Definition of a laser threshold

Gunnar Björk,[1,2] Anders Karlsson,[1] and Yoshihisa Yamamoto[2,3]
[1] Department of Electronics, Royal Institute of Technology, Electrum 229, S-164 40 Kista, Sweden
[2] ERATO Quantum Fluctuation Project, Edward L. Ginzton Laboratory, Stanford University, Stanford, California 94305
[3] NTT Basic Research Laboratories, Atsugi-shi, Kanagawa 243, Japan
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We propose that the threshold of a laser is more appropriately described by the pump power (or current) needed to bring the mean cavity photon number to unity, rather than the conventional "definition" that it is the pump power at which the optical gain equals the cavity loss. In general the two definitions agree to within a factor of 2, but in a class of microcavity lasers with high spontaneous emission coupling efficiency and high absorption loss, the definitions may differ by several orders of magnitude. We show that in this regime the laser undergoes a transition from a linear (amplifier) behavior to a nonlinear (oscillatory) behavior at our proposed threshold pump rate. The photon recycling resulting from the high spontaneous emission coupling efficiency and high absorption may in this case result in lasing without population inversion, and coherent light is generated via "loss saturation" instead of gain saturation. This mechanism for lasing without inversion is very different from lasing without inversion using a radiation trapped state.

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I. INTRODUCTION

In recent years the interest in microcavity masers and lasers has increased. The idea behind these devices is that if the cavity volume is reduced to about one wavelength cubed, only one, or at most a few, modes will interact with the atoms constituting the gain medium. One consequence of this interaction with few modes is that dissipation (i.e., coupling to the radiation mode continuum), which washes away the features of the coherent interaction between the field and the gain medium, is kept to a minimum. Therefore microcavity devices, and micromasers in particular, have been used to probe the quantum-mechanical features of the light-matter interaction [1,2]. In contrast, most of the features of the coherent interaction between light and matter are washed out by intrinsic averaging over all quantum-mechanical phases in masers of macroscopic dimensions and with some loss [3]. In this paper we demonstrate that in microcavity semiconductor lasers, some unusual features are retained even if the phase of atomic dipole moment is averaged over (physically this is accomplished by phonon scattering). We shall concentrate on continuously pumped optical devices, but some of our conclusions may be applicable to micromasers, although the micromasers usually are pumped and probed in a very different fashion using Rydberg atoms in flight.

One advantage of having only one or a few optical modes interacting with the gain medium is that very little power is carried away by spontaneous emission in spurious modes and hence the threshold power of microcavity lasers can be very low [4–7]. In the extreme case, when only a single electromagnetic mode interacts with the gain material, the input power versus output power curve is ideally a straight line. This result is easy to understand, because if radiative recombination dominates, and all the radiation goes into a single mode, the device quantum efficiency is always close to unity. Hence every electron is converted to a photon which is detected, irrespective of pump power. Due to the absence of any signature of the threshold in the input power versus output power characteristics, such as a kink or "knee," some of the early investigators dubbed them "thresholdless lasers" [4,5]. In this paper we will argue that every microcavity laser has a threshold, but sometimes that threshold is reached even before the gain medium is population inverted.

Conventionally, the threshold is taken as the pump power (injection current) needed to make the gain of the optical mode equal the cavity losses. This is not a definition, and it cannot be, because the optical gain is always smaller than the cavity loss because of the spontaneous emission [this can be seen from (2) below]. Nonetheless, the conventional threshold "definition" works in macroscopic lasers since in lasers with large cavity volume ($\gg \lambda^3$), the spontaneous emission contributes only a negligible amount of power to the lasing mode just above the threshold. The small spontaneous emission coupling coefficient β is also the reason the gain is pinned above threshold (gain saturation) at a level very close to that of the optical loss.

