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# Production (in Mg Vapor) and Loss (in H<sub>2</sub> Gas) of 1- to 42-keV/Nucleon  $X$   $^1\Sigma_g^+$  and  $c$   $^3\Pi_u$ Hydrogen Molecules

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We have investigated collisions which result in the formation and destruction of the  $({1s\sigma^2})X^1\Sigma^*$  ground state and the  $({1s\sigma^2\pi})c^3\Pi_{\nu}$  long-lived electronically excited state of molecular hydrogen. The molecules were formed by electron capture by  $1-$  to  $42$ -keV/nucleon  $H_2^*$ or D<sub>2</sub><sup>+</sup> ions in Mg vapor. The resulting yields of H<sub>2</sub>(<sup>1</sup> $\Sigma_r^+$ ), H<sub>2</sub>(<sup>3</sup> $\Pi_u$ ), H, and H<sup>+</sup> are reported as a function of Mg-target thickness. Also reported are cross sections for total loss and ionization of the  ${}^{1}\Sigma_{\rho}^{*}$  and  ${}^{3}\Pi_{\mu}$  states of H<sub>2</sub> in collisions with H<sub>2</sub> gas.

### I. INTRODUCTION

In recent years a large amount of experimental work has been devoted to measuring excited-state populations of energetic atomic-hydrogen and helium beams, and their destruction and formation cross sections. Much less is known about energetic excited- molecular hydrogen beams. The present work extends the field of measurement of keV-energy heavy-particle collisions involving excited states to the  $n = 2$  united-atom equivalent state of molecular hydrogen. In this work we quantitatively analyze by beam-attenuation techniques the collisional formation and loss of the  $(1s\sigma^2)X^1\Sigma^*$  ground state and the  $(1s\sigma2p\pi)c^{3}\Pi_{u}$  long-lived electronically excited state of energetic molecular hydrogen. The energy range of the present work is from 1- to 42  $keV/nucleon$ . The  $H_2$  molecules were produced from energetic  $H_2^*$  ions by electron capture from  $H_2$  or  $N_2$  gas or from Mg vapor. According to the Massey hypothesis<sup>1</sup> the maximum cross section for charge exchange is large when the energy defect is small. (The energy defect is the difference between the internal energy of the initial and final states of the collision particles. ) In the case of the charge-exchange process of interest in the present experiment, i.e.,  $H_2^+ + X \rightarrow H_2 + \cdots$ , the energy defect for electron capture into  $n = 2$  bound states of  $H_2$  is about 4 eV for a Mg-vapor target and about 12 eV for a  $H<sub>2</sub>$  or  $N<sub>2</sub>$  gas target. Therefore, at low energies where the Massey criterion is valid, we expect Mg vapor to be a more effective charge-exchange medium for capture into excited states than  $H_2$  or  $N_2$ gas.

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The desirability of metal vapors as charge-exchange media for the formation of excited hydrogen atoms has been experimentally verified by many groups (see, for example, Refs. 2-6), A theoretical survey of electron capture into excited states of atomic hydrogen by ground-state elements has been performed by Hiskes.<sup>7</sup> From these results Hiskes has estimated up to 50% electron capture into the  ${}^{3}$ II<sub>u</sub> state of H<sub>2</sub> by passage of H<sub>2</sub><sup>+</sup> through Mg vapor.<sup>8</sup> Based on the above considerations, Mg vapor was used to attempt to enhance the  ${}^{3}$ H<sub>u</sub> excited-state population of  $H_2$  beams. In order to compare the metal-vapor results with common gases we also used  $H_2$  and  $N_2$  targets.

The experimental technique used for the analysis of the hydrogen molecules is the beam-attenuation technique introduced by Gilbody  $et\ al.^{9}$  for metastable He atoms. Accurate absolute measurements can be obtained by this method only if a beam of fast projectiles contains sufficient excited particles, with long enough lifetimes and large enough cross sections to ensure observable changes in the colli-



Nuclear separation (A)

FIG. 1. Potential-energy diagram of  $H_2$  and  $H_2^*$  showing several excited states of  $H_2$ . The hatched area is the Franck-Condon classical region of transitions from the ground state  $(v=0)$  of H<sub>2</sub>. Several vibrational levels are indicated by dashes. The potential-energy curves shown are based on calculations cited throughout this work.

sional attenuation of the beam when compared with a beam consisting almost exclusively of ground-state atoms or molecules. These conditions are satisfied in the present experiment for  $H_2$  molecules.

We have measured the fraction of  $H_2$  molecules that are produced in the  ${}^{3}$ II<sub>u</sub> state by electron capture in Mg vapor at various target thicknesses.

We have also measured the total-loss cross sections  $(H_2 \rightarrow H_2^+, 2H, H + H^+, 2H^+)$  and the ionization cross sections  $(H_2 + H_2^+)$  for both the  ${}^1\Sigma^*$  (ground) state and the  ${}^{3}$ II<sub>u</sub> state in collisions with H<sub>2</sub> gas. Preliminary results were reported in Ref. 10.

#### II. HYDROGEN MOLECULE

The structure of the hydrogen molecule has been studied in detail both theoretically and experimentally (Herzberg<sup>11</sup> tabulated a large number of references on this topