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PHYSICAL REVIEW A VOLUME 5, NUMBER 4 APRIL 1972

# Role of Electrostriction, Absorption, and the Electrocaloric Effect in the Stimulated Scattering of Light

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(Received 16 August 1971)

The modified hydrodynamic equations describing the behavior of a fluid under the influence of an optical electric field are solved. The contributions to stimulated scattering arising from electrostriction, absorptive heating, and the electrocaloric effect are exactly described. Previously unreported terms are given which significantly modify both the Rayleigh and Brillouin scattering. In particular, the experimentally observed stimulated Rayleigh scattering in non-absorbing media is shown to arise as much from electrostriction as from the electrocaloric effect, while anti-Stokes stimulated Brillouin scattering is predicted in a medium near its critical point. The possibility of observing additional scattering contributions proportional to $(\partial\epsilon/\partial T)_\rho$ is also discussed.

## I. INTRODUCTION

Recent theoretical interest in stimulated Rayleigh scattering (SRS) and stimulated Brillouin scattering (SBS) has concentrated on establishing the profile of the nonlinear gain as a function of frequency shift.[1-9] The general theory describing the gain profile in an absorbing medium was originated by Herman and Gray[1] and developed by Starunov and Fabelinskii,[3,4] who found that important modifications to that profile are indicated when the electrocaloric effect is taken into account.

In this paper we follow the above-mentioned authors in solving the hydrodynamic equations in the steady-state small-signal approximation,[4,10] i.e., assuming that the gain per wavelength of light is small and that the absorption coefficient for hypersound is large compared with the gain. However, in solving these equations without further approximation, we have found new terms arising from electrostriction which contribute significantly to both SRS and SBS. The nature of the scattering contribution arising from the change in dielectric constant with temperature at constant density[4,7] is also described and the possibility of observing such a contribution is discussed.

In order to clarify the role of the various physical effects involved in stimulated scattering, we present our results in a form in which the contributions arising from electrostriction, absorption, and the electrocaloric effect are clearly distinguished. For the same reason we present our calculations in terms of the actual changes of density, temperature, and dielectric constant of the medium rather than in terms of the more conventional nonlinear polarizability.[10]

In order to relate our results to an experimental situation, convolution with the laser line is of course necessary.[1] This can easily be done but has been omitted here as it adds nothing to the understanding of the physical processes involved.

## II. HYDRODYNAMIC AND ELECTROMAGNETIC EQUATIONS

The stimulated scattering of light arises from the modification of the dielectric constant of a medium by the high electric fields of a laser light beam. The changes in dielectric constant responsible for Rayleigh and Brillouin scattering are due to changes in the density and temperature of the medium. These are described by the linearized hydrodynamic equations,[1,4,11,12]

$$\frac{\partial^2\rho}{\partial t^2}-\frac{v^2}{\gamma}\nabla^2\rho-\frac{(\eta+\frac{4}{3}\eta')}{\rho_0}\frac{\partial}{\partial t}\nabla^2\rho-\frac{v^2}{\gamma}\beta\rho_0\nabla^2 T=-\operatorname{div}f\,, \tag{1}$$

$$\rho_0 c_v\frac{\partial T}{\partial t}-K\nabla^2 T-\frac{c_v(\gamma-1)}{\beta}\frac{\partial\rho}{\partial t}=\frac{dQ}{dt}\,, \tag{2}$$

where the notation is standard, $\rho$ and $T$ being the density and temperature of the medium, and $\rho_0$ and $T_0$ their unperturbed values, respectively. $v$ is the adiabatic velocity of sound, $\eta$ and $\eta'$ the shear and bulk viscosities, respectively. $\beta$ is the coefficient of thermal expansion, $K$ the thermal conductivity, $c_v$ the specific heat at constant volume, and $\gamma$ the ratio of the specific heats. $f$ is the force density at any point and, in a medium under the influence of an electric field, $E$ is given by[13]

$$f=\frac{Y}{8\pi}\operatorname{grad}E^2-\frac{E^2}{8\pi}\left(\frac{\partial\epsilon}{\partial T}\right)_\rho\operatorname{grad}T\,, \tag{3}$$

where $\epsilon$ is the dielectric constant of the medium and $Y$ is the electrostrictive constant $\rho_0(\partial\epsilon/\partial\rho)_T$.

It will be seen that neglecting the second term of this equation a sinusoidal modulation of temperature is indicated in the stimulated scattering situation. Hence, it may be shown that we are justified in neglecting this term provided that

$$\frac{T-T_0}{T_0}\ll\rho_0\left(\frac{\partial\epsilon}{\partial\rho}\right)_T\Big/T_0\left(\frac{\partial\epsilon}{\partial T}\right)_\rho$$

(This condition represents a small-signal approximation.) Then we have

$$\operatorname{div}f=(Y/8\pi)\nabla^2E^2\,.$$

$Q$ is the heat per unit volume supplied to the medium which, under the influence of absorption and the electrocaloric effect, is given by[4,13]

$$\frac{dQ}{dt}=\frac{1}{4\pi}n_0c\,\alpha E^2-\frac{1}{8\pi}T_0\left(\frac{\partial\epsilon}{\partial T}\right)_p\frac{dE^2}{dt}\,, \tag{4}$$

where $n_0$ is the refractive index of the unperturbed medium, $c$ is the velocity of light, and $\alpha$ is the light intensity absorption coefficient.

