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Radiative Transition Rates from the $2p3p\ ^3P$ and $2p3d\ ^{1,3}D$ States of the Helium Isoelectronic Sequence*

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The Hylleraas-Scherr-Knight variation-perturbation method is applied to the calculation of oscillator strengths for the dominant transitions from the metastable $2p3p\ ^3P$ and $2p3d\ ^{1,3}D$ states of helium and the heliumlike ions. The results explain the absence of the $1s2p\ ^3P - 2p3p\ ^3P$ transition of He I from recent beam-foil excitation spectra.

I. INTRODUCTION

Many atoms possess rydberg series of doubly excited states which, although imbedded in the autoionizing continuum, are prevented from auto-ionizing by angular momentum and parity selection rules. Such states may be termed "metastable" since their lifetimes are determined by the relatively slow process of spontaneous emission of radiation, rather than autoionization. The $2p^2\ ^3P$ metastable state of helium is the most extensively studied.[1-3] A recent remeasurement of its energy[4] has corrected an earlier error and brought the theoretical and experimental energies into good agreement. Progress in the theoretical study of multiply excited states has been reviewed by Holøien.[5]

In previous papers,[2,6] we have applied the Hylleraas-Scherr-Knight $1/Z$ expansion perturbation technique to the calculation of the energies of the metastable $2p^2\ ^3P$, $2p3p\ ^1P$, $2p3p\ ^3P$, $2p3d\ ^1D$, and $2p3d\ ^3D$ states of the helium isoelectronic sequence. Direct variational bounds have also been obtained for the neutral helium eigenvalues as a check on the accuracy of the perturbation expansions. In this paper, we evaluate the oscillator strengths for those transitions from the $2p3p\ ^3P$, $2p3d\ ^3D$, and $2p3d\ ^1D$ states which are expected to be dominant. Results for the corresponding transitions

from the $2p^2\ ^3P$ and $2p3p\ ^1P$ states have been reported previously.[2] Transitions involving the above states have been observed in beam-foil excitation spectra[7] and are of interest in the study of solar flares and the solar corona.[8,9]

The computational method is briefly summarized in Sec. II and the results presented in Sec. III.

II. COMPUTATIONAL METHOD

The Hylleraas-Scherr-Knight variation-perturbation method used in the present work has been described previously[2,6] and is only briefly summarized here. In units of Z^2 a.u., the Hamiltonian for a two-electron atom may be written in the form $H = H_0 + Z^{-1}V$, where H_0 is a sum of hydrogen-atom Hamiltonians and $V = 1/r_{12}$. The above partition of H leads to the following perturbation expansions for the eigenfunctions and eigenvalues:

$$\Psi = \Psi_0 + \sum_{n=1} \Psi_n Z^{-n}, \tag{1}$$

$$E = E_0 + \sum_{n=1} E_n Z^{-n}. \tag{2}$$

The perturbed functions Ψ_n are expressed in terms of a truncated set of N orthonormal functions φ_i, which are constructed from linear combinations of N correlated functions of the form

$$\chi_j = (1 \pm P_{12}) \, r_1^a \, r_2^b \, r_{12}^c \, e^{-\alpha r_1 - \beta r_2} \, \mathcal{Y}_{L \, l_1 l_2}^M (\hat{r}_1, \hat{r}_2) , \qquad (3)$$

where

$$\mathcal{Y}_{L \, l_1 l_2}^M (\hat{r}_1, \hat{r}_2) = \sum_{m_1, m_2} \langle l_1 m_1 l_2 m_2 | LM \rangle$$

$$\times Y_{l_1}^{m_1} (\hat{r}_1) \, Y_{l_2}^{m_2} (\hat{r}_2) . \qquad (4)$$

H_0 is diagonalized in the basis set φ_i such that

$$\langle \varphi_i | H_0 | \varphi_j \rangle = \epsilon_i \delta_{ij} . \qquad (5)$$

