to the second line in f_3 means an interchange of identical particles. Therefore the second term in (B1) is also symmetric. Thus $\overleftrightarrow{P}_w$ is a symmetric tensor.

*Also at Facultad de Ciencias, Universidad Nacional Autónoma de México, México.

[1]J. H. Irving and J. G. Kirkwood, J. Chem. Phys. 18, 817 (1950).

[2]S. T. Choh, Ph.D. thesis (University of Michigan, 1958) (unpublished).

[3]N. N. Bogoluybov, *Problems of a Dynamical Theory in Statistical Mechanics*, translated by E. K. Gora, Vol. I of *Studies in Statistical Mechanics*, edited by J. de Boer and G. E. Uhlenbeck (North-Holland, Amsterdam, 1962).

[4]M. S. Green, J. Chem. Phys. 25, 836 (1956).

[5]E. G. D. Cohen, Physica 28, 1025 (1962); 28, 1045 (1962); J. Math. Phys. 4, 183 (1963).

[6]M. S. Green and R. A. Piccirelli, Phys. Rev. 132, 1388 (1963).

[7]L. S. García-Colín, M. S. Green, and F. Chaos, Physica 32, 450 (1966).

[8]M. H. J. J. Ernst, Ph.D. thesis (University of Amsterdam, 1965) (unpublished); Physica 32, 209 (1966).

[9]H. Margenau and N. R. Kestner, *Theory of Intermolecular Forces* (Pergamon, Oxford, 1969).

[10]D. Rosen, J. Chem. Phys. 21, 1007 (1953).

[11]A. Shostak, J. Chem. Phys. 23, 1808 (1955).

[12]R. F. W. Bader, O. A. Novaro, and V. Beltrán-López, Chem. Phys. Letters 8, 568 (1971).

[13]O. A. Novaro and V. Beltrán-López, J. Chem. Phys. (to be published).

[14]J. V. Sengers, Phys. Rev. Letters 15, 515 (1965); Phys. Fluids 9, 1685 (1966).

[15]E. G. D. Cohen, in *Lectures in Theoretical Physics*, edited by W. E. Britten (Gordon and Breach, New York, 1967), Vol. 9 C.

[16]M. H. J. J. Ernst. Physica 32, 209 (1966).

[17]E. Braun, following paper, Phys. Rev. A 5, 1947 (1972).

PHYSICAL REVIEW A VOLUME 5, NUMBER 4 APRIL 1972

Nonequilibrium Statistical Mechanics of Systems Interacting with Nonadditive Forces. II. Kinetic Equation and Transport Coefficients

E. Braun*

Reactor, Centro Nuclear, Comisión Nacional de Energía Nuclear, México 18 *D. F.*, *México*

(Received 4 October 1971)

A kinetic equation is set up for a system of particles interacting with nonadditive intermolecular forces. Bogolyubov's functional assumption is used. After linearizing in the gradients, the kinetic equation is solved by a Chapman-Enskog method. Using the expressions for the stress tensor and heat current obtained in an earlier paper, the contributions of nonadditive forces to the shear and bulk viscosities and thermal conductivity are explicitly obtained. The results obtained are independent of density expansions.

I. INTRODUCTION

In an earlier paper[1] we obtained the hydrodynamical equations of a system of particles interacting with nonadditive intermolecular forces. Explicit expressions for the stress tensor and heat current were given, in terms of the intermolecular potential.

It is the purpose of this paper to obtain general expressions for the linear transport coefficients of a system of particles which interact with nonadditive forces. We obtain these expressions making Bogolyubov's assumption,[2] namely, that the distribution functions of more than one particle are functionals of the single-particle distribution. Thus, no expansion as power series in the density is used. Therefore the results that are obtained are independent of whether the density expansions exist or not. In this paper we generalize to our case the method proposed by García-Colín, Green, and Chaos[3] of obtaining linear transport coefficients without recourse to density expansions.

In Sec. II we start from Liouville's equation to obtain the generalization of the Bogolyubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy to the case of systems that interact with nonadditive forces. Taking the first equation of this hierarchy and making the Bogolyubov functional assumption, we obtain the kinetic equation. We then proceed to linearize the kinetic equation in the gradients of the system.

In Sec. III we solve the linearized kinetic equation by the usual Chapman-Enskog method.

In Sec. IV we use the expressions for the stress tensor and heat current obtained in I, together with Bogolyubov's functional assumption and the solution of the linearized kinetic equation, to compute the transport coefficient of this system, namely, the shear and bulk viscosities and thermal conductivity. We find the explicit contributions to these coefficients due to nonadditive forces.

II. KINETIC EQUATION

Let us consider a one-component system composed of N particles of mass m enclosed in a volume V. The Hamiltonian of the system is taken in the form

$$H=\sum_{i}^{N} \frac{p_i^2}{2m}+\frac{1}{2} \sum_{i \neq j}^{N}\sum \varphi_{ij}+\frac{1}{6} \sum_{i\neq j\neq k}\sum^{N}\sum w_{ijk} . \quad (2.1)$$

For the meaning of the symbols the reader is referred to I.

The Liouville equation of this system is

$$\frac{\partial F_N}{\partial t}=[H; F_N] , \quad (2.2)$$

with $[a;\ b]$ denoting the Poisson bracket of a and b. Integrating this equation over the arguments $x_{s+1}, \ldots, x_N$, we find, in the usual way, the generalization of the BBGKY hierarchy, namely, that

$$\frac{\partial f_s}{\partial t}=[H_s; f_s]+\int\left[\sum_{i=1}^{s} \varphi(|\vec{q}_i-\vec{q}_{s+1}|) +\frac{1}{2} \sum_{i\neq j}^{s}\sum w(\vec{q}_i, \vec{q}_j, \vec{q}_{s+1}); f_{s+1}\right] dx_{s+1} +\iint\left[\sum_{i=1}^{s} w(\vec{q}_i, \vec{q}_{s+1}, \vec{q}_{s+2}); f_{s+2}\right] dx_{s+1}\,dx_{s+2} , \quad (2.3)$$

with $s=1, 2, \ldots$. Here the function f_s denotes the reduced distribution function of s particles, and H_s, the Hamiltonian of s particles, is given by

$$H_s(x_1, \ldots, x_s) =\sum_{i=1}^{s} \frac{p_i^2}{2m}+\frac{1}{2} \sum_{i\neq j}^{s}\sum \varphi_{ij}+\frac{1}{6} \sum_{i\neq j\neq k}\sum^{s}\sum w_{ijk} . \quad (2.4)$$

It should be mentioned that in obtaining the hierarchy given by Eq. (2.3) we have taken the limit $N\to\infty$, $V\to\infty$, N/V finite. In particular for $s=1$, Eq. (2.3) becomes

$$\frac{\partial f_1(\vec{p},\vec{q};t)}{\partial t}+\frac{\vec{p}}{m}\cdot\vec{\nabla}_q f_1(\vec{p},\vec{q};t) =\int dx_2 [\vec{\nabla}_q \varphi(|\vec{q}-\vec{q}_2|)]\cdot[\vec{\nabla}_p f_2(\vec{p},\vec{q},x_2;t)] +\iint dx_2\,dx_3 [\vec{\nabla}_q w(\vec{q}_1,\vec{q}_2,\vec{q}_3)] \cdot[\vec{\nabla}_p f_3(\vec{p},\vec{q},x_2,x_3;t)] . \quad (2.5)$$

Now we make Bogolyubov's functional assumption namely, that the distribution functions of more than one particle are time-independent functionals of the one-particle distribution function

$$f_s(x_1, \ldots, x_s; t)=f_s(x_1, \ldots, x_s\,|f_1(x;t)), \quad s\geq 2. \quad (2.6)$$

This assumption will be valid only in the so-called "kinetic stage" of the evolution of the system towards its equilibrium state. By substituting Eq. (2.6) into the BBGKY hierarchy, Eq. (2.3), and eliminating the time derivative of f_1 by means of the equation corresponding to $s=1$, one can, in principle, obtain the explicit form of f_s as given by Eq. (2.6). This problem and those related to the existence of the kinetic stage for a system interacting with nonadditive forces will be discussed in a subsequent paper. For the time being we will assume the the f_s are determined.

Substituting Eq. (2.6) into Eq. (2.5), we obtain the equation

$$\frac{\partial f_1}{\partial t}+\frac{\vec{p}}{m}\cdot\vec{\nabla}_q f_1=\int dx_2[\vec{\nabla}_q \varphi(|\vec{q}-\vec{q}_2|)]\cdot[\vec{\nabla}_p f_2(x, x_2|f_1)] +\iint dx_2\,dx_3[\vec{\nabla}_q w(\vec{q},\vec{q}_2,\vec{q}_3)]\cdot[\vec{\nabla}_p f_3(x, x_2, x_3|f_1)]\equiv\Psi(x|f_1) . \quad (2.7)$$

This is the kinetic equation for a system of particles interacting with nonadditive forces. Here we have denoted $x\equiv(\vec{p}, \vec{q})$. $\Psi(x|f_1)$ is a nonlocal functional of f_1.