Now, if we consider a steady-state backscattering situation with a laser beam of frequency $\omega_L$, wave vector $k_L$, and amplitude $A_L$, and a scattered wave of frequency $\omega_s$, wave vector $-k_S$, and amplitude $A_s$, $E^2$ contains traveling wave components at the sum, difference, and doubled frequencies in addition to a constant term. But the hydrodynamic equations are linear in $\rho$, $T$, and $E^2$ so each component of $E^2$ generates a variation of $\rho$ and $T$ and hence of $\epsilon$ with its own frequency and wave vector. Solution of these equations will show that only the term in $(\omega_L-\omega_S)$ can significantly modulate the density and temperature of the medium. Inserting the traveling wave component of $(\epsilon-\epsilon_0)$ with this frequency into the nonlinear Maxwell's equation[10]

$$\nabla^2E+\frac{n_0^2}{c^2}\frac{\partial^2E}{\partial t^2}=-\frac{1}{c^2}\frac{\partial^2(\epsilon-\epsilon_0)E}{\partial t^2}\,, \tag{5}$$

we find that, in the small-signal approximation, the spatial variation of the backscattered beam is described by

$$\frac{\partial A_s}{\partial Z}=\frac{k_s}{4n_0^2}\epsilon''A_L\,. \tag{6}$$

Here we define $\epsilon''$, $\rho''$, and $T''$ as the amplitudes of those parts of the variation of dielectric constant, density, and temperature which are out of phase with the component of $E^2$ producing them, and $\epsilon'$, $\rho'$ and $T'$ as the in-phase amplitudes, respectively. (The in-phase change of dielectric constant causes a change in the propagation constant but not in the amplitude of the scattered beam.[10])

Now we have

$$\epsilon''=\left(\frac{\partial\epsilon}{\partial\rho}\right)_T\rho''+\left(\frac{\partial\epsilon}{\partial T}\right)_\rho T''\,. \tag{7}$$

and therefore the gain in intensity of the backscattered beam is given by

$$G=-\frac{2}{A_s}\frac{dA_s}{dz}=-\frac{k_s}{2n_0^2}\left[\left(\frac{\partial\epsilon}{\partial\rho}\right)_T\rho''+\left(\frac{\partial\epsilon}{\partial T}\right)_\rho T''\right]\frac{A_L}{A_s}\,. \tag{8}$$

Thus, in order to obtain the stimulated scattering gain profile we must determine the expressions for $\rho''$ and $T''$ caused by a component of $E^2$ whose frequency, wave vector, and amplitude we define as $\omega$, $k$, and $\Lambda$, respectively.

## III. CALCULATION OF GAIN PROFILE

Inserting expressions for $\rho$, $T$, and $E^2$ with the above frequency and wave vector into the hydrodynamic equations and equating coefficients of like terms we find in the steady-state small-signal approximation that

$$\left(\frac{\omega_B^2}{\gamma}-\omega^2\right)\rho'-\Gamma_B\omega\rho''+\frac{\omega_B^2}{\gamma}\beta\rho_0 T'=k^2\frac{Y\Lambda}{8\pi}, \tag{9}$$

$$\left(\frac{\omega_B^2}{\gamma}-\omega^2\right)\rho''+\Gamma_B\omega\rho'+\frac{\omega_B^2}{\gamma}\beta\rho_0 T''=0\ . \tag{10}$$

$$-\omega T''+\frac{\gamma\Gamma_R}{2}T'+\frac{(\gamma-1)\omega}{\beta\rho_0}\rho''=\frac{nc\alpha\Lambda}{4\pi\rho_0 c_v}, \tag{11}$$

$$+\omega T'+\frac{\gamma\Gamma_R}{2}T''-\frac{(\gamma-1)\omega}{\beta\rho_0}\rho'=-\frac{T_0(\partial\epsilon/\partial T)_\rho\omega\Lambda}{8\pi\rho_0 c_v}, \tag{12}$$

where

$$\omega_B=kv,\quad \Gamma_B=(\eta+\tfrac{4}{3}\eta')\frac{k^2}{\rho_0},\quad \Gamma_R=\frac{2Kk^2}{\rho_0 c_p}\ .$$

Hence we have

$$\rho''=-\frac{\Lambda Y}{8\pi v^2}\left(\frac{\gamma(1-\mathfrak{a})\left[1+\dfrac{\gamma\Gamma_R\Gamma_B}{2\omega_B^2}-\dfrac{\omega^2}{\omega_B^2}\right]-(1-\mathcal{E})\left[1-\dfrac{\gamma\omega^2}{\omega_B^2}-\dfrac{2\Gamma_B}{\Gamma_R}\dfrac{\omega^2}{\omega_B^2}\right]}{1+\dfrac{4\omega^2}{\Gamma_R^2}\left[\left(1-\dfrac{\omega^2}{\omega_B^2}\right)^2+\dfrac{\Gamma_B^2+\frac{1}{4}\gamma^2\Gamma_R^2}{\omega_B^2}\dfrac{\omega^2}{\omega_B^2}+\dfrac{\Gamma_R\Gamma_B}{\omega_B^2}\left((\gamma-1)-\dfrac{\gamma\Gamma_R}{2\Gamma_B}+\dfrac{\gamma^2\Gamma_R\Gamma_B}{4\omega_B^2}\right)\right]}\right)\frac{2\omega}{\Gamma_R} \tag{13}$$

