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Ultrasonic Attenuation in Liquid ^{4}He under Pressure*

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In an attempt to help resolve previously observed discrepancies between the theory for ultrasonic attenuation and the data at the vapor pressure, we have extended our measurements to higher pressures. We report data on the temperature dependence of the attenuation from 0.1 to 1.0 K at frequencies of 15, 45, 105, and 256 MHz for pressures between 0 and 24.7 atm. At low pressures or temperatures, the data resemble the theoretical predictions, but the measured attenuation is about twice that predicted by most theories. At higher pressures and for temperatures above about 0.3 K, the attenuation shows a sharp reduction below the value extrapolated from low temperatures according to previous theories. It has recently been suggested that this unexpected behavior is due to a restriction on the number of thermal phonons that are able to play a part in the attenuation process at higher temperatures and pressures.

INTRODUCTION

At very low temperatures, where there is a negligible roton population, the attenuation of sound in liquid ^{4}He is thought to arise from three-phonon events. In the limit $\omega\tau_{pp} \gg 1$, where ω is the sound angular frequency and τ_{pp} is the appropriate thermal-phonon relaxation time, the attenuation has generally been thought to be given by[1]

$$\alpha(\omega, T) = \frac{\pi^2}{30}\frac{(u+1)^2}{\rho}\frac{k_B^4}{\hbar^3 c^6}\,\omega T^4 \times [\tan^{-1}(2\omega\tau_{pp}) - \tan^{-1}(3\gamma\bar{p}^2\omega\tau_{pp})], \qquad (1)$$

where ρ is the density, c is the sound velocity, $u \equiv (\rho/c)(\partial c/\partial\rho)$ is the Grüneisen constant, $\bar{p} \equiv 3k_B T/c$ is the average thermal-phonon momentum, and γ is the dispersion constant defined by the relation $\epsilon = cp(1-\gamma p^2)$, where ϵ and p are the energy and momentum, respectively, of an elementary excitation.

In previous experiments at the vapor pressure[2] the measured temperature dependence of the attenuation was in qualitative agreement with the T^4 law but was larger than that predicted by Eq. (1) by about a factor of 2. The frequency dependence, measured in the range 12–208 MHz, was more complex; for temperatures below about 0.3 K an approximately linear behavior was observed while for temperatures between 0.3 and 0.4 the frequency dependence fell below linear above 36 MHz.

Several attempts to explain the previous data have involved the assumption that γ is negative (anomalous dispersion). Maris and Massey[3] pointed out that for negative γ the limit $|3\gamma\bar{p}^2\omega\tau_{pp}| \gg 1$ results in twice the usually expected magnitude of the attenuation according to Eq. (1) (the arctangent functions are then additive). A lack of knowledge of τ_{pp}, however, made it unclear whether this suggestion could be applied to our data. Very recently, Maris[4] presented a calculation which assumes anomalous dispersion but differs from Eq. (1) in that no relaxation-time approximation is made for the three-phonon collision process. Preliminary comparisons at 0.35 K with our previous data show fairly good agreement both in magnitude and frequency dependence.

The present investigation was initiated in order to test how well the theories work at elevated pressure. A preliminary report of this work has appeared previously.[5]

EXPERIMENTAL TECHNIQUE

The sound cell with which these data were taken was attached to the copper mixing chamber of a ^{3}He-^{4}He dilution refrigerator which regularly cools below 20 mK. Temperatures were determined by measuring the susceptibility of single crystals of cerium magnesium nitrate which were thermally anchored to the sound cell via coil foil. To ensure thermal contact between the liquid ^{4}He being measured and the copper walls of the sound cell to which the magnetic thermometer was attached, the sound cell had added to it a chamber filled with sintered copper in contact with the sample liquid. The sound was generated and detected by separate 15-MHz unloaded x-cut quartz transducers. The transducers were separated by a 2.02-cm-long hollow quartz spacer. The phase and amplitude of the received sound signal can be compared with a signal coming from the oscillator in order to determine both changes in the time delay through the liquid (due to changes in the sound velocity) and changes in the attenuation through the liquid. Details of the sound cell and the ultrasonic comparator have been described previously.[6]

EXPERIMENTAL RESULTS

Figures 1–4 show our ultrasonic-attenuation measurements at frequencies of 15, 45, 105, and 256 MHz, respectively. At each frequency a number

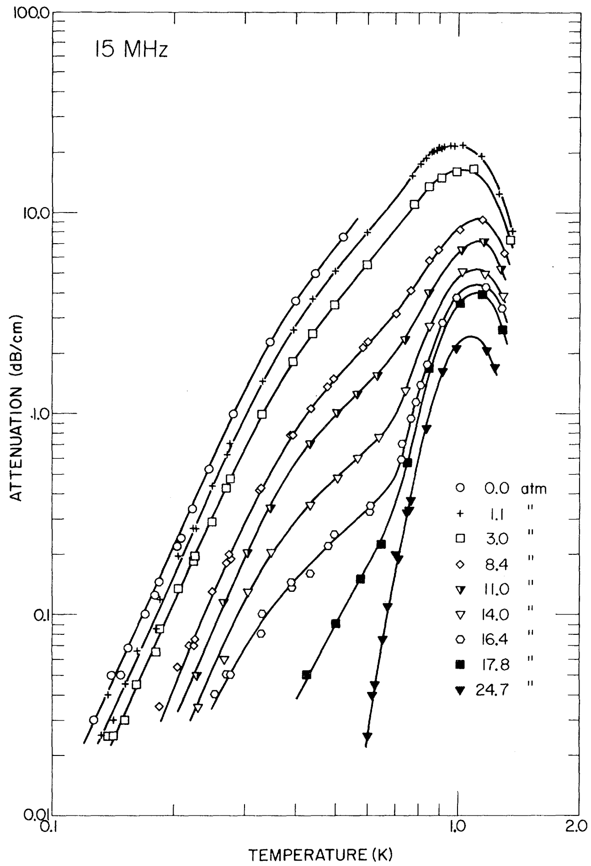

FIG. 1. Temperature dependence of the measured attenuation for a frequency of 15 MHz at various values of pressure. The solid lines are smooth curves through the data.

of attenuation-vs-temperature isobars are shown for pressures up to 24.7 atm. The data at 105 MHz have been reported previously.[5] Earlier measurements[7,8] of the attenuation under pressure have shown the general decrease in attenuation with increasing pressure that we have observed. However, these previous measurements did not extend above 15 MHz in frequency and did not show the features to be discussed for our data. We have compared our data with the theory of Khalatnikov and Chernikova[9] using the data of Van den Meijdenberg, Taconis, and De Bruyn Ouboter[10] for the pressure dependence of the roton parameters and using our previous ultrasonic measurements[11] for the pressure dependence of ρ, c, and u. Figure 5 shows curves of the theory at 105 MHz and various pressures corresponding to some of the experimental curves. These curves are representative of the theory for the entire frequency and pressure range of our experiment. They show a well-defined T^4 temperature dependence at low temperatures and a peak in the attenuation at around 1 K. Our data, however, show this behavior only at low pressures and even then the measured attenuation is consistently larger than that predicted by theory. At higher pressures the data show a sharp departure from theory; in the vicinity of 0.5 K the higher pressure data show a shoulder followed by a rapid increase of the attenuation with temperature until the vicinity of the maximum is reached at about 1 K. At the highest pressure (24.7 atm) only the very rapid rise below the maximum is seen; neither the T^4 dependence nor the shoulder could be resolved in the present experiment.

Tables I–IV give most of our data in numerical form. The accuracy with which the attenuation is given in the tables (± 0.005 dB/cm) is only meaningful at the lower values of attenuation. At attenuations above 10 dB/cm, signal-to-noise problems limited our accuracy to several percent in the attenuation measurements. Because the data are presented on logarithmic scales, it is very important that the zero of attenuation be accurately known. In the present experiment this was empirically de-

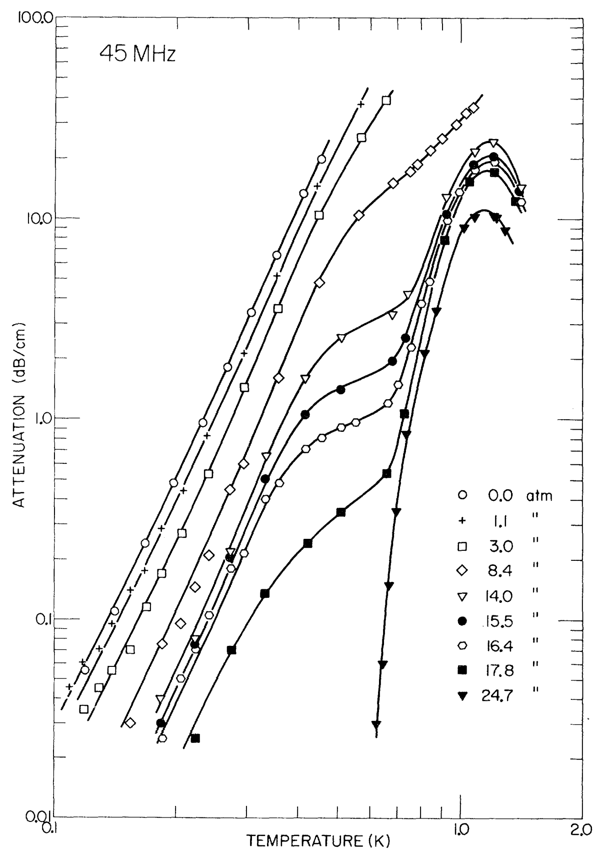

FIG. 2. Temperature dependence of the measured attenuation for a frequency of 45 MHz at various values of the pressure. The solid lines are smooth curves through the data.

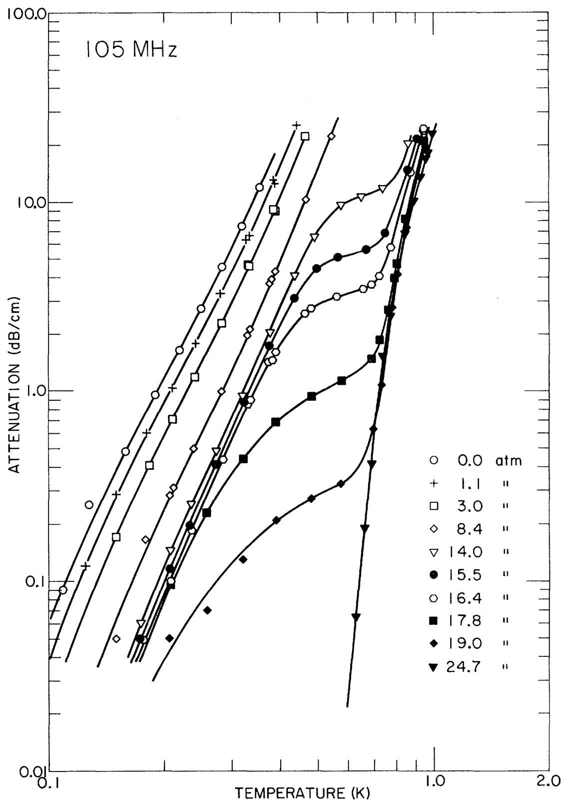

FIG. 3. Temperature dependence of the measured attenuation for a frequency of 105 MHz at various values of the pressure. The solid lines are smooth curves through the data.

termined for each of the curves shown by finding that value which made the low-temperature part of the curve most nearly linear (on a log-log plot) and/or consistent with the curves for other pressures. This seldom involved a correction of more than 0.01 dB/cm to the data. The accuracy quoted in the tables for the temperature (± 0.1 mK) is at best useful only as a measure of the relative temperatures of the points. The absolute accuracy of our temperature calibration is estimated to be a few percent while our temperature resolution is everywhere better than 1%. Our pressures were measured on a fused quartz bourdon gauge.[12] Although they are quoted to only ± 0.1 atm, in fact the pressures were held to ± 0.001 atm during the taking of data at any of the given pressures.

DISCUSSION

In the time since the original presentation[5] of parts of this data, several features of the data have been more or less successfully accounted for theoretically.

Klein and Wehner[13,14] have considered the pressure dependence of our new data in the low-temperature T^4 region. They do not account for the magnitude of the data but they observe that the pressure dependence of the attenuation is not completely explained by the pressure dependence of u, ρ, and c; they conclude that, in addition, a negative value of γ combined with its pressure dependence as measured by Phillips *et al.*[15] is necessary to explain the data. Since Maris[4] has had some success in accounting for the magnitude of our zero-pressure data, it seems reasonable to anticipate that his calculations will yield equally good agreement with our high-pressure data.