By linearizing the functional Ψ one obtains the following linearized kinetic equation to first order in the gradients[3]:

$$\frac{\partial f_1}{\partial t}+\frac{\vec{p}}{m}\cdot\vec{\nabla}_q f_1=\Psi(x\,|f_1(\vec{q})) +\int dx'\,\Psi'(x, x'\,|f_1(\vec{q}))(\vec{q}\,'-\vec{q})\cdot\left(\frac{\partial f_1}{\partial \vec{q}\,'}\right)_{\vec{q}\,'=\vec{q}} . \quad (2.8)$$

In this expression $\Psi(x\,|f_1(\vec{q}))$ is evaluated for the local distribution function $f_1(\vec{q})$, and $\Psi'(x, x'|f_1(\vec{q}))$ denotes the functional derivative of Ψ taken at the point $x'=(\vec{q}\,', \vec{p}\,')$ and evaluated for the local distribution function $f_1(\vec{q})$.

We would like to stress the fact that the linearized kinetic equation given by Eq. (2.8) was obtained without making any density expansion.

III. SOLUTION OF LINEARIZED KINETIC EQUATION

We proceed now to solve the linearized kinetic equation that was obtained in Sec. II. We will use the method of Chapman and Enskog. This method will be valid only in the "hydrodynamic stage," in which the only variables that change in time are the macroscopic variables that describe the system. Thus, we make the assumption that the one-particle distribution function f_1 is a time-independent functional of the macroscopic variables, namely, of the average concentration $n(\vec{q};t)$, the average local velocity $\vec{u}(\vec{q};t)$, and the average energy density $\epsilon(\vec{q}\,|f_1)$. Therefore, one has that

$$f_1(x;t)=f_1(x\,|\,n(\vec{q};t),\ \vec{u}(\vec{q};t),\ \epsilon(\vec{q}\,|f_1)), \tag{3.1}$$

where n, $\vec{u}$, and ϵ are given by [see Eqs. (3.6), (3.7) and (3.27) of I]

$$n(\vec{q};t)=\int f_1(x;t)\,d\vec{p}\,, \tag{3.2}$$

$$\vec{u}(\vec{q};t)=(1/n)\int d\vec{p}\,(\vec{p}/m)f_1(x;t), \tag{3.3}$$

and

$$\epsilon(\vec{q}\,|f_1)=(1/2m)\int d\vec{p}\ \mathcal{P}^2 f_1(x;t)+\tfrac{1}{2}\int d\vec{p}\,d\vec{p}_2\,d\vec{R}\,\varphi(R)f_2(\vec{q},\vec{q}+\vec{R},\vec{p},\vec{p}_2\,|f_1)$$
$$+\tfrac{1}{6}\int d\vec{p}\,d\vec{p}_2\,d\vec{p}_3\,d\vec{r}\,d\vec{R}\,w(\vec{r},\vec{R})f_3(\vec{q},\vec{q}-\vec{R},\vec{q}-\vec{R}+\vec{r},\vec{p},\vec{p}_2,\vec{p}_3\,|f_1)\,. \tag{3.4}$$

Here $\vec{\mathcal{P}}=\vec{p}-m\vec{u}$ is the thermal momentum.

If the system under consideration is not far from the equilibrium state, we can write for the one-particle distribution function

$$f_1=f_1^{(0)}(1+\Phi)\,, \tag{3.5}$$

where $f_1^{(0)}$ denotes the one-particle distribution function in equilibrium and Φ represents the separation from the equilibrium state. The function Φ will be taken up to linear terms in the gradients of the macroscopic variables.

Substituting Eq. (3.5) into Eqs. (2.8) and (3.2)–(3.4), we find to zero order in the gradients

$$\Psi(x\,|f_1^{(0)}(\vec{q}))=0\,, \tag{3.6}$$

$$n(\vec{q};t)=\int d\vec{p}\,f_1^{(0)}(x;t), \tag{3.7}$$

$$\vec{u}(\vec{q};t)=(1/n)\int d\vec{p}\,(\vec{p}/m)\,f_1^{(0)}(x;t), \tag{3.8}$$

and

$$\epsilon(\vec{q}\,|f_1)=\epsilon(\vec{q}\,|f_1^{(0)}(\vec{q}))=(1/2m)\int d\vec{p}\ \mathcal{P}^2 f_1^{(0)}(x;t)+\tfrac{1}{2}\int d\vec{p}\,d\vec{p}_2\,d\vec{R}\,\varphi(R)f_2(\vec{q},\vec{q}+\vec{R},\vec{p},\vec{p}_2\,|f_1^{(0)}(\vec{q}))$$
$$+\tfrac{1}{6}\int d\vec{p}\,d\vec{p}_2\,d\vec{p}_3\,d\vec{r}\,d\vec{R}\,w(\vec{r},\vec{R})f_3(\vec{q},\vec{q}-\vec{R},\vec{q}-\vec{R}+\vec{r},\vec{p},\vec{p}_2,\vec{p}_3\,|f_1^{(0)}(\vec{q}))\,. \tag{3.9}$$

In Appendix A we show that the solution to Eq. (3.6) is given by the local Maxwellian distribution function

$$f_1^{(0)}(x;t)=\frac{n}{(2\pi m\theta)^{3/2}}\exp\left(-\frac{(\vec{p}-m\vec{u})^2}{2m\theta}\right). \tag{3.10}$$

Here n and $\vec{u}$ are the local values of the particle density and average velocity, respectively. Equation (3.9) together with Eq. (3.10) will give us the relation between the energy density and the temperature θ.

If we now substitute Eq. (3.10) into the hydrodynamical equations obtained in I [see Eqs. (3.5), (3.19) and (3.26) of I], we obtain the Euler equations, valid to zero order in the gradients,

$$\frac{\partial n}{\partial t}=-\vec{\nabla}_q\cdot(n\vec{u}), \tag{3.11}$$

$$nm\,\frac{D\vec{u}}{Dt}=-\vec{\nabla}_q p\,, \tag{3.12}$$

$$n\,\frac{D}{Dt}\left(\frac{\epsilon}{n}\right)=-p\,\vec{\nabla}_q\cdot\vec{u}\,. \tag{3.13}$$

In these equations we used the fact that $\rho=nm$. The local equilibrium pressure, denoted by p, is

$$p=n\theta-\tfrac{1}{6}\int d\vec{p}\,d\vec{p}_2\,d\vec{R}\,R\,\varphi'(R)f_2(\vec{q},\vec{q}+\vec{R},\vec{p},\vec{p}_2\,|f_1^{(0)}(\vec{q}))-\tfrac{1}{9}\int d\vec{p}\,d\vec{p}_2\,d\vec{p}_3\,d\vec{r}\,d\vec{R}\,\vec{R}\cdot[\vec{\nabla}_R w(\vec{r},\vec{R})]$$
$$\times f_3(\vec{q},\vec{q}+\vec{R},\vec{q}+\vec{R}-\vec{r},\vec{p},\vec{p}_2,\vec{p}_3\,|f_1^{(0)}(\vec{q}))\,. \tag{3.14}$$

In equilibrium, the equation of state has this form. The contribution of nonadditive forces is given by the last term on the right-hand side. This expression for the equation of state may, of course, also be obtained directly from the partition function of the system in equilibrium.

Substituting again Eq. (3. 5) into Eqs. (2. 8) and (3. 2)–(3. 4) and keeping terms that are linear in the gradients, we obtain the equation

$$\frac{\partial f_1^{(0)}}{\partial t}+\frac{\vec{p}}{m}\cdot\vec{\nabla}_q f_1^{(0)}-\int dx'\,\Psi'(x,x'|f_1^{(0)}(\vec{q}))\times(\vec{q}'-\vec{q})\cdot\left(\frac{\partial f_1^{(0)}(x')}{\partial\vec{q}'}\right)_{\vec{q}'=\vec{q}}=\int d\vec{p}'\,\Psi'(x,\vec{p}'|f_1^{(0)}(\vec{q}))f_1^{(0)}(\vec{q},\vec{p}')\,\Phi(\vec{q},\vec{p}') \qquad (3.15)$$

and the subsidiary conditions

$$\int d\vec{p}f_1^{(0)}(\vec{p})\,\Phi(\vec{p})=0\,, \qquad (3.16)$$

$$\int d\vec{p}\frac{\vec{p}}{m}f_1^{(0)}(\vec{p})\,\Phi(\vec{p})=0\,, \qquad (3.17)$$

and

$$\int d\vec{p}'\,\epsilon'(\vec{q},\vec{p}'|f_1^{(0)}(\vec{q}))f_1^{(0)}(\vec{q},\vec{p}')\,\Phi(\vec{q},\vec{p}')+\int dx'\,\epsilon'(\vec{q},x'|f_1^{(0)}(\vec{q}))\times(\vec{q}'-\vec{q})\cdot\left(\frac{\partial f_1^{(0)}}{\partial\vec{q}'}\right)_{\vec{q}'=\vec{q}}=0\,. \qquad (3.18)$$

Equation (3. 15) is an inhomogeneous integral equation for Φ. The solutions of this integral equation must satisfy the conditions given by Eqs. (3. 16)–(3. 18).

One can eliminate $(\partial f_1^{(0)}/\partial t)$ in the left-hand side of Eq. (3. 15) by means of the Euler equations. This is a well-known procedure and we just quote the result (see Ref. 3):

$$\vec{D}(\vec{p})\cdot\vec{\nabla}_q\ln n+\vec{G}(\vec{p})\cdot\vec{\nabla}_q\ln\theta+\overleftrightarrow{A}(\vec{p}):\vec{\nabla}_q\vec{u}+B(\vec{p})\,\vec{\nabla}_q\cdot\vec{u}=\int d\vec{p}'\,\Psi'(x,\vec{p}'|f_1^{(0)}(\vec{q}))\times f_1^{(0)}(\vec{p}')\,\Phi(\vec{p}')\,, \qquad (3.19)$$

with

$$\vec{D}(\vec{p})=f_1^{(0)}(\vec{p})\left(1-\frac{1}{n\kappa\theta}\right)\frac{\vec{\mathcal{P}}}{m}-\int dx'\,\Psi'(x,x'|f_1^{(0)}(\vec{q}))(\vec{q}'-\vec{q})f_1^{(0)}(\vec{p}')\,, \qquad (3.20)$$

$$\vec{G}(\vec{p})=f_1^{(0)}(\vec{p})\left(\frac{\mathcal{P}^2}{2m\theta}-\frac{3}{2}-\frac{\beta}{n\kappa}\right)\frac{\vec{\mathcal{P}}}{m}-\int dx'\,\Psi'(x,x'|f_1^{(0)}(\vec{q}))(\vec{q}'-\vec{q})\times\left(\frac{\mathcal{P}'^2}{2m\theta}-\frac{3}{2}\right)f_1^{(0)}(\vec{p}')\,, \qquad (3.21)$$

$$\overleftrightarrow{A}(\vec{p})=\frac{f_1^{(0)}(\vec{p})}{m\theta}\,\mathcal{P}^0\mathcal{P}-\frac{1}{\theta}\int dx'\,\Psi'(x,x'|f_1^{(0)}(\vec{q}))\,S^0S\,f_1^{(0)}(\vec{p}')\,, \qquad (3.22)$$

$$B(\vec{p})=Lf_1^{(0)}(\vec{p})-\frac{1}{3\theta}\int dx'\,\Psi'(x,x'|f_1^{(0)}(\vec{q}))\,\vec{\mathcal{P}}'\cdot(\vec{q}'-\vec{q})f_1^{(0)}(\vec{p}')\,. \qquad (3.23)$$

In these expressions we have set

$$\kappa^{-1}=n\left(\frac{\partial p}{\partial n}\right)_\theta\,, \qquad (3.24)$$

$$\beta=\kappa\left(\frac{\partial p}{\partial\theta}\right)_n\,, \qquad (3.25)$$

$$L=\left(1-\frac{\mathcal{P}^2}{3m\theta}\right)\left(1-\frac{3}{2}\frac{\beta}{n\kappa c_v}\right)\,, \qquad (3.26)$$

$$c_v=\frac{1}{n}\left(\frac{\partial\epsilon}{\partial\theta}\right)_n\,, \qquad (3.27)$$

and

$$\mathcal{P}^0\mathcal{P}=\vec{\mathcal{P}}\vec{\mathcal{P}}-\tfrac{1}{3}\mathcal{P}^2\overleftrightarrow{I}\,, \qquad (3.28)$$

$$S^0S=\tfrac{1}{2}[(\vec{q}'-\vec{q})\vec{\mathcal{P}}'+\vec{\mathcal{P}}'(\vec{q}'-\vec{q})]-\tfrac{1}{3}\vec{\mathcal{P}}'\cdot(\vec{q}'-\vec{q})\overleftrightarrow{I}\,. \qquad (3.29)$$

are symmetric traceless tensors. Here $\overleftrightarrow{I}$ is the unit tensor.

Due to the fact that we are dealing with a one-component gas, no diffusion is present. Therefore, the coefficient of $\vec{\nabla}_q \ln n$ must be equal to zero, i. e.,

$$\vec{D}(\vec{p})=0\,. \qquad (3.30)$$

This is discussed in Appendix B.

In order to obtain the solution of the integral equation, Eq. (3. 19), we mention the following properties of the nonsymmetrical kernel $\Psi'(x,\vec{p}'|f_1^{(0)}(\vec{q}))$ (the proof for our case is exactly the same as the one given in Ref. 3): (a) The right eigenfunctions with zero eigenvalue of the kernel are 1, $\vec{p}$, and p^2. (b) The left eigenfunctions with zero eigenvalue of the kernel are 1, $\vec{\mathcal{P}}$, and $E'(x'|f_1^{(0)}(\vec{q}))$. This last quantity is the functional derivative of the total energy E, evaluated for the local Maxwellian distribution function $f_1^{(0)}(\vec{q})$.

These two properties of the kernel establish as consequence the existence of a solution of the integral equation, Eq. (3.19), and the fact that this solution is undetermined up to an arbitrary linear combination of the five solutions to the homogeneous equation. However, this undeterminacy is resolved with the aid of the subsidiary conditions given by Eqs. (3.16)–(3.18). Therefore, we may write the solution of the integral equation, Eq. (3.19) as follows:

$$\Phi(\vec{p}) = \mathcal{G}\,(\mathcal{P}^2)\,\vec{\mathcal{P}}\cdot\vec{\nabla}_q \ln\theta + \mathcal{A}\,(\mathcal{P}^2)\,\mathcal{P}^0\mathcal{P}:\vec{\nabla}_q\vec{u} + \mathcal{B}\,(\mathcal{P}^2)\,\vec{\nabla}_q\cdot\vec{u}\,. \qquad (3.31)$$

The scalar functions $\mathcal{G}$, $\mathcal{A}$, and $\mathcal{B}$ satisfy the following integral equations:

$$f_1^{(0)}(\vec{p})\left(\frac{\mathcal{P}^2}{2m\theta}-\frac{3}{2}-\frac{\beta}{n\kappa}\right)\frac{\vec{\mathcal{P}}}{m} - \int dx'\,\Psi'(x,\,x'\,|\,f_1^{(0)}(\vec{q}))\,(\vec{q}\,'-\vec{q})\left(\frac{\mathcal{P}'^2}{2m\theta}-\frac{3}{2}\right) f_1^{(0)}(\vec{p}\,')$$

$$= \int d\vec{p}\,'\,\Psi'(x,\,\vec{p}\,'\,|\,f_1^{(0)}(\vec{q}))\,\vec{\mathcal{P}}'f_1^{(0)}(\vec{p}\,')\,\mathcal{G}\,(\mathcal{P}'^2)\,, \qquad (3.32)$$

$$\frac{f_1^{(0)}(\vec{p})}{m\theta}\,\mathcal{P}^0\mathcal{P} - \frac{1}{\theta}\int dx'\,\Psi'(x,\,x'\,|\,f_1^{(0)}(\vec{q}))\,S^0S\,f_1^{(0)}(\vec{p}\,') = \int d\vec{p}\,'\,\Psi'(x,\,\vec{p}\,'\,|\,f_1^{(0)}(\vec{q}))\,\mathcal{P}'^0\mathcal{P}'f_1^{(0)}(\vec{p}\,')\,\mathcal{A}\,(\mathcal{P}'^2), \qquad (3.33)$$

$$Lf_1^{(0)}(\vec{p}) - \frac{1}{3\theta}\int dx'\,\Psi'(x,\,x'\,|\,f_1^{(0)}(\vec{q}))\,\vec{\mathcal{P}}'\cdot(\vec{q}\,'-\vec{q})f_1^{(0)}(\vec{p}\,') = \int d\vec{p}\,'\,\Psi'(x,\,\vec{p}\,'\,|\,f_1^{(0)}(\vec{q}))\,f_1^{(0)}(\vec{p}\,')\,\mathcal{B}\,(\mathcal{P}'^2)\,. \qquad (3.34)$$

Also, the functions $\mathcal{G}$, $\mathcal{A}$, and $\mathcal{B}$ must satisfy the following subsidiary conditions in order to ensure unicity of the solution:

$$\int d\vec{p}\,f_1^{(0)}(\vec{p})\,\mathcal{P}^2\,\mathcal{G}\,(\mathcal{P}^2)=0\,, \qquad (3.35)$$

$$\int d\vec{p}\,f_1^{(0)}(\vec{p})\,\mathcal{B}\,(\mathcal{P}^2)=0\,, \qquad (3.36)$$

and

$$\int d\vec{p}\,'\,\epsilon'(\vec{q},\,\vec{p}\,'\,|\,f_1^{(0)}(\vec{q}))\,f_1^{(0)}(\vec{p}\,')\,\mathcal{B}\,(\mathcal{P}'^2) = -\frac{1}{6\theta}\int d\vec{p}\,d\vec{p}_2\,d\vec{R}\,dx'\,\varphi(R)\,f_2'\,(\vec{q},\,\vec{q}+\vec{R},\,\vec{p},\,\vec{p}_2,\,x'\,|\,f_1^{(0)}(\vec{q}))$$

$$\times\,\vec{\mathcal{P}}'\cdot(\vec{q}\,'-\vec{q})f_1^{(0)}(\vec{p}\,') - \frac{1}{18\theta}\int d\vec{p}\,d\vec{p}_2\,d\vec{p}_3\,d\vec{r}\,d\vec{R}\,dx'\,w(\vec{r},\vec{R})$$

$$\times f_3'(\vec{q},\vec{q}-\vec{R},\vec{q}-\vec{R}+\vec{r},\vec{p},\vec{p}_2,\vec{p}_3,x'\,|\,f_1^{(0)}(\vec{q}))\,\vec{\mathcal{P}}'\cdot(\vec{q}\,'-\vec{q})f_1^{(0)}(\vec{p}\,')\,. \qquad (3.37)$$

For our purposes, we will assume that the scalar functions $\mathcal{G}$, $\mathcal{A}$, and $\mathcal{B}$ are determined. Thus, the solution to the linearized kinetic equation [see Eqs. (3.5) and (3.31)], which depend on $\mathcal{G}$, $\mathcal{A}$, and $\mathcal{B}$ will be considered as known.

It should be mentioned that the $\mathcal{G}$, $\mathcal{A}$, and $\mathcal{B}$ functions that we defined above, are not the same $\mathcal{G}$, $\mathcal{A}$, and $\mathcal{B}$ functions defined in Ref. 3. In fact, although the integral equations which the functions satisfy have the same formal structure, the kernels are not the same. In our case the kernels take into account the nonadditivity of the intermolecular forces.

In Sec. IV we will obtain the transport coefficients of the system.

IV. TRANSPORT COEFFICIENTS

We are now in a position to evaluate the transport coefficients of the system. We will start with the expressions for the stress tensor and heat current obtained in I [see Eqs. (3.20) and (3.28) of I]. Using Bogolyubov's functional assumption [see Eq. (2.6)] and Eq. (3.1), these quantities take the following form.

The stress tensor is

$$\overleftrightarrow{P} = \overleftrightarrow{P}_\kappa + \overleftrightarrow{P}_\varphi + \overleftrightarrow{P}_w\,, \qquad (4.1)$$

with

$$\overleftrightarrow{P}_\kappa = \frac{1}{m}\int d\vec{p}\,\vec{\mathcal{P}}\,\vec{\mathcal{P}}\,f_1(\vec{q},\vec{p}\,|\,n,\vec{u},\epsilon)\,, \qquad (4.2)$$

$$\overleftrightarrow{P}_\varphi = -\tfrac{1}{2}\int d\vec{p}\,d\vec{p}_2\,d\vec{R}\,\frac{\vec{R}\vec{R}}{R}\,\varphi'(R) \times f_2(\vec{q},\vec{q}+\vec{R},\vec{p},\vec{p}_2\,|\,f_1)\,, \qquad (4.3)$$

$$\overleftrightarrow{P}_w = -\tfrac{1}{3}\int d\vec{p}\,d\vec{p}_2\,d\vec{p}_3\,d\vec{r}\,d\vec{R}\,\vec{R}\,[\vec{\nabla}_R\,w(\vec{r},\vec{R})]$$

$$\times f_3(\vec{q}, \vec{q}-\vec{R}, \vec{q}-\vec{R}+\vec{r}, \vec{p}, \vec{p}_2, \vec{p}_3 \,|\, f_1) . \qquad (4.4)$$

For the heat current we have

$$\vec{\mathcal{J}} = \vec{\mathcal{J}}_\kappa + \vec{\mathcal{J}}_\varphi^{(1)} + \vec{\mathcal{J}}_\varphi^{(2)} + \vec{\mathcal{J}}_w^{(1)} + \vec{\mathcal{J}}_w^{(2)} , \qquad (4.5)$$

with

$$\vec{\mathcal{J}}_\kappa = \int d\vec{p}\, \frac{\vec{\mathcal{P}}}{m} \frac{\mathcal{P}^2}{2m} f_1(\vec{q}, \vec{p} \,|\, n, \vec{u}, \epsilon) , \qquad (4.6)$$

$$\vec{\mathcal{J}}_\varphi^{(1)} = \frac{1}{2} \int d\vec{p}\, d\vec{p}_2\, d\vec{R}\, \frac{\vec{\mathcal{P}}}{m}\, \varphi(R) f_2(\vec{q}, \vec{q}+\vec{R}, \vec{p}, \vec{p}_2 \,|\, f_1), \qquad (4.7)$$

$$\vec{\mathcal{J}}_\varphi^{(2)} = -\frac{1}{4} \int d\vec{p}\, d\vec{p}_2\, d\vec{R}\, \varphi'(R) \frac{\vec{R}\vec{R}}{R} \cdot \left[\frac{\vec{\mathcal{P}}}{m} + \frac{\vec{\mathcal{P}}_2}{m}\right] \times f_2(\vec{q}, \vec{q}+\vec{R}, \vec{p}, \vec{p}_2 \,|\, f_1) , \qquad (4.8)$$

$$\vec{\mathcal{J}}_w^{(1)} = \frac{1}{6} \int d\vec{p}\, d\vec{p}_2\, d\vec{p}_3\, d\vec{r}\, d\vec{R}\, \frac{\vec{\mathcal{P}}}{m}\, w(\vec{r}, \vec{R}) \times f_3(\vec{q}, \vec{q}-\vec{R}, \vec{q}-\vec{R}+\vec{r}, \vec{p}, \vec{p}_2, \vec{p}_3 \,|\, f_1) , \qquad (4.9)$$

$$\vec{\mathcal{J}}_w^{(2)} = -\frac{2}{q} \int d\vec{p} d\vec{p}_2 d\vec{p}_3 d\vec{r} d\vec{R}\, \vec{R}\, [\vec{\nabla}_R w(\vec{r}, \vec{R})] \cdot \left[\frac{\vec{\mathcal{P}}}{m} + \frac{\vec{\mathcal{P}}_2}{m} + \frac{\vec{\mathcal{P}}_3}{m}\right] f_3(\vec{q}, \vec{q}-\vec{R}, \vec{q}-\vec{R}+\vec{r}, \vec{p}, \vec{p}_2, \vec{p}_3 | f_1) . \qquad (4.10)$$

We now substitute in these expressions $f_1 = f_1^{(0)} \times (1+\Phi)$, expand in powers of the gradients, and use the relations

$$f_i(\cdots | f_1) = f_i(\cdots | f_1^{(0)}(\vec{q})) + \int d\vec{p}\,' f_i'(\cdots, \vec{p}\,' | f_1^{(0)}(\vec{q})) f_1^{(0)}(\vec{p}\,') \Phi(\vec{q}, \vec{p}\,') + \int dx' f_i'(\cdots, x' | f_1^{(0)}(\vec{q}))(\vec{q}\,' - \vec{q}) \cdot \left(\frac{\partial f_1^{(0)}(x')}{\partial \vec{q}\,'}\right)_{\vec{q}\,'=\vec{q}} , \quad i = 2, 3 \qquad (4.11)$$

$$\frac{\partial f_1^{(0)}}{\partial \vec{q}} = f_1^{(0)}(\vec{p}) \left[\vec{\nabla}_q \ln n + \left(\frac{\mathcal{P}^2}{2m\theta} - \frac{3}{2}\right) \vec{\nabla}_q \ln\theta + \frac{\vec{\mathcal{P}}}{\theta} \cdot \vec{\nabla}_q \vec{u}\right], \qquad (4.12)$$

and Eq. (3.31), to find the following results for the stress tensor (see Appendix C for the details):

$$\overleftrightarrow{P} = p\overleftrightarrow{I} - 2\eta_1(\overleftrightarrow{D} - \tfrac{1}{3} \vec{\nabla}_q \cdot \vec{u}\, \overleftrightarrow{I}) - 2\eta_2 \vec{\nabla}_q \cdot \vec{u}\, \overleftrightarrow{I} . \qquad (4.13)$$

The local equilibrium pressure p is given by Eq. (3.14), and the deformation tensor $\overleftrightarrow{D}$ with components D_{ij} by

$$D_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial q_j} + \frac{\partial u_j}{\partial q_i}\right) . \qquad (4.14)$$

The coefficient of shear viscosity η_1 is

$$\eta_1 = \eta_\kappa^{(1)} + \eta_{\varphi(1)}^{(1)} + \eta_{\varphi(2)}^{(1)} + \eta_{w(1)}^{(1)} + \eta_{w(2)}^{(1)} , \qquad (4.15)$$

with

$$\eta_\kappa^{(1)} = -\frac{1}{15m} \int d\vec{p}\, \mathcal{P}^4 f_1^{(0)}(\vec{p})\, \mathcal{A}(\mathcal{P}^2) , \qquad (4.16)$$

$$\eta_{\varphi(1)}^{(1)} = +\frac{1}{20} \int d\vec{p} d\vec{p}_2 d\vec{p}\,' d\vec{R}\, \frac{\varphi'(R)}{R} \times [(\vec{R} \cdot \vec{\mathcal{P}}\,')^2 - \tfrac{1}{3} R^2 \mathcal{P}'^2]\, f_1^{(0)}(\vec{p}\,') \mathcal{A}(\mathcal{P}'^2) \times f_2'(\vec{q}, \vec{q}+\vec{R}, \vec{p}, \vec{p}_2, \vec{p}\,' \,|\, f_1^{(0)}(\vec{q})) , \qquad (4.17)$$

$$\eta_{\varphi(2)}^{(1)} = \frac{1}{20\theta} \int d\vec{p} d\vec{p}_2 d\vec{R} dx' \frac{\varphi'(R)}{R} [\vec{R} \cdot (\vec{q}\,' - \vec{q}) \vec{R} \cdot \vec{\mathcal{P}}\,' - \tfrac{1}{3} R^2 (\vec{q}\,' - \vec{q}) \cdot \vec{\mathcal{P}}\,'] f_1^{(0)}(\vec{p}\,') \times f_2'(\vec{q}, \vec{q}+\vec{R}, \vec{p}, \vec{p}_2, x' | f_1^{(0)}(\vec{q})) , \qquad (4.18)$$

$$\eta_{w(1)}^{(1)} = \tfrac{1}{15} \int d\vec{p} d\vec{p}_2 d\vec{p}_3 d\vec{p}\,' d\vec{r} d\vec{R} g_1(\vec{r}, \vec{R}) \times [(\vec{R} \cdot \vec{\mathcal{P}}\,')^2 - \tfrac{1}{3} R^2 \mathcal{P}'^2] f_1^{(0)}(\vec{p}\,') \mathcal{A}(\mathcal{P}'^2) \times f_3'(\vec{q}, \vec{q}-\vec{R}, \vec{q}-\vec{R}+\vec{r}, \vec{p}, \vec{p}_2, \vec{p}_3, \vec{p}\,' | f_1^{(0)}(\vec{q})) , \qquad (4.19)$$

$$\eta_{w(2)}^{(1)} = \frac{1}{30\theta} \int d\vec{p} d\vec{p}_2 d\vec{p}_3 d\vec{r} d\vec{R} dx' g_1(\vec{r}, \vec{R}) \times [\vec{R} \cdot (\vec{q}\,' - \vec{q}) \vec{R} \cdot \vec{\mathcal{P}}\,' - \tfrac{1}{3} R^2 (\vec{q}\,' - \vec{q}) \cdot \vec{\mathcal{P}}\,'] f_1^{(0)}(\vec{p}\,') \times f_3'(\vec{q}, \vec{q}-\vec{R}, \vec{q}-\vec{R}+\vec{r}, \vec{p}, \vec{p}_2, \vec{p}_3, x' | f_1^{(0)}(\vec{q})) . \qquad (4.20)$$

In these expressions $g_1(\vec{r}, \vec{R})$ is [see Eqs. (2.11) and (A13) of I]

$$g_1(\vec{r}, \vec{R}) = -\frac{\csc\theta}{Rr} \frac{\partial w}{\partial \theta} , \qquad (4.21)$$

where θ is the angle between the vectors $\vec{r}$ and $\vec{R}$.

The contributions to the shear viscosity due to nonadditive forces is given by Eqs. (4.19) and (4.20). There will also be contributions due to the nonadditive forces to the other terms of the shear viscosity, because the kernel of the integral equations [Eq. (3.33)] that the function $\mathcal{A}$ satisfies contains the nonadditive forces. Moreover, the functional derivative f_2' will also contain contributions of nonadditive forces.

The coefficient of bulk viscosity η_2 is given by

$$\eta_2 = \eta_\kappa^{(2)} + \eta_{\varphi(1)}^{(2)} + \eta_{\varphi(2)}^{(2)} + \eta_{w(1)}^{(2)} + \eta_{w(2)}^{(2)} , \tag{4.22}$$

with

$$\eta_\kappa^{(2)} = -\frac{1}{3m}\int d\vec{p}\, \mathcal{P}^2 f_1^{(0)}(\vec{p})\mathcal{B}(\mathcal{P}^2) , \tag{4.23}$$

$$\eta_{\varphi(1)}^{(2)} = \tfrac{1}{12}\int d\vec{p}\, d\vec{p}_2\, d\vec{R}\, d\vec{p}'\, R\, \varphi'(R) f_1^{(0)}(\vec{p}')\, \mathcal{B}(\mathcal{P}'^2) \times f_2'(\vec{q}, \vec{q}+\vec{R}, \vec{p}, \vec{p}_2, \vec{p}' \,|\, f_1^{(0)}(\vec{q})) , \tag{4.24}$$

$$\eta_{\varphi(2)}^{(2)} = \frac{1}{36\theta}\int d\vec{p}\, d\vec{p}_2\, d\vec{R}\, dx'\, R\, \varphi'(R)\, \vec{\mathcal{P}}' \cdot (|\vec{q}' - \vec{q}) f_1^{(0)}(\vec{p}') \times f_2'(\vec{q}, \vec{q}+\vec{R}, \vec{p}, \vec{p}_2, x' \,|\, f_1^{(0)}(\vec{q})) , \tag{4.25}$$

$$\eta_{w(1)}^{(2)} = \tfrac{1}{9}\int d\vec{p}\, d\vec{p}_2\, d\vec{p}_3\, d\vec{p}'\, d\vec{r}\, d\vec{R}\, R^2 g_1(\vec{r}, \vec{R}) f_1^{(0)}(\vec{p}') \times \mathcal{B}(\mathcal{P}'^2) f_3'(\vec{q}, \vec{q}-\vec{R}, \vec{q}-\vec{R}+\vec{r}, \vec{p}, \vec{p}_2, \vec{p}_3, \vec{p}' \,|\, f_1^{(0)}(\vec{q})) , \tag{4.26}$$

$$\eta_{w(2)}^{(2)} = \tfrac{1}{27}\int d\vec{p}\, d\vec{p}_2\, d\vec{p}_3\, d\vec{r}\, d\vec{R}\, dx'\, R^2 g_1(\vec{r}, \vec{R})\, \vec{\mathcal{P}}' \cdot (\vec{q}' - \vec{q}) \times f_1^{(0)}(\vec{p}') f_3'(\vec{q}, \vec{q}-\vec{R}, \vec{q}-\vec{R}+\vec{r}, \vec{p}, \vec{p}_2, \vec{p}_3, x' \,|\, f_1^{(0)}(\vec{q})) . \tag{4.27}$$

The contribution to the bulk viscosity due to nonadditive forces is expressed in Eqs. (4.26) and (4.27), and is also present in the function $\mathcal{B}$, and in the functional derivative f_2'.

For the heat current $\vec{\mathcal{J}}$ we find the following results:

$$\vec{\mathcal{J}} = -\lambda \vec{\nabla}_q \theta , \tag{4.28}$$

where the coefficient of thermal conductivity λ is given by

$$\lambda = \lambda_\kappa + \lambda_{\varphi(1)}^{(1)} + \lambda_{\varphi(2)}^{(1)} + \lambda_{\varphi(1)}^{(2)} + \lambda_{\varphi(2)}^{(2)} + \lambda_{w(1)}^{(1)} + \lambda_{w(2)}^{(1)} + \lambda_{w(1)}^{(2)} + \lambda_{w(2)}^{(2)} . \tag{4.29}$$

