

Properties of Electron Surface States on Liquid Helium

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Methods have been developed for depositing electrons on the surface of liquid helium, either from the gas side or from the liquid side. In a suitable applied electric field they escape from the surface into the gas and the rate of escape has been measured. Electrons inside the surface escape by a thermally activated process. Those outside the surface escape much more rapidly by what appears to be a tunneling process. Analysis of the equilibrium between bound and free electrons identifies important features of the binding which have not previously been considered.

I. INTRODUCTION

The idea that an electron could form a surface state outside some liquids was first proposed by Cole and Cohen¹ for He, H₂, D₂, and Ne and independently by Shikin² for He. The conditions for these surface states are that the liquid must have a negative electron affinity to prevent the electron from entering the liquid and that the dielectric image force be large enough to produce a bound state.

There is also a surface state for an electron *inside* these liquids. Near the liquid surface an electron is repelled by the image force. But if an electric field is added of such polarity as to move the electron toward the surface, it will give a potential minimum just *inside* the liquid surface.

In this paper we shall describe measurements of the lifetimes of electrons in surface states inside and outside liquid ⁴He. A preliminary account of these measurements has been published.³ The conclusion is that the two surface states are different.

In an applied electric field the lifetime of the electron on the surface depends on whether it is inside or outside the surface and on the polarity of the field. In the state *outside* the liquid, the electron is stable indefinitely in a field so directed as to move it from the vapor toward the liquid, and will remain on the surface for hours. A field of opposite polarity, drawing the electron away into the gas, will remove it in a time of 10⁻⁴ to 10⁻⁶ sec depending on the magnitude of the field and the temperature. An electron *inside* the liquid will be pulled into the vapor by a field in a time of some seconds, depending weakly on the magnitude of the field and strongly on the temperature. There is no experimental information available for the lifetime of an electron inside the surface against a field drawing it into the interior of the liquid, but it is probably relatively short, if any bound state exists.

In Sec. II we give a brief description of the ori-

gin of these surface states. In Sec. III we describe the equipment and measurement procedure. Section IV gives the results, while Sec. V is a discussion of these results and a comparison of the the measured surface-state lifetimes with theoretical estimates.

II. ORIGIN OF SURFACE STATES

A detailed discussion of electrons in the surface states of liquid helium may be found in Refs. 1 and 2. Therefore, we shall give only a brief outline.

A. Outside States

An electron outside liquid helium experiences an attractive image or polarization force. If z is the distance between the electron in the gas and the liquid surface, then the image potential is

$$V(z) = -\frac{\alpha e^2}{z}, \quad \alpha = \frac{1}{4} \frac{\epsilon_l - \epsilon_g}{\epsilon_l + \epsilon_g}, \quad (1)$$

where ϵ_l is the static dielectric constant of the liquid and ϵ_g that of the gas. The quantity α is the strength of the image charge. Because of the short-range repulsive interaction between the electron and helium, the electron must gain an energy of about 1 eV to enter the liquid.^{4,5} Thus it is attracted by the image force, but remains outside the liquid. Of course there is some wave function tailing into the liquid, but for helium this is minimal. The authors of Refs. 1 and 2 make the assumption that $V(z)$ given by Eq. (1) is the only potential in the z direction and the further assumption that the potentials in the x and y directions play the role of giving the electron an effective mass. Thus the motion in the x, y plane is described by a two-dimensional Bloch state and is localized near the liquid surface. The potential for the electron is shown in Fig. 1 with the assumption that the liquid presents an infinite barrier to the electron. The electron energy spectrum is

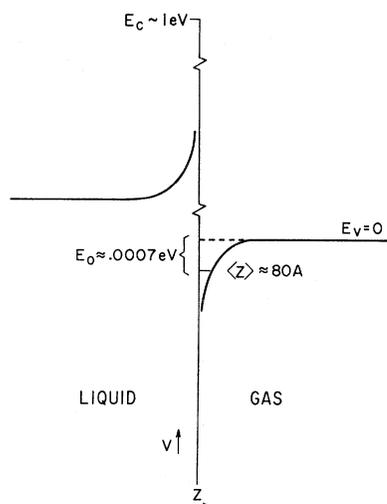


FIG. 1. Sketch of the potential that an electron sees near the surface of liquid helium. Potential energy V is plotted vertically and the z coordinate is plotted horizontally. E_c is the conduction-band energy, E_0 the energy of the image-potential ground state, $\langle z \rangle$ the average value of z in the ground state, and E_V the vacuum level.

$$E = \hbar^2 K_{\parallel}^2 / 2m_{\parallel} - \alpha^2 m e^4 / 2n^2 \hbar^2, \quad (2)$$

where K_{\parallel} and m_{\parallel} are the wave vector and effective mass in the x, y plane, n is an integer, and m is the free electron mass. The first term represents the nearly free-electron spectrum for the x, y motion. The second term is the energy of localization caused by the image potential. This energy spectrum is that of a hydrogen atom whose proton charge is αe . For ${}^4\text{He}$ the ground hydrogenic state is about 0.0007 eV and the mean value of the z coordinate is about 70 Å. We shall henceforth refer to this state as the "outside" state.

B. Inside States

Not only can an electron be localized in the gas outside of liquid helium, but there exists a state inside the liquid, localized near the surface. Because of the short-range repulsion between an electron and liquid helium, the ground state is an electron in a bubble of about 20-Å radius.⁵ As this charged bubble nears the liquid surface, there is an image force of roughly the same magnitude, but of opposite sign from that on an electron outside the interface. If an electric field is applied so as to force the electron bubble toward the surface, the potential seen by the bubble is that shown in Fig. 1. The electron is localized in this potential minimum in the z direction but is free to move parallel to the surface. We shall refer to this surface state as the "inside" state. In both surface states, the electron can escape from the surface

into the gas region. It is the purpose of this paper to describe measurements of the rate of escape of electrons from these states into the gas.

III. EXPERIMENT: EQUIPMENT

The apparatus and circuit used are shown schematically in Fig. 2. The measurement cell is a cylindrical vessel about 10 cm high by 6 cm in diameter having glass walls and metal ends. The inner surface of the glass has a transparent conductive coating connected electrically to the ends of the container. There are two 3-cm-diam brass plates mounted, one fixed and the other moveable along the vertical axis. The upper plate, mounted on a stainless-steel tube, moves vertically at 1 cps. The plate separation during a cycle varies from 1.5 to 2.0 cm. The cell is immersed in a glass liquid-helium Dewar. Helium gas is passed through a cold trap at 77°K and then condensed into the cell until the liquid level is about 1 cm above the bottom plate. The temperature of the helium is changed by pumping on the liquid helium in the glass Dewar, and the temperature is determined by measuring the helium vapor pressure.

The moveable upper plate is used to measure the surface potential of the liquid when electrons are present, as in the Kelvin method for determining work functions.⁶ This plate is connected to a Keithley electrometer, which in turn is connected to a recorder.

To deposit electrons on the surface of the liquid from above, the bottom plate is biased at say +200 V with the upper plate near ground potential. The potential that an electron sees near the liquid surface is shown in Fig. 3(a). Several kilovolts, the exact value depending on the gas pressure, are then applied to the negative-corona discharge wire at the upper right-hand side of the cell. Electrons produced by the discharge are transported through the gas as free electrons, since helium has no stable negative ions. They are drawn toward the lower plate and stop at the liquid surface, where they are confined by the electrostatic potential to lie in the area vertically over the plate. The field caused by the sheet of charge which builds up at the surface decreases the field between the surface and

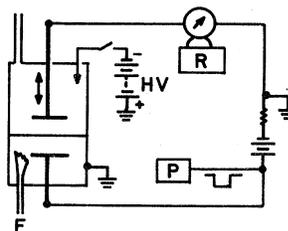


FIG. 2. Schematic of apparatus and measuring circuit: F, filament; P, electronic pulser for applying negative pulses to bottom plate; R, recorder driven by electrometer output; HV, voltage for corona discharge. The moving plate oscillates at 1 cps.

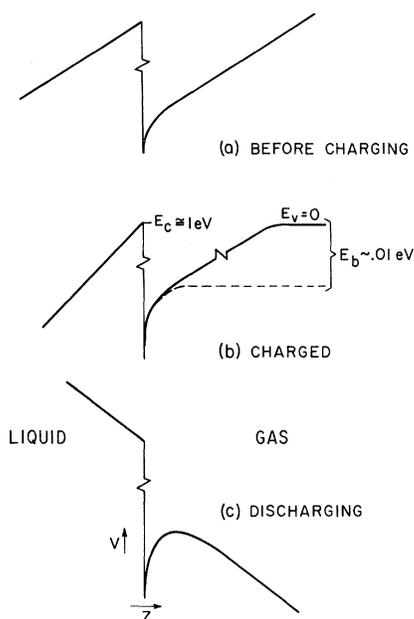


FIG. 3. Sketch of the potential that an electron outside liquid helium sees during the different experimental operations. Potential energy is plotted vertically and the z coordinate horizontally. In (a) and (b) the bottom plate is positive; for (c) it is negative. The dotted curve in (b) is under the assumption that the electrons are distributed as a sheet in the plane of the liquid. The solid curve results when the electrons are localized in the plane of the liquid. E_c is the conduction band, E_v the vacuum, and E_b the increased binding due to the electric field in the gas region.

the upper plate and increases that between the surface and the lower plate until the potential of the surface reaches that of the upper plate. Any more charge landing on the surface would make the surface more negative than the upper plate. The resulting field would draw electrons away from the surface into the gas. Under these conditions, as we will show, the electrons leave the surface in 100 μ sec or less and would not be present for the subsequent operations in the experiment.

An illustration of the effect of electrons on the surface of the liquid helium is shown in Fig. 4. Here the depression of the liquid surface caused by the condenser force between the surface layer of electrons and the bottom plate is shown as a circular indentation above the bottom plate.⁷

Figure 5(a) shows a recorder tracing describing the charging and discharging operations. The moving plate originally senses the voltage difference of 200 V applied between upper and lower plates. After the electrons from the corona discharge have landed, the recorder signal decreases, since the surface potential is nearly equal to that of the moving plate. The corona discharge is now shut off for

the remainder of the experiment. As long as the positive voltage is maintained on the bottom plate, the electrons remain on the surface. To measure the rate of escape from the surface, a negative voltage pulse is applied to the bottom plate so that electrons are forced from the liquid into the gas. The pulse is longer than the electron transit time through the gas, but shorter than the time to remove all the electrons from the surface. The transit time is determined by decreasing the pulse-width until no electrons can be collected during a pulse. These transit times are consistent with the electron drift velocity measured in ^4He gas by Levine and Sanders.⁸ The potential that the electron sees is shown in Fig. 3(c). In this case, the potential is composed of the image potential and the potential due to the applied electric field.

Figure 5(a) shows the recorder tracing for the

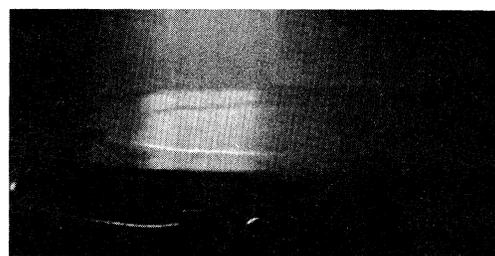
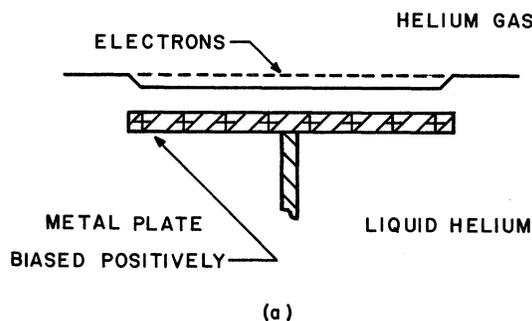


FIG. 4. (a) Schematic side view of liquid-helium surface, electrons on the surface, and metal plate below the surface. (b) Photograph of depression on the surface due to condenser force exerted by electrons. The photograph is taken from an angle about 10° above the horizontal. The circular depression can be seen over the plate; the plate below the surface can be seen through the side wall of the Dewar. The plate is 1.0 mm below the surface and has a diameter of 3.0 cm. The voltage between the sheet of electrons and the plate is 200 V, giving a field of 2000 V/cm. The temperature is 3.7 $^\circ\text{K}$ and the depth of the depression is about 0.13 mm, in agreement with our theoretical estimate (Ref. 7) for the depth.

