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Relativistic Streaming Effects on Electromagnetic Instability in Magnetoplasmas

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The electromagnetic waves propagating transverse to the direction of streaming of the contrastreaming plasmas in the presence of a uniform magnetic field are investigated. For electron plasmas these waves become unstable only for some bounded values of the streaming velocity U_0, namely, $U_{\min} < U_0 < U_{\max}$. For relativistic streaming velocities, the electron thermal effects are destabilizing whereas for nonrelativistic streaming these are stabilizing unless $c^2k^2/\omega_{pe}^2 < \frac{1}{3}$ and $k^2V_{te}^2/\Omega_e^2 \ll 1$ (k being the characteristic wave number, V_{te} the electron thermal velocity, Ω_e the electron cyclotron frequency, and ω_{pe} the electron plasma frequency). The magnetic field however reduces the region of instability no matter what the streaming velocity is. The effect of nonrelativistic-ion streaming is destabilizing while that of the ion temperature is stabilizing. In the presence of a strong magnetic field, the effect of relativistic-ion streaming is also stabilizing.

I. INTRODUCTION

The plasmas streaming with a relative velocity exhibit the well-known two-stream (TS) instability whenever this relative velocity exceeds a certain critical velocity. Buneman[1] showed that besides this TS instability, there exists a transversel instability in plasmas with wave propagation transverse to the direction of the relative streaming. Momota[2] presented the criterion for the existence of these transversel instabilities in cold as well as warm plasmas and showed that the thermal effects were stabilizing. Lee[3] studied this instability for counter-streaming plasmas in the presence of the magnetic field and found that both the magnetic field and the thermal effects were stabilizing. Buti and Lakhina[4] modified Lee's[3] criterion for instability and concluded that the magnetic field was always stabilizing while the temperature can be destabilizing in some regions. Recently, Tzoar and Yang[5] have discussed this problem for the case of large k, i.e., $\lambda = k^2V_{te}^2/\Omega_e^2 \gg 1$. They find that the spectrum of the unstable k has a lower as well as upper limit and that the magnetic field reduces this unstable region. Lee and Bornatici[6] have reported that for anisotropic-temperature plasmas the perpendicular temperature is stabilizing whereas the parallel temperature is destabilizing. The transverse instabilities have also been studied for collisional plasmas[7] and for plasmas in which both the electrons and the ions are streaming.[8,9]

The above-mentioned investigations were carried out for nonrelativistic streaming velocities; but recently the production of intense relativistic electron streams has been reported[10] and it has stimulated the investigations of the instabilities for relativistic streaming.[11,12] Here we have studied the transversel instabilities in plasmas which are contrastreaming with any arbitrary velocity (relativistic or nonrelativistic) in the presence of external uniform magnetic field. We find that for very low streaming velocities as well as for extreme relativistic streaming velocities, the waves propagating transverse to the direction of streaming, are stable i.e., the bounded region $U_{\min} < U_0 < U_{\max}$ is an unstable one. The increase in the magnetic field increases $U_{\min}$, decreases $U_{\max}$, and consequently reduces the region of instability. In the relativistic regime of streaming velocities, the electron thermal effects are found to be destabilizing whereas for low streaming[4] these are stabilizing unless $c^2k^2/\omega_{pe}^2 < \frac{1}{3}$ and $k^2V_{te}^2/\Omega_e^2 \ll 1$.

II. DISPERSION RELATION

Let us consider two identical homogeneous plasmas contrastreaming along the direction of an external uniform magnetic field $\vec{B}_0$ which we take along the z axis. Each plasma consists of electrons and ions which are streaming with velocities U_{0e} and U_{0i}, respectively. For small perturbations the motion of the particles, which are streaming with relativistic velocities, is governed by the linearized relativistic Vlasov equation,[13] namely,

$$\frac{\partial f_j^\alpha}{\partial t} + \frac{\vec{p}}{m_j\gamma_j}\cdot\frac{\partial f_j^\alpha}{\partial \vec{x}} + e_j\left(\vec{E}_1 + \frac{\vec{p}\times\vec{B}_1}{m_j c\gamma_j}\right)\cdot\frac{\partial f_{0j}^\alpha}{\partial \vec{p}} + \frac{e_j}{m_j c\gamma_j}(\vec{p}\times\vec{B}_0)\cdot\frac{\partial f_j^\alpha}{\partial \vec{p}} = 0 , \quad (1)$$

where $\gamma_j = (1 + p^2/m_j^2c^2)^{1/2}$. f_j^α, $\vec{E}_1$, and $\vec{B}_1$ are the perturbed distribution functions, electric field, and the magnetic field, respectively. α labels the two streams and j refers to the species, i.e., $e_j = +e$ for ions and $-e$ for electrons. f_{0j}^α is the equilibrium distribution function which for the case of nonrelativistic plasmas with relativistic streaming is given by[13]

$$f_{0j}^\alpha = NA\exp[-(\vec{V} - \vec{U}_{0j}^\alpha)^2/2V_{tj}^{\alpha 2}] , \quad (2)$$

where

$$A = \frac{a_j^\alpha \exp[-a_j^\alpha(1 - U_{0j}^{\alpha 2}/2c^2)]}{4\pi c^3\gamma_{0j}^{\alpha 2}K_2(a_j^\alpha/\gamma_{0j}^\alpha)} , \quad (3)$$

with $a_j^\alpha = m_jc^2/kT_j^\alpha$, $\gamma_{0j}^\alpha = (1 - U_{0j}^{\alpha 2}/c^2)^{-1/2}$, and K_2 is the Bessel function of second order for a purely imaginary arguments.

For streaming along the direction of the magnetic field, Eq. (1) along with the Maxwell's equations, gives the dispersion relation[13]

$$|\overleftrightarrow{R}| = 0 , \quad (4)$$

where

$$\overleftrightarrow{R} = (c^2k^2 - \omega^2)\overleftrightarrow{I} - c^2\vec{k}\vec{k} \mp \sum_{\alpha=1}^{2}\sum_j \frac{i\omega}{N}\omega_{pj}^2 \times\int d\vec{p}\,\vec{p}\int_{\pm\infty}^{\phi} d\phi'\,\frac{G}{\Omega_j}\left[\frac{\partial f_{0j}^\alpha}{\partial \vec{p}'} + \vec{k}\left(\vec{p}'\times\frac{\partial f_{0j}^\alpha}{\partial \vec{p}'}\right)\Big/ m_j\omega\gamma_j\right] , \quad (5)$$

with

$$\ln G = \pm(i/m_j\Omega_j[k_\parallel p\cos\theta - m_j\omega\gamma_i)(\phi - \phi') + k_\perp p\sin\theta(\sin\phi - \sin\phi')] , \quad (6)$$

where $k_\parallel$ and $k_\perp$ are the components of the wave vector $\vec{k}$ parallel and perpendicular to $\vec{B}_0$. $\omega_{pj} = (4\pi Ne^2/m_j)^{1/2}$ and $\Omega_j = |e|B_0/m_jc$ are the plasma and the cyclotron frequencies, respectively. The upper and the lower signs in Eqs. (5) and (6) correspond to the ions and the electrons, respectively.

Following Buti and Lakhina,[4] we can show that for nonrelativistic ($\gamma = 1$) identical contrastreaming plasmas and for wave propagation transverse to the direction of the magnetic field, i.e., $\vec{k} = k\hat{e}_x$, the dispersion relation takes the form

$$c^2k^2 - \omega^2 - \omega\sum_j \frac{\omega_{pj}^2}{\Omega_j}\beta_j\sum_{n=-\infty}^{\infty}\frac{I_n(\lambda_j)e^{-\lambda_j}}{(n - \omega/\Omega_j)} - \sum_j \beta_j\frac{\omega_{pj}^2}{\Omega_j^2}U_{0j}^2k^2\sum_{n=-\infty}^{\infty}\frac{I_n(\lambda_j)e^{-\lambda_j}}{[1 - (n - \omega/\Omega_j)^2]} = 0 , \quad (7)$$

where $\beta_j = A(2\pi V_{tj}^2)^{3/2}$, $\lambda_j = k^2V_{tj}^2/\Omega_j^2$, and I_n is the Bessel function of first kind and of imaginary argument. By using some Bessel-function identities,[14] the dispersion relation (7) can be transformed to a more convenient form as

$$c^2k^2 - \omega^2 + \sum_j \beta_j\omega_{pj}^2 + 2\sum_j \omega_{pj}^2\beta_j\left(1 + \frac{U_{0j}^2}{V_{tj}^2}\right)\cdot\sum_{n=1}^{\infty}\frac{n^2\Omega_j^2 I_n(\lambda_j)e^{-\lambda_j}}{(\omega^2 - n^2\Omega_j^2)} = 0 . \quad (8)$$

Equation (8) can be studied numerically for the growth rate of the instability. Before doing so we shall, however, discuss some special cases where it is possible to obtain simple analytical expressions for the growth rate.