Jäckle and Kehr[16] have suggested an explanation for the shoulder in our data. They assume that the phonon spectrum is initially curved upwards as in Fig. 6 (where the effect is exaggerated). For this situation there will be a momentum q_c above which thermal phonons cannot absorb an ultrasonic phonon by the three-phonon process. This momentum is the point on the spectrum where the group velocity v of a thermal phonon is equal to the velocity c of

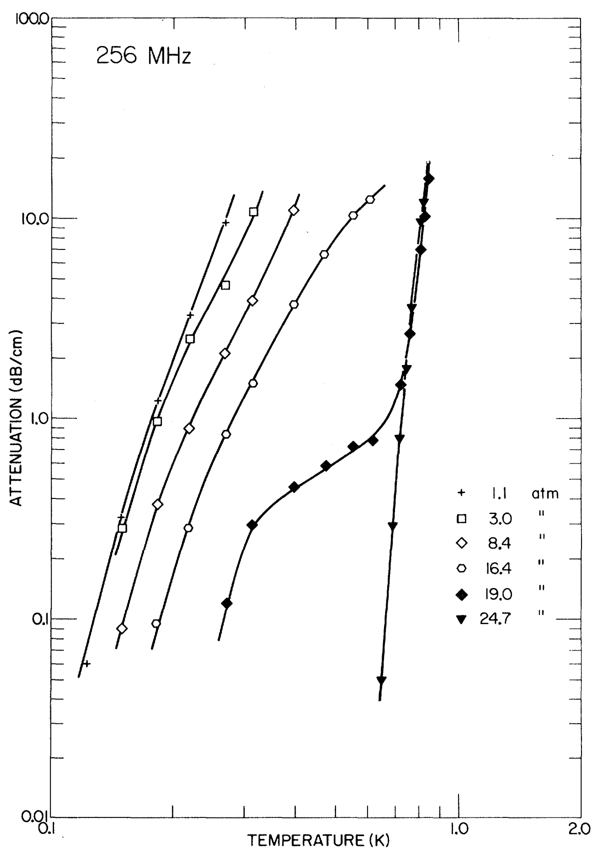

FIG. 4. Temperature dependence of the measured attenuation for a frequency of 256 MHz at various values of the pressure. The solid lines are smooth curves through the data.

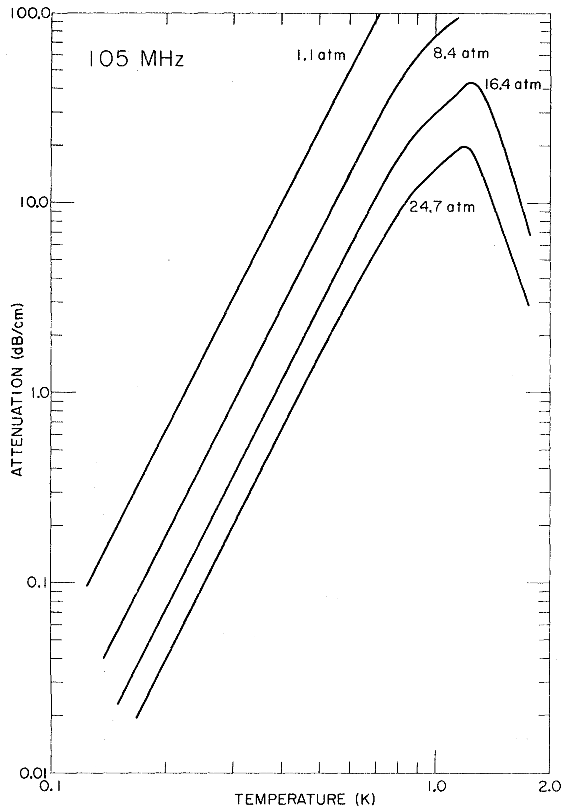

FIG. 5. Temperature dependence of the attenuation as calculated from the theory of Khalatnikov and Chernikova for 105 MHz and for pressures corresponding to some of the experimental values.

an ultrasonic phonon ($q \approx 0$). When a temperature is reached where a significant fraction of the thermal phonons have momenta above q_c, then the attenuation due to the three-phonon process will begin to be suppressed. An additional assumption is made that q_c decreases considerably with pres-

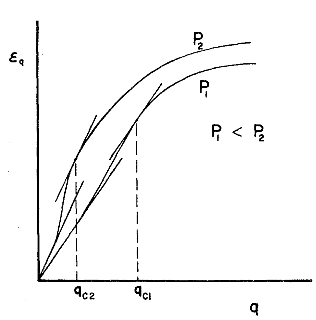

FIG. 6. Possible form of anomalous dispersion (greatly exaggerated) at two different pressures. q_c is the momentum above which thermal phonons cannot absorb an acoustic phonon by the three-phonon process.