For a microcavity laser, in which most of the spontaneous emission goes into a single mode, the conventional threshold "definition" sometimes fails. In these lasers, several transitions that usually are associated with lasing (such as the increase of the temporal coherence of the optical field, the increase of the quantum efficiency, the pinning of the population inversion, and the resonance peak of the intensity noise) take place at pump powers that may be several orders of magnitude below the conventional threshold. What uniquely characterizes the transitions is that they all occur at the point where the

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stimulated emission starts to overtake the spontaneous emission. For any mode and any linear gain medium, this will happen when the mean photon number in the mode is unity. We propose that this is a more suitable definition of the threshold than the conventional "definition" for the reason stated above, and for the fact that the proposed threshold could be experimentally assessed, whereas the conventional threshold condition really cannot even be reached at any finite pump rate. In the following we will refer to the threshold definition

$$P_{th} \equiv 1, \tag{1}$$

where P_{th} is the mean photon number in the lasing mode at threshold as the *quantum threshold condition* since it is more intimately coupled to the microscopic emission processes than the conventional definition.

In the paper it shall be shown that this definition is valid for a larger class of lasers than the conventional definition. The quantum threshold is associated with the changes in physical characteristics of the system mentioned above, even when $\beta \to 1$. In this regime all transitions are smoother than in the small β regime if plotted or measured as a function of the pump power. (If plotted versus the mean cavity photon number the transitions are smooth in both regimes.) The transitions, and hence the threshold, become "fuzzy," and it has been argued that since the transitions are no longer "sharp," the concept of a threshold is not meaningful. Taken to the extreme, one may then argue that as $\beta \to 1$, the device becomes thresholdless. Our viewpoint is exactly the opposite. We believe that as the transitions become more "fuzzy" it becomes so much more important to have a well *defined* threshold. The definition should also allow one to experimentally verify whether one is operating below or above threshold. It is inarguably so that the device characteristics in a $\beta \approx 1$ laser will not differ much as the pump power is varied from, say 10% below to 10% above the quantum threshold power. Nonetheless, the proposed definition will always be in the zone separating the linear (amplifier) device regime from the nonlinear (laser) operating regime irrespective of β. Sufficiently far from the quantum threshold the characteristics of the device will always be linear for smaller pump powers and nonlinear for higher pump powers. Hence a "thresholdless" laser does not exist, or at least it does not if the laser gain medium qualitatively follows, e.g., (4) below.

In [6] we stated that in a microcavity laser where the absorption rate is higher than the cavity loss rate ($\xi > 1$, see below), the quantum threshold condition (1) may not be suitable since it was found that it is possible in some circumstances to reach (1) before the population is inverted. In this paper we retract this statement. Even though the population is not inverted, we may observe all transitions associated with lasing once the quantum threshold is reached, and above the quantum threshold the device will be nonlinear. Lasing without population inversion contradicts the classical threshold, per definition. On the contrary, the quantum threshold condition encompasses the lasing without population inversion regime as a special case, and therefore this condition is more general. We would like to stress that in most cases, and specifically in the macroscopic laser regime, the quantum threshold condition and the classical threshold condition coincide within a factor of 2 [6]. However, in this report we focus on the regime when the two definitions disagree, i.e., the lasing without inversion regime. Note that the present mechanism for lasing without population inversion is different from the mechanism utilizing a quantum-mechanical interference between two upper or lower laser levels in a radiation trapped state [9,10].

As an example of a transition that may occur before the medium is inverted, we will show that it is possible to get amplitude squeezing in noninverted (formally absorbing) devices. We will also discuss how lasing and squeezing without inversion can be detected, for it is obviously not sufficient to observe the output field, since that only tells us whether or not the device is lasing, not whether the gain medium is population inverted. We propose using the sign of the quantum correlation between the photon flux and the carrier number as a criterion for inversion. The correlation is positive in a noninverted laser and becomes negative when the population becomes inverted [11–13].