A. High-Frequency Instability ($\omega \sim \Omega_e$)

In this case the contribution due to ions can be neglected, and the dispersion relation given by Eq. (8), for the case of strong magnetic field, i.e., $\lambda_e \ll 1$, simplifies to

$$\omega^4 - p\omega^2 + q\Omega_e^2 = 0 , \quad (9)$$

where

$$p = (c^2k^2 + \omega_{pe}^2\beta_e + \Omega_e^2) \quad (10)$$

and

$$q = c^2k^2 + \beta_e\omega_{pe}^2 - \beta_e\omega_{pe}^2(1 + U_{0e}^2/V_{te}^2)\lambda_e . \quad (11)$$

According to Eq. (9), ω^2 is given by

$$\omega^2 = \tfrac{1}{2}p \pm \tfrac{1}{2}(p^2 - 4q\Omega_e^2) . \quad (12)$$

From Eq. (12) it is obvious that ω^2 will be negative, which corresponds to a purely growing wave, if $q < 0$, i.e., if

$$k^2 > k_c^2 \,, \tag{13}$$

where

$$k_c^2 = \Omega_e^2 \left[(V_{te}^2 + U_{0e}^2) - c^2 \Omega_e^2/(\beta_e \omega_{pe}^2)\right]^{-1} \,. \tag{14}$$

In order to be consistent with the assumption of strong magnetic field, i.e., $\lambda_e \ll 1$, for instability, we must satisfy the inequality

$$1 \ll \Omega_e^2/k^2 V_{te}^2 \ll \beta_e (U_{0e}^2/c^2)(\omega_{pe}^2/k^2 V_{te}^2) \,. \tag{15}$$

The numerical evaluation of (15) shows that this inequality can not be satisfied for relativistic streaming, and thus the system would be stable for these relativistic streaming velocities. However, this can be satisfied for nonrelativistic U_{0e}, and if we make use of the inequality given by (13), the instability criterion can be rewritten as

$$1 \ll \Omega_e^2/k^2 v_{te}^2 < (1 + U_{0e}^2/V_{te}^2) \,, \tag{16}$$

which can be satisfied for $U_{0e}^2 \gg V_{te}^2$.

The growth rates for wave numbers corresponding to Eq. (13) or Eq. (16) is given by

$$\mathrm{Im}\omega = \left[\tfrac{1}{2}(p^2 + 4\chi\Omega_e^2)^{1/2} - \tfrac{1}{2}p\right]^{1/2} \,, \tag{17}$$

where

$$\chi = -q > 0 \,. \tag{18}$$

Note that Eq. (17) has a meaning only for $U_{0e}^2/c^2 \ll 1$, in which case β_e tends to unity and χ increases with the increase of U_{0e} so that for nonrelativistic U_{0e} the growth rate is enhanced by the increase of U_{0e}. Since χ decreases with the increase of the magnetic field, the magnetic field will suppress the growth rate.

B. Low-Frequency Instability ($\omega \gtrsim \Omega_i$)

For the case of low frequencies and for $\lambda_i = (kR_i)^2 \ll 1$, where $R_i = V_{ti}/\Omega_i$ is the ion Larmor radius, the dispersion relation of Eq. (8) reduces to

$$\omega^4 - p\omega^2 + Q\Omega_i^2 = 0 \,, \tag{19}$$

where

$$P = (q + \beta_i \omega_{pi}^2 + \Omega_i^2) \,, \tag{20}$$

$$Q = \left[q + \beta_i \omega_{pi}^2 - \beta_i \omega_{pi}^2 (1 + U_{0i}^2/V_{ti}^2)\lambda_i\right] \,, \tag{21}$$

and q is given by Eq. (11). Equation (19) for ω^2 gives

$$\omega^2 = \tfrac{1}{2}P \pm \tfrac{1}{2}(P^2 - 4Q\Omega_i^2)^{1/2} \,. \tag{22}$$