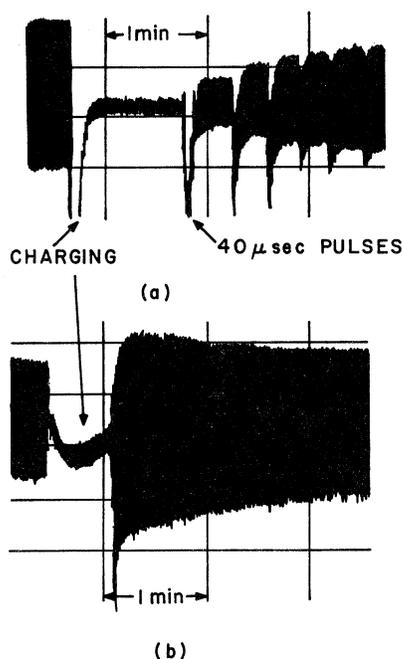


FIG. 5. (a) Recorder tracing showing signal before and after charging. The height of the dark recorder tracing is proportional to the voltage variation at the electrometer input as the plate moves up and down. Before charging, this indicates the flow of charge under constant applied voltage as the plate spacing changes and gives a relatively large signal. After charging, the signal is much smaller, indicating that the potential of the surface after the arrival of electrons is nearly equal to that of the upper plate. After this, the applied voltage is shorted to ground for $40 \mu\text{sec}$ by means of a pulse. The signal amplitude is greater after the pulse, indicating that some of the electrons have escaped from the surface, changing the surface potential. Subsequent pulses result in the escape of more electrons, providing a quantitative measure of the lifetime of electrons on the surface. (b) Inside electrons similar to (a) except electrons are deposited from below by means of a heated W filament immersed in the liquid. During charging the recorder gain is reduced by a factor of 10 and then restored to its original value at the end of the charging period. The applied voltage (upper plate positive) is left on at all times. The relatively slow escape of electrons from the surface is indicated by the decreasing amplitude of the signal with time.

discharge operation using $40\text{-}\mu\text{sec}$ pulses. After the pulse, the bottom plate is again positive. The amplitude of the recorder tracing indicates the number of electrons which have left the surface. In this manner, the amount of charge Δq leaving the surface in a time interval Δt can be measured. This operation is repeated until all the charge is removed from the surface.

To put electrons on the surface from below, we used thermionic emission from a 0.003-in. thori-

ated-tungsten filament immersed in the liquid. This is an effective electron source as shown by Spangler and Hereford.⁹ The filament is shown schematically in Fig. 2. Voltage was again applied between top and bottom plates, but this time the top plate was positive so that the electrons were drawn upward to the surface. Electrons collect at the surface, and some are localized there until the surface potential is equal to that of the negative bottom plate. Then the filament current is shut off and the voltage left on between the plates so that there is a field drawing electrons into the gas. As they escape, the change in surface potential is indicated on a recorder tracing as shown in Fig. 5(b). We are able to obtain a continuous recorder trace in this case, since the lifetime for these inside states is the order of seconds or longer.

IV. RESULTS

A. Outside States

Figure 6 shows the amount of charge remaining as a function of time in a field of the polarity shown in Fig. 3(c). The charge was placed on the helium surface from the gas at 2.4°K with an initial density of $1.25 \times 10^8 \text{ cm}^{-2}$. The data were obtained by applying negative pulses to the bottom plate or by simply connecting top and bottom plates together (zero-voltage pulse). Each point gives the fraction of the initial charge density remaining, and the t axis shows the total integrated time that the pulses have been applied. The circles are for the bottom plate at -50 V and the squares for the bottom plate shorted to the top plate. The straight lines obey the law $q/q_0 = e^{-t/\tau}$. Here τ is the surface lifetime. For the -50-V pulse the exponential decay law is reasonably well obeyed. For the zero-voltage pulse this decay law is not obeyed. This result has a reasonable explanation when we recall that the field that a surface electron experiences is

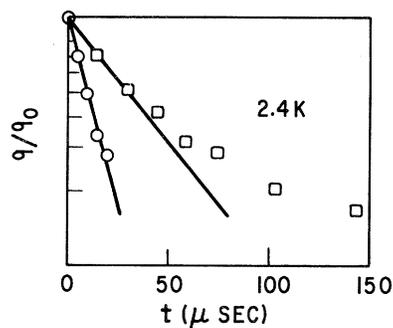


FIG. 6. Normalized surface-charge density as a function of time that the bottom plate is negative or grounded. The circles are for the bottom plate at 50 V , and the squares for the bottom plate at ground.