TABLE I. Temperature dependence of the sound attenuation in liquid ^{4}He under pressure at 14.92 MHz.

Temp. (mK)	Attenuation (dB/cm)
$P = 0.0$ atm *A*	
148.4	0.050
180.2	0.125
209.8	0.240
245.7	0.530
282.0	1.000
347.4	2.275
401.1	3.635
448.6	4.975
523.5	7.580
B	
127.5	0.030
140.5	0.050
155.1	0.068
170.4	0.100
185.0	0.146
204.8	0.220
223.4	0.336
$P = 1.1$ atm *A*	
132.4	0.025
141.1	0.030
151.7	0.045
162.1	0.065
184.9	0.120
205.8	0.195
223.7	0.270
249.2	0.435
276.0	0.705
B	
227.4	0.270
271.6	0.630
331.0	1.430
394.2	2.610
499.2	5.095
595.8	7.950
771.8	15.170
809.8	17.450
833.3	18.835
860.3	20.010
869.0	20.195
869.0	20.310
889.1	20.760
898.4	21.010
905.5	21.010
910.2	21.110
922.4	21.160
955.6	21.260
980.1	21.360
1026.7	21.710
1134.8	19.060
1259.1	12.260

TABLE I. (*Continued*)

Temp. (mK)	Attenuation (dB/cm)
$P=3.0$ atm	
A	
141.7	0.025
151.7	0.030
161.8	0.045
184.8	0.085
205.6	0.135
224.1	0.185
249.0	0.290
276.8	0.475
B	
226.2	0.195
270.5	0.425
330.1	0.990
394.2	1.810
497.1	3.470
595.8	5.510
780.5	10.995
C	
137.7	0.025
181.1	0.065
849.7	13.535
912.6	14.945
994.2	15.990
1098.7	16.425
1347.6	7.300
$P=8.4$ atm	
A	
184.0	0.035
204.5	0.055
224.6	0.070
249.0	0.130
277.1	0.190
B	
225.9	0.075
270.5	0.180
329.8	0.425
393.3	0.785
495.6	1.495
598.9	2.285
766.6	4.110
C	
217.8	0.070
275.1	0.200
327.9	0.420
389.8	0.785
477.8	1.365
583.7	2.150
705.5	3.150
849.7	5.765
898.4	6.565
1008.7	8.260
1150.0	9.195
1301.8	6.235

TABLE I. (*Continued*)

Temp. (mK)	Attenuation (dB/cm)
$P=11.0$ atm	
229.3	0.050
267.0	0.115
305.8	0.205
349.1	0.340
435.0	0.710
505.8	1.015
565.5	1.260
635.3	1.555
741.9	2.355
849.7	4.025
1023.7	6.570
1157.7	7.175
1282.5	5.225
$P=14.0$ atm	
229.3	0.035
266.6	0.060
305.8	0.130
348.4	0.205
433.4	0.350
505.8	0.480
568.3	0.600
638.8	0.765
741.9	1.305
853.9	2.730
1029.8	5.075
1169.4	4.945
1292.1	3.885
$P=16.4$ atm	
A	
271.0	0.050
330.4	0.100
390.7	0.135
495.6	0.250
609.5	0.345
729.3	0.705
766.6	0.945
B	
276.2	0.050
327.3	0.080
389.8	0.145
481.1	0.220
606.3	0.325
723.2	0.595
791.2	1.140
809.8	1.380
841.4	1.755
915.1	2.825
994.2	3.745
1169.4	4.255
1277.7	3.325

TABLE I. (*Continued*)

Temp. (mK)	Attenuation (dB/cm)
$P = 17.8$ atm	
425.9	0.050
501.4	0.090
575.0	0.150
647.2	0.225
750.0	0.570
849.7	1.685
1011.7	3.540
1292.1	2.610
1146.1	3.910
$P = 24.7$ atm	
597.9	0.025
625.0	0.045
652.1	0.075
671.2	0.110
709.8	0.190
764.9	0.370
837.3	0.855
915.1	1.635
994.2	2.120
1181.5	2.075
1232.1	1.705

sure. For pressures below 10 atm, q_c must be above the range of thermal phonons ($c_1 q_{c1} > 3k_B T$), while at the highest pressures, q_c must be within this range ($c_2 q_{c2} \lesssim 3k_B T$) so that the attenuation is suppressed. Jäckle and Kehr give the attenuation for the restricted three-phonon process as