and

$$\rho_0\beta T''=\frac{\Lambda Y}{8\pi v^2}\left(\frac{\gamma(1-\mathfrak{a})\left[\left(1-\dfrac{\omega^2}{\omega_B^2}\right)\left(1-\dfrac{\gamma\omega^2}{\omega_B^2}\right)+\dfrac{\gamma\Gamma_B^2}{\omega_B^2}\dfrac{\omega^2}{\omega_B^2}\right]-(1-\mathcal{E})\left[\left(1-\dfrac{\gamma\omega^2}{\omega_B^2}\right)^2+2(\gamma-1)\dfrac{\Gamma_B}{\Gamma_R}\dfrac{\omega^2}{\omega_B^2}+\gamma^2\dfrac{\Gamma_B^2}{\omega_B^2}\dfrac{\omega^2}{\omega_B^2}\right]}{1+\dfrac{4\omega^2}{\Gamma_R^2}\left[\left(1-\dfrac{\omega^2}{\omega_B^2}\right)^2+\dfrac{\Gamma_B^2+\frac{1}{4}\gamma^2\Gamma_R^2}{\omega_B^2}\dfrac{\omega^2}{\omega_B^2}+\dfrac{\Gamma_R\Gamma_B}{\omega_B^2}\left((\gamma-1)-\dfrac{\gamma\Gamma_R}{2\Gamma_B}+\dfrac{\gamma^2\Gamma_R\Gamma_B}{4\omega_B^2}\right)\right]}\right)\frac{2\omega}{\Gamma_R}, \tag{14}$$

where $\mathfrak{a}$ represents the effect of light absorption and is given by

$$\mathfrak{a}=\frac{4n_0 c\,\alpha\beta v^2}{Y\gamma c_p\Gamma_R}, \tag{15}$$

while $\mathcal{E}$ represents the result of the electrocaloric effect and is given by

$$\mathcal{E}=(\gamma-1)\left(1-\frac{(\partial\epsilon/\partial T)_\rho}{\beta\gamma}\right), \tag{16}$$

where the thermodynamic relationship $\gamma-1=v^2T_0\beta^2/c_p$ has been used. (Typical values of the parameters in these equations are given in Table I.)

The expressions for $\rho''$ and $T''$ have maxima in the regions $\omega\sim\pm\frac{1}{2}\Gamma_R$ and $\omega\sim\pm\omega_B$ and are always very small when $\omega\gg\omega_B$. Thus the only component of $E^2$ which can induce a significant modulation of the properties of the medium is that for which $\omega=\omega_L-\omega_s$, $k=k_L+k_s$, and $\Lambda=A_LA_s$.

Furthermore, under most experimental conditions we have

$$\left(\frac{\partial\epsilon}{\partial T}\right)_\rho T''\ll\left(\frac{\partial\epsilon}{\partial\rho}\right)_T\rho'',\quad \left(\frac{\partial\epsilon}{\partial T}\right)_\rho\ll\beta Y,$$

so we shall at first consider only those terms of Eqs. (8) and (16) which are independent of $(\partial\epsilon/\partial T)_\rho$. The additional contributions arising from the remaining terms are considered in Sec. V.

Thus using Eqs. (13) and (8) and separating the contributions arising from different physical effects, we find that

| Electrostrictive terms | Absorptive terms | Electrocaloric terms |
|---|---|---|

$$G=\frac{k_sY^2A_L^2}{16\pi n_0^2\rho_0 v^2}\left[\frac{\omega_B}{\Gamma_B}\left(1+(\gamma-1)\frac{\Gamma_R}{2\Gamma_B}+\frac{\gamma^2\Gamma_R^2}{4\omega_B^2}\right)F_1-\mathfrak{a}\gamma^2\frac{\Gamma_R^2}{4\Gamma_B\omega_B}F_1-(\gamma-1)\frac{\omega_B}{\Gamma_B}\left(1+(\gamma-1)\frac{\Gamma_R}{2\Gamma_B}\right)F_1\right.$$
$$\left.+\frac{\Gamma_R}{4\Gamma_B}\left(\gamma-1+\frac{\gamma\Gamma_R\Gamma_B}{2\omega_B^2}\right)F_2-\mathfrak{a}\frac{\gamma\Gamma_R}{4\Gamma_B}\left(1+\frac{\gamma\Gamma_R\Gamma_B}{2\omega_B^2}\right)F_2+(\gamma-1)\frac{\Gamma_R}{4\Gamma_B}F_2\right], \tag{17}$$

where

$$F_1 = 4\frac{\Gamma_B^2}{\Gamma_R^2}\frac{\omega^3}{\omega_B^3}\Bigg/\left\{1+4\frac{\omega^2}{\Gamma_R^2}\left[\left(1-\frac{\omega^2}{\omega_B^2}\right)^2+\frac{\Gamma_B^2+\frac{1}{4}\gamma^2\Gamma_R^2}{\omega_B^2}\frac{\omega^2}{\omega_B^2}+\frac{\Gamma_R\Gamma_B}{\omega_B^2}\left((\gamma-1)-\frac{\gamma\Gamma_R}{2\Gamma_B}+\gamma^2\frac{\Gamma_R\Gamma_B}{4\omega_B^2}\right)\right]\right\}, \tag{18}$$

$$F_2 = \left(1-\frac{\omega^2}{\omega_B^2}\right)\frac{\Gamma_B\omega}{\Gamma_R^2}\Bigg/\left\{1+4\frac{\omega^2}{\Gamma_R^2}\left[\left(1-\frac{\omega^2}{\omega_B^2}\right)^2+\frac{\Gamma_B^2+\frac{1}{4}\gamma^2\Gamma_R^2}{\omega_B^2}\frac{\omega^2}{\omega_B^2}+\frac{\Gamma_R\Gamma_B}{\omega_B^2}\left((\gamma-1)-\frac{\gamma\Gamma_R}{2\Gamma_B}+\gamma^2\frac{\Gamma_R\Gamma_B}{4\omega_B^2}\right)\right]\right\}. \tag{19}$$

Under the approximations $\Gamma_R \ll \Gamma_B$ and $\Gamma_R\Gamma_B \ll \omega_B^2$ these expressions reduce to

$$F_1 = 4\frac{\Gamma_B^2}{\Gamma_R^2}\frac{\omega^3}{\omega_B^3}\Bigg/\left\{1+4\frac{\omega^2}{\Gamma_R^2}\left[\left(1-\frac{\omega^2}{\omega_B^2}\right)^2+\left(\frac{\Gamma_B\omega}{\omega_B^2}\right)^2\right]\right\}, \tag{20}$$

$$F_2 = 4\left(1-\frac{\omega^2}{\omega_B^2}\right)\frac{\Gamma_B\omega}{\omega_B^2}\Bigg/\left\{1+4\frac{\omega^2}{\Gamma_R^2}\left[\left(1-\frac{\omega^2}{\omega_B^2}\right)^2+\left(\frac{\Gamma_B\omega}{\omega_B^2}\right)^2\right]\right\}. \tag{21}$$

The reduced expression for $F_1$ which is significant only in the region $\omega \sim \pm\omega_B$ and describes the gain profile usually associated with SBS in a nonabsorbing medium[1,3,4,15] is illustrated in Fig. 1. The expression for $F_2$, also illustrated in Fig. 1, is significant in both the regions $|\omega| \lesssim \Gamma_R$ and $\omega \sim \pm\omega_B$ and represents the profile associated with the thermal Brillouin and Rayleigh scattering in absorbing media.[1,3,4]

The profiles described by the exact expressions differ from these in that the widths of the Brillouin lines are increased and that of the Rayleigh line reduced by terms proportional to $\Gamma_R/\Gamma_B$ and $\Gamma_R\Gamma_B/\omega_B^2$, respectively. The maxima of these lines are, respectively, reduced and increased by a similar factor. These corrections are negligible in liquids but in gases even under conditions in which the hydrodynamic description is valid[2,3,16,17] $\Gamma_R$ and $\Gamma_B$ may approach $\omega_B$[3,18] and the exact expressions must be used (see Table I).

However, under the above approximation, we may consider separately the functions $F_{2R}$ and $F_{2B}$ representing $F_2$ in the regions $|\omega| \lesssim \Gamma_R$ and $\omega \sim \pm\omega_B$. These correspond, respectively, to the profiles usually associated with SRS and stimulated thermal Brillouin scattering[1] (STBS). Then we have

Electrostrictive terms / Absorptive terms / Electrocaloric terms

$$\begin{aligned} G = \frac{k_2 Y^2 A_L^2}{16\pi n_0^2\rho_0 v^2}\Bigg\{ & \frac{\omega_B}{\Gamma_B}[F_1] && -\frac{\alpha\gamma^2\Gamma_R^2}{4\omega_B\Gamma_B}[F_1] && -(\gamma-1)\frac{\omega_B}{\Gamma_B}[F_1] \\ & +\frac{(\gamma-1)}{2}\left[\frac{\Gamma_R}{2\Gamma_B}F_{2R}\right] && -\frac{\alpha\gamma}{2}\left[\frac{\Gamma_R}{2\Gamma_B}F_{2R}\right] && +\frac{(\gamma-1)}{2}\left[\frac{\Gamma_R}{2\Gamma_B}F_{2R}\right] \\ & +\frac{(\gamma-1)}{4}\frac{\Gamma_R}{\Gamma_B}[F_{2B}] && -\frac{\alpha\gamma}{4}\frac{\Gamma_R}{\Gamma_B}[F_{2B}] && +\frac{(\gamma-1)}{4}\frac{\Gamma_R}{\Gamma_B}[F_{2B}]\Bigg\}, \end{aligned} \tag{22}$$

TABLE I. Parameters governing the profile of stimulated scattering of ruby laser light in hydrodynamic media.