P_{12} in (3) indicates the interchange of labels 1 and 2, with the plus sign referring to singlet states and the minus sign to triplet states. The scale factors α and β are set equal to their hydrogenic values; for example, $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ for the $2p3d \; {}^{1,3}D$ states. Then, as a consequence of the diagonalization (5), one of the φ_i, say, φ_s, is automatically the exact hydrogenic eigenfunction Ψ_0 with eigenvalue $\epsilon_s = E_0$, and the remaining functions $\varphi_i (i \neq s)$ form a synthetic representation of the remaining spectrum of H_0. It can be shown from a variational principle that the Ψ_n and E_n are determined by the recursion relations

$$| \Psi_n \rangle = \sum_{i \neq s}^N \left(\frac{\langle \varphi_i | V | \Psi_{n-1} \rangle}{\epsilon_s - \epsilon_i} - \sum_{p=1}^{n-1} \frac{E_p \langle \varphi_i | \Psi_{n-p} \rangle}{\epsilon_s - \epsilon_i} \right) | \varphi_i \rangle , \qquad (6)$$

$$E_n = \langle \Psi_{n-1} | V | \Psi_0 \rangle - \sum_{p=1}^{n-1} E_p \langle \Psi_{n-p} | \Psi_0 \rangle , \qquad (7)$$

provided that all the lower order Ψ_i, E_i, $i = 1, \ldots,$ $n - 1$, are known exactly. With a finite basis set, the lower-order solutions are not known exactly, but the above equations provide useful approximations to Ψ_n and E_n (but not bounds beyond E_2) up to some finite order n_{max}. The nonrelativistic perturbation expansions (1) and (2) summed up to order n_{max} become increasingly accurate with increasing Z and are asymptotically correct. An advantage of the $1/Z$ expansion procedure over a direct variational calculation is that results for an entire isoelectronic sequence are obtained by a single diagonalization of H_0 in the finite basis set.

III. CALCULATED OSCILLATOR STRENGTHS

We are concerned with the calculation of oscillator strengths for the dominant transitions from the $2p3p \; {}^3P$, $2p3d \; {}^3D$, and $2p3d \; {}^1D$ states. The transitions considered are

$$2p3p \; {}^3P \rightarrow 1s2p \; {}^3P, \; 1s3p \; {}^3P, \; 1s4p \; {}^3P,$$

$$2p3d \; {}^3D \rightarrow 1s3d \; {}^3D, \; 2p^2 \; {}^3P, \; 2p3p \; {}^3P,$$

$$2p3d \; {}^1D \rightarrow 1s3d \; {}^1D, \; 2p3p \; {}^1P.$$

In addition, there are one-electron transitions to doubly excited *autoionizing* states, but these should be less important than transitions to singly excited

states since the energy separations are much smaller in the former case.

The "length" form of the oscillator strength for a transition between two atomic states labeled by angular momentum quantum numbers LM and $L'M'$ and having energies E and E' is defined as[10]

$$f_l(\gamma LM \rightarrow \gamma' L'M')$$

$$= \tfrac{2}{3} (E' - E) \, | \langle \gamma' L'M' | \textstyle\sum_i \vec{r}_i | \gamma LM \rangle |^2 \qquad (8)$$

in atomic units. The sum runs over all the electrons in the atom. An averaged oscillator strength $\bar{f}$ which is independent of magnetic quantum number is obtained by averaging over the initial-state orientation degeneracy and summing over the final-state degeneracy. It is first convenient to introduce reduced matrix elements through the relation involving the 3-j symbol[11]

$$\langle \gamma' L'M' | \textstyle\sum_i r_i^q | \gamma LM \rangle$$

$$= (-1)^{L'-M'} \begin{pmatrix} L' & 1 & L \\ -M' & q & M \end{pmatrix} (\gamma' L' \| \textstyle\sum_i \vec{r}_i \| \gamma L), \qquad (9)$$

with $r^{\pm 1} = \mp (x \pm iy)/\sqrt{2}$, $r^0 = z$. With the aid of a sum rule for the 3-j symbols, the averaged oscillator strength can be written

$$\bar{f}_l (\gamma L \rightarrow \gamma' L') = \frac{2(E' - E)}{3(2L+1)} \, | (\gamma' L' \| \textstyle\sum_i \vec{r}_i \| \gamma L) |^2 . \qquad (10)$$

The results are presented in terms of the above reduced matrix elements and averaged oscillator strengths.

For exact initial- and final-state wave functions, the length form given by Eq. (10) is formally identical to the velocity form

$$\bar{f}_v(\gamma L \rightarrow \gamma' L') = \frac{2(E' - E)^{-1}}{3(2L+1)} \, | (\gamma' L' \| \textstyle\sum_i \vec{p}_i \| \gamma L) |^2 , \qquad (11)$$

where $\vec{p}_i = (\hbar/i) \nabla_i$. The length and velocity transition integrals $R = (\gamma' L' \| \textstyle\sum_i \vec{r}_i \| \gamma L)$ and $P = (\gamma' L' \| \textstyle\sum_i \vec{p}_i \| \gamma L)$ have the Z^{-1} expansions

$$R = Z^{-1} \sum_{n=0}^{\infty} R_n Z^{-n} , \qquad (12)$$

$$P = Z \sum_{n=0}^{\infty} P_n Z^{-n} , \qquad (13)$$

with the *exact* expansion coefficients being related through

$$P_n = \frac{i}{\hbar} \sum_{m=0}^{n} R_{n-m} \Delta E_m \qquad (14)$$

as a consequence of the equivalence of Eqs. (10) and (11). ΔE_m in (14) is the mth coefficient in the Z^{-1} expansion of $E' - E$. With the approximate perturbed eigenfunctions obtained by expansion in

a finite basis set, (14) is not satisfied exactly (except in lowest nonvanishing order) and thus provides a check on the accuracy of the calculation. We denote by $R_n^{(1)}$ the R expansion coefficients calculated directly from the matrix elements of $\sum \vec{r}_i$, and by $R_n^{(v)}$ the coefficients calculated indirectly from the matrix elements of $\sum \vec{p}_i$ and the use of Eq. (14). Similarly, $P_n^{(v)}$ denotes the P expansion coefficients calculated directly from the matrix elements of $\sum \vec{p}_i$ and $P_n^{(1)}$ those calculated indirectly from the $R_n^{(1)}$ and Eq. (14). The above four sets of expansion coefficients each yield slightly different results for the oscillator strengths. One could also eliminate the energy difference between (10) and (11) to obtain the intermediate form for the oscillator strength

$$\bar{f}\left(\gamma L \rightarrow \gamma' L'\right) = \frac{2}{3(2L+1)}\,|RP|\,, \qquad (15)$$

thus generating several more possible procedures.

It is not clear which of the above procedures is the most accurate. Starace[12] argues that the length formulation is preferable if the wave functions are exact eigenfunctions of an approximate nonlocal Hamiltonian, but he specifically excludes correlated variational wave functions. In the latter case, Dalgarno and Lewis[13] and Crossley[14] argue that for large excitation energies the velocity form is preferable, while Stewart[15] has shown that for transitions between adjacent states involving no change in principal quantum number, the length form is preferable. We have adopted the procedure of calculating the oscillator strengths in the length formulation, with the transition integrals R obtained by summing the $R_n^{(v)}$ for transitions in which the principal quantum number changes and by summing the $R_n^{(1)}$ for transitions in which the principal quantum number does not change.

The reduced $R_n^{(1)}$ and $R_n^{(v)}$ transition integral expansion coefficients calculated from 50-term basis sets, as discussed in Sec. II, are given in Table I. The relative phases are internally consistent and correspond to radial wave functions which are positive near the origin. The zero-order coefficients are related to the one-electron hydrogenic reduced transition integrals by[11]

$$\left(\gamma' l_1' l_2 L' \| \sum_i \vec{r}_i \| \gamma l_1 l_2 L\right)$$

$$= (-1)^{l_1' + l_2 + L + 1}\left[(2L+1)(2L'+1)\right]^{1/2}$$

$$\times \begin{Bmatrix} l_1' & L' & l_2 \\ L & l_1 & 1 \end{Bmatrix}\left(\gamma' l_1' \| \vec{r} \| \gamma l_1\right). \qquad (16)$$

A comparison of the $R_n^{(1)}$ and $R_n^{(v)}$ in Table I indicates that the accuracy varies considerably from transition to transition. The neutral helium oscillator strengths calculated from the truncated ex-

TABLE I. Electric dipole reduced transition integral expansions.[a]

Order	1s2p 3P-2p3p 3P	1s3p 3P-2p3p 3P	1s4p 3P-2p3p 3P	2p² 3P-2p3d 3D
0	-0.51669	-1.29027	0.00000	-8.2238
	-0.51669	-1.29027	0.00000	-8.2238
1	0.62332	-0.21186	-0.33944	-0.4037
	0.62349	-0.21536	-0.34717	-0.4036
2	0.83795	0.02627	-0.24393	2.2576
	0.83787	0.03487	-0.22492	2.2587
3	0.65075	0.40291	-0.08497	2.8142
	0.65490	0.39094	-0.10257	2.8140
4	0.31499	0.58033	-0.03708	1.9665
	0.31070	0.59326	-0.08501	1.9791
5	-0.01135	0.54568	-0.29859	0.6815
	-0.01176	0.55245	-0.20077	0.6775
6	-0.15775	0.39178	-0.27426	0.0461
	-0.14231	0.37623	-0.15268	0.0613
7	-0.10841	0.09111	···	0.4751
	-0.09739	0.10881	···	0.4738
8	-0.16261	-0.03574	···	1.4647
	-0.13819	0.00869	···	1.4537
9	···	-0.57299	···	2.4277
	···	-0.62883	···	2.2368
	2p3p 3P-2p3d 3D	2p3p 1P-2p3d 1D	1s3d 1D-2p3d 1D	1s3d 3D-2p3d 3D
0	12.3238	12.3238	-1.66573	-1.66573
	12.3236	12.3235	-1.66573	-1.66573
1	10.2498	9.0039	0.07448	0.03730
	10.2542	9.0048	0.08347	0.03908
2	12.7032	8.0555	0.26362	-0.04908
	12.6693	8.0529	0.25394	-0.04736
3	12.7187	7.6641	-0.20432	0.03003
	12.8288	7.6693	-0.21201	0.02197
4	10.8103	7.3500	0.00928	0.07430
	10.6355	7.3586	-0.00279	0.06630
5	7.7642	6.8461	-0.18575	0.00816
	8.1217	6.8742	-0.15699	0.01974
6	5.5383	6.0260	-0.15642	-0.06770
	6.0997	6.0334	-0.19500	-0.05198
7	5.4518	4.7931	-0.10393	-0.07278
	5.4294	4.7689	-0.13295	-0.08120
8	7.5644	3.2092	-0.14652	0.03628
	6.4057	3.2181	-0.14282	-0.04032
9	9.1916	9.9709	-0.07363	0.13504
	···	···	-0.06926	0.00852

[a] $R = Z^{-1}(R_0 + R_1 Z^{-1} + R_2 Z^{-2} + \cdots)A_0$, where Z is the nuclear charge and a_0 is the Bohr radius. For each pair of numbers, the first is $R_{n1}^{(1)}$ and the second is $R_n^{(v)}$ as defined in the text.

pansions

$$\bar{f}_l = \frac{2\Delta E}{3(2L+1)}\left|\sum_{n=0}^{n\max} R_n^{(1)} Z^{-(n+1)}\right|^2 \qquad (17)$$

and

$$\bar{f}_v = \frac{2\Delta E^{-1}}{3(2L+1)}\left|\sum_{n=0}^{n\max} P_n^{(v)} Z^{-(n-1)}\right|^2 \qquad (18)$$

are compared in Table II. Although the above two

TABLE II. Comparison of f_l and f_v for the He I transitions as defined by Eqs. (17) and (18).

Transition	n_{max} [a]	f_l	f_v
$1s2p\,^3P\text{-}2p3p\,^3P$	8	0.000 89	0.000 90
$1s3p\,^3P\text{-}2p3p\,^3P$	9	0.135 73	0.135 76
$1s4p\,^3P\text{-}2p3p\,^3P$	6	0.005 38	0.005 35
$2p^2\,^3P\text{-}2p3d\,^3D$	9	0.454	0.451
$2p3p\,^3P\text{-}2p3d\,^3D$	9	0.255	0.257
$2p3p\,^1P\text{-}2p3d\,^1D$	9	0.3890	0.3883
$1s3p\,^1D\text{-}2p3d\,^1D$	9	0.126 88	0.126 80
$1s3d\,^3D\text{-}2p3d\,^3D$	9	0.136 11	0.136 14

[a] n_{max} is the highest order retained in perturbation summations (17) and (18).

forms tend to be the most discrepant of the various possible formulations discussed earlier, the comparison in Table II probably overestimates the accuracy by about an order of magnitude. The agreement for the $1s3d\,^{1,3}D$ - $2p3d\,^{1,3}D$ transitions is particularly good.

The energy differences, absorption oscillator strengths, and emission Einstein A coefficients are given in Table III for the isoelectronic sequences up to $Z = 18$. Although the nonrelativistic results become increasingly accurate with increasing Z, it is important to consider relativistic corrections to the energy differences for large Z, particularly for transitions between adjacent states. The effect is to increase the decay rate through the ΔE^3 dependence of the A coefficient. Relativistic and retardation contributions to the transition integral itself are probably not important for $Z \lesssim 25$.

IV. DISCUSSION OF RESULTS

Berry et $al.$[7] have observed several transitions from the doubly excited states of He I, including the $1s2p\,^1P$ - $2p3p\,^1P$ and $1s3p\,^1P$ - $2p3p\,^1P$ transitions at 295.2 and 309.0 Å, respectively. For the corresponding triplet transitions, they observe only a single line at 306 ± 1 Å which they tentatively identify as the $1s3p\,^3P$ - $2p3p\,^3P$ transition predicted to be at 305.7 Å.[6] The unexpected absence of the $1s2p\,^3P$ - $2p3p\,^3P$ transition at 291.1 Å from their spectra is understandable from our calculated oscillator strengths. The transition integral is anomalously small (and inaccurate) due to severe numerical cancellation in the summation of the Z^{-1} expansion series given in Table I. The oscillator strength in Table II is nearly an order of magnitude smaller than that for the two-electron jump $1s4p\,^3P$- $2p3p\,^3P$, even though the latter is forbidden for simple product-type eigenfunctions and proceeds only through correlation effects. The lifetime of the $2p3p\,^3P$ state is then determined primarily by the $1s3p\,^3P$ - $2p3p\,^3P$ transition. In contrast,

TABLE III. Absorption oscillator strengths and spontaneous emission rates.

	$1s2p\,^3P\text{-}2p3p\,^3P$		
Z	ΔE (a.u.)	$f(i \to j)$	$A(j \to i)$ [a] (sec^{-1})
2	1.5654	0.0009	7.1×10^7
3	3.6541	0.0032	1.37×10^9
4	6.6342	0.0081	1.15×10^{10}
5	10.5040	0.0116	4.12×10^{10}
6	15.2629	0.0141	1.05×10^{11}
7	20.9110	0.0158	2.22×10^{11}
8	27.4480	0.0172	4.15×10^{11}
9	34.8741	0.0182	7.11×10^{11}
10	43.1890	0.0190	1.14×10^{12}
11	52.3929	0.0197	1.74×10^{12}
12	62.4857	0.0203	2.54×10^{12}
13	73.4674	0.0208	3.60×10^{12}
14	85.3380	0.0212	4.95×10^{12}
15	98.0975	0.0215	6.65×10^{12}
16	111.7459	0.0218	8.75×10^{12}
17	126.2832	0.0221	1.13×10^{13}
18	141.7094	0.0223	1.44×10^{13}
	$1s3p\,^3P\text{-}2p3p\,^3P$		
2	1.4903	0.136	9.69×10^9
3	3.3569	0.147	5.34×10^{10}
4	5.9738	0.147	1.69×10^{11}
5	9.3409	0.146	4.10×10^{11}
6	13.4579	0.145	8.46×10^{11}
7	18.3250	0.145	1.56×10^{12}
8	23.9421	0.144	2.65×10^{12}
9	30.3092	0.143	4.23×10^{12}
10	37.4263	0.143	6.43×10^{12}
11	45.2935	0.143	9.40×10^{12}
12	53.9106	0.142	1.33×10^{13}
13	63.2777	0.142	1.83×10^{13}
14	73.3948	0.142	2.45×10^{13}
15	84.2619	0.142	3.23×10^{13}
16	95.8790	0.141	4.18×10^{13}
17	108.2461	0.141	5.32×10^{13}
18	121.3632	0.141	6.68×10^{13}
	$1s4p\,^3P\text{-}2p3p\,^3P$		
Z	ΔE (a.u.)	$f(i \to j) \cdot Z^2$	$A(j \to i)$ (sec^{-1})
2	1.4647	0.021	3.69×10^8
3	3.2548	0.016	5.88×10^8
4	5.7465	0.014	8.99×10^8
5	8.9396	0.013	1.29×10^9
6	12.8340	0.012	1.75×10^9
7	17.4298	0.012	2.29×10^9
8	22.7270	0.011	2.91×10^9
9	28.7256	0.011	3.60×10^9
10	35.4255	0.011	4.36×10^9
11	42.8268	0.011	5.20×10^9
12	50.9296	0.011	6.12×10^9
13	59.7337	0.010	7.10×10^9
14	69.2392	0.010	8.16×10^9
15	79.4460	0.010	9.30×10^9
16	90.3543	0.010	1.05×10^{10}
17	101.9640	0.010	1.18×10^{10}
18	114.2750	0.010	1.32×10^{10}

TABLE III. *(Continued)*

$2p^2\,{}^3P-2p3d\,{}^3D$

Z	ΔE (a.u.)	$f(i \to j)$	$A(j \to i)$ (sec^{-1})
2	0.1512	0.454	2.00×10^8
3	0.4395	0.690	2.57×10^9
4	0.8664	0.796	1.15×10^{10}
5	1.4321	0.853	3.37×10^{10}
6	2.1365	0.890	7.83×10^{10}
7	2.9799	0.915	1.57×10^{11}
8	3.9621	0.932	2.82×10^{11}
9	5.0831	0.946	4.71×10^{11}
10	6.3430	0.957	7.42×10^{11}
11	7.7419	0.965	1.12×10^{12}
12	9.2796	0.972	1.61×10^{12}
13	10.9562	0.978	2.26×10^{12}
14	12.7716	0.983	3.09×10^{12}
15	14.7260	0.987	4.13×10^{12}
16	16.8192	0.991	5.41×10^{12}
17	19.0513	0.994	6.96×10^{12}
18	21.4224	0.997	8.82×10^{12}

$2p3p\,{}^3P-2p3d\,{}^3D$

Z	ΔE (a.u.)	$f(i \to j) \cdot Z$	$A(j \to i)$ (sec^{-1})
2	0.0085	0.510	3.53×10^5
3	0.0165	0.387	6.74×10^5
4	0.0245	0.345	9.98×10^5
5	0.0325	0.325	1.33×10^6
6	0.0406	0.314	1.66×10^6
7	0.0487	0.307	2.00×10^6
8	0.0568	0.302	2.34×10^6
9	0.0648	0.298	2.68×10^6
10	0.0729	0.295	3.02×10^6
11	0.0810	0.293	3.37×10^6
12	0.0891	0.291	3.71×10^6
13	0.0972	0.289	4.05×10^6
14	0.1053	0.288	4.40×10^6
15	0.1134	0.287	4.74×10^6
16	0.1215	0.286	5.09×10^6
17	0.1296	0.285	5.43×10^6
18	0.1377	0.284	5.77×10^6

$2p3p\,{}^1P-2p3d\,{}^1D$

Z	ΔE (a.u.)	$f(i \to j) \cdot Z$	$A(j \to i)$ (sec^{-1})
2	0.0164	0.778	2.03×10^6
3	0.0298	0.611	3.49×10^6
4	0.0424	0.547	4.74×10^6
5	0.0548	0.514	5.95×10^6
6	0.0671	0.494	7.15×10^6
7	0.0794	0.480	8.34×10^6
8	0.0916	0.470	9.52×10^6
9	0.1039	0.463	1.07×10^7
10	0.1161	0.457	1.19×10^7
11	0.1283	0.453	1.31×10^7
12	0.1405	0.449	1.42×10^7
13	0.1527	0.446	1.54×10^7
14	0.1649	0.443	1.66×10^7
15	0.1771	0.441	1.78×10^7
16	0.1893	0.439	1.89×10^7
17	0.2015	0.437	2.01×10^7
18	0.2136	0.436	2.13×10^7