$$\alpha(\omega,\ p,\ T) = A\omega T^4 F(z)/F(\infty)\ , \tag{2}$$

where $z = cq_c/(k_B T)$ and $F(z) = \int_0^z dx\, x^4 f(x)[1+f(x)]$; $f(x)$ is the Bose function. At low temperatures ($k_B T \ll cq_c$) this becomes $\alpha = A\omega T^4$, which is equivalent to Eq. (1). For high temperatures ($k_B T \gg cq_c$) the three-phonon process is restricted, and one obtains

$$\alpha = \tfrac{1}{3}A(cq_c/k_B)^3 \omega T \tag{3}$$

Many of our curves do show such a linear temperature dependence in the vicinity of the shoulder. Jäckle and Kehr also treat the rapid rise of the attenuation above the shoulder. They consider the effect of thermal-phonon lifetime on the three-phonon process in the region above q_c where the process is not favorable because of an energy deficit. In the temperature region above 0.6 K, where the number of rotons is increasing very rapidly, they find that the attenuation, although strongly depressed, is rising very steeply with temperature. Figure 7 shows their fit to our data at a pressure of 16.4 atm for frequencies of 15 and 105 MHz. The fitted curve for 105 MHz also includes a contribution from roton viscosity. Although the fit

TABLE II. Temperature dependence of the sound attenuation in liquid ^{4}He under pressure at 44.95 MHz.

Temp. (mK)	Attenuation (dB/cm)
$P = 0.0$	
120.0	0.055
142.3	0.110
168.7	0.240
198.3	0.480
234.2	0.960
269.7	1.815
308.3	3.395
354.9	6.570
413.6	13.365
456.9	19.930
$P = 1.1$ atm	
118.3	0.060
139.4	0.095
167.9	0.175
208.5	0.440
238.9	0.825
294.3	2.125
355.6	5.175
445.7	14.605
568.3	37.275
$P = 3.0$ atm	
118.8	0.035
139.4	0.055
169.6	0.115
207.3	0.270
240.5	0.535
295.6	1.430
356.7	3.560
449.2	10.435
570.2	25.290
658.3	38.830
$P = 8.4$ atm	
A	
206.0	0.095
242.2	0.210
294.6	0.600
359.0	1.605
451.5	4.780
564.6	10.430
684.5	15.070
754.9	17.130
785.8	18.680
843.5	21.940
903.1	25.140
980.1	29.590
1036.0	33.540
1078.1	35.890
B	
155.1	0.030
186.0	0.075
223.4	0.145
272.7	0.445

TABLE II. (*Continued*)

Temp. (mK)	Attenuation (dB/cm)
$P=14.0$ atm	
184.0	0.040
223.4	0.080
273.3	0.220
334.3	0.660
416.1	1.610
511.1	2.575
680.4	3.360
746.7	4.265
929.9	12.910
1084.9	21.760
1206.2	24.260
1413.9	14.400
$P=15.5$ atm	
185.0	0.030
224.9	0.075
274.0	0.205
333.3	0.505
417.1	1.050
511.1	1.405
683.1	1.945
735.6	2.555
929.9	10.475
1081.5	18.655
1214.7	20.355
1408.1	13.865
$P=16.4$ atm *A*	
206.0	0.050
242.2	0.105
294.3	0.215
360.5	0.480
457.5	0.805
552.0	0.970
704.0	1.495
756.5	2.280
1088.3	17.545
997.1	13.620
932.4	9.820
841.4	4.870
806.0	3.800
B	
186.0	0.025
224.9	0.070
274.0	0.180
332.3	0.400
417.1	0.715
511.1	0.910
664.7	1.205
1219.0	19.360
1413.9	12.210

TABLE II. (*Continued*)

Temp. (mK)	Attenuation (dB/cm)
$P=17.8$ atm	
224.9	0.025
274.9	0.070
331.0	0.135
421.7	0.240
509.6	0.345
659.6	0.535
729.3	1.070
917.5	7.815
1058.2	15.370
1219.0	17.020
1369.0	12.270
$P=24.7$ atm *A*	
696.9	0.350
740.3	0.850
1088.3	10.240
1020.7	9.070
875.6	3.470
821.4	2.140
B	
622.7	0.030
643.6	0.060
664.7	0.150
1210.5	10.370
1232.1	10.220
1292.1	8.770

is not equally good at both frequencies it is certainly apparent that this explanation of the shoulder in our data has great merit and should be pursued.