| | $\Gamma_R$ [a] (Mc/sec) | $\Gamma_B$ [a] (Mc/sec) | $\omega_B$ [a] (Mc/sec) | $\gamma-1$ | $\alpha/\alpha$ (cm) | $\beta$ ($10^{-3}\,°K^{-1}$) | $Y$ [b] | $(\partial\epsilon/\partial T)_\rho$ [c] / $\beta Y$ |
|---|---|---|---|---|---|---|---|---|
| Carbon tetrachloride | 18 | 630 | 3990 | 0.46 | 1010 | 1.18 | 1.33 | 0.028 |
| Acetone | 21 | 270 | 4660 | 0.4 | 788 | 1.32 | 1.00 | ... |
| Methanol | 18[d] | 300 | 4640 | 0.21 | 895 | 1.18 | 0.90 | ... |
| Carbon disulphide | 37 | 65 | 5490 | 0.55 | 425 | 1.14 | 2.10 | 0.055 |
| Benzene | 27 | 350 | 5050 | 0.4 | 470 | 1.18 | 1.53 | 0.046 |
| Ethyl ether | 16 | 290 | 4000 | 0.39 | 833 | 1.51 | 0.97 | ... |
| Water at 20 °C | 26 | 380 | 5590 | 0.006[d] | 109 | 0.20 | 0.90 | 0.84 |
| Water at 4 °C | 26 | 380 | 5590 | 0 | 0 | 0 | 0.90 | $(\partial\epsilon/\partial T)_\rho = -0.000\,045$[d] |
| Fused silica | 22[d] | 82 | 24900 | 0.000 02[d] | 56 | 0.0012 | 1.33 | 42.5[d] |
| Hydrogen at 5 atm | 3150[d] | 1080[d] | 3780[d] | 0.41 | 1300 | 3.41 | 0.0014 | ... |

[a]Herman and Gray, Ref. 1. [b]Vuks, Ref. 14. [c]Batra and Enns, Ref. 7. [d]Calculated.

where each term in square brackets is a frequency profile with a maximum of unity.

## IV. DISCUSSION

Equation (22) shows that the total gain profile is a sum to which *each* physical effect makes contributions with *all three* types of profile $F_1$, $F_{2R}$, and $F_{2B}$. The gain components existing in a typical experimental situation are shown in Fig. 2. The absorptive contributions are of course proportional to the absorption coefficient and can therefore be of any chosen magnitude. The curves in Fig. 2 show the absorptive contributions occurring when the absorption coefficient is such that the maximum gain of the absorptive contribution to the SRS is equal to that of the electrostrictive contribution to SBS. Under these circumstances the orders of magnitude of the contributions fall into the three groups shown.

Certain of the terms of Eq. (22) describe scattering gains previously discussed by other workers. The term $(\omega_B/\Gamma_B)[F_1]$ gives the usual expression for SBS in a nonabsorbing medium.[1,3,4,15] The terms $-\frac{1}{2}\alpha\gamma[(\Gamma_R/2\Gamma_B)F_{2R}]$ and $-\frac{1}{4}\alpha\gamma(\Gamma_R/\Gamma_B)[F_{2B}]$ describe the SRS and STBS due to absorption first predicted by Herman and Gray.[1] The term $\frac{1}{2}(\gamma-1)\times[(\Gamma_R/2\Gamma_B)F_{2R}]$ arising from the electrocaloric effect represents the expression used by Starunov to describe SRS in a nonabsorbing medium.[3,4] The same author has also predicted a reduction of the SBS gain by a factor of $2-\gamma$ as a result of the electrocaloric effect.[3,4] This is represented by the negative term $-(\gamma-1)(\omega_B/\Gamma_B)[F_1]$, which has the same frequency profile as that of the conventional SBS. Under normal experimental conditions the values of $\gamma-1$ (Table I) are such as to reduce the SBS gain by up to one-half. It has not been previously noted, however, that under certain conditions, particularly in the region of the critical point, $\gamma-1$ may exceed unity thus reversing the usual shape of the SBS gain profile and causing anti-Stokes SBS. The small term $\frac{1}{4}(\gamma-1)(\Gamma_R/\Gamma_B)[F_{2B}]$ representing an electrocaloric modification of the Brillouin lineshape has also been mentioned by Starunov.[3,4]

The remaining terms have not been previously reported, but as can be seen in Fig. 2, these make a significant contribution to the gain profile.[19]

The term $\frac{1}{2}(\gamma-1)[(\Gamma_R/2\Gamma_B)F_{2R}]$, arising from electrostriction, gives a contribution to SRS identical to that produced by the electrocaloric effect. The SRS gain in nonabsorbing media should therefore be twice that expected from the electrocaloric effect alone. Thus, it is incorrect to interpret the experimentally observed SRS in nonabsorbing media[3,18,20,21] as resulting solely from the electrocaloric effect. Similarly, the electrostrictive term $\frac{1}{4}(\gamma-1)(\Gamma_R/\Gamma_B)[F_{2B}]$ doubles Starunov's expression for the modification of the Brillouin lineshape in nonabsorbing media. The remaining absorptive term $-(\gamma^2\Gamma_R^2/4\omega_B\Gamma_B)[F_1]$ slightly affects