Here

$$\lambda_\kappa = -\frac{1}{6m^2\theta}\int d\vec{p}\, \mathcal{P}^4 f_1^{(0)}(\vec{p})\, \mathcal{G}(\mathcal{P}^2) , \tag{4.30}$$

$$\lambda_{\varphi(1)}^{(1)} = -\frac{1}{6m\theta}\int d\vec{p}\, d\vec{p}_2\, d\vec{p}'\, d\vec{R}\, \varphi(R)\, \vec{\mathcal{P}}\cdot\vec{\mathcal{P}}' f_1^{(0)}(\vec{p}') \times \mathcal{G}(\mathcal{P}'^2) f_2'(\vec{q}, \vec{q}+\vec{R}, \vec{p}, \vec{p}_2, \vec{p}' \,|\, f_1^{(0)}(\vec{q})) , \tag{4.31}$$

$$\lambda_{\varphi(2)}^{(1)} = -\frac{1}{12m^2\theta^2}\int d\vec{p}\, d\vec{p}_2\, d\vec{R}\, dx'\, \varphi(R)\, \vec{\mathcal{P}} \cdot (\vec{q}' - \vec{q})\mathcal{P}'^2 \times f_1^{(0)}(\vec{p}') f_2'(\vec{q}, \vec{q}+\vec{R}, \vec{p}, \vec{p}_2, x' \,|\, f_1^{(0)}(\vec{q})), \tag{4.32}$$

$$\lambda_{\varphi(1)}^{(2)} = \frac{1}{12m\theta}\int d\vec{p}\, d\vec{p}_2\, d\vec{p}'\, d\vec{R}\, \frac{\varphi'(R)}{R}\, \vec{R}\cdot(\vec{\mathcal{P}}+\vec{\mathcal{P}}_2)\, \vec{R}\cdot\vec{\mathcal{P}}' \times f_1^{(0)}(\vec{p}')\, \mathcal{G}(\mathcal{P}'^2) f_2'(\vec{q}, \vec{q}+\vec{R}, \vec{p}, \vec{p}_2, \vec{p}' \,|\, f_1^{(0)}(\vec{q})) , \tag{4.33}$$

$$\lambda_{\varphi(2)}^{(2)} = \frac{1}{24m^2\theta^2}\int d\vec{p}\, d\vec{p}_2\, d\vec{R}\, dx'\, \frac{\varphi'(R)}{R}\, \vec{R}\cdot(\vec{\mathcal{P}}+\vec{\mathcal{P}}_2) \times \vec{R}\cdot(\vec{q}' - \vec{q})\, \mathcal{P}'^2 f_1^{(0)}(\vec{p}') \times f_2'(\vec{q}, \vec{q}+\vec{R}, \vec{p}, \vec{p}_2, x' \,|\, f_1^{(0)}(\vec{q})) , \tag{4.34}$$

$$\lambda_{w(1)}^{(1)} = -\frac{1}{18m\theta}\int d\vec{p}\, d\vec{p}_2\, d\vec{p}_3\, d\vec{r}\, d\vec{R}\, d\vec{p}'\, w(\vec{r}, \vec{R}) \times \vec{\mathcal{P}}\cdot\vec{\mathcal{P}}' f_1^{(0)}(\vec{p}')\, \mathcal{G}(\mathcal{P}'^2) \times f_3'(\vec{q}, \vec{q}-\vec{R}, \vec{q}-\vec{R}+\vec{r}, \vec{p}, \vec{p}_2, \vec{p}_3, \vec{p}' \,|\, f_1^{(0)}(\vec{q})) , \tag{4.35}$$

$$\lambda_{w(2)}^{(1)} = -\frac{1}{36m^2\theta^2}\int d\vec{p}\, d\vec{p}_2\, d\vec{p}_3\, d\vec{r}\, d\vec{R}\, dx'\, w(\vec{r}, \vec{R}) \times \vec{\mathcal{P}}\cdot(\vec{q}' - \vec{q})\, \mathcal{P}'^2 f_1^{(0)}(\vec{p}') \times f_3'(\vec{q}, \vec{q}-\vec{R}, \vec{q}-\vec{R}+\vec{r}, \vec{p}, \vec{p}_2, \vec{p}_3, x' \,|\, f_1^{(0)}(\vec{q})) , \tag{4.36}$$

$$\lambda_{w(1)}^{(2)} = \tfrac{2}{27}\int d\vec{p}\, d\vec{p}_2\, d\vec{p}_3\, d\vec{r}\, d\vec{R}\, d\vec{p}'\, [\vec{\nabla}_r\, w(\vec{r}, \vec{R})]\cdot(\vec{\mathcal{P}}+\vec{\mathcal{P}}_2+\vec{\mathcal{P}}_3) \times \vec{r}\cdot\vec{\mathcal{P}}'\, f_1^{(0)}(\vec{p}')\, \mathcal{G}(\mathcal{P}'^2) f_3'(\vec{q}, \vec{q}-\vec{R}, \vec{q}-\vec{R}+\vec{r}, \vec{p}, \vec{p}_2, \vec{p}_3, \vec{p}' \,|\, f_1^{(0)}(\vec{q})) , \tag{4.37}$$

$$\lambda_{w(2)}^{(2)} = \tfrac{1}{27}\int d\vec{p}\, d\vec{p}_2\, d\vec{p}_3\, d\vec{r}\, d\vec{R}\, dx'\, [\vec{\nabla}_r\, w(\vec{r}, \vec{R})]\cdot(\vec{\mathcal{P}}+\vec{\mathcal{P}}_2+\vec{\mathcal{P}}_3) \times \vec{r}\cdot(\vec{q}' - \vec{q})\, \mathcal{P}'^2 f_1^{(0)}(\vec{p}') f_3'(\vec{q}, \vec{q}-\vec{R}, \vec{q}-\vec{R}+\vec{r}, \vec{p}, \vec{p}_2, \vec{p}_3, x' \,|\, f_1^{(0)}(\vec{q})) . \tag{4.38}$$

The contributions to the thermal conductivity due to nonadditive forces is given by Eqs. (4.35)–(4.38), and is also manifested in $\mathcal{G}$ and in f_2'.

It should be mentioned that the expressions obtained above for the transport coefficients are general, in the sense that they are valid independently of density expansions.

In a forthcoming publication we will discuss the density expansions of the transport coefficients, and obtain the effect of the nonadditivity on the triplecollision part of these quantities. In doing that we will solve explicitly the integral equations, Eqs. (3.32)–(3.34) for the scalar functions $\mathcal{G}$, $\mathcal{A}$, and $\mathcal{B}$.

ACKNOWLEDGMENTS

The author would like to thank F. Chaos for several relevant discussions and A. Flores for reading the manuscript.

APPENDIX A

In this Appendix we show that the local Maxwellian distribution function given by Eq. (3.10) is the solution of Eq. (3.6).

In fact, we use the relations[3]

$$f_2(x, x_2 | f_1^{(0)}(\vec{q})) = f_{1\,eq}^{(0)}(\vec{p})\, f_{1\,eq}^{(0)}(\vec{p}_2)\, G_2(|\vec{q}-\vec{q}_2|) \tag{A1}$$

and

$$f_3(x, x_2, x_3 | f_1^{(0)}(\vec{q})) = f_{1\,eq}^{(0)}(\vec{p})\, f_{1\,eq}^{(0)}(\vec{p}_2)\, f_{1\,eq}^{(0)}(\vec{p}_3) \times G_3(\vec{q}_2-\vec{q}, \vec{q}_3-\vec{q}) , \tag{A2}$$

where G_2 and G_3 are the pair- and triple-correlation functions, respectively, and $f_{1\,eq}^{(0)}$ is the true equilibrium distribution function. Substituting these expressions into the left-hand side of Eq. (3.6), and using Eq. (2.7) we find that

$$\Psi(x | f_1^{(0)}(\vec{q})) = \int d\vec{p}_2\, d\vec{R}\, \frac{\varphi'(R)}{R}\, \vec{R}\cdot [\vec{\nabla}_p f_{1\,eq}^{(0)}(\vec{p})]\, f_{1\,eq}^{(0)}(\vec{p}_2)\, G(R)$$
$$- 2\int\!\!\int d\vec{p}_2\, d\vec{p}_3\, d\vec{R}\, d\vec{r}\, [\vec{R}\, g_1(\vec{r},\vec{R}) + \vec{r}\, g_2(\vec{r},\vec{R})]\cdot [\vec{\nabla}_p f_{1\,eq}^{(0)}(\vec{p})]\, f_{1\,eq}^{(0)}(\vec{p}_2)\, f_{1\,eq}^{(0)}(\vec{p}_3)\, G_3(\vec{r},\vec{R}) . \tag{A3}$$

In this expression we also used Eqs. (2.8) and (2.11) of I.

The first term in Eq. (A3) vanishes because the integrand is odd in $\vec{R}$. In the second term of Eq. (A3) we note the following properties of G_3:

$$G_3(\vec{r},\vec{R}) = G_3(-\vec{r}, -\vec{R}) . \tag{A4}$$

The same relations hold for the g functions [see Eqs. (2.5), (A13), and (A14) of I]. Therefore, the two remaining integrands in Eq. (A3) are odd in $\vec{R}$ and in $\vec{r}$, respectively. Thus the right-hand side of Eq. (A3) vanishes.

