Screening effects in pair production by 10-MeV photons on uranium atoms

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With a partial-wave interpolation method we calculated the positron energy spectra of pair production in the field of uranium atoms for photons of energy $k=10$ MeV, permitting a comparison of theory with the recent experiment. Our results show that the atomic-electron screening effect decreases the cross section for the main region of the positron energy spectra which contributes to the total pair-production cross section, and the effect can be as large as about 9% for the region of low positron energies which contributes little to the total pair-production cross section. The screening effect to the total pair-production cross section is about 3.5%. Our results also indicate that the approximate treatment of screening through energy-shift screening theory becomes inadequate at this intermediate photon energy, while the approximate treatment of screening through form-factor screening theory is quite good. Our result for the total pair-production cross section is compared with the experimental result.

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In recent years [1,2] there has been interest in the calculation of accurate cross sections of electron-positron pair production by intermediate-energy photons in the field of the atomic nucleus. There is a need for an accurate calculation of pair-production cross sections for intermediate-energy photons in the field of atoms. We have reported a study of this problem in a preliminary note [3]. In this report we wish to present results on the positron energy spectra of pair production by photons of energy $k=10$ MeV, obtained with the same partial-wave interpolation method used in our previous work [3], which is an extension of our previous numerical partial-wave calculation techniques which utilize interpolation in partial-wave cross sections [4]. Our results for the total pair-production cross section permit a comparison of theory with the recent experiment [5] of Sherman *et al.* Electron and positron wave functions are obtained in partial-wave series by numerically solving the radial Dirac equation. Photon wave functions are also expanded in partial-wave series. The angular integrals of the pair-production matrix elements are performed analytically, while the radial integrals are calculated numerically and then summed numerically over the partial series. The target atom is assumed to be unpolarized and is described by a central potential with a point nucleus.

The unpolarized pair-production energy spectrum has the form [3]

$$\sigma(E_+)=Z^{-2}d\sigma/dE_+=\sum_{l_1,l_2}\sigma_{l_1,l_2}(E_+)=\sum_{l_1}\sigma_{l_1}(E_+)\ , \quad (1)$$

where E_+ is the positron energy, Z is the atomic number of the atom, and l_1 and l_2 are the orbital angular momentum quantum numbers of the positron and the electron, respectively. The total pair-production (TP) cross sections are obtained by integration of the energy spectra

$$\sigma(\mathrm{TP})=\int_1^{k-1}\frac{d\sigma}{dE_+}dE_+\ . \quad (2)$$

It is well known that many partial waves are needed for obtaining accurate positron energy spectra of pair production by intermediate-energy photons with the partial-wave method. In Fig. 1 we show the variation of the pair-production partial cross section $\sigma_{l_1}(E_+)$ as a function of l_1 for the cases $Z=92$, $k=10$ MeV, and [3]

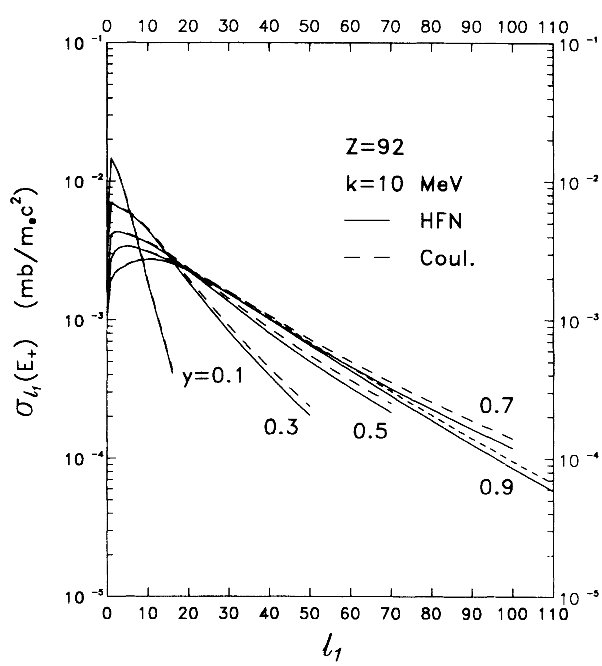

FIG. 1. The variation of the partial cross section $\sigma_{l_1}(E_+)$ of the positron energy spectrum as a function of the orbital angular-momentum quantum number l_1 of the positron for the cases $Z=92$, $k=10$ MeV, and [3] $y=(E_+-1)/(k-2)=0.1$, 0.3, 0.5, 0.7, and 0.9. The symbols Coul. and HFN refer to the point-Coulomb potential, and the Hartree-Fock-Slater potential with the exchange term omitted, respectively.

TABLE I. Comparisons of unpolarized pair-production cross section $\sigma_C(E_+)$ by photons of energy $k=10$ MeV in the field of atomic nucleus with $Z=92$ between the results of Wright, Sud, and Kosik (WSK) for the point-Coulomb potential, and our results calculated with the partial-wave interpolation method, also for the point-Coulomb potential. Here, E_+ is the positron energy in units of MeV.

	$\sigma_C(E_+)$	$(\mu b/m_e c^2)$
E_+	WSK	This work
0.75	23.63	23.66
1.00	55.24	55.36
5.00	121.4	121.5
9.00	96.34	96.27
9.25	83.85	83.19

$$y=(E_+-1)/(k-2)=0.1,0.3,0.5,0.7,0.9 .$$

Here the pair-production partial cross sections are calculated numerically both with Hartree-Fock-Slater potential with the exchange term omitted [6] (designated as HFN potential, solid line) and with the point-Coulomb potential (dashed line). Our results show that the partial cross section $\sigma_{l_1}(E_+)$ is a smoothly varying function of l_1 as $l_1>10$. A modified partial-wave method is therefore possible, which directly calculates a finite set of l values on a grid in l whose spacing increases with l, and then interpolates the intermediate terms.

With the partial-wave interpolation method we have obtained the positron energy spectra of pair production $\sigma(E_+)$ for incident photons of energy $k=10$ MeV and the element of atomic number $Z=92$. In Table I, we show comparison of unpolarized pair-production cross section $\sigma_C(E_+)$ by photons of energy $k=10$ MeV in the field of atomic nucleus with $Z=92$ between the results of Wright, Sud, and Kosik (WSK) [1] for the point-Coulomb potential and our results calculated with the partial-wave interpolation method, also for the point-Coulomb potential. The agreement is very good.

In Table II, we give our results $\sigma(E_+)$ calculated with the partial-wave interpolation method for the cases $Z=92$, $k=10$ MeV, with the point-Coulomb potential (σ_C) and with the HFN potential (σ_{HFN}). Our results show that the atomic-electron screening effect decreases the cross section for the main region of the positron energy spectra, which contributes to the total pair-production cross section, and the effect can be as large as 9% for the region of low positron energies, which contributes little to the total pair-production cross section. In Table II, we show comparisons of our results σ_{HFN} with the results calculated with the energy-shift screening theory (EST) [7] (σ_{EST}). Our results indicate that the approximate treatment of screening through energy-shift screening theory becomes inadequate for $k=10$ MeV. This can be understood qualitatively from Fig. 2, where we show the potential difference V_S-V_C between the screened (HFN) and the point-Coulomb potential. The difference remains the same only at small distances. For $k=10$ MeV, the minimum momentum transfer $q_{\min}$ is about 0.1 and the corresponding distance $r_{\max}=1/q_{\min}$, the shapes of point-Coulomb and energy-shifted screened wave functions are not close, and thus the energy-shift screening theory fails here. At high energies we could expect a good estimate of atomic-electron screening in the Born approximation [8] and describe the screening by a form factor. In Table II, we show also comparisons of our results $\sigma_{\text{HFN}}/\sigma_C$ with the results σ_{BF}/σ_{BH} calculated with the form-factor screening theory, where σ_{BH} and σ_{BF} are the unpolarized pair-production cross section $\sigma(E_+)$ calculated with the Born approximation for the point-Coulomb potential and for the screened case, respectively. Our results indicate that the approximate treatment of screening through the form-factor screening theory is quite good.

In Table III, we present a comparison of the total pair-production cross section for the case $Z=92$ and $k=10$ MeV between our results calculated with the partial-wave interpolation method for the point-Coulomb potential σ_C(TP) and for the HFN potential σ_{HFN}(TP); the result calculated with the Born approximation

TABLE II. Comparisons of unpolarized pair-production cross section $\sigma(E_+)$ by photons of energy $k=10$ MeV in the field of atoms with $Z=92$ among our results calculated with the partial-wave interpolation method for the point-Coulomb potential (σ_C), and for the Hartree-Fock-Slater potential with the exchange term omitted (σ_{HFN}); the results calculated with the Born approximation for the point-Coulomb potential (σ_{BH}) and for the screened cases (σ_{BF}) calculated with the form-factor screening theory; and the results calculated with the energy-shift screening theory (σ_{EST}). Here [3] $y=(E_+-1)/(k-2)$; σ_{BH}, σ_C, and σ_{HFN} are in units of $\mu b/m_e c^2$.

y	σ_{BH}	σ_C	σ_{HFN}	$\sigma_{\text{HFN}}/\sigma_C$	$\sigma_{\text{EST}}/\sigma_C$	σ_{BF}/σ_{BH}
0.026 62	54.02	23.66	25.68	1.09	1.12	0.988
0.054 47	78.45	55.36	55.60	1.004	1.04	0.985
0.1	103.44	83.46	81.87	0.981	1.01	0.979
0.3	137.39	117.6	112.9	0.960	1.00	0.964
0.5	140.28	121.5	116.0	0.955	1.00	0.960
0.7	137.39	122.7	117.8	0.960	1.00	0.964
0.9	103.44	108.7	105.9	0.974	0.996	0.979
0.945 53	78.45	96.27	93.82	0.975	0.990	0.985
0.973 38	54.02	83.19	80.74	0.971	0.986	0.988

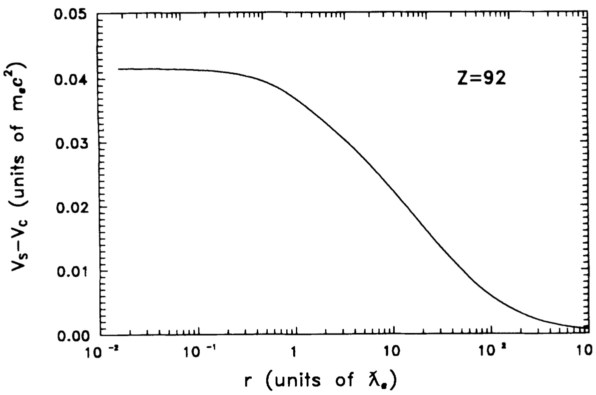

FIG. 2. Difference $V_S - V_C$ between the screened (HFN) and the point-Coulomb potential as a function of distance r in units of the Compton electron wavelength λ_e.

σ_{BH}(TP) for the point-Coulomb potential [8]; and the result of Wright, Sud, and Kosik $\sigma_{\rm WSK}$(TP) for the point-Coulomb potential. The agreement between our point-Coulomb result σ_C(TP) and the result $\sigma_{\rm WSK}$(TP) is very good, while the Born approximation is overestimated. The screening effect to the total pair-production cross section is about 3.5%. With the form-factor screening theory we find the screening effect to the total pair-production cross section is about 3.2%, which agrees quite well with our calculated result with the partial-wave interpolation method.

In Table III, we also show a comparison between our screened result $\sigma_{\rm HFN}$(TP) and the experimental result of Sherman *et al.* $\sigma_{\rm expt}$(TP) [5] divided by the radiative correction [9] $f_{\rm rad} \cong 1.0122$. There is a 3% discrepancy between $\sigma_{\rm HFN}$(TP) and $\sigma_{\rm expt}$(TP). The experimental result $\sigma_{\rm expt}$(TP) is obtained from the measurement of the total cross section $\sigma_{\rm tot}$ for photon absorption by an atom. From the tabulation of Hubbell, Gimm, and Øverbø [2], we see that at $k = 10$ MeV the contribution of total pair-production cross section to the $\sigma_{\rm tot}$ is about 74% and that of the incoherent scattering cross section to the $\sigma_{\rm tot}$ is about 23%. Our calculation is based on the assumption that the target atom is described by a central potential with a point nucleus. Does the discrepancy come from this assumption? Since the form-factor screening theory works well for the cases we considered here, we calculated the atomic-electron screening effect $(1-\sigma_{BF}/\sigma_{BH})$ with the form-factor theory for a finite nucleus and find $(1-\sigma_{BF}/\sigma_{BH})=3.2\%$, which is the same as that obtained for a point nucleus. This can be understood qualitatively by the fact that at high energies the total pair-production cross section is dominated by small angles involving small momentum transfer to the atom. Finite nuclear size effects become important only in the region of large momentum transfer. Thus the finite nuclear size effect is very small for the energy we considered in this report.

TABLE III. Comparisons of total pair-production (TP) cross section by photons of energy $k = 10$ MeV in the field of atoms with $Z = 92$ among our results calculated with the partial-wave interpolation method for the point-Coulomb potential σ_C(TP), and for the Hartree-Fock-Slater potential with the exchange term omitted $\sigma_{\rm HFN}$(TP); the result calculated by the Born approximation σ_{BH}(TP); the result of Wright, Sud, and Kosik $\sigma_{\rm WSK}$(TP) for the point-Coulomb potential; and the experimental result of Sherman *et al.* $\sigma_{\rm expt}$(TP) divided by the radiative correction $f_{\rm rad} \cong 1.0122$. Here, the σ(TP) is in units of b/atom.

σ_{BH}(TP)	σ_C(TP)	$\sigma_{\rm HFN}$(TP)	$\sigma_{\rm WSK}$(TP)	$\sigma_{\rm expt}$(TP)
17.93	15.97	15.41	15.91	15.91±0.091

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