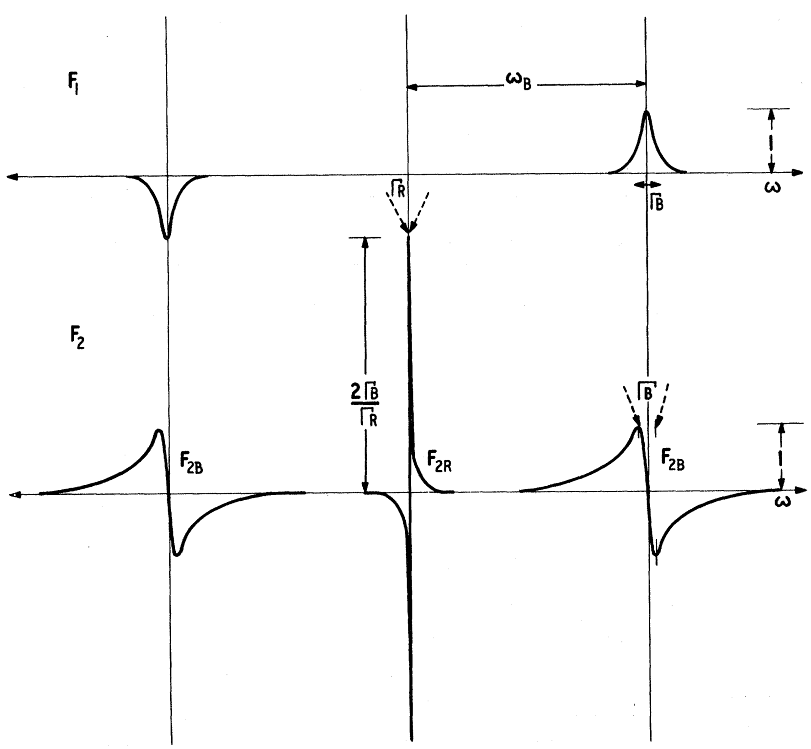


FIG. 1. Frequency profiles $F_1$ and $F_2$ governing stimulated scattering in hydrodynamic media.

the magnitude of the SBS gain but is negligible under normal experimental conditions.

## V. FURTHER EFFECTS DEPENDENT ON $(\partial\epsilon/\partial T)_\rho$

In the preceding analysis we have neglected the second term of Eq. (8) which represents the effect of the change of dielectric constant with temperature at constant density. This term is always small but, as will be shown, can under certain conditions give rise to observable steady-state scattering phenomena. Now, the electrocaloric contributions to $\rho''$ and $T''$ themselves contain terms in $(\partial\epsilon/\partial T)_\rho$ [see Eq. (13), (14), and (16)], so in order to make these terms explicit we define $\rho_1''$, $T_1''$ as the values of $\rho''$, $T''$ which would occur if $(\partial\epsilon/\partial T)_\rho$ were zero and $\rho''_{1\,\mathrm{cal}}$, $T''_{1\,\mathrm{cal}}$ as those parts of $\rho_1''$ and $T_1''$ resulting from the electrocaloric effect. Then we have

$$\rho'' = \rho_1'' - \frac{(\partial\epsilon/\partial T)_\rho}{\beta Y}\,\rho''_{1\,\mathrm{cal}}\,, \qquad T'' = T_1'' - \frac{(\partial\epsilon/\partial T)_\rho}{\beta Y}\,T''_{1\,\mathrm{cal}}\,.$$

Therefore, using Eq. (7), we have

$$\epsilon'' = \frac{Y}{\rho_0}\left[\rho_1'' + \left(\frac{(\partial\epsilon/\partial T)_\rho}{\beta Y}\right)(\rho_0\beta T_1'' - \rho''_{1\,\mathrm{cal}}) - \left(\frac{(\partial\epsilon/\partial T)_\rho}{\beta Y}\right)^2 \rho_0\beta T''_{1\,\mathrm{cal}}\right]. \tag{23}$$

Hence, using Eqs. (8), (13), (14), and (23) we may derive the complete expression for the steady-state gain existing in a hydrodynamic medium:

Electrostrictive terms | Absorptive terms | Electrocaloric terms

$$\begin{aligned}
G = \frac{k_S Y^2 A_L^2}{16\pi n_0^2 \rho_0 v^2} & \left(\left[\frac{\omega_B}{\Gamma_B}\left(1 + \frac{(\gamma-1)}{2}\,\frac{\Gamma_R}{\Gamma_B} + \frac{\gamma^2\Gamma_R^2}{4\omega_B^2}\right)F_1 + \frac{\Gamma_R}{4\Gamma_B}\left(\gamma - 1 + \frac{\gamma\Gamma_R\Gamma_B}{2\omega_B^2}\right)F_2\right.\right. \\
& - \frac{\alpha\gamma^2\Gamma_R^2}{4\omega_B\Gamma_B}F_1 - \frac{\alpha\gamma\Gamma_R}{4\Gamma_B}\left(1 + \frac{\gamma\Gamma_R\Gamma_B}{2\omega_B^2}\right)F_2 \\
& \left. - (\gamma-1)\frac{\omega_B}{\Gamma_B}\left(1 + \frac{(\gamma-1)}{2}\,\frac{\Gamma_R}{\Gamma_B}\right)F_1 + \frac{(\gamma-1)}{4}\,\frac{\Gamma_R}{\Gamma_B}F_2\right] \\
& + \frac{(\partial\epsilon/\partial T)_\rho}{\beta Y}\left\{(\gamma-1)\left(1 + (\gamma-1)\frac{\omega_B}{2\Gamma_B}\right)\frac{\omega_B}{\Gamma_B}F_1 - (\gamma-1)\frac{\Gamma_R}{4\Gamma_B}F_2\right. \\
& + \frac{\alpha\gamma^2\Gamma_R}{2\omega_B}F_1 - \alpha\gamma(\gamma-1)\frac{\Gamma_R}{4\Gamma_B}F_2 - \frac{\alpha\gamma^2\Gamma_R}{4\Gamma_B}\left(1 - \frac{\omega^2}{\omega_B^2}\right)F_2 \\
& - (\gamma-1)\frac{\omega_B}{\Gamma_B}\left[(2-\gamma)\left(1 + (\gamma-1)\frac{\Gamma_R}{2\Gamma_B}\right) - \gamma^2\frac{\Gamma_R\Gamma_B}{2\omega_B^2}\right]F_1 \\
& - (\gamma-1)(2-\gamma^2)\frac{\Gamma_R}{4\Gamma_B} \\
& \left. - (\gamma-1)\gamma^2\,\frac{\Gamma_R}{4\Gamma_B}\left(1 - \frac{\omega^2}{\omega_B^2}\right)F_2\right\} \\
& + \left(\frac{(\partial\epsilon/\partial T)_\rho}{\beta Y}\right)^2\left\{(\gamma-1)\frac{\omega_B}{\Gamma_B}\left[(\gamma-1) + (\gamma-1)^2\,\frac{\Gamma_R}{2\Gamma_B} + \gamma^2\,\frac{\Gamma_R\Gamma_B}{2\omega_B^2}\right]F_1\right. \\
& - (\gamma-1)(\gamma^2-1)\frac{\Gamma_R}{4\Gamma_B}F_2 \\
& \left.\left. + (\gamma-1)\,\frac{\gamma^2\Gamma_R}{4\Gamma_B}\left(1 - \frac{\omega^2}{\omega_B^2}\right)F_2\right\}\right).
\end{aligned} \tag{24}$$

In the region of SRS (i.e., where $|\omega| \lesssim \Gamma_R$), providing $\Gamma_R \ll \Gamma_B$ and $\Gamma_R\Gamma_B \ll \omega_B^2$, this expression reduces to

$$\begin{aligned}
G = \frac{k_s Y^2 A_L^2}{16\pi n_0^2 \rho_0 v^2} & \left\{(\gamma-1) + \frac{\gamma}{4}\,\frac{\Gamma_R\Gamma_B}{\omega_B^2} - \frac{\alpha\gamma}{2}\right. \\
& - \tfrac{3}{2}(\gamma-1)\left(\frac{(\partial\epsilon/\partial T)_\rho}{\beta Y}\right) + \frac{\alpha\gamma}{2}\left(\frac{(\partial\epsilon/\partial T)_\rho}{\beta Y}\right) \\
& \left. + \frac{(\gamma-1)}{2}\left(\frac{(\partial\epsilon/\partial T)_\rho}{\beta Y}\right)^2\right\}\left[\frac{\Gamma_R}{2\Gamma_B}F_{2R}\right]
\end{aligned} \tag{25}$$

Now in most media

$$\left(\frac{(\partial\epsilon/\partial T)_\rho}{\beta Y}\right)$$

is small and the change in SRS gain amounts to a few percent as described by Batra and Enns,[7] who derived an expression equivalent to the term

$$\frac{\alpha\gamma}{2}\left(\frac{(\partial\epsilon/\partial T)_\rho}{\beta Y}\right).$$

In certain media, however (the most common being water at 4 °C and silica), $\beta$ is very small and the usual SRS contributions disappear. In this case

$$\left(\frac{(\partial\epsilon/\partial T)_\rho}{\beta Y}\right)$$

may be much greater than unity, so that these media offer the possibility of observing stimulated scattering arising solely from the change of refractive index with temperature at constant density. This type of scattering is nevertheless very weak. For example, the resulting gain only exceeds that of SRS in a typical nonabsorbing liquid for a colored sample of water in which $\alpha \gg 0.1\ \mathrm{cm}^{-1}$. As SRS in a nonabsorbing medium is itself quite difficult to detect,[3,18,20,21] it seems unlikely that a scattering