APPENDIX B

In this Appendix we will discuss the vanishing of $\vec{D}(\vec{p})$, given by Eq. (3.20). It can be shown that $\vec{D}(\vec{p})$ can be written in the form[3]

$$\vec{D}(\vec{p}) = \frac{f_1^{(0)}}{3n\theta}\, \frac{\vec{\varphi}}{m} \int d\vec{p}_3\, dx'\, \Psi'(x_3, x' | f_1^{(0)}(\vec{q})) \times \vec{p}_3\cdot (\vec{q}'-\vec{q})\, f_1^{(0)}(\vec{p}') - \int dx'\, \Psi'(x, x' | f_1^{(0)}(\vec{q}))\, (\vec{q}'-\vec{q})\, f_1^{(0)}(\vec{p}') . \tag{B1}$$

By substituting the explicit form of the functional Ψ [see Eq. (2.7)] we find that the integral

$$\int dx'\Psi'(x, x' | f_1^{(0)}(\vec{q}))(\vec{q}'-\vec{q}) f_1^{(0)}(\vec{p}') = \vec{\nabla}_p\cdot \overleftrightarrow{G} , \tag{B2}$$

where the tensor $\overleftrightarrow{G}$ is given by

$$\overleftrightarrow{G} = \int d\vec{p}_2 d\vec{R} dx' \frac{\varphi(R)}{R}\, \vec{R}(\vec{q}'-\vec{q}) \times f_2'(x, x_2, x' | f_1^{(0)}(\vec{q}))\, f_1^{(0)}(\vec{p}')$$
$$- 2\int d\vec{p}_2 d\vec{p}_3 d\vec{r} d\vec{R} dx' [\vec{\nabla}_r w(\vec{r}, \vec{R})](\vec{q}'-\vec{q}) \times f_3'(x, x_2, x_3, x' | f_1^{(0)}(\vec{q})) f_1^{(0)}(\vec{p}') . \tag{B3}$$

By the argument given in Appendix A of Ref. 3, we have

$$\int dx' f_2'(x, x_2, x' | f_1^{(0)}(\vec{q}))(\vec{q}'-\vec{q}) f_1^{(0)}(\vec{p}') = a\vec{R} + b\vec{p} + c\vec{p}_2 , \tag{B4}$$

where a, b, and c are scalar functions of R, p, p_2, n, and θ. Using the same argument we obtain

$$\int dx' f_3'(x, x_2, x_3, x' | f_1^{(0)}(\vec{q}))(\vec{q}'-\vec{q}) f_1^{(0)}(\vec{p}') = d\vec{p} + e\vec{p}_2 + f\vec{p}_3 + j\vec{r} + k\vec{R} . \tag{B5}$$

Here d, e, f, j, and k are again scalar functions of $\vec{R}$, $\vec{r}$, p, p_2, p_3, n, and θ. Therefore,

$$\overleftrightarrow{G} = \int d\vec{p}_2 d\vec{R} \frac{\varphi'(R)}{R}\, \vec{R}(a\vec{R} + b\vec{p} + c\vec{p}_2)$$
$$- 2\int d\vec{p}_2 d\vec{p}_3 d\vec{r} d\vec{R} (\vec{R} g_1(\vec{r}, \vec{R}) + \vec{r} g_2(\vec{r}, \vec{R})) \times (d\vec{p} + e\vec{p}_2 + f\vec{p}_3 + j\vec{r} + k\vec{R}). \tag{B6}$$

Here we used Eq. (2.11) of I.

The first term of Eq. (B6) was worked out in Ref. 3, so we shall just quote the result. We proceed to analyze the second term. For this purpose we consider the following integral:

$$\overleftrightarrow{M} = \int d\vec{r} d\vec{R} \vec{R} \vec{R} \vec{R}\, F(\vec{r}, \vec{R}) , \tag{B7}$$

with F having the properties

$$F(\vec{r}, \vec{R}) = F(\vec{R}, \vec{r}) , \quad F(\vec{r}, \vec{R}) = F(-\vec{r}, -\vec{R}) . \tag{B8}$$

Integrating in (B7) first over $\vec{r}$, we find that

$$H(\vec{R}) \equiv \int d\vec{r}\, F(\vec{r}, \vec{R}) = \int d\vec{r}\, F(-\vec{r}, -\vec{R}) = \int d\vec{r}\, F(\vec{r}, -\vec{R}) = H(-\vec{R}) .$$

Thus

$$H(\vec{R}) = H(|\vec{R}|) . \qquad \text{(B9)}$$

Therefore,

$$\overleftrightarrow{M} = \int d\vec{R}\, \vec{R}\vec{R}\, H(|\vec{R}|) = \tfrac{1}{3} \int d\vec{R}\, R^2 H(|\vec{R}|) \overleftrightarrow{I} = \tfrac{1}{3} \int d\vec{r}\, d\vec{R}\, R^2 F(\vec{r}, \vec{R}) \overleftrightarrow{I} . \qquad \text{(B10)}$$

Here $\overleftrightarrow{I}$ is the unit tensor. Analogously, we find that

$$\int d\vec{r}\, d\vec{R}\, \vec{R}\vec{r}\; F(\vec{r}, \vec{R}) = \tfrac{1}{3} \int d\vec{r}\, d\vec{R}\, \vec{R} \cdot \vec{r}\, F(\vec{r}, \vec{R}) \overleftrightarrow{I} . \qquad \text{(B11)}$$

Using these results, we can express G in the following form:

$$\overleftrightarrow{G} = G(p) \overleftrightarrow{I} , \qquad \text{(B12)}$$

where the scalar function $G(p)$ is

$$G(p) = \tfrac{1}{3} \int d\vec{p}_2 d\vec{R}\, R\varphi'(R)\, a(R, p, p_2) - \tfrac{2}{3} \int d\vec{p}_2 d\vec{p}_3 d\vec{r}\, d\vec{R}\, [R^2 k g_1 + r^2 j g_2 + \vec{R} \cdot \vec{r}\, (k g_2 + j g_1)] . \qquad \text{(B13)}$$

Therefore, combining Eqs. (B1), (B2), and (B12) we find that

$$\vec{D}(\vec{p}) = + \frac{f_1^{(0)}(\vec{p})}{n\theta} \frac{\vec{\mathcal{P}}}{m} \int d\vec{p}_3\, G(p_3) + \vec{\nabla}_p G(p) . \qquad \text{(B14)}$$

We now prove a theorem which is a generalization of the theorem given in Appendix A of Ref. 3. The theorem is $\vec{D}(\vec{p}) = 0$ if

$$a(R, p, \vec{p}_2) = \mathcal{K}(R, p_2) f_1^{(0)}(\vec{p}),$$
$$k(p, p_2, p_3, \vec{r}, \vec{R}) = \mathcal{J}(\vec{r}, \vec{R}, p_2, p_3) f_1^{(0)}(\vec{p}), \qquad \text{(B15)}$$
$$j(p, p_2, p_3, \vec{r}, \vec{R}) = \mathcal{L}(\vec{r}, \vec{R}, p_2, p_3) f_1^{(0)}(\vec{p}).$$

Here $\mathcal{K}, \mathcal{J}, \mathcal{L}$ are arbitrary functions of their arguments such that

$$\int d\vec{p}_2 d\vec{R}\, R\varphi'(R)\, \mathcal{K}(R, p_2) \neq 0 ,$$
$$\int d\vec{p}_2 d\vec{p}_3 d\vec{r}\, d\vec{R}\, [R^2 \mathcal{J} g_1 + r^2 \mathcal{L} g_2 + \vec{R} \cdot \vec{r}\, (\mathcal{J} g_2 + \mathcal{L} g_1)] \neq 0 . \qquad \text{(B16)}$$

In fact, if Eqs. (B15) hold, we have from Eq. (B14) that

$$\vec{D}(\vec{p}) = \left(\frac{1}{3} \int d\vec{p}_2 d\vec{R}\, R\varphi'(R) \mathcal{K}(R, p_2) - \frac{2}{3} \int d\vec{p}_2 d\vec{p}_3 d\vec{r}\, d\vec{R}\, [R^2 \mathcal{J} g_1 + r^2 \mathcal{L} g_2 + \vec{R} \cdot \vec{r}\, (\mathcal{J} g_2 + \mathcal{L} g_1)] \right) \left[\frac{f_1^{(0)}(\vec{p})}{n\theta} \frac{\vec{\mathcal{P}}}{m} \int d\vec{p}_3 f_1^{(0)}(\vec{p}_3) + \vec{\nabla}_p f_1^{(0)}(\vec{p}) \right] = 0, \qquad \text{(B17)}$$

because the last bracket vanishes.

The reciprocal of this theorem is given in Ref. 3.

Thus, it is enough to show that a, k, and j are proportional to $f_1^{(0)}(\vec{p})$ in order to prove that $\vec{D}(\vec{p}) = 0$. Without a density expansion of f_2 and f_3 we have not been able to show these properties of a, k, and j. In a forthcoming paper, in which we discuss the density expansions of f_2 and f_3, we will show that a, k, and j are proportional to $f_1^{(0)}(\vec{p})$, and thus, that $\vec{D}(\vec{p}) = 0$.

