Exclusion of intrinsically classical domains and the problem of quasiclassical emergence

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One difficulty with the correspondence principle is its vagueness. To what should the quantum theory correspond in the quasiclassical domain? Here we show that, whatever it is, it cannot be Hamilton's equations. This is done using Weinberg's generalized nonlinear quantum theory [S. Weinberg, Ann. Phys. (N.Y.) **194**, 336 (1989)] by exploiting the fact that it contains an *exact* copy of classical dynamics [K.R.W. Jones, Phys. Rev. D **45**, R2590 (1992)]. An enlarged dynamical theory incorporating mixed quantum and classical interactions is shown to have some desirable properties in relation to measurement. By studying this system, we show that the existence of observable physical domains obeying intrinsically classical laws would violate the uncertainty principle, thereby ruling out an entire class of such larger theories. We interpret this result as a demonstration that the correspondence principle is essentially approximate. Further, the given exclusion is suggestive as a guide to physical models of quasiclassical emergence in a scenario based upon environmental noise and stochastic reduction.

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I. INTRODUCTION

In attempts to solve the measurement problem many authors have been prepared to consider altering the quantum theory [1–13]. Here we consider the idea of *intrinsic classical domains* [14–16] as a possible physical source of nonquantized environmental noise, and show that these would permit violation of the uncertainty principle. The argument employs the nonlinear mathematics of Weinberg [17,18], since this formalism allows for a simple treatment of mixed classical and quantum systems [19].

From the logical inconsistency exhibited we conclude that there can be no domain of *linear* quantum theory where the classical results become exact, or where even the properties of isolated classical systems are shown. While this is not a surprising conclusion, it is quite definitive, since we are able to show clearly, and in complete generality, the *precise* nature of the changes to quantum theory that would be required to obtain such an exact recovery. On this basis it is argued that there are indeed some major difficulties with the Copenhagen intepretation, if one considers quantum theory to be universally applicable [20].

The discussion proceeds first from consideration of a plausible class of measurement models through to the introduction of a generalized mathematical framework that *might* realize such a scheme, and then to the exclusion of our imagined scenario [21]. This need not exclude the class of models at the point of our introduction, but it shows clearly the nature of the problems one must solve to complete them as fundamental physics, rather than as convenient phenomenological models to describe quantum measurements [22–25].

Thereafter we interpret the existence of classical modes within the Weinberg formalism as a generic statement of the approximate physics that underlies semiclassical methods, and are thus able to associate nonlinear quantum mathematics with the subject of quantum approximation theory [26]. This observation has important implications for the constraint of any physical nonlinearity and the design of empirical exclusions. On this point we conclude with an example that is chosen to highlight the key difficulty, and which may, in turn, suggest a future direction for research on these and related topics.

II. THE MOTIVATION OFFERED BY STOCHASTIC REDUCTION

In the realist view of state vectors and measurements, one seeks an evolution equation for pure states including noise terms that generate localization, collapse, jumps, or other effects that may serve as models for measurement behavior. Ghirardi *et al.* [6], Pearle [7], Diósi [8], and Gisin [9], among others [11–13], have found model equations which exhibit collapse with the correct quantum mechanical probabilities.

The success of such models might lead one to try and build a larger theory by using intrinsically classical degrees of freedom as a source of deterministically random noise [16]. Then one might have some hope of solving the measurement problem, by introducing a potentially testable element. To see why, consider the generic reduction equation:

$$i\hbar\frac{d}{dt}|\psi\rangle = \left\{\hat{H}_f + \hat{H}_e(\psi,\psi^*;\eta(t))\right\}|\psi\rangle, \qquad (1)$$

where $\hat{H}_f$ is the free Hamiltonian of a quantum system and $\hat{H}_e(\psi,\psi^*;\eta(t))$ represents a collapse-inducing nonlinear stochastic operator term, which depends upon a collection of noisy environmental parameters, denoted $\eta(t)$.

Since we are following the line that $\eta(t)$ is due to a noisy interaction, and is not the result of genuine instantaneous quantum jumps [6], we would need to obtain the noisy operator $\hat{H}_e(\psi, \psi^*; \eta(t))$ from interactions with unobserved degrees of freedom, such as a dissipative heat bath [27]. However, quantum interactions entangle states. Therefore (1) is not derivable from quantum theory proper. One would have to assume that the heat bath does not entangle, which is in conflict with quantum universality.