II. LANGEVIN EQUATIONS

The quantum Langevin equations for the carrier number N and the photon number P in a semiconductor diode laser can be written [14]

$$\frac{dP}{dt} = [g - \gamma]P + \frac{\beta}{\tau_{sp}}N + F_P, \tag{2}$$

$$\frac{dN}{dt} = R_p - \frac{N}{\tau_{sp}} - \frac{N}{\tau_{nr}} - gP + F_N. \tag{3}$$

Here R_p is the injected number of carriers, g is the optical power gain, and γ is the photon loss rate, all per unit time. τ_{sp} is the radiative spontaneous recombination time, τ_{nr} the nonradiative recombination time, and β is the fraction of spontaneous emission that enters the lasing mode. F_P and F_N are Langevin fluctuation forces driving the photon and carrier fluctuations, respectively, whose spectral density can be calculated using the fluctuation dissipation theorem [14,15]. Using a linear gain model and the relation linking the spontaneous and stimulated emission rates (Einstein's A and B coefficients), the gain g may be written as

$$g = \frac{\beta}{\tau_{sp}}(N - N_t), \tag{4}$$

where N_t is the transparency carrier number. Using (2) and (4), it is easy to derive the mean carrier number as a function of the mean photon number

$$N = N_t \frac{P}{1+P}\left(1 + \frac{1}{\xi}\right) \quad . \tag{5}$$

Here, the dimensionless parameter $\xi = \beta N_t/\gamma\tau_{sp}$ has been introduced. ξ is the mean cavity photon number at transparency, and $\xi > 1$ indicates an absorption rate $\beta N_t/\tau_{sp}$ larger than the cavity loss rate γ. From the equation it is easy to see that N increases approximately linearly with P until P equals unity, then it becomes clamped. This indicates that at the photon number unity, the system undergoes a transition from the linear operating regime to the nonlinear regime [8]. It should be noted that (5) is a very general equation. For any laser parameters, the transition always takes place at $P = 1$, and the only dependence in (5) on cavity loss, spontaneous emission coupling factor, and spontaneous emission lifetime is via ξ. Hence the onset of clamping of the carrier number at the quantum threshold is also valid for the ideal $\beta \to 1$ microcavity laser. It is also easy to see that as long as P is smaller than ξ, N is smaller than N_t. That is, in the region $1 \le P \le \xi$, the population is noninverted but the device is operating in a saturated, nonlinear regime. A ξ larger than unity should be possible to obtain using cavity QED effects, increasing β faster than decreasing the cavity volume [16,17]. We will explore this regime of lasing without inversion below. Before doing that, it should be pointed out that a large ξ is often not desirable in a laser since it raises the threshold pump rate, broadens the laser linewidth (at a given pump rate), and increases the population inversion factor $n_{sp} = N/(N - N_t)$, which, using (5), high above threshold can be expressed $n_{sp} = 1 + \xi$.

To calculate measurable quantities, we need the cavity boundary condition

$$I = \gamma_e P - F_v\,, \tag{6}$$

relating the external photon flux I to the cavity photon number P, the output coupling losses γ_e, and the vacuum fluctuations F_v impinging on the output coupling mirror [15]. For clarity, we shall express γ_e as $\eta_e\gamma$ where $\eta_e = \gamma_e/\gamma$ is the output coupling efficiency of the mode. The total free carrier to photon quantum efficiency below threshold can be calculated from (3) and (5) as

$$\eta = \frac{\beta\eta_e}{1 + \vartheta + \xi(1 - \beta + \vartheta)}\,, \tag{7}$$

where we have used the dimensionless parameter $\vartheta = \tau_{sp}/\tau_{nr}$. It is seen that in order to have a high quantum efficiency when ξ is higher than unity, two conditions must be fulfilled. First, $1 - \beta$ must be smaller than $1/\xi$. In addition, τ_{nr} must be larger than $\xi\tau_{sp}$. It is seen that the emission and subsequent reabsorption of the photons when ξ is high, the photon recycling, augments the requirements for a high spontaneous emission coupling ratio and a long nonradiative recombination time. This is easy to understand. Every time a photon is reabsorbed, the excitation has a new chance to recombine nonradiatively, or radiatively into nonlasing modes. To give an example, assume that we have a $\beta = 1$ laser with a nonradiative lifetime 1000 times longer than the spontaneous lifetime. If the photon is reabsorbed on the average, say, 1000 times, the probability of having the excitation decay nonradiatively is still about $1 - 0.999^{1000} \approx 0.632$.