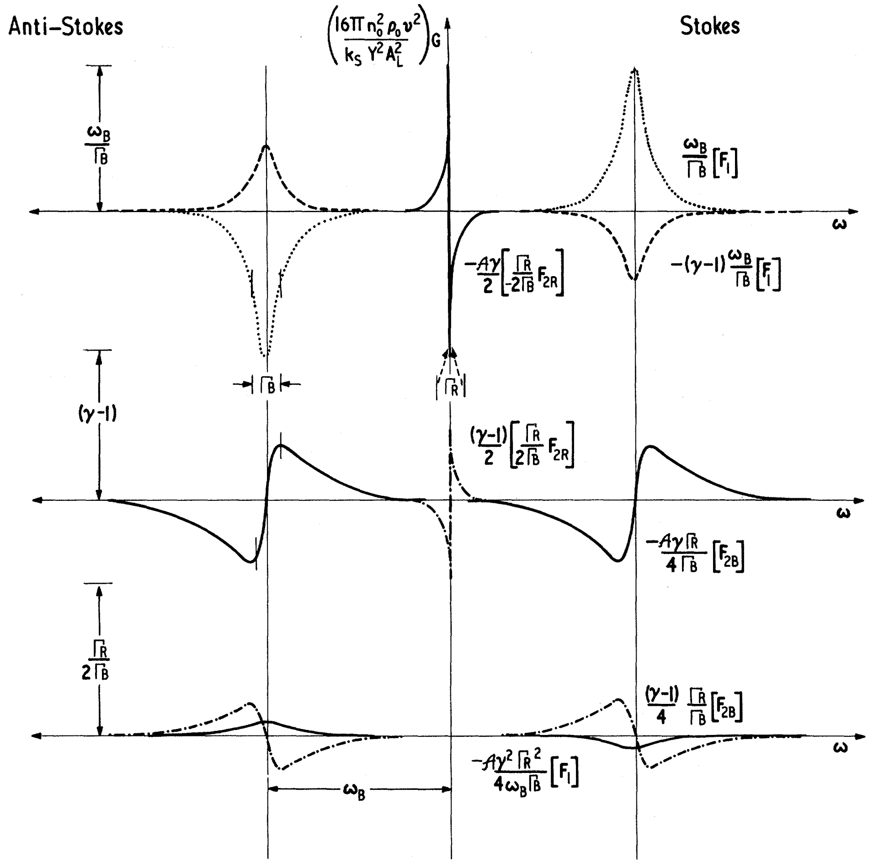


FIG. 2. Contributions to stimulated scattering gain as a function of frequency shift in a typical liquid ($CCl_4$). Dotted line, contributions arising from electrostriction; dashed line, contribution arising from the electrocaloric effect; dot-dashed line, superposition of the above notations used where these two contributions are equal; solid line, contributions arising from absorption.

effect of this magnitude could be directly observed in such a highly absorbing medium.

This stimulated scattering process could however be investigated using the probe scattering technique.[22,23] In this technique a continuous laser beam is directed at the stimulated scattering region in such a direction as to be incident, at the Bragg angle, on the induced modulation of the dielectric constant. As the probe beam may be of completely different frequency and direction from those of the scattering beams, this method allows extremely sensitive detection of such modulations. In water at 4 °C, for example, with $\alpha = 0.1$ cm$^{-1}$ and scattering beams of 1 cm diam having powers of 100 and 10 MW in the forward and backward directions, respectively, the reflectivity experienced by the probe beam would be about $10^{-5}$, which should be readily detectable.

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PHYSICAL REVIEW A VOLUME 5, NUMBER 4 APRIL 1972

# Relativistic Streaming Effects on Electromagnetic Instability in Magnetoplasmas

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(Received 2 June, 1971)

The electromagnetic waves propagating transverse to the direction of streaming of the contrastreaming plasmas in the presence of a uniform magnetic field are investigated. For electron plasmas these waves become unstable only for some bounded values of the streaming velocity $U_0$, namely, $U_{\min} < U_0 < U_{\max}$. For relativistic streaming velocities, the electron thermal effects are destabilizing whereas for nonrelativistic streaming these are stabilizing unless $c^2k^2/\omega_{pe}^2 < \frac{1}{3}$ and $k^2V_{te}^2/\Omega_e^2 \ll 1$ ($k$ being the characteristic wave number, $V_{te}$ the electron thermal velocity, $\Omega_e$ the electron cyclotron frequency, and $\omega_{pe}$ the electron plasma frequency). The magnetic field however reduces the region of instability no matter what the streaming velocity is. The effect of nonrelativistic-ion streaming is destabilizing while that of the ion temperature is stabilizing. In the presence of a strong magnetic field, the effect of relativistic-ion streaming is also stabilizing.

## I. INTRODUCTION

The plasmas streaming with a relative velocity exhibit the well-known two-stream (TS) instability whenever this relative velocity exceeds a certain critical velocity. Buneman[1] showed that besides this TS instability, there exists a transversel instability in plasmas with wave propagation transverse to the direction of the relative streaming. Momota[2] presented the criterion for the existence of these transversel instabilities in cold as well as warm plasmas and showed that the thermal effects were stabilizing. Lee[3] studied this instability for counter-streaming plasmas in the presence of the magnetic field and found that both the magnetic field and the thermal effects were stabilizing. Buti and Lakhina[4] modified Lee's[3] criterion for instability and concluded that the magnetic field was always stabilizing while the temperature can be destabilizing in some regions. Recently, Tzoar and Yang[5] have discussed this problem for the case of large $k$, i.e., $\lambda = k^2V_{te}^2/\Omega_e^2 \gg 1$. They find that the spectrum of the unstable $k$ has a lower as well as upper limit and that the magnetic field reduces this unstable region. Lee and Bornatici[6] have reported that for anisotropic-temperature plasmas the perpendicular temperature is stabilizing whereas the parallel temperature is destabilizing. The transverse instabilities have also been studied for collisional plasmas[7] and for plasmas in which both the electrons and the ions are streaming.[8,9]

The above-mentioned investigations were carried out for nonrelativistic streaming velocities; but recently the production of intense relativistic electron streams has been reported[10] and it has stimulated the investigations of the instabilities for relativistic streaming.[11,12] Here we have studied the transversel instabilities in plasmas which are contrastreaming with any arbitrary velocity (relativistic or nonrelativistic) in the presence of external uniform magnetic field. We find that for very low streaming velocities as well as for extreme relativistic streaming velocities, the waves propagating transverse to the direction of stream-