Therefore if we take a literal view of (1) as being an equation that demands a new kind of nonentangling interaction, then we may start to ask the question, what could it be? The goal is not to prejudge the issue, but rather to examine the options to see if some candidate presents itself. If one can find a good, i.e., predictive, candidate, then one might hope to test the new form of interaction without having to do an experiment that is designed to probe measurements. We simply use the measurement problem as a guide.

III. WHY CONSIDER INTRINSICALLY CLASSICAL DOMAINS?

The idea that part of the world might be intrinsically classical is an obvious one. It is suggested immediately by Bohr's phrase "describable in classical terms," with reference to measuring instruments [28]. Further, it gives the simplest possible correspondence principle.

As anyone familiar with semiclassical computations knows, the essence of such methods is to replace some operators by c numbers, usually via the device of taking expectations [26]. The advantage is that the approximated degree of freedom will no longer entangle, because quantum correlations are neglected, making the problem easier to solve.

Looking at (1), where there is a pure state coupled to the outside world, which remains a pure state, it is obvious that a mixed quantum and semiclassical theory would present one possible scenario for a fundamentally different, and larger, quantum theory, where equations like (1) are admissible. We would simply widen the theory to embrace them.

We would then assert that some sources of physical interaction are this way, and I do not mean approximately. The idea has been explored in some depth by Primas [14], who considered the exclusion of this possibility to be of some importance.

I take it up here for a simple reason. The proposed generalized theory of Weinberg [17] is inclusive of the standard theory, with entanglement, but it also allows for the unified treatment of intrinsically classical modes that are nonentangling. It has already been the subject of stringent tests [29–36] which show that if physical nonlinearities exist they must be extremely weak for an isolated atomic system. Thus we may ask if any place remains to look for them, and what physical motivation there may be for them.

As I have shown elsewhere, we can obtain, within the Weinberg formalism, a complete embedding of the entire formalism of classical Hamiltonian mechanics [15]. Therefore, if we work upon excluding this option, one has, at the very least, shown that a very large body of possible ideas, all based upon modes that obey Hamilton's equations, are unphysical. A strong negative result like this may point the way to a better idea.

IV. THE WEINBERG FORMALISM

Originally the Weinberg formalism [17] was proposed as a foil to test the superposition principle. However, since that time it has been recognized that this very same formalism actually lies behind many semiclassical methods. Here we will exploit this fact to construct a model universe that contains mixed classical and quantal modes.

The formalism resembles Hamiltonian classical mechanics. It is based upon the algebra of real-valued functionals $h(\Psi, \Psi^*)$ of complex wave functions Ψ, that are homogeneous of degree one [37]. The Lie bracket

$$[g, h]_W = \frac{\delta g}{\delta \Psi}\frac{\delta h}{\delta \Psi^*} - \frac{\delta h}{\delta \Psi}\frac{\delta g}{\delta \Psi^*} \tag{2}$$

plays the role of the quantum commutator, where $\delta/\delta\Psi$ is a functional derivative, and the computation involves contraction against the wave function coordinates [15]. The equation of motion reads

$$i\hbar\frac{dg}{dt} = [g, h]_W, \tag{3}$$

of which the generalized wave equation

$$i\hbar\frac{d\Psi}{dt} = [\Psi, h]_W = \frac{\delta h}{\delta \Psi^*} \tag{4}$$

is a special case. Linear quantum theory is the restriction to bilinear functionals. These can be characterized as expectation values: $h(\Psi, \Psi^*) = \langle\Psi|\hat{H}|\Psi\rangle$. The norm functional $n = \langle\Psi|\Psi\rangle$ acts as a unit element, and satisfies $dn/dt = 0$.

Weinberg's proposal has been tested to very high precision in nuclear spin precession experiments [29–36]. Bollinger *et al.* [30] have bounded the relative magnitude of nonlinear self-energy terms in freely precessing $^9Be^+$ nuclei at less than 4×10^{-27}. Other attempts to exclude nonlinearity have focused upon the theoretical problems posed by interbranch communication in Einstein-Podolsky-Rosen experiments [38,39], thermodynamic constraints [40], and the lack of any plausible, consistent, and inclusive statistical interpretation for the nonlinear theory [41,42], when posed in a completely abstract setting [43].

All this work was guided by the assumption that any physical nonlinearity would have to be a weak perturbation. Subsequently, it was discovered [15] that Weinberg's theory contains a copy of Hamiltonian classical mechanics, realized via the functional ansatz:

$$h^c(\Psi, \Psi^*) = nH(\langle\hat{Q}\rangle, \langle\hat{P}\rangle), \tag{5}$$

where $\langle\cdots\rangle = \langle\Psi|\cdots|\Psi\rangle/n$, so that homogeneity restrictions are satisfied. The role of classical canonical coordinates is played by the c-number quantities

$$Q(t) = \langle\psi|\hat{Q}|\psi\rangle \text{ and } P(t) = \langle\psi|\hat{P}|\psi\rangle, \quad (6)$$

which induce a nonperturbative nonlinearity. The evolution of Ψ is nondispersive for all $H(Q,P)$, and (6) follow the classical trajectories for all $\hbar > 0$ [19].

For comparison, the ordinary quantum theory is obtained via the ansatz [17]

$$h^q(\Psi,\Psi^*) = \langle\Psi|\hat{H}(\hat{Q},\hat{P})|\Psi\rangle. \quad (7)$$

Comparing (5) and (7), one could view this inclusion of exact classical dynamics within the Weinberg formalism as the semiclassical approximation [26]

$$\langle\Psi|H(\hat{Q},\hat{P})|\Psi\rangle \approx nH(\langle\hat{Q}\rangle,\langle\hat{P}\rangle).$$

Alternatively, (5) might have fundamental content. We should remember that the historical motivation for research into nonlinearity has been to locate a wave equation whose solutions behave like classical particles [1–4].

Since we cannot test quantum theory in the classical domain the problem assumes some urgency. Plausible arguments might be advanced either way; the problem is to show exactly why such an equation cannot be physically correct. Since the Copenhagen interpretation assumes, implicitly, that there do exist devices with the properties of classical systems, and because the mathematics of such systems is actually nonlinear, we will answer, thereby, the question of whether this is a satisfactory formulation for quantum mechanics as a universally valid theory. Either strictly classical devices do not exist, or the theory is not linear.

To examine the fundamental option, imagine, for the sake of argument, a universe with intrinsic classical and quantal domains [15,16], see Fig. 1. As we show, intrinsic classical domains have the special property that they remain disentangled from the quantum domains, even in the presence of mutual interaction. The quantum state of the universe, $|\Psi_U\rangle$, would, if once factorized, remain forever so:

$$|\Psi_U(t)\rangle = |\psi_c(t)\rangle \otimes |\psi_q(t)\rangle. \quad (8)$$

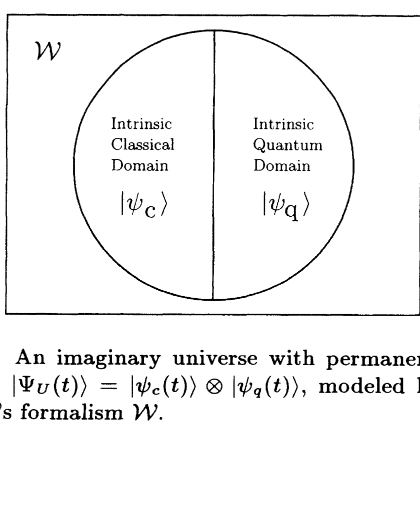

FIG. 1. An imaginary universe with permanently factorized state $|\Psi_U(t)\rangle = |\psi_c(t)\rangle \otimes |\psi_q(t)\rangle$, modeled here within Weinberg's formalism $\mathcal{W}$.

Here $|\psi_c\rangle$ denotes the pure classical part, and $|\psi_q\rangle$ the pure quantum part. Although the latter might become internally entangled, the former factorizes completely, at all times.

To show this, we compare (5) and (7), and consider the ansatz

$$h^{(q,c)}(\Psi,\Psi^*) \equiv \langle\Psi|\hat{H}(\hat{q},\hat{p},\langle\hat{Q}\rangle,\langle\hat{P}\rangle)|\Psi\rangle \quad (9)$$

as a candidate Weinberg functional describing interaction between an intrinsic quantum system, $(\hat{q},\hat{p})$, and an intrinsic classical system, $(\hat{Q},\hat{P})$.

From this we obtain the wave equation

$$\begin{aligned} i\hbar\frac{d}{dt}|\Psi\rangle &= \frac{\delta h^{(q,c)}}{\delta\Psi^*} \\ &= \hat{H}(\hat{q},\hat{p},\langle\hat{Q}\rangle,\langle\hat{P}\rangle)|\Psi\rangle + \langle\hat{H}_Q\rangle(\hat{Q}-\langle\hat{Q}\rangle)|\Psi\rangle \\ &\quad + \langle\hat{H}_P\rangle(\hat{P}-\langle\hat{P}\rangle)|\Psi\rangle, \end{aligned} \quad (10)$$

where we have used $\delta\langle\hat{A}\rangle/\delta\Psi^* = (\hat{A}-\langle\hat{A}\rangle)/n|\Psi\rangle$, valid for linear operators $\hat{A}$ [15].

In (10) there are no operators mixing components of Ψ in the Hilbert space of $(\hat{Q},\hat{P})$, with those in the Hilbert space of $(\hat{q},\hat{p})$. Thus property (8) is verified. As a further check we compute the Weinberg bracket of a pair of mixed functionals:

$$\begin{aligned} [g^{(q,c)},h^{(q,c)}]_W/i\hbar &= n\langle[\hat{G},\hat{H}]\rangle/i\hbar \\ &\quad + n\left\{\partial_Q\langle\hat{G}\rangle\partial_P\langle\hat{H}\rangle - \partial_P\langle\hat{G}\rangle\partial_Q\langle\hat{H}\rangle\right\}. \end{aligned} \quad (11)$$

This bracket contains an ordinary commutator part and a Poisson bracket contribution. Thus it follows that pure classical functionals obey classical equations, while pure quantum observables obey quantum equations. Thus (9) is a reasonable postulate.

To fix physical ideas we might imagine the considered classical domain to be some (any) field whose interaction is only ever important for macroscopic systems.

V. MEASURABILITY OF INTRINSIC CLASSICAL MODES

Consider now the measurability properties of intrinsic classical domains. Although the canonical coordinates (6) were defined using quantum operators, we cannot use the usual probability interpretation. To see this, recall that $P(q) = \Psi(q)\Psi^*(q)$ obeys two constraints in the standard theory [44]. First, $n = \int P(q)\,dq$ is conserved by unitarity. Second, the quantity

$$j = \frac{\hbar}{2mi}\left(\Psi^*\partial_q\Psi - \Psi\partial_q\Psi^*\right)$$

may be interpreted as a probability current, and obeys the continuity equation

$$\frac{\partial P}{\partial t} + \text{div}j = 0.$$

This property depends upon the appearance of second order spatial derivatives (which induce dispersion). For the classical wave equation, one finds that

$$\frac{\partial P}{\partial t} + \frac{\langle \hat{P} \rangle}{M} (\Psi^* \partial_q \Psi + \Psi \partial_q \Psi^*) = 0.$$

The second term cannot be written as a divergence, since it is first order in spatial derivatives. Although the norm is preserved, the classical wave equation allows no probability current. All solutions are rigid, showing no change in shape, Fig. 2. As such, the wave function of an intrinsically classical degree of freedom would have no physical significance [45].

Thus it is inconsistent to apply the uncertainty relations to (6) for an intrinsic classical domain. If we view (5) as fundamental, one must acknowledge that there is no intrinsic probability interpetation to draw upon. One would have to bootstrap this to the quantum domain using an idea like (1) [46].

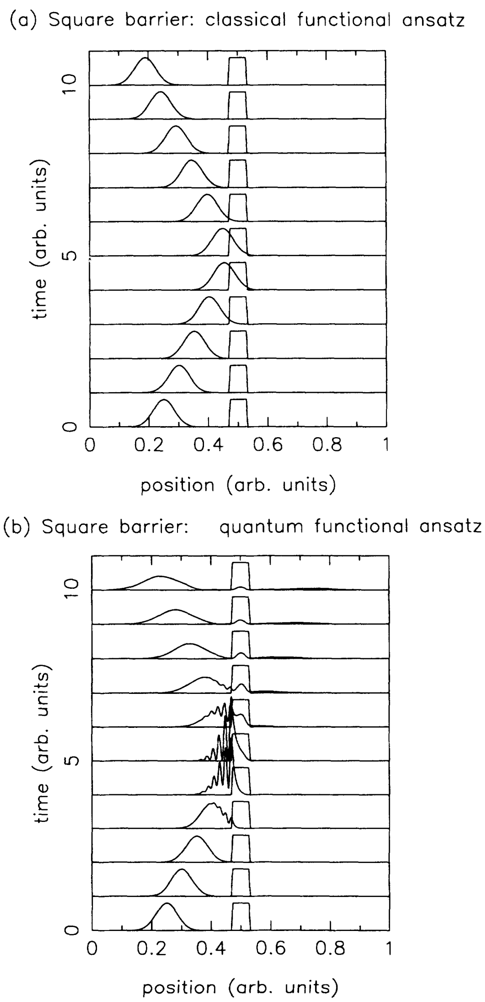

FIG. 2. Numerical simulation of the square barrier problem using (a) the classical ansatz and (b) the quantal ansatz. Note the rigidity and nondispersive nature of the classical evolution. Parameters were chosen for the tunneling regime, with initial wave-packet kinetic energy set at 90% of the barrier height (which has a small slope so that its derivative exists). See Ref. [63].

VI. VIOLATION OF THE UNCERTAINTY PRINCIPLE

Granted this constraint, assume that a single observable intrinsic classical degree of freedom exists. Consider a von Neumann measurement-type coupling [47] between a classical meter variable and a quantum particle,

$$h^{(q,c)}(\Psi, \Psi^*) = \left\langle \Psi \middle| \frac{\hat{p}^2}{2m} + \frac{\langle \hat{P} \rangle^2}{2M} + g\langle \hat{P} \rangle \hat{q} \middle| \Psi \right\rangle. \quad (12)$$

Here $(\hat{Q}, \hat{P})$ are the meter variables, M its mass, $(\hat{q}, \hat{p})$ are the particle variables, m its mass, and g is the coupling strength.

Using the identity

$$\frac{d}{dt}|\Psi(t)\rangle = \left(\frac{d}{dt}|\psi_c(t)\rangle\right) \otimes |\psi_q(t)\rangle + |\psi_c(t)\rangle \otimes \left(\frac{d}{dt}|\psi_q(t)\rangle\right),$$

one can separate (10) into two coupled, but disentangled wave equations:

$$i\hbar \frac{d}{dt}|\psi_c(t)\rangle = \left\{ \frac{\langle \hat{P} \rangle^2}{2M} + \frac{\langle \hat{P} \rangle}{M}(\hat{P} - \langle \hat{P} \rangle) + g\langle \hat{q} \rangle (\hat{P} - \langle \hat{P} \rangle) \right\} |\psi_c(t)\rangle, \quad (13)$$

$$i\hbar \frac{d}{dt}|\psi_q(t)\rangle = \left\{ \frac{\hat{p}^2}{2m} + g\langle \hat{P} \rangle \hat{q} \right\} |\psi_q(t)\rangle. \quad (14)$$

Thus it is consistent to suppose that $Q(t)$ and $P(t)$ can be monitored without need of any collapse postulate (recall that one needs this in the standard theory to fix a definite branch for a quantum entangled superposition). From (11) we obtain the auxiliary equations

$$\frac{d\langle \hat{P} \rangle}{dt} = 0, \quad \frac{d\langle \hat{Q} \rangle}{dt} = \frac{\langle \hat{P} \rangle}{M} + g\langle \hat{q} \rangle,$$
$$\frac{d\langle \hat{q} \rangle}{dt} = \frac{\langle \hat{p} \rangle}{m}, \quad \frac{d\langle \hat{p} \rangle}{dt} = -g\langle \hat{P} \rangle. \quad (15)$$

These can be solved independently of (13) and (14) (the solutions for moments are identical to pure quantum theory, or pure classical theory). Doing this, we find

$$\langle \hat{P} \rangle(t) = \langle \hat{P} \rangle(0), \quad (16)$$

$$\langle \hat{Q} \rangle(t) = \langle \hat{Q} \rangle(0) + \frac{\langle \hat{P} \rangle(0)}{M} t + g\langle \hat{q} \rangle(0) t + \frac{g\langle \hat{p} \rangle(0)}{2m} t^2 - \frac{g^2 \langle \hat{P} \rangle(0)}{6m} t^3, \quad (17)$$

$$\langle \hat{p} \rangle(t) = \langle \hat{p} \rangle(0) - g\langle \hat{P} \rangle(0) t, \quad (18)$$

$$\langle \hat{q} \rangle(t) = \langle \hat{q} \rangle(0) + \frac{\langle \hat{p} \rangle(0)}{m} t - \frac{g\langle \hat{P} \rangle(0)}{2m} t^2. \quad (19)$$

Now we choose $Q(0) = 0$ and $P(0) = 0$, and examine (17) to see that

$$\langle \hat{q} \rangle(0) = Q'(0)/g \text{ and } \langle \hat{p} \rangle(0) = mQ''(0)/g.$$

Since no measurability constraints apply to $Q(t)$, the conjugate parameters $(\langle\hat{q}\rangle(0), \langle\hat{p}\rangle(0))$ could be precisely measured for a single particle. Thus the measured dispersion (zero) would violate the uncertainty principle: $\Delta^2 q \Delta^2 p \geq \hbar^2/4$.

To summarize, if we suppose that (5) has fundamental content, then the probability interpretation cannot be used in classical domains. However, the disentangling property of a mixed universe would offer immediate hope to solve the measurement problem via a fundamentally derived stochastic reduction equation [5–9]. Following this through, one exhibits a violation of the uncertainty principle. The option is therefore excluded.

VII. DISCUSSION

At first sight this result appears innocuous, especially given the well-known arguments of Heisenberg [48]. However, there is a new element, unknown 65 years ago, namely the exact recovery of Hamiltonian classical mechanics at all nonzero $\hbar$. This merits some discussion, since it is obvious that (5) is of a generality such that it demands some physical interpretation, when it is not fundamental. Clearly, it is just an approximation. However, its exactitude begs a deeper inquiry. We think it may be viewed as a possible explanation for the mathematical origins of the structure of classical mechanics, as being a very special kind of approximation offered by the equations of quantum theory.

The key is that the factorization ansatz ignores the form of the wave function, thus making it a redundant part of the physical description. Through this specific form of approximation the classical concepts of separable systems and nonentangling noise find their origin as being very nearly a property of quantum theory in the macroscopic domain.

This was precisely the state of our prequantum understanding. In this respect it is significant to note that the exact Eherenfest equations

$$\frac{d\langle\hat{p}\rangle}{dt} = -H_q(\langle\hat{q}\rangle, \langle\hat{p}\rangle) \text{ and } \frac{d\langle\hat{q}\rangle}{dt} = +H_p(\langle\hat{q}\rangle, \langle\hat{p}\rangle) \quad (20)$$

are the *only* possible autonomous approximation to quantum theory. When expectation values are carried into the arguments of our Hamiltonian, the detailed form of ψ is made redundant, so that only the initial values of $\langle\hat{q}\rangle$ and $\langle\hat{p}\rangle$ are needed, being now a set of c-number canonical coordinates, just like in ordinary classical dynamics.

In this way of viewing the situation we may understand why classical mechanics is both a workable quantum approximation, and further, exactly why it is fundamentally incorrect in leaving out ψ, i, and the physical constant $\hbar$, while at the same time neglecting the measurability constraints that apply to the aforementioned approximate initial conditions.

One further understands, upon the basis of our exclusion, just why there can be no simple combination of physical parameters that defines the classical limit. Although it has been common practice to consider the limit $\hbar \to 0$ as the link between classical and quantum theory, one must not be blinded to the fact that classical theory is perfectly well defined, and would present a good approximation, in a universe where $\hbar$ assumes any non-zero value. To illustrate, we combine (5) and (7) to identify the dimensionless number:

$$\Delta_{QC} = \frac{\langle\hat{H}(\hat{Q}, \hat{P})\rangle - H(\langle\hat{Q}\rangle, \langle\hat{P}\rangle)}{H(\langle\hat{Q}\rangle, \langle\hat{P}\rangle)}, \quad (21)$$

as a simple measure of the degree to which the replacement of a linear equation by its nonlinear approximation may be in error.

For the harmonic oscillator this reads $\Delta_{QC} = \hbar\omega/2E_C$, with ω the resonant frequency, and E_C the classical energy. In the case of a pendulum we have $\omega = (g/\ell)^{1/2}$, and $E_c = mg\ell\theta^2_{\max}/2$, with ℓ its length, $\theta_{\max}$ the maximum amplitude, m the bob mass, and g the acceleration due to gravity. Then $\Delta_{QC} = \hbar/mg^{1/2}\ell^{3/2}\theta^2_{\max}$. For $m = 1$ kg, $\ell = 1$ m, $\theta_{\max} = 0.1$ rad, we find $\Delta_{QC} \approx 2.1\times10^{-32}$, decreasing further as m increases, but never vanishing. However, if we set $\hbar$ to zero the illusion of perfect agreement is manufactured. The smallness of this number serves to warn us of the subtlety of forming an adequate empirical exclusion of quantum nonlinearities.

On this point, recall Bollinger *et al.*'s [30] bound of 4×10^{-27} for the relative size of the Weinberg nonlinearity in the nuclear spin precession experiments. Obviously, this elegant experiment is both a fundamental, and, we think, decisive exclusion of nonlinearity for electromagnetism. Since other sources of nonlinearity might remain, it is helpful to exclude (5) by an argument that is nonspecific about the possible interaction. There is no single experiment that could do this for us with any confidence.

While this is very comforting, there remain some delicate issues in the final exclusion of the nonlinear option. For example, once we have excluded any possibility of an exact domain wherein classical devices exist, there is an obvious difficulty of principle for the Copenhagen interpretation [20]. This is particularly acute in the matter of cosmology, at which juncture we have no recourse to an observer outside of the considered system.

Recall that Bohr's stance on measurements presumes the existence of physical domains "describable in classical terms" [28]. This assumption is in accord with common sense. To maintain consistency, Bohr denied physical reality to the wave function. It is, in the Born interpretation, merely a bookkeeping device for computing the probability that a point particle will be found at some given location [44]. This position would be acceptable if quantum theory recovered the quasiclassical concept of separable systems as the underlying basis for the bootstrap of physical recording devices.

However, one now sees that the ideal of an exact classical limit is never joined. Clearly, it is an unphysical idealization whose key property of separability places it squarely outside the present quantum theory. While we may be complacent that the theory is excellent, we may not be complacent that this fact follows because nature behaves just so.

In answer to this dilemma, some authors contend that decoherence is the solution [49]. Certainly it is true that this scenario establishes conditions under which it is impossible in principle to tell if a measurement or collapse event has actually occurred. However, in spite of the attractive nature of the minimalist stance that we should change nothing, one is equally committed, thereby, to examine no further the options for a theory that might well refute this position. This may obstruct progress if the problem is a real one.

Furthermore, one cannot, as Bell often complained [50], defend the view, as logical, that the mere vanishing of off-diagonal terms in a density matrix establishes that a quantum measurement has occurred. One has then to explain why, in other circumstances, this step would lead to erroneous conclusions; most clearly in the case of photon echoes [51]. Equally, it is a peculiar activity to "derive" an equation of type (1), for which quantum theory allows no fundamental possibility, while declaring that nothing has been changed [52].

This subterfuge was analyzed recently by Bonci *et al.* [53], who showed it to be an unreliable consequence of some of the semiclassical approximations that are used to simplify calculations in the treatment of quantum systems that are coupled to baths. In light of this, and the present work, we feel that it is quite pointless to defend, as a proof of the consistency of linear quantum theory, any argument that employs methods that destroy the linearity of said theory. Obviously the factorization approximation lies in this category.

In summary, we feel that, all things considered, the prospect one must face is that the Everett interpretation may well survive as the future self-consistent formulation of quantum theory that is best suited to cosmological studies [54].

Having said that, the author finds this prospect rather chilling. It is disturbing to the pragmatic view that "things do happen," and would appear to be a scientific dead end in so far as it is not testable within itself. One might, one day, refute such a view by finding a theory that enlarged quantum mechanics, agreed with experiment, and said more. As argued most eloquently by Bohm and Bub [5], the nature of such experiments cannot be imagined within the confines of a theory which admits no logical possibility for them.

VIII. OPEN QUESTIONS

The present argument attempts to rule out a broad class of possible enlarged theories that admit fundamental nonentangling interactions for the purpose of explaining quasiclassical behavior and measurements. The argument definitely does not rule out Eqs. (1) as a way to solve the problem. Its main use, for this research, is to draw a plausible link between the concepts of noise, nonlinearity, and nonentangling interactions. Further, it promotes the uncertainty principle to the status of a useful sieve.

The important feature of such logical discussions is to identify their loopholes. The most obvious one is that we have assumed a scenario where the quantum noise is to be traced to an environmental interaction. This is perhaps inessential, although I believe it is the most fruitful in suggestive power for the purpose of locating candidate theories.

I would stress, however, that theories such as that of Ghirardi *et al.* [6] do not adhere to this assumption. Rather, they invoke a new process of spontaneous localization, i.e., a genuine jumpiness at the root of the quantum theory. Recently, they have sought to link this with some novel ideas of Diósi [8] on stochastic gravitational localization.

They argue that for Diósi's idea a free parameter is needed for self-consistency. On this basis they conclude that a new constant of nature is a necessary feature for a unified description of micro and macro physics. This may be so, but it would be preferable, we feel, to continue the search for a constrained theory. The problem with free theories is just that they are not predictive. They may offer some phenomenological guide, but they are not very convincing in themselves. It is hoped, therefore, that we can do better.

The possibility of jumps aside, the second obvious assumption to question is that our classical modes should obey Hamilton's equations. Certainly that is an obvious choice, however, it was not physically motivated. It is just a convenient mathematically inspired guess. Since a nonentangling interaction is the major requirement for stochastic models we might seek to furnish these from another source.

Here one must consider very carefully just what is intended by a modification to the quantum theory. Presumably one means that there is something left out of the present theory which one is attempting to incorporate.

At the fundamental level one would normally associate such a change with the discussion of physical forces, of which we know there are only four—(1) strong force, (2) weak force, (3) electromagnetism, and (4) gravity. The first three are directly testable at the fundamental level. The first two are very effectively screened from any direct role in quasiclassical physics. In the case of electromagnetism, any possible nonlinearity is very effectively probed by the elegant experiments of Bollinger *et al.* [30] and others [31–34].