In [6], it was shown that the pump rate needed to satisfy the quantum threshold condition can be expressed

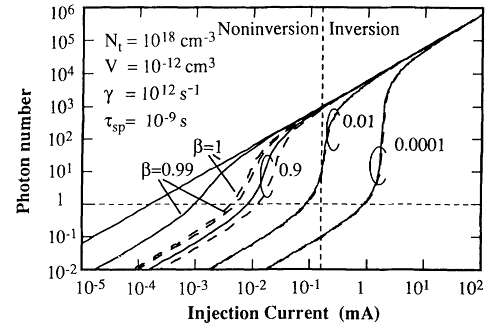

FIG. 1. Mean cavity photon number versus injection current for a $\xi = 10^3\beta$ microcavity laser. The solid lines are for $\vartheta = 0$, i.e., $\tau_{nr} = \infty$, and the dashed lines are for $\vartheta = 0.05$, i.e., $\tau_{nr} = 20\tau_{sp}$. The mean output power is found by multiplying the photon number with the photon energy and the output coupling rate $\gamma\eta_e$.

$$R_{p,th} = \frac{\gamma}{2\beta}[(1 + \beta + \vartheta) + \xi(1 - \beta + \vartheta)]\,. \tag{8}$$

In Fig. 1 the mean photon number versus injection current of a microcavity laser with $\xi = 10^3\beta$ is plotted. The vertical dashed line separates the regions of population inversion and noninversion, and clearly, the threshold jump (the kink in the curve) can appear well before population inversion is established. The dashed lines are plots of the same quantity for $\vartheta = 0.05$. Even at this relatively small value of nonradiative recombination, a distinct threshold jump appears even when $\beta \to 1$, decreasing the difference between the transparency current and the threshold current. From (8) it is clear that increasing β, in this case, above 0.95, will only marginally affect the threshold current. The reason is that in this regime the dominant loss process below threshold is nonradiative recombination, and not spontaneous emission into nonlasing modes.

III. SQUEEZING WITHOUT INVERSION

In this section we will, as one example, show how the output photon flux intensity noise changes (undergoes a transition) as the pump power is increased. To see a clear transition it is necessary to drive the laser with a quiet current, with sub-Poissonian electron count statistics. If one drives the laser with a Poissonian current, no transition can be seen as the current exceeds the quantum threshold [8,18].

It is well known that intensity noise reduction below the shot noise is closely associated with high quantum efficiency. In order to transfer the quiet statistics of the injection current to the photon flux it is detrimental that both the conversion process and the subsequent detection have a high quantum efficiency. In Fig. 2 we have solved the linearized fluctuation equations (using the formalism in [14,15]) and plotted the low-frequency [$\ll 1/(\xi\tau_{sp})$] intensity noise spectral density. While it is true that

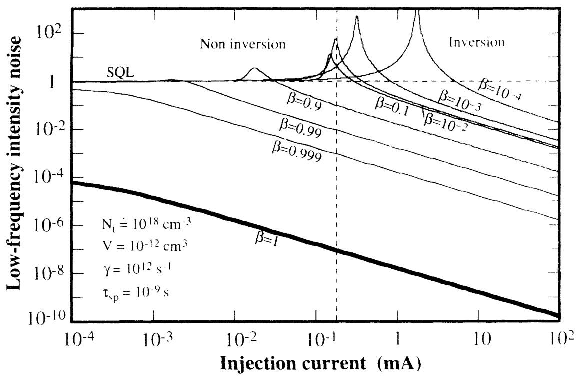

FIG. 2. Low-frequency intensity noise (relative to the shot noise limit) versus injection current. The injection current has been assumed to be free of noise.