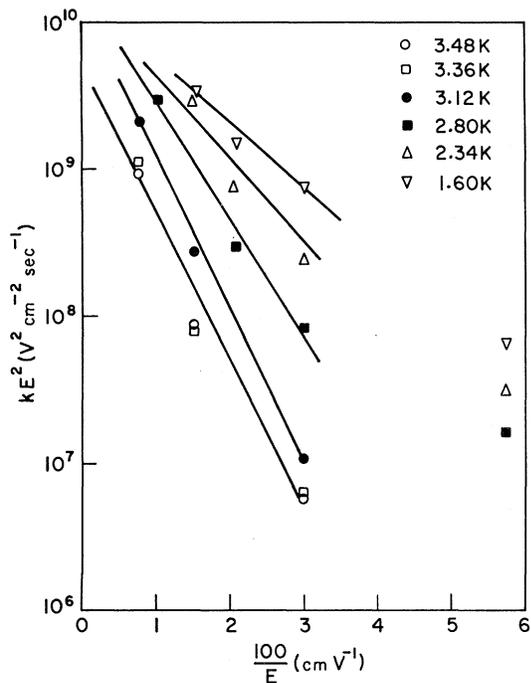


FIG. 7. Product of rate constant k for escape of electrons from the outside states into the gas and the square of the applied electric field E as a function of the reciprocal of the applied field.

composed of two parts: the applied field E_A and the field E_S caused by all the other surface electrons. This latter field is given by $E_S = 2\pi\sigma$. Therefore, as the charge on the surface decreases, so does E_S . For the data in Fig. 6, $E_S = 125$ V/cm when $q = q_0$ and $E_A = 33$ V/cm. Since we find that τ increases with decreasing E , the nonexponential decay is understandable.

We find, however, that τ increases with increasing temperature. This is just opposite to the effect one would expect for thermal activation over the barrier [Fig. 3(c)]. Thus we might expect that the electrons leave the surface by tunneling through the barrier. To verify this idea, we construct the plot in Fig. 7. Here k is the rate constant which is the reciprocal of τ . An expression for tunneling in the potential shown by Fig. 3(c) can be derived and is of the form $k \propto E^{-2} e^{-A/E}$. Therefore, we plot $\ln(kE^2)$ vs $1/E$ in Fig. 7. Here E is the applied field. The data are plotted for different temperatures to show the temperature dependence of k .

B. Inside States

For the electrons placed on the surface from the liquid, we found the same dependence of q on t as for the outside states. However, here τ is much larger than for the outside states and has a strong

temperature dependence. The surface lifetime for these inside states is plotted vs $1/T$ in Fig. 8. The applied field is 100 V/cm. The straight line through the data points is the function $1/\tau = (1/\tau_0) \times e^{-\Theta/T}$ where $1/\tau_0 = 10^7$ and $\Theta = 27^\circ\text{K}$.

V. DISCUSSION

A. Outside States

Before discussing the experimental data, it is worthwhile to sketch some of the general properties to be expected of a system of electrons in the outside surface states.

We shall analyze the equilibrium between free and bound electrons, assuming the theoretical^{1,2} binding energy of 0.7 meV and that lateral motion along the surface is that of a free particle. It turns out that, with these assumptions, for our experimental conditions, nearly all the electrons would be free in the gas rather than bound to the surface. It will be shown, on the other hand, how the experimental results require that the electrons be bound rather than free. This contradiction can be most simply resolved by assuming that the lateral motion along the surface is restricted rather than that of a free particle.

According to the original discussions of Cole and Cohen¹ and Shikin,² the electrons on the surface move as free particles in the x and y directions parallel to the surface. Motion in the z direction includes both bound and continuum states. In the bound states the motion is restricted to the hydrogenic orbit, localized near the surface for the ground state, but extending more and more into the gas for higher quantum numbers. Above the ionization limit (0.7 meV above the ground state), the motion in the z direction, like that in the x and y directions, is that of a free particle with the entire gas volume available to it. In general, entropy considerations favor free states and energy considerations favor bound states. In what follows,

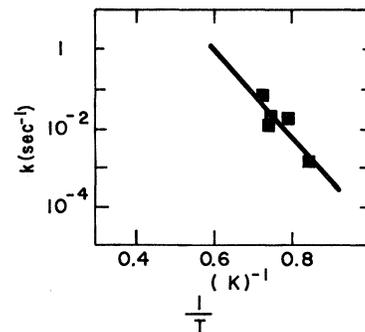


FIG. 8. Rate constant for the escape of inside electrons from the surface as a function of reciprocal temperature. Applied voltage 110 V.

we analyze the equilibrium between free and bound electrons and show that, with only the above assumptions,^{1,2} the majority would be free, rather than bound, at all temperatures above 1 °K. Our observations, on the other hand, indicate that they are indeed localized at the surface. We shall discuss the new features of the problem that resolve these differences.