Table III. *(Continued)*

$1s3d\,{}^1D - 2p3d\,{}^1D$

Z	ΔE (a.u.)	$f(i \to j)$	$A(j \to i)$ (sec^{-1})
2	1.4918	0.127	9.07×10^9
3	3.3508	0.130	4.69×10^{10}
4	5.9586	0.132	1.51×10^{11}
5	9.3161	0.134	3.73×10^{11}
6	13.4235	0.135	7.80×10^{11}
7	18.2807	0.135	1.45×10^{12}
8	23.8879	0.136	2.49×10^{12}
9	30.2451	0.136	4.00×10^{12}
10	37.3522	0.136	6.12×10^{12}
11	45.2093	0.137	8.97×10^{12}
12	53.8164	0.137	1.27×10^{13}
13	63.1735	0.137	1.76×10^{13}
14	73.2806	0.137	2.37×10^{13}
15	84.1377	0.137	3.12×10^{13}
16	95.7447	0.137	4.05×10^{13}
17	108.1018	0.137	5.16×10^{13}
18	121.2089	0.138	6.49×10^{13}

$1s3d\,{}^3D - 2p3d\,{}^3D$

Z	ΔE (a.u.)	$f(i \to j)$	$A(j \to i)$ (sec^{-1})
2	1.4963	0.136	9.79×10^9
3	3.3654	0.137	4.98×10^{10}
4	5.9843	0.137	1.58×10^{11}
5	9.3531	0.137	3.86×10^{11}
6	13.4719	0.138	8.02×10^{11}
7	18.3407	0.138	1.49×10^{12}
8	23.9594	0.138	2.54×10^{12}
9	30.3282	0.138	4.07×10^{12}
10	37.4469	0.138	6.22×10^{12}
11	45.3157	0.138	9.11×10^{12}
12	53.9344	0.138	1.29×10^{13}
13	63.3032	0.138	1.78×10^{13}
14	73.4219	0.138	2.39×10^{13}
15	84.2907	0.138	3.15×10^{13}
16	95.9094	0.138	4.09×10^{13}
17	108.2781	0.138	5.21×10^{13}
18	121.3969	0.138	6.55×10^{13}

[a]$A(j \to i) = \Delta E^2 (g_i/g_j) f(i \to j) \times 3.2130 \times 10^{10}$ sec^{-1}, where g_i and g_j are the statistical weights.

the oscillator strengths for both the $1s2p\,{}^1P - 2p3p\,{}^1P$ and $1s3p\,{}^1P - 2p3p\,{}^1P$ transitions are large and of roughly equal magnitude.[2]

The line[7] observed at 306 ± 1 Å may also contain a contribution from the $1s3d\,{}^1D - 2p3d\,{}^1D$ transition[6] at 305.4 Å since the experimental resolution is not sufficient to separate the two possible contributions. One would further expect the $1s3d\,{}^3D - 2p3d\,{}^3D$ transition[6] at 304.5 Å to be present since the companion $2p^2\,{}^3P - 2p3d\,{}^3D$ transition predicted to be at 3014 Å has been observed in unpublished work by Berry. Higher-resolution wavelength and lifetime measurements would make possible a positive identification of these transitions.

The selection rule preventing autoionization ap-

plies rigorously only in the limit of exact LS coupling. Holøien[5] has pointed out that some of the fine-structure levels of the doubly excited metastable states are coupled directly with the continuum by the spin-orbit, spin-other-orbit, and spin-spin interactions, thereby making auto-

ionization possible. Although the autoionization rate for the He I states is small compared to radiative decay, the rate increases rapidly with increasing nuclear charge and eventually leads to the selective depopulation of some of the triplet fine-structure levels.

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