum fluctuations and of radiation reaction on the spontaneous excitation and deexcitation of an accelerated atom.

The contribution of the vacuum fluctuations to the rate of change of the atomic Hamiltonian becomes with (26)

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{VF} = \frac{\mu^2}{8\pi}\sum_b \omega_{ab}|\langle a|R_2^f(0)|b\rangle|^2 \times\sum_{k=-\infty}^{\infty}\int_{-\infty}^{\infty}du\left[\frac{e^{i\omega_{ab}u}}{(u+\frac{2\pi i}{a}k+2i\epsilon)^2} + \frac{e^{i\omega_{ab}u}}{(u+\frac{2\pi i}{a}k-2i\epsilon)^2}\right]. \quad (55)$$

Again, we have extended the range of integration to infinity for sufficiently long times. It is interesting to compare (55) with the corresponding expression (37) for the inertial atom. There, only the term $k=0$ was present. The remaining terms describe the modification due to the acceleration. The integrals can be calculated using the residue theorem, leading to a geometric series for the k summation. The resulting expression for the rate of change of atomic excitation energy caused by vacuum fluctuations is

$$\begin{aligned}\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{VF} = -\frac{\mu^2}{2\pi}\Bigg[&\sum_{\omega_a>\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2 \times\left(\frac{1}{2}+\frac{1}{e^{\frac{2\pi}{a}\omega_{ab}}-1}\right) \\ &-\sum_{\omega_a<\omega_b}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2 \times\left(\frac{1}{2}+\frac{1}{e^{\frac{2\pi}{a}|\omega_{ab}|}-1}\right)\Bigg].\end{aligned} \quad (56)$$

We note the appearance of the thermal terms in addition to the inertial vacuum fluctuation terms $\frac{1}{2}$. As for an inertial atom, vacuum fluctuations tend to excite an accelerated atom in the ground state and deexcite it in the excited state. Both processes are supported with equal magnitude and are enhanced by the thermal terms compared to the inertial case.

Turning to the contribution of radiation reaction, we find with (27)

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{RR} = i\frac{\mu^2}{8\pi}\sum_b \omega_{ab}|\langle a|R_2^f(0)|b\rangle|^2 \times\int_{-\infty}^{\infty}du\frac{a}{\sinh\frac{a}{2}u}\,\delta(u)\,e^{i\omega_{ab}u}. \quad (57)$$

After the evaluation of the integral, this becomes

$$\left\langle\frac{dH_A(\tau)}{d\tau}\right\rangle_{RR} = -\frac{\mu^2}{2\pi}\left(\sum_{\omega_a>\omega_b}\frac{1}{2}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2 + \sum_{\omega_a<\omega_b}\frac{1}{2}\omega_{ab}^2|\langle a|R_2^f(0)|b\rangle|^2\right). \quad (58)$$

It is remarkable that this is the same expression as (40) for an inertial atom, leading again always to a loss of energy. A similar situation is known in classical electrodynamics: a uniformly accelerated charge on the trajectory (52) is not subject to a radiation reaction force, although radiation is emitted [27]. Thus the fact that the contribution of radiation reaction is not changed in the uniformly accelerated case is perhaps a property of the particular trajectory (52).

Finally, we add the contributions of vacuum fluctuations (56) and radiation reaction (58) to obtain the total rate of change of the atomic excitation energy:

$$\left\langle \frac{dH_A}{d\tau} \right\rangle_{\text{tot}} = \frac{\mu^2}{2\pi} \left[- \sum_{\omega_a > \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \times \left(1 + \frac{1}{e^{\frac{2\pi}{a}\omega_{ab}} - 1}\right) + \sum_{\omega_a < \omega_b} \omega_{ab}^2 |\langle a|R_2^f(0)|b\rangle|^2 \frac{1}{e^{\frac{2\pi}{a}|\omega_{ab}|} - 1} \right]. \tag{59}$$

For an atom in the excited state, only the term $\omega_a > \omega_b$ contributes. It describes the *spontaneous emission* of an accelerated atom. Compared to an inertial atom, it is modified by the appearance of the thermal term. If, however, the atom is in the ground state, there is a nonzero contribution from the term $\omega_a < \omega_b$. For an atom in its ground state in uniformly accelerated motion through the Minkowski vacuum there is no perfect balance between vacuum fluctuations and radiation reaction. Accordingly, transitions to the excited state $[\langle dH_A(\tau)/d\tau\rangle_{\text{tot}} > 0]$ become possible even in the vacuum. This *spontaneous excitation* is the Unruh effect [13], which has now been traced back to the interplay between the two underlying physical effects. The often stated conjecture that the Unruh effect goes back to the vacuum fluctuations has been made precise in this sense.

Note that for an atom in the ground state, the atomic energy can only increase. A loss of energy from the ground state, which would have fatal consequences for the stability of the atom, is not possible according to (59).