Because the electron density is low, Boltzmann statistics can be used. Motion in the x and y directions is identical for free and bound electrons, so it is sufficient to analyze motion along the z direction alone. In such a system¹⁰ the number N_i of electrons in any quantum state i of energy E_i is related to the total number of electrons N and the partition function Q by the expression

$$N_i/N = (1/Q) e^{-E_i/kT}. \quad (3)$$

Q may be decomposed into a sum Q_b over the bound states and a sum Q_c over the continuum states: $Q = Q_b + Q_c$. The energy zero is chosen to be that of an electron at rest in the gas, far from the surface. Q is then the familiar translational partition function for one-dimensional motion in a container of length L plus a sum over the hydrogenic bound states. Thus we have

$$Q = \left(\frac{2\pi mkT}{h^2} \right)^{1/2} L + \sum_{n=1}^{\infty} \exp\left(\frac{-E_0}{n^2 kT} \right). \quad (4)$$

Here E_0 is the hydrogenic ground-state energy and

$$E_0 = -\alpha^2 me^4 / 2\hbar^2 \cong -0.7 \text{ meV}. \quad (5)$$

The summation in (4) is divergent and raises an interesting question which has been discussed for the analogous case of the partition function of the hydrogen atom.¹¹ The apparent paradox is resolved on considering that, if a_0 is the radius of the ground-state hydrogenic orbit, the orbital radius of the state with quantum number n is $a_0 n^2$. Any finite container precludes states of very high quantum number. For physical consistency, the summation must be cut off when the length L of the container is equal to the orbital radius. Thus we have

$$L = a_0 n_{\max}^2. \quad (6)$$

Since a_0 is 70 Å and L is of order 1 cm, $n_{\max} \approx 1200$. At 2 °K we have $Q_b \approx 1230$, $Q_c = 2 \times 10^5$, and $N_b/N \approx 10^{-4}$. Thus the majority of the surface states would be ionized at this temperature and the electrons distributed throughout the available gas volume. Even at 1 °K, only 10% of the electrons would be in the lowest bound state, and it would require a substantially lower temperature for complete occupation of the bound states. According to this model, then, the electrons would have been distributed throughout the gas at the temperatures of our experiments,

rather than localized on the surface.

The observation of the surface depression shown in Fig. 4 indicates, however, that they are, indeed, close to the surface. The steady force exerted on the liquid by the electrons can be understood as the result of either of two possible situations. The first and simplest is that they are in "direct contact" with the liquid in the usual macroscopic sense of the word. This means that they are so close to the surface that they interact with it continuously through short-range repulsive forces. This happens when they remain at an average distance a_0 from the surface which is as close as they can get. Alternatively, they might be free in the gas, confined to a region near the surface of thickness t by the electrostatic potentials in the actual experiment, including the field from the positively charged plate and the space-charge field due to the electrons themselves. In this case, not being in "direct contact" with the liquid in the above sense, they could exert a force on it only by the integrated effect of successive collisions, as a gas exerts a force on the walls of its container. For the photograph of Fig. 4 there were 10^9 electrons/cm² of surface, and the level of the liquid was depressed approximately 1×10^{-2} cm, corresponding to a pressure of about 1 dyn/cm². If the electrons were free in the gas, but confined by electrostatic potentials to remain within a layer of thickness t adjacent to the surface, then the number ν of electrons/cm³ would be $10^9/t$. The pressure p exerted by them is given by the ideal-gas equation:

$$p = \nu kT = 10^9 kT/t. \quad (7)$$

We then ask what value of t is required to make the electron density such that the pressure exerted by it is 1 dyn/cm². For the temperature of 3.7 °K, used for the experiment of Fig. 4, the required value of t is 50 Å. This is of the same order of magnitude as a_0 . Again we arrive at the conclusion that the electrons must be localized at the surface. In addition to this argument, we have observed a lifetime which is determined by the interaction of the electron with the surface, again implying that it is on the surface. To reconcile these experimental observations with our above conclusion that the image-force model alone does not predict localization on the surface at these temperatures, we must consider other features of the experimental system.