One root of Eq. (22) is always negative, independent of the sign of P, provided $Q<0$. However, for $P<0$, i.e., $\chi > (\Omega_i^2 + \beta_i \omega_{pi}^2)$, where $\chi = -q$, we shall have a low-frequency instability along with the high-frequency instability. In fact $\chi > 0$ ensures that the high-frequency waves are unstable (cf. Sec. II A). The typical growth rates associated with this low-frequency instability are of the order of $(\Omega_i^2 + \beta_i \omega_{pi}^2)^{1/2}$.

For $\chi < 0$, the high-frequency waves are stable but the low-frequency instability can persist provided $Q<0$, i.e.,

$$q + \beta_i \omega_{pi}^2 - \beta_i \omega_{pi}^2 (1 + U_{0i}^2/V_{ti}^2)\lambda_i < 0 \,, \tag{23}$$

with $q > 0$. In fact this is the situation where the low-frequency instability is most important.

Equation (23) along with the assumption of $\lambda_i \ll 1$ requires that for instability

$$\beta_i U_{0i}^2 \gg \frac{m}{M}\beta_e \left\{ V_{ti}^2 \frac{M^2}{m^2} \left[(V_{te}^2 + U_{0e}^2) - c^2 \Omega_e^2(\beta_e \omega_{pe}^2)\right] \right\} \,. \tag{24}$$

Now from $q>0$ we observe that for k to be real, the term in the square bracket in Eq. (24) must be positive. For the case of low ion thermal velocities Eq. (24) can be easily satisfied even when $[(V_{te}^2 + U_{0e}^2) - c^2\Omega_e^2/(\beta_e \omega_{pe}^2)]$ is very small. It is interesting to note that for relativistic-ion streaming, the condition (24) is violated due to the fact that for $U_{0i}/c \to 1$, β_i tends to zero whereas β_e is restricted to some minimum value to ensure the reality of k. Hence for relativistic-ion streaming the instability can not occur.

When the requirements for the instability, i.e., Eqs. (23) and (24) are satisfied the growth rate corresponding to Eq. (22) is given by

$$\mathrm{Im}\omega = \left[\tfrac{1}{2}(P^2 + 4Q_0\Omega_i^2)^{1/2} - \tfrac{1}{2}P\right]^{1/2} \,, \tag{25}$$

where $Q_0 = -Q$. For nonrelativistic-ion streaming β_i tends to unity and Q_0 increases with the increase in U_{0i} and decreases with the increase in T_i. Hence for nonrelativistic-ion streaming, U_{0i} has a destabilizing effect and T_i has a stabilizing effect on the growth rate. For $U_{0i}/c \to 1$, β_i decreases rapidly with U_{0i}, and so Q_0 is decreased; hence for extreme relativistic-ion streaming the system will be stable against low-frequency instability. The effect of magnetic field on this instability is stabilizing as seen from Eqs. (21) and (25).

C. Cold Ions ($T_i = 0$)

The analysis of Secs. II A and II B was restricted to the strong magnetic fields and small wave numbers, i.e., $\lambda_j \ll 1$. These restrictions are not necessary if one considers the case of cold ions streaming with nonrelativistic velocities. The dispersion relation for this case reduces to (for low frequencies)

$$\omega^4 - P'\omega^2 + Q' = 0 \,, \tag{26}$$

where

$$P' = (C_0 + \Omega_i^2 + \omega_{pi}^2) \,, \tag{27}$$

$$Q' = \left[(C_0 + \omega_{pi}^2)\Omega_i^2 - k^2 U_{0i}^2 \omega_{pi}^2\right] \,, \tag{28}$$

and

$$C_0 = c^2k^2 + \beta_e\,\omega_{pe}^2 - \beta_e\,\omega_{pe}^2(1 + U_{0e}^2/V_{te}^2)\,[1 - I_0(\lambda_e)\,e^{-\lambda_e}]\,. \tag{29}$$

In Sec. III we shall show that $C_0 < 0$ is the necessary criterion for the high-frequency instability. In the absence of high-frequency instability, the low-frequency instability can exist if $Q' < 0$, i.e., if $U_{0i}^2 > U_*^2$, where

$$U_*^2 = (C_0 + \omega_{pi}^2)\,\Omega_i^2/k^2\omega_{pi}^2\,, \tag{30}$$

with $C_0 > 0$.