the linearized rate equations are not accurate when the photon number is close to unity, the error is only of the order of 10% [18]. Since the effects we consider here span over several orders of magnitude the error is small in comparison and the analysis justified. It is assumed that the injection current has no noise and that $\eta_e = 1$ and $\vartheta = 10^{-7}$. The parameters are such that $\xi \approx 10^3\beta$. The finite squeezing at low frequencies when $\beta = 1$ is due to the nonvanishing nonradiative recombination assumed. With $\vartheta = 0$ a $\beta = 1$ laser would exhibit perfect low-frequency squeezing at all pump rates. It may be surprising that if β decreases from unity to 0.999 almost all squeezing below threshold disappears. The reason is, as illustrated in the preceding section, that the photon recycling increases the spontaneous emission into nonlasing modes and therefore decreases the quantum efficiency as manifested by (7). In Fig. 2 it is also evident that in lasers with $\beta < 1$ there is an intensity noise peak at a distinct injection current. The location of this peak corresponds well to the proposed quantum threshold.

In Fig. 3 the intensity noise spectral density normalized to the standard quantum limit is plotted. Since $\beta = 1$, $\eta_e = 1$, and $\vartheta = 10^{-7}$ has been assumed, (almost) every injected electron gives rise to an emitted photon. In such a system the photon flux and injection current must be (nearly) identical in the low-frequency range. (A Poissonian current will give rise to a Poissonian photon flux, as demonstrated in [8] and [18].) The thick line in the previous figure can be seen reproduced at the extreme left (at 10^{-6} GHz) in this figure. It is also seen that the absorption induced photon recycling slows the system down considerably below threshold. The squeezing cutoff frequency is approximately $1/(\xi\tau_{sp})$.

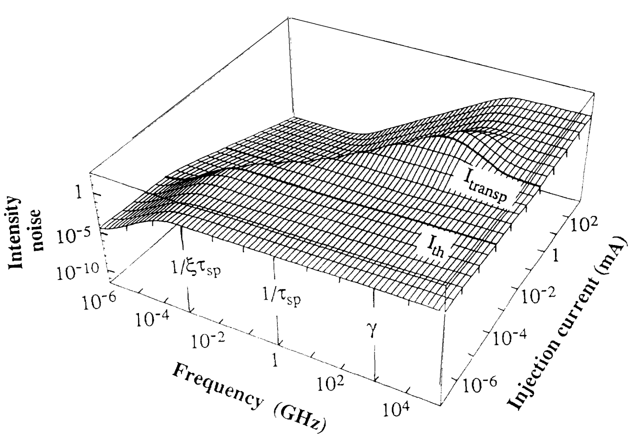

FIG. 3. Intensity noise (relative to the shot noise) versus frequency and injection current. The parameters are the same as in the previous figure.

When the quantum threshold current is reached, the system response grows faster, and the squeezing bandwidth increases. However, there is no excess noise at any frequency in this loss saturation dominated regime. At yet higher injection currents (about 0.1 mA), there is a relaxation oscillation noise peak. This is because at this point the stimulated emission and absorption become equal, so there is neither loss nor gain saturation. Since the system is such that there is an (almost) one-to-one correspondence between injected carriers and emitted photons, at low frequencies there is always squeezing. At this pump rate the typical time scale of the system (indicated by, e.g., the squeezing bandwidth) is $1/\tau_{sp}$. At still higher pump rates the stimulated emission starts to dominate, the gain saturation quenches the relaxation oscillations, and the time scale of the system (squeezing bandwidth) approaches the cavity loss rate γ. (The squeezing floor at high pumping is due to the nonvanishing nonradiative recombination assumed.)

IV. PROPOSAL FOR AN EXPERIMENTAL TEST: CARRIER NUMBER AND THE PHOTON FLUX CORRELATION

In the two preceding sections we showed that as a $\beta \approx 1$, $\xi \gg 1$ laser is driven above the quantum threshold it will exhibit characteristics that are nonlinear and laserlike. We have argued that even if the population is noninverted, the output light will have all the characteristics of light from an ordinary laser and therefore the quantum threshold condition is more appropriate than the ordinary condition. However, since it is predicted that all signatures of the output field are similar in the noninverted and inverted regimes, it will be difficult to determine whether the population is inverted or not from simply observing the output. In this section we propose that the low-frequency correlation between the carrier number and the photon flux $C_{I,N}$ could provide a means to qualitatively separate the inverted from the noninverted regime to support or disprove our prediction of inversionless lasing. For the interesting regimes of ξ and β, the in-phase correlation changes sign from positive to negative when the population becomes inverted.