Recently Feenberg[17] has shown that for van der Waals forces the elementary excitation spectrum of liquid ^{4}He may contain both odd and even powers of the momentum with the exception of a p^2 term. Molinari and Regge[18] and later Barucchi, Ponzano, and Regge[19] have suggested that a p^2 term may also be present; no detailed calculations based on these suggestions have been performed to date.

CONCLUSIONS

Although many aspects of our data remain to be fully explained, it appears that considerable progress has been made recently toward a better understanding of the processes involved in the sound attenuation in liquid He. One common point that emerges not only from the calculation of Klein and Wehner[14] concerning the pressure dependence of our data,

TABLE III. Temperature dependence of the sound attenuation in liquid ^{4}He under pressure at 105.05 MHz.

Temp. (mK)	Attenuation (dB/cm)
$P=0.0$	
109.3	0.090
128.0	0.250
159.4	0.475
189.1	0.945
219.6	1.635
250.1	2.730
282.3	4.525
317.9	7.450
355.6	11.965
$P=1.1$ atm	
124.8	0.120
149.7	0.285
180.2	0.600
209.8	1.030
240.5	1.775
279.5	3.240
332.6	6.610
386.7	12.475
$P=3.0$ atm	
149.7	0.170
183.1	0.405
211.1	0.705
240.2	1.175
281.1	2.260
331.0	4.560
386.7	8.955
$P=8.4$ atm *A*	
149.7	0.050
178.3	0.165
211.1	0.310
237.9	0.500
282.0	0.990
332.3	2.090
387.6	4.275
B	
327.3	1.980
378.7	3.910
464.9	10.230
544.1	22.045
$P=14.0$ atm	
174.7	0.060
207.3	0.145
235.6	0.255
272.9	0.485
318.8	0.935
375.0	2.040
436.1	4.060
490.0	6.535
571.1	9.635
650.9	10.655
737.1	11.735
860.3	20.235

TABLE III. (*Continued*)

Temp. (mK)	Attenuation (dB/cm)
$P=15.5$ atm	
173.8	0.050
207.3	0.115
234.0	0.195
272.5	0.410
323.0	0.875
375.0	1.730
436.1	3.100
497.8	4.445
564.6	5.115
664.7	5.575
753.2	6.855
856.0	14.675
907.8	21.425
$P=16.4$ atm *A*	
176.5	0.050
212.4	0.140
236.4	0.185
282.5	0.435
333.3	0.880
388.9	1.600
B	
327.3	0.850
380.3	1.460
461.2	2.545
559.1	3.150
720.2	4.025
871.2	14.340
940.0	23.980
773.5	5.745
655.8	3.475
$P=17.8$ atm	
207.3	0.095
256.6	0.230
320.0	0.435
388.0	0.685
480.5	0.935
575.9	1.125
688.6	1.485
724.7	1.865
761.5	2.685
802.3	4.700
843.5	8.105
947.8	20.860
$P=19.0$ atm	
206.0	0.050
258.6	0.070
320.0	0.130
389.8	0.210
480.5	0.275
574.0	0.325
694.1	0.625
730.9	1.075
782.3	2.770
804.1	4.110
849.7	7.310
953.0	19.670

TABLE III. (*Continued*)

Temp. (mK)	Attenuation (dB/cm)
$P=24.7$ atm	
630.7	0.065
663.4	0.190
690.0	0.410
789.4	3.915
969.1	18.120
997.1	22.990
955.6	16.940
927.4	13.540
851.8	7.955
841.4	6.800
889.1	10.190
740.3	1.545