Thus if one is serious about testing the nonlinear theory further [55], it would seem pointless to tinker with any of these interactions. One cannot change them without upsetting the agreement with prior results. However, in the matter of gravity things are different. We do not yet know how it must be treated quantum mechanically.

Moreover, it is clear that gravity does have an important effect on macroscopic bodies. The reason for this may be traced to its unscreenable nature, which property sets it apart from the other fundamental interactions.

Thus it is plausible to suppose that the many-particle Schrödinger equation may well incorporate a nonlinearity that depends upon the density and arrangement of a number of individual particles. It is true that the effect must generally be very weak, but then we see from (21) that a weak nonlinearity need not be wholly insignificant.

Speculations of this kind have been raised often [56].

However, on the basis of the present investigation, one has a rather better idea of what such a nonlinearity must look like if we are to have any hope of building a consistent nonlinear theory. It must admit nonentangling noise, and it must not be in conflict with the Heisenberg uncertainty principle [48]. The most obvious candidate is that suggested by Møller [57], Rosenfeld [58], and Kibble [59]; namely, the equations of semiclassical gravity

$$G_{\mu\nu} = 8\pi \langle T_{\mu\nu} \rangle. \tag{22}$$

Certainly this involves the notion of a disentangled classical environment, now space time, left unquantized. The equations are nonperturbative, and would reflect a nontrivial change to quantum theory that is most significant for an intense quantum field, while remaining inclusive of the linear theory as an isolated microsystem limit. Moreover, the nature of the nonlinearity is such as to allow for inherently noisy behavior in a complicated quantum mechanical system [60]. In this regard, the arguments given by Page and Geilzer [61] and Eppley and Hannah [62] for the necessity of quantizing the gravitational metric merit closer analysis to see if they have any loopholes.

In conclusion, the central significance of this result is to establish, via the inconsistency of (5), that the quantum mechanical correspondence principle is essentially approximate. In the past, the analytical intractability of the general quantum many-body problem may have offered refuge to those who might consider this question to have been of an undecidable nature, and thus of no consequence. I do not believe that this position remains tenable.

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interest of this dynamical system lies in its mathematical properties. See Ref. [19].
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[43] In Ref. [17] Weinberg suggested that the functionals $a(\psi,\psi^*)$ should be interpreted as average values. He then attempted to obtain an induced statistical interpetation from the chain of functional products a, $a\star a$, $a\star a\star a, \ldots$. The problem he ran into rests with the nonassociative nature of the $\star$ product in the nonlinear regime. This leads to nonuniqueness in the induced probability density, unless $a\,\star$ commutes with all of its powers. However, this condition fixes the ordinary linear theory [K.R.W. Jones, Ann. Phys. (N.Y.) (to be published)], so that Weinberg's idea does not extend beyond it. This suggests that the general formalism is not compatible with quantum statistics. One must look to a particular theory to have any hope of consistency.
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[46] One feels this would be a necessary feature of any successful nonlinear theory. Since it has no intrinsic statistical interpetation one must recover the standard one as a consequence of a physical theory of this kind. In other words, a correct nonlinear theory must solve the measurement problem, otherwise past results are nonrecoverable and the theory is wrong, cf. Ref. [43].
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[56] See, for example, R. Penrose, in *Quantum Concepts in Space and Time*, edited by C.J. Isham and R. Penrose (Oxford University Press, Oxford, 1986), and references therein. Most proponents of stochastic reduction have suggested the gravitational field, see, e.g., P. Pearle, Int. J. Theor. Phys. **18**, 489 (1979); L. Diósi, Phys. Rev. A **40**, 1165 (1989), and references therein. While these views are unfashionable they have the logical merit of introducing

the physical constant G, as a possible arbiter of nonlinear effects.

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[60] It has been stressed by Ford and co-workers that linear quantum evolution is not complex enough to replicate dynamical chaos. See J. Ford, G. Mantica, and G.H. Ristow, Physica D **50**, 493 (1991); J. Ford and M. Ilg, Phys. Rev. A **45**, 6165 (1992). Since Weinberg's formalism contains an exact copy of Hamiltonian classical mechanics via (5), it is obvious that nonlinear quantum theory is not so restricted. Clearly, that is important in the context of quantum statistical mechanics and the measurement problem.

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[63] O. Bonfim (private communication) performed these numerical simulations first to our knowledge. The algorithm we used is a variant of the split operator fast Fourier transform method [C. Leforestier *et al.*, J. Comput. Phys. **94**, 59 (1991)]. The general solution of (5) requires only the classical trajectories, see Ref. [19].