The low-frequency correlation between the carrier number and the photon flux, with possible applications, were discussed in [11–13], but not in the context of lasing without population inversion. Figure 4 shows the real part of the low-frequency correlation between the carrier number and the photon flux $C_{I,N}$. Note that even if the

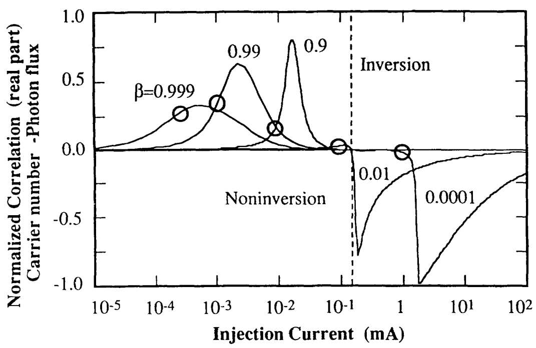

FIG. 4. Normalized correlation (real part) between carrier number and photon flux versus injection current. The pump current has been assumed to be perfectly noise suppressed. The circles on the lines show the threshold current for the corresponding value of β.

real part of the correlation function between the carrier number and the photon flux is low (it disappears for a pump noise suppressed $\beta = 1, \vartheta = 0, \eta_e = 1$ laser), the imaginary part due to carrier storage effects can remain high [13]. As shown, in the microcavity lasing without population inversion, the positive correlation is enhanced above lasing threshold roughly to the point when the medium becomes inverted. The positive correlation is due to photon recycling, that is, an increase in the cavity photon number above the average value increases the number of carriers since the absorption is larger than the stimulated emission. In this regime, "loss" is saturated due to the onset of absorption assisted photon recycling. On the other hand, in the (normal) regime of lasing with population inversion, the correlation becomes negative, since an increase in the photon number results in a decrease of the carrier number. This is ordinary "gain saturation." In principle, an experimental mean to identify lasing without inversion in a microcavity would be to measure a positive correlation between the carrier number and the photon flux in the region above lasing threshold. The threshold in turn could be identified by monitoring the intensity noise of the laser as a function of the injection current.

Experimental verification of the departure from the classical threshold, and hence of squeezing without inversion and positive carrier number-photon flux above threshold, may be realized in the not so distant future. In [19] a GaAs hemispherical laser with a β of 0.05 to 0.1 was reported, and in [20] a β value of 0.2–0.3 was reported for an InGaAs microdisc laser. These microcavities had a cavity decay rate γ of about 10^{12} s^{-1}, a spontaneous emission lifetime τ_{sp} shorter than 1 ns, and a near unity internal quantum efficiency. If such microcavities have, say, five quantum wells of 200 Å thickness and 7 μm diameter and these quantum wells have a transparency carrier density of 10^{17} cm^{-3}, the value of ξ is about 50, and the effect is within reach, at least for the laser with highest β.

V. CONCLUSIONS

In summary, we have proposed a quantum threshold condition which is valid for a larger class of lasers than is the conventional classical definition. In most cases the two definitions predict the same threshold power within a factor of 2 (the quantum threshold always being the lower one). The condition predicts that the threshold in some circumstances can be reached before the lasing medium is population inverted. In such $\beta \approx 1$, $\xi \gg 1$ lasers, the quantum threshold is substantially lower than that predicted with the conventional definition. It was shown that in reaching the quantum threshold the laser went from a linear (light emitting diode) operating regime to a nonlinear (laser) regime. However, in the noninverted laser loss saturation rather than gain saturation was the source of nonlinearity. Finally, an experimental test of lasing without inversion was proposed. It was shown that the in-phase carrier number-intensity noise correlation changes sign as the population is inverted, and monitoring the intensity noise or the spectral linewidth simultaneously with the correlation would establish experimental proof for operation in this new regime.

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