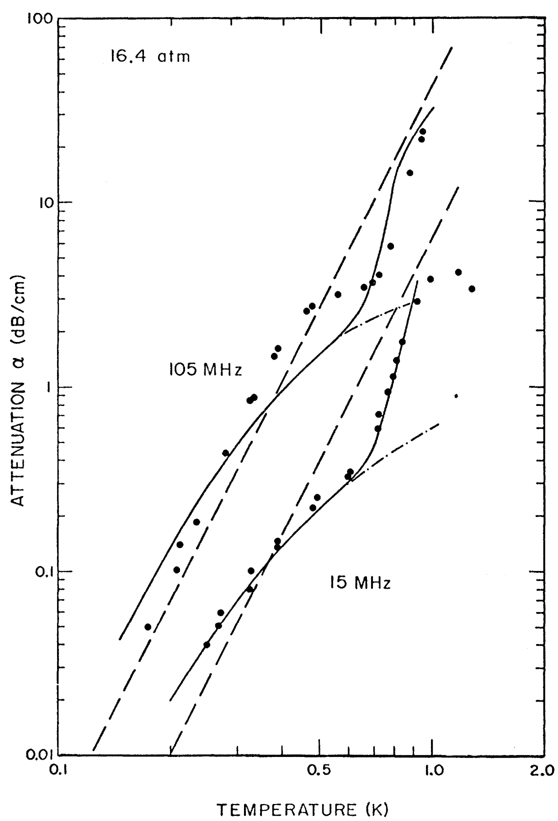

FIG. 7. Theory of Jäckle and Kehr (Ref. 16). Attenuation at 16.4-atm pressure and for frequencies of 15 (lower curves) and 105 MHz (upper curves). Solid points: experimental measurements. Dashed lines: basic three-phonon result $\propto \omega T^4$. Dash-dotted lines: restricted three-phonon process. Full line for 15 MHz: sum of restricted three-phonon and lifetime-induced attenuation. Full line for 105 MHz: the same, plus contribution of the roton viscosity.

TABLE IV. Temperature dependence of the sound attenuation in liquid ^{4}He under pressure at 256.22 MHz.

Temp. (mK)	Attenuation (dB/cm)
$P=1.1$ atm	
123.0	0.060
149.0	0.320
183.1	1.220
220.5	3.290
268.9	9.540
$P=3.0$ atm	
149.7	0.285
182.1	0.970
220.5	2.505
268.4	4.640
314.4	10.720
$P=8.4$ atm	
150.3	0.090
183.1	0.375
219.1	0.890
268.2	2.115
313.3	3.890
395.1	10.990
$P=16.4$ atm	
182.1	0.095
217.8	0.285
269.5	0.835
312.5	1.490
395.1	3.725
469.3	6.615
552.8	10.315
606.3	12.365
$P=19.0$ atm	
270.8	0.120
312.7	0.295
396.5	0.455
475.2	0.585
551.1	0.725
617.1	0.785
721.7	1.475
759.9	2.655
809.8	6.995
825.3	10.145
845.5	15.845
$P=24.7$ atm	
648.4	0.050
690.0	0.295
717.2	0.805
746.7	1.785
768.3	3.605
809.8	9.715
823.3	12.065

but also from the calculation by Maris[4] of the magnitude and frequency dependence of our zero-pressure data and from the explanation by Jäckle of the shoulder in our data, is that all these theories require an assumption of anomalous dispersion to account for our data. This has to be interpreted as very strong evidence for anomalous dispersion. Whether this dispersion should be described by a quadratic or a cubic term in the energy spectrum remains to be determined; more detailed theoretical calculations using both descriptions would be very desirable.

*Based on work performed under the auspices of the U. S. Atomic Energy Commission.

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Liquid-Helium Configuration around a Metastable Excited Helium Atom†

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The pair distribution for normal He atoms in the vicinity of a metastable $2\,^3S$ He atom is calculated in the zero-temperature limit, from first principles, using a variational Jastrow wave function and the Percus-Yevick integral equations for fluid mixtures. From this pair function, the "bubble radius," coordination number, and energy per metastable atom are calculated.

Spectroscopic studies of the infrared emission and absorption spectra of neutral localized excitations, created in liquid helium by electron beams with an energy of the order of 10^2 keV, have identified these as metastable excited states of He atoms and He_2 molecules shifted only very slightly in energy from their free atomic or molecular values.[1-3] For the atomic states, Hickman and Lane, by extending the calculations of Jortner *et al.*[4] for an excess electron in liquid helium to include the interaction of a helium core with both the excited electron and the liquid, have shown these observations to be consistent with the existence of a cavity surrounding the excited atom.[5]

The purpose of this paper is to study these "bubble states" from first principles, for the special case of metastable $2\,^3S$ atoms (hereafter symbolized by He*) which have been observed at concentrations greater than 10^{12} atoms cm^{-3} by the Rice University group.[1,2] The calculation presented here could be extended to other excitations if the pair interaction between such excitations and normal He atoms were known.

The Hamiltonian for a system of N_1 normal and