We may point out that the instability condition of Eq. (30) can be very easily satisfied over a large range of the magnetic field values even when U_{0e} vanishes. For $U_{0e} = 0$, instability criterion (30) goes over to

$$\begin{aligned} U_{0i}^2 &> (m/M)\,[c^2k^2/\omega_{pe}^2 + I_0(\lambda_e)\,e^{-\lambda_e} + m/M\,](\Omega_e^2/k^2) \\ &= (m/M)\,\Omega_e^2/k^2 \end{aligned} \tag{31}$$

because $I_0(\lambda_e)\,e^{-\lambda_e} \le 1$ and for transverse instabilities $c^2k^2/\omega_{pe}^2 \lesssim 1$. When the instability requirements of Eq. (31) is satisfied, the corresponding growth rates can be calculated by solving Eq. (27) for ω^2 and are given by

$$\mathrm{Im}\,\omega = [\tfrac{1}{2}(P'^2 + 4\chi')^{1/2} - \tfrac{1}{2}P'\,]^{1/2}\,, \tag{32}$$

where

$$\chi' = -Q'\,.$$

From Eq. (32) we observe that for low electron streamings, C_0 decreases with the increase of U_{0e} and also by the decrease of T_e but nevertheless remains positive. Hence, for small values of U_{0e}, U_{0e} and T_e will enhance and suppress the instability, respectively. For extreme relativistic streamings $C_0 \approx c^2k^2$ and so in this regime U_{0e} and T_e have no effect on the low-frequency instability. For intermediate U_{0e}, high-frequency instability is excited as C_0 no longer remains positive. In Sec. III we find that the qualitative behavior of this instability remains unchanged for $T_i \ne 0$ (cf. Table I).

III. ANALYSIS OF THE GENERAL DISPERSION RELATION

In this section we shall first find the general criterion for the onset of instability and then present some numerical results for the growth rate of instability. We rewrite Eq. (8) as

$$L(\omega^2) = R(\omega^2)\,, \tag{33}$$

where

$$L(\omega^2) = \omega^2 - c^2k^2 - \textstyle\sum_j \beta_j\,\omega_{pj}^2 \tag{34}$$

and

$$R(\omega^2) = 2\sum_j \beta_j\,\omega_{pj}^2\left(1 + \frac{U_{0j}^2}{V_{tj}^2}\right)\sum_{n=1}^{\infty}\frac{n^2\,\Omega_j^2 I_n(\lambda_j)e^{-\lambda_j}}{(\omega^2 - n^2\Omega_j^2)}\,. \tag{35}$$

Following Hamasaki,[15] we observe that the purely growing waves will be excited if $L(0) > R(0)$, i.e., if

$$c^2k^2 + \sum_j \beta_j\,\omega_{pj}^2 - \sum_j \beta_j\,\omega_{pj}^2\left(1 + \frac{U_{0j}^2}{V_{tj}^2}\right)\sigma_j < 0\,, \tag{36}$$

where

$$\sigma_j = [1 - I_0(\lambda_j)\,e^{-\lambda_j}\,]\,. \tag{37}$$

For nonrelativistic streamings, i.e., $U_{0j}^0/c^2 \ll 1$, Eq. (36) which expresses the general criterion for instability goes over to that of Buti and Lakhina.[4,9] The instability conditions discussed in Secs. II A, II B, and II C are just the special cases of Eq. (36). Moreover for high frequencies where ion dynamics can be neglected, Eq. (36) reduces to $C_0 < 0$ as mentioned in Sec. II C.

Incidently from instability criterion given by Eq. (36) we can find out the critical value of the plasma density or plasma frequency above which the instability can occur. For this we write Eq. (36) as $\omega_{pc} < \omega_{pe}$, where

$$\begin{aligned} \omega_{pc}^2 = c^2k^2\,[\beta_e(1 + U_{0e}^2/V_{te}^2)\,\sigma_e + \beta_i\,(m/M)(1 + U_{0i}^2/V_{ti}^2)\,\sigma_i \\ - \beta_e - (m/M)\,\beta_i\,]^{-1}\,, \end{aligned} \tag{38}$$

ω_{pc} being the critical plasma frequency. We may

TABLE I. Variation of $(\mathrm{Im}\,\omega/\omega_{pe})^2$ for $a_e = 120$, $\lambda_e = 0.1$, $c^2k^2/\omega_{pe}^2 = 0.1$, and $T_i/T_e = 0.001$ and 0.01.