The electric field from the positive plate below the surface of the liquid is sufficient to localize the electrons on the surface of the liquid under one condition. This condition is that, with respect to motion in the xy plane, they behave as localized charges and not as delocalized Bloch waves. This is contrary to the assumption of the image-force model.^{1,2} For discrete electron charges on the

surface, the positive field from the plate is not canceled completely by the electrons at the plane of the surface, but extends into the gas, out to distances of order a , the lateral distance between electrons.¹² To discuss this more clearly, we review the steps during charging and discharging of the surface and examine the field in the gas near the surface at each step, with reference to Fig. 3. In Fig. 3(a) the field is shown before charging and during the early stages of charging. As electrons from the discharge arrive at the surface, the field in the gas *far from the surface* is reduced and finally goes to zero. If the electrons behave as discrete charges, localized in the xy plane, they cannot cancel the field from the positive plate at distances closer to the surface than a . In our experiments, a is typically 10^{-4} cm and the field from the positive plate around 100 V/cm. This field extends into the gas, as shown in Fig. 3(b), for a distance of 10^{-4} cm, giving a potential barrier E_b of about 10^{-2} eV. This is 14 times as large as the barrier due to the image force alone and is sufficient to keep all the electrons in bound states at the highest temperature of our experiments. Moreover, they are concentrated in the lowest bound state up to 4 °K. Localization in the x and y directions seems to be a necessary and sufficient condition for localization in the z direction over our experimental temperature range. This finding is consistent with the model of electron crystallization on the surface which has been proposed earlier.¹³ Lateral Coulomb repulsion can give a two-dimensional array of electrons situated at the points of a hexagonal lattice. In order to abandon the idea that the electrons are localized in the xy plane, we would have to assume that the binding force is much greater than the theoretical estimates. There is no evident reason to expect a large error in this quantity. Moreover, strong binding of the electron to the surface would reduce the tunneling probability to insignificant values.

With this background we can discuss the measurements of the surface-state lifetime. The electrons leave the surface for the gas during the time that a pulse is applied of polarity such that the plate in the liquid is negative with respect to the surface. The corresponding potential near the surface is shown in Fig. 3(c). To leave the surface, the electron can either be thermally excited over the barrier or can tunnel through it. For thermal excitation, the rate constant should increase strongly with increasing temperature. The data, however, show the opposite behavior. There is a decrease of the rate constant with temperature for any given value of the applied field. We conclude, therefore, that tunneling is the more likely escape mechanism.

It is straightforward to calculate the tunneling

probability for the simple image-force model in analogy with calculations that have been made for field ionization of the hydrogen atom.¹⁴ The result for the tunneling probability W as a function of the field F is

$$W = \left(\frac{16m^5 \alpha^8 e^{14}}{\hbar^{11} F^2} \right) \exp \left(\frac{-2m^2 \alpha^3 e^5}{3\hbar^4 F} \right). \quad (8)$$

Comparing this expression with the data in Fig. 7, we find that the theoretical expression gives a rate constant several orders of magnitude larger than that observed and contains no temperature dependence, though one was observed. We see no obvious modification that would improve agreement with experiment. It appears that a satisfactory theory will have to take account of the detailed interactions between electrons and the surface and possibly of the interactions among electrons. This is consistent with our findings above on the localization of electrons. It has been shown² that an electron deforms the liquid surface locally and this is probably important.

B. Inside States

The rate constant for the escape of electrons from the inside states into the gas indicates a thermally activated process with an activation energy U/k of 27 °K. This agrees with measurements of the activation energy for current flow across the surface^{15,16} and appears to be explained¹⁷ by a barrier due to the superposition of applied field and image field.

VI. SUMMARY

An experimental method has been developed to study the properties of electrons on the surface of liquid helium. The properties are very different depending on whether the electrons are deposited on the surface from above (outside state) or are brought to the surface from inside the liquid (inside states). In both cases, lifetimes were measured for the escape of electrons from the surface into the gas in an applied electric field. A simple thermodynamic analysis indicates that, at temperatures above 1 °K, the discrete nature of the electron charge is essential for its binding to the surface. This appears to be important also for an understanding of the kinetics of its escape from the surface.