VII. EVOLUTION OF THE ATOMIC POPULATION, EINSTEIN COEFFICIENTS

Analogous to the procedure in Sec. V, we can simplify Eq. (59) to obtain a differential equation for $\langle H_A\rangle$,

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle = -\frac{\mu^2}{4\pi}\omega_0 \left[\frac{1}{4}\omega_0 + \left(\frac{1}{2} + \frac{1}{e^{\frac{2\pi}{a}\omega_0} - 1} \right) \langle H_A(\tau)\rangle \right]. \tag{60}$$

Its solution gives the time evolution of the mean atomic excitation energy

$$\langle H_A(\tau)\rangle = -\frac{1}{2}\omega_0 + \frac{\omega_0}{e^{\frac{2\pi}{a}\omega_0} + 1} + \left(\langle H_A(0)\rangle + \frac{1}{2}\omega_0 - \frac{\omega_0}{e^{\frac{2\pi}{a}\omega_0} + 1} \right) \times \exp\left[-\frac{\mu^2}{4\pi}\omega_0 \left(\frac{1}{2} + \frac{1}{e^{\frac{2\pi}{a}\omega_0} - 1} \right) \tau \right]. \tag{61}$$

We see that the atom evolves with a modified decay parameter towards the equilibrium value

$$\langle H_A\rangle = -\frac{1}{2}\omega_0 + \frac{\omega_0}{e^{\frac{2\pi}{a}\omega_0} + 1}, \tag{62}$$

representing a thermal excitation with temperature $T = a/2\pi$ above the ground state. It is remarkable that the atom obeys Fermi-Dirac statistics in thermal equilibrium. This is due to the fermionic nature of a two-level system (for example, the atomic raising and lowering operators obey the anticommutation relation $\{R_+, R_-\} = 1$).

The identification of the Einstein coefficient is more complicated than in the inertial case, since we have now two competing processes that both occur spontaneously. There are two Einstein coefficients $A_\downarrow$ and $A_\uparrow$, which describe the transition rates corresponding to these processes.

Einstein coefficients are defined with respect to rate equations for the atomic populations. Consider therefore an ensemble of N atoms. Let N_1 denote the number of atoms in the ground state, N_2 the number in the excited state, with $N = N_1 + N_2$. The rate equations describing the two spontaneous processes above are

$$\frac{dN_2}{d\tau} = -\frac{dN_1}{d\tau} = A_\uparrow N_1 - A_\downarrow N_2 \tag{63}$$

with

$$\langle H_A\rangle = \frac{1}{N}\left(-\frac{1}{2}\omega_0 N_1 + \frac{1}{2}\omega_0 N_2 \right). \tag{64}$$

The solution of the rate equations

$$\langle H_A(\tau)\rangle = -\frac{1}{2}\omega_0 + \omega_0 \frac{A_\uparrow}{A_\uparrow + A_\downarrow} + \left(\langle H_A(0)\rangle + \frac{1}{2}\omega_0 - \frac{A_\uparrow}{A_\uparrow + A_\downarrow}\omega_0 \right) \times e^{-(A_\uparrow + A_\downarrow)\tau} \tag{65}$$

shows also an equilibrium state different from the ground state and a modified decay constant. Comparing (61) and (65), we can identify the Einstein coefficients $A_\downarrow$ and $A_\uparrow$ for an accelerated atom:

$$A_\downarrow = \frac{\mu^2}{8\pi}\omega_0 \left(1 + \frac{1}{e^{\frac{2\pi}{a}\omega_0} - 1} \right), \qquad A_\uparrow = \frac{\mu^2}{8\pi}\omega_0 \frac{1}{e^{\frac{2\pi}{a}\omega_0} - 1}. \tag{66}$$

The coefficient $A_\downarrow$ for spontaneous emission from an accelerated atom can be compared to its inertial value (46).

We see that the rate of spontaneous emission is enhanced by the thermal contribution. The transition rate $A_{\uparrow}$ for the spontaneous excitation, on the other hand, is given by (46) weighted with the thermal factor. It vanishes as $a \to 0$.

VIII. CONCLUSIONS

We have studied atoms in arbitrary stationary motion through the vacuum of a photon field. For an atom observable G, the total rate $\frac{dG}{d\tau}$ with regard to the proper time τ of the atom can be split into two distinct rates going back to vacuum fluctuations acting on the atom and to radiation reaction. This splitting is unique if one demands that the two processes should separately have a physical meaning. Whether spontaneous processes may occur for the different states of the moving atoms depends on the balance between the two physical processes. The corresponding symmetric correlation function and linear susceptibility of the field are simply obtained by calculating the respective special relativistic Green's functions with respect to the proper time along the atom's trajectory.

We computed the two contributions to the variation of the mean energy of a state of a two-level atom for uniform motion and for constant acceleration. For the ground state of an atom in uniform motion, the influence of vacuum fluctuations is exactly canceled by the radiation reaction. If an atom moves with constant acceleration, this perfect balance is disturbed. The radiation reaction remains unchanged, but the vacuum fluctuations are modified by an additional term with thermal spectrum of temperature $T = \hbar a/2\pi ck = 4 \times 10^{-20}$K $(a/9.81\ \mathrm{m\,s^{-2}})$. This has two consequences: spontaneous excitation from the ground state becomes possible (the Unruh effect) and the Einstein coefficient for spontaneous emission from the upper state is changed. These effects are now traced back quantitatively to the two underlying physical processes.

The calculations presented above can be easily transcribed to other physical situations. The change to another atom trajectory in Minkowski space is obvious. In addition, continuous efforts over the past two decades have provided us with a plethora of worked out vacuum expectation values of products of free-field operators (Green's functions) for different fields in various curved space-times and in situations with boundaries. We mention black holes [28], moving mirrors [25], cosmic strings [29], and Robertson-Walker universes [25]. In all these cases, the two distinct effects underlying spontaneous processes of atoms on certain trajectories can be worked out following closely the scheme presented above.

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