APPENDIX C

In this Appendix we will sketch how one can obtain the contribution to the transport coefficients labeled in the text with the index w [see Eqs. (4.19), (4.20), (4.26), (4.27), and (4.35)–(4.38)]. The other contributions to the transport coefficients are formally obtained in Ref. 3.

Let us consider first, the tensor $\overleftrightarrow{P}_w$. Substituting Eqs. (4.11), (3.31), and (4.12) into Eq. (4.4) we find that

$$\overleftrightarrow{P}_w = -\frac{1}{3} \int d\vec{p}\, d\vec{p}_2 d\vec{p}_3 d\vec{r}\, d\vec{R}\, \vec{R}\vec{R}\, (\vec{\nabla}_R w) \Big\{ f_3(\cdots | f_1^{(0)}(\vec{q})) + \int d\vec{p}\,' f_3'(\ldots, \vec{p}\,' | f_1^{(0)}(\vec{q})) f_1^{(0)}(\vec{p}\,') \times [\mathcal{G}(\mathcal{P}'^2) \vec{\mathcal{P}}' \cdot \vec{\nabla}_q \ln\theta + \mathcal{A}(\mathcal{P}'^2) \mathcal{P}'^0 \mathcal{P}' : \vec{\nabla}_q \vec{u} + \mathcal{B}(\mathcal{P}'^2) \vec{\nabla}_q \cdot \vec{u}] + \int dx' f_3' (\ldots, x' | f_1^{(0)}(\vec{q}))\, (\vec{q}\,' - \vec{q}) f_1^{(0)}(\vec{p}\,') \cdot \left[\vec{\nabla}_q \ln n + \left(\frac{\mathcal{P}'^2}{2m\theta} - \frac{3}{2} \right) \vec{\nabla}_q \ln\theta + \frac{\vec{\mathcal{P}}'}{\theta} \cdot \vec{\nabla}_q \vec{u} \right] \Big\} . \qquad \text{(C1)}$$

The first term in (C1) gives

$$-\tfrac{1}{9} \int d\vec{p}\, d\vec{p}_2 d\vec{p}_3 d\vec{r}\, d\vec{R}\, \vec{R} \cdot (\vec{\nabla}_R w) f_3(\cdots | f_1^{(0)}(\vec{q})) \overleftrightarrow{I} , \qquad \text{(C2)}$$

and this is the contribution to the local equilibrium pressure [see Eqs. (3.14) and (4.13)].

The second term in Eq. (C1) vanishes. In fact, if we use Eq. (2.10) of I, one obtains that this term is of the form

$$[\int d\vec{p}\,d\vec{p}_2\,d\vec{p}_3\,d\vec{r}\,d\vec{R}\,(\cdots)\;\vec{R}\vec{R}\vec{\mathcal{P}}'\,]\cdot\vec{\nabla}_q \ln\theta. \qquad \text{(C3)}$$

However, the bracket must be an isotropic tensor of order three. Thus, it must be of the form of a scalar times the tensor ϵ_{ijk},[4] which is an antisymmetric tensor in the first two indices. But the integral is symmetric in the first two indices. Therefore the only solution is a scalar equal to zero. Using the same argument, the coefficients of $\vec{\nabla}_q \ln n$ and $\vec{\nabla}_q \ln\theta$ in Eq. (C1) also vanish.

The third term in Eq. (C1) is

$$[-\tfrac{2}{3}\int d\vec{p}\,d\vec{p}_2\,d\vec{p}_3\,d\vec{r}\,d\vec{R}\,d\vec{p}'f_3'\,(\cdots,\vec{p}'\,|\,f_1^{(0)}(\vec{q}))f_1^{(0)}(\vec{p}')\,\mathcal{A}(\mathcal{P}'^2)g_1(\vec{r},\vec{R})\vec{R}\vec{R}\,\mathcal{P}'^0\mathcal{P}'\,]:\vec{\nabla}_q\vec{u}\,. \qquad \text{(C4)}$$

If we make

$$\vec{R}\vec{R}=R^0R+\tfrac{1}{3}R^2\overleftrightarrow{I}, \qquad \text{(C5)}$$

where R^0R is a traceless symmetric tensor, we find that the part containing $\frac{1}{3}R^2\overleftrightarrow{I}$ must vanish. Therefore, we are left with

$$[-\tfrac{2}{3}\int d\vec{p}\,d\vec{p}_2\,d\vec{p}_3\,d\vec{r}\,d\vec{R}\,d\vec{p}'(\cdots)R^0R\;\mathcal{P}'^0\mathcal{P}'\,]:\vec{\nabla}_q\vec{u}$$

and the tensor in the bracket must be a symmetric isotropic tensor of order four, i.e., of the form[4]

$$\lambda\delta_{ij}\delta_{kl}+\mu\,(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})\,. \qquad \text{(C6)}$$

In the usual way, we find (see Ref. 3) that this term is

$$-2\eta_{w(1)}^{(1)}\,[\overleftrightarrow{D}-\tfrac{1}{3}\vec{\nabla}_q\cdot\vec{u}\overleftrightarrow{I}\,], \qquad \text{(C7)}$$

with $\eta_{w(1)}^{(1)}$ given by Eq. (4.19).

With the same argument it is found that the fourth term in Eq. (C1) is

$$-2\eta_{w(1)}^{(2)}\,\vec{\nabla}_q\cdot\vec{u}\overleftrightarrow{I}, \qquad \text{(C8)}$$

with $\eta_{w(1)}^{(2)}$ given by Eq. (4.26).

In the last term of Eq. (C1) we use Eq. (C5) and

$$(\vec{q}'-\vec{q})\vec{\mathcal{P}}'=S^0S+\tfrac{1}{3}\,\vec{\mathcal{P}}'\cdot(\vec{q}'-\vec{q})\overleftrightarrow{I}+\overleftrightarrow{A}, \qquad \text{(C9)}$$

where S^0S is a symmetric traceless tensor [see Eq. (3.29)] and $\overleftrightarrow{A}$ an antisymmetric tensor. Thus, the last term in Eq. (C1) is

$$[-(2/3\theta)\int d\vec{p}\,d\vec{p}_2\,d\vec{p}_3\,d\vec{r}\,d\vec{R}\,dx'f_3'\,(\ldots,x'\,|\,f_1^{(0)}(\vec{q}))g_1(\vec{r},\vec{R})f_1^{(0)}(\vec{p}')\,(R^0R+\tfrac{1}{3}R^2\overleftrightarrow{I})(S^0S+\tfrac{1}{3}\vec{\mathcal{P}}'\cdot(\vec{q}'-\vec{q})\overleftrightarrow{I}+\overleftrightarrow{A})]:\vec{\nabla}_q\vec{u}\,. \qquad \text{(C10)}$$

The contribution of $\overleftrightarrow{A}$ vanishes.[3] The contribution of the term containing $R^0R\overleftrightarrow{I}$ is of the form

$$[\int d\vec{p}\,d\vec{p}_2\,d\vec{p}_3\,d\vec{r}\,d\vec{R}\,dx'(\cdots)R^0R\,]\vec{\nabla}_q\cdot\vec{u}.$$

Thus the bracket must be an isotropic tensor of order two. But the only isotropic tensor of order two is proportional to $\overleftrightarrow{I}$. Therefore the bracket must vanish. By the same argument the contribution of $S^0S\overleftrightarrow{I}$ vanishes. The contribution of the term containing $\overleftrightarrow{I}\overleftrightarrow{I}$ is

$$-2\,\eta_{w(2)}^{(2)}\,\vec{\nabla}_q\cdot\vec{u}\overleftrightarrow{I},$$

with $\eta_{w(2)}^{(2)}$ given by Eq. (4.27). Finally, the contribution of the term containing $R^0R\;S^0S$ must be an isotropic tensor of order four, which gives

$$-2\,\eta_{w(2)}^{(1)}\,[\overleftrightarrow{D}-\tfrac{1}{3}\vec{\nabla}_q\cdot\vec{u}\overleftrightarrow{I}\,],$$

with $\eta_{w(2)}^{(1)}$ given by Eq. (4.20).

The explicit expressions for the heat current are obtained using the same arguments as those given above.

---

*Also at Facultad de Ciencias, Universidad Nacional Autónoma de México, México.

[1]E. Braun, preceding paper, Phys. Rev. A 5, 1941 (1972), hereafter referred to as I. In the present paper we use the same notation as in I.

[2]N. N. Bogolyubov, *Problems of a Dynamical Theory in Statistical Physics*, Vol. I of *Studies in Statistical Mechanics*, edited by J. de Boer and G. E. Uhlenbeck translated by E. K. Gora (North Holland, Amsterdam, 1962).

[3]L. S. García-Colín, M. S. Green, and F. Chaos, Physica 32, 450 (1966).

[4]H. Jeffreys, *Cartesian Tensors* (Cambridge U.P., England, 1963).