U_{0i}^2/V_{te}^2 → U_{oe}/c ↓	$(\mathrm{Im}\,\omega/\omega_{pe})^2$ 0.01 T_i/T_e		0.1 T_i/T_e		0.5 T_i/T_e	
	0.001	0.01	0.001	0.01	0.001	0.01
0.01	0.17×10^{-8}	0.0	0.42×10^{-7}	0.33×10^{-7}	0.23×10^{-6}	0.21×10^{-6}
0.1	0.22×10^{-8}	0.0	0.48×10^{-7}	0.39×10^{-7}	0.26×10^{-6}	0.25×10^{-6}
0.2	0.56×10^{-8}	0.11×10^{-8}	0.83×10^{-7}	0.73×10^{-7}	0.43×10^{-6}	0.42×10^{-6}
0.3	0.17×10^{-6}	0.16×10^{-6}	0.17×10^{-5}	0.17×10^{-5}	0.83×10^{-5}	0.83×10^{-5}
0.4	0.48×10^{-2}	0.48×10^{-2}	0.48×10^{-2}	0.48×10^{-2}	0.48×10^{-2}	0.48×10^{-2}
0.5	0.83×10^{-2}	0.83×10^{-2}	0.83×10^{-2}	0.83×10^{-2}	0.83×10^{-2}	0.83×10^{-2}
0.6	0.30×10^{-2}	0.30×10^{-2}	0.30×10^{-2}	0.30×10^{-2}	0.30×10^{-2}	0.30×10^{-2}
0.7	0.45×10^{-7}	0.35×10^{-7}	0.53×10^{-6}	0.51×10^{-6}	0.26×10^{-5}	0.26×10^{-5}
0.8	0.42×10^{-7}	0.32×10^{-7}	0.45×10^{-6}	0.44×10^{-6}	0.23×10^{-5}	0.22×10^{-5}
0.9	0.42×10^{-7}	0.32×10^{-7}	0.45×10^{-6}	0.44×10^{-6}	0.23×10^{-5}	0.22×10^{-5}
0.95	0.42×10^{-7}	0.32×10^{-7}	0.45×10^{-6}	0.44×10^{-6}	0.23×10^{-5}	0.22×10^{-5}

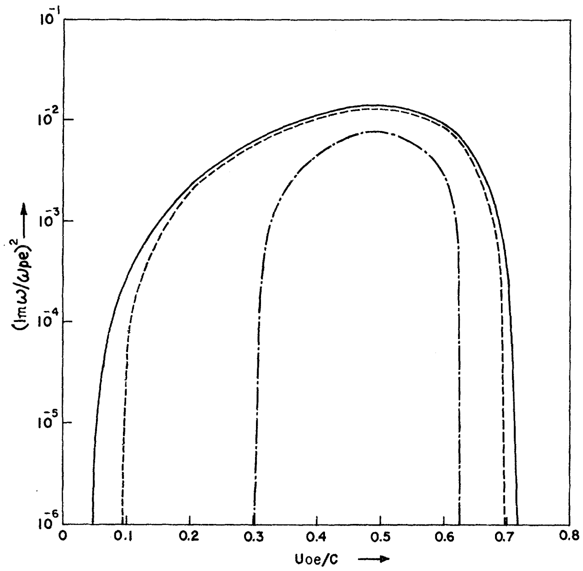

FIG. 1. Variation of $(\text{Im}\,\omega/\omega_{pe})^2$ vs U_{0e}/c for $c^2k^2/\omega_{pe}^2 = 0.1$, $a_e = 120$, and $m/M = 0$ for $\lambda_e = 0.1$ (dot-dashed line), 1.0 (dotted line), and 10.0 (solid line), respectively.