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Phononlike Surface Modes in Two-Component Plasmas and in Systems with Short-Range Interactions*

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Surface modes in a homogeneous system with a sharp boundary are discussed using the collisionless Boltzmann equation for the Wigner distribution function. In a one-component system of fermions interacting via a Yukawa potential, we show that there is an acoustic surface mode. In the limit of long wavelengths, it becomes indistinguishable from the analogous bulk zero-sound mode. For a system of electrons and ions, we find a low-frequency in-phase surface mode, in addition to a high-frequency out-of-phase optical surface mode. For long wavelengths, the low-frequency mode is phononlike with a speed identical to that given by the Bohm-Staver expression.

I. INTRODUCTION

In recent years there has been considerable interest in the properties of collective modes which are localized at surfaces. Since the surface of a real physical system is usually diffuse, a detailed investigation of surface collective modes involves the determination of the dynamic response of an inhomogeneous many-body system. In view of the complexity of this problem, it is useful to consider a model where surface diffuseness is neglected and the inhomogeneous system is represented by a collection of fermions confined to a half-space by an infinite potential barrier. This model is characterized by the static Wigner distribution function

$$f_0(\vec{p}, \vec{R}) = f(\epsilon_p) \Theta(z), \quad (1.1)$$

where $f(\epsilon_p)$ is the Fermi distribution function and $\Theta(z)$ is the step function. The great advantage of such a model (which will be referred to as the semiclassical half-space model) is that the condition for a surface collective mode can be formulated in terms of bulk properties.

In the case of a bounded electron gas, the condi-

tion for a surface mode which one obtains using the semiclassical half-space model is¹⁻³

$$-1 = 2k_{||} \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \frac{1}{k_z^2 + k_{||}^2} \frac{1}{\epsilon_B(\vec{k}, \omega)}, \quad (1.2)$$

where $\vec{k} \equiv (k_{||}, k_z)$ is a three-dimensional vector with components $(k_{||}, k_z)$ parallel and normal to the plane surface, and $\epsilon_B(\vec{k}, \omega)$ is the bulk dielectric function. Equation (1.2) gives the dispersion relation of surface plasmons for the model system. Unfortunately, it appears that surface-plasmon dispersion is sensitive to the surface diffuseness.⁴ As a result, the dispersion resulting from the k dependence of $\epsilon_B(\vec{k}, \omega)$ in (1.2) cannot by itself be considered significant.

In this paper we consider acoustic (or phononlike) modes in one- and two-component systems using the semiclassical half-space model. We expect this model to provide an adequate description in the limit of long wavelengths since, in this case, the modes involve density fluctuations which extend far into the bulk. In Sec. II, we give a derivation of the analogous condition to (1.2) for a system of par-

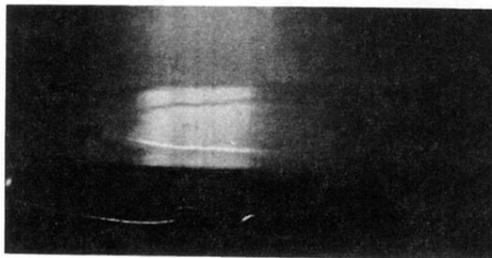
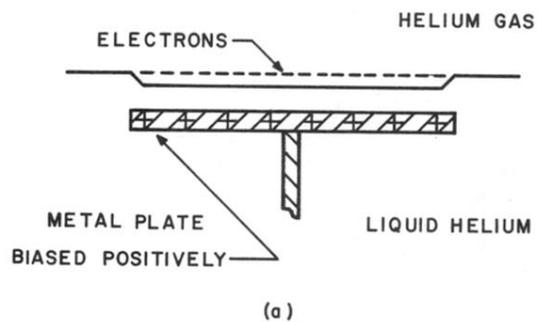


FIG. 4. (a) Schematic side view of liquid-helium surface, electrons on the surface, and metal plate below the surface. (b) Photograph of depression on the surface due to condenser force exerted by electrons. The photograph is taken from an angle about 10° above the horizontal. The circular depression can be seen over the plate; the plate below the surface can be seen through the side wall of the Dewar. The plate is 1.0 mm below the surface and has a diameter of 3.0 cm. The voltage between the sheet of electrons and the plate is 200 V, giving a field of 2000 V/cm. The temperature is 3.7 °K and the depth of the depression is about 0.13 mm, in agreement with our theoretical estimate (Ref. 7) for the depth.