point out that the critical plasma frequency for the high-frequency instability is higher than that for the low-frequency instability. Just to have an idea of the critical plasma frequencies, we find that for $a_e = 120$, $\lambda_e = 0.1$, and $U_{0e}/c = 0.3$, the high-frequency instability is excited when $\omega_{pc}^2 = 162\,\Omega_e^2$, whereas for the same values of a_e, λ_e, U_{0e}/c, and $T_i/T_e = 0.001$, the low-frequency instability is excited when (a) $\omega_{pc}^2 = 7.14\Omega_e^2$ for $U_{0i}^2/V_{te}^2 = 0.01$, and (b) $\omega_{pc}^2 = 0.15\Omega_e^2$ for $U_{0i}^2/V_{te}^2 = 0.5$. The critical plasma frequencies appearing in the above example are easily attainable in the laboratory plasmas. In general, the inequality $\omega_{pc} < \omega_{pe}$ can be satisfied for quite a few astrophysical plasmas as well as for the laboratory plasmas.

Equation (33) was solved on IBM 1620, and the growth rates for $m/M = 0$ are shown in Figs. 1–3. For nonvanishing m/M and for nonrelativistic U_{0i}, the growth rates are given in Table I. The case of relativistic-ion streaming could not be handled on the computer for the parameters of interest. This case has been dealt with in Sec. II B.

From Fig. 1–3 we note that for electron plasmas the instability can be excited only for some bounded values of the streaming velocities. The increase of the magnetic field suppresses the growth of the instability and also reduces the range of streaming velocities in which the instability is excited. This is in complete agreement with our conclusions of Sec. II A.

The high growth rates $\gtrsim 0.5\,w_p$ obtained by Ignat and Hirshfield[11] (Fig. 5) for relativistic streaming velocities are the ones for longitudinal electrostatic instability ($p \ll 1$). Our results could be compared with their results for $p \gg 1$; in which case our growth rates corresponding to the parameters considered by them are comparable to the ones obtained by them.

It is interesting to note that the streaming velocity has an upper bound for instability. This can be explained as follows: The electromagnetic instability is excited due to the bunching of the current (as pointed out by Momota[2]). The bunching is caused by the Lorentz force $\vec{J}\times\vec{B}$ with $\vec{J} = ne\vec{U}$ as the current density. This bunching of the current causes the perturbed magnetic field to grow and thereby makes the system unstable. For nonrelativistic streaming velocities, the local density of the charged particles remains unchanged by increasing $\vec{U}$; so $\vec{J}$ increases with the increase in U and this enhances the bunching. However, in the relativistic regime $U/c \sim 1$, the local density decreases sharply as U increases, thereby decreasing J and consequently the bunching is reduced.

From Fig. 2 we observe that the instability can

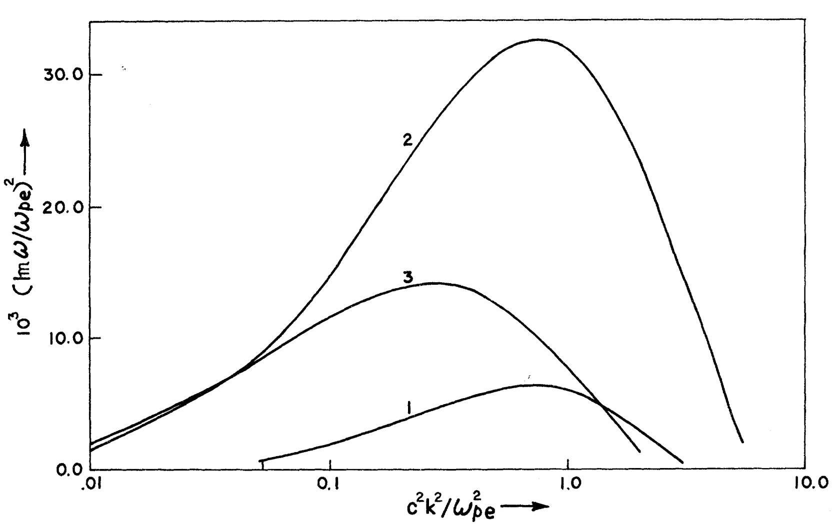

FIG. 2. Variation of $(\text{Im}\,\omega/\omega_{pe})^2$ vs c^2k^2/ω_{pe}^2 for $a_e = 100$, $\Omega_e^2/\omega_{pe}^2 = 0.001$, and $m/M = 0$. Curves 1, 2, and 3 are for $U_{0e}/c = 0.2$, 0.5, and 0.6, respectively.

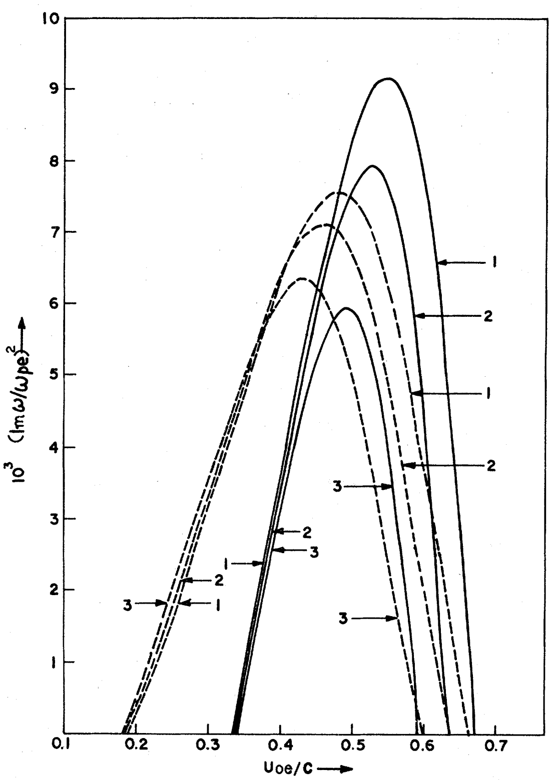

FIG. 3. Variation of $(\mathrm{Im}\,\omega/\omega_{pe})^2$ vs U_{0e}/c for $\Omega_e^2/\omega_{pe}^2 = 0.01$ and $m/M = 0$. Solid curves are for $c^2k^2/\omega_{pe}^2 = 0.1$ and dashed curves for $c^2k^2/\omega_{pe}^2 = 0.5$. Curves 1, 2, and 3 are for $a_e = 80$, 100, and 140, respectively. The scale for the dashed curves along the y axis has been reduced by a factor of 4.

persist only in a bounded unstable region in k space. The increase of the magnetic field reduces the unstable region (this is not shown in Fig. 2).

From Fig. 3 we note that in the relativistic regime of electron streaming, the electron thermal effects are always destabilizing whereas on the nonrelativistic side they may or may not be destabilizing. In fact, in the nonrelativistic regime,[4] lectron thermal effects are always stabilizing unless $c^2k^2/\omega_{pe}^2 < \frac{1}{3}$ and $\lambda_e \ll 1$. This follows directly from the instability criterion (36).

When $U_{0i} = 0$, the effect of ion temperature on the instability criterion (36) and on the growth rate is negligible. From Table I we, however, note that for $U_{0i} \neq 0$ but nonrelativistic, some purely growing waves with very small growth rates are excited whenever the electron streaming U_{0e} is such that the system is stable against the high-frequency instability (cf. Fig. 1). From Table I it is obvious that the low-frequency instability is enhanced by the ion streaming and suppressed by the increase of ion temperature; this is in complete agreement with the results of Sec. II B and II C for nonrelativistic U_{0i}. From Table I we also note that when U_{0e} is such that the high-frequency instability is excited; U_{0i} and T_i have negligible effect on the growth rates as one would have expected.

IV. CONCLUSIONS

A system comprising of homogeneous contra-streaming magnetoplasmas is susceptible to both the high-frequency and the low-frequency transversel instabilities. The high-frequency instability can be excited only for some bounded values of the electron streaming velocity, U_{0e}. When the streaming velocities are in the relativistic regime, the effect of electron temperature is destabilizing whereas for low streaming velocities the effect is stabilizing unless $c^2k^2/\omega_{pe}^2 < \frac{1}{3}$ and $\lambda_e \ll 1$. This instability is not affected appreciably by ion streaming U_{0i} and ion temperature T_i, but is suppressed by the magnetic field.

For nonrelativistic U_{0i}, the low-frequency instability is enhanced by the ion streaming and is suppressed by ion temperature and also by the magnetic field. For the case of strong magnetic fields the low-frequency instability can be excited only for some bounded values of U_{0i}. Moreover, the low-frequency instability can be easily excited at low plasma densities whereas the high-frequency instability requires comparatively larger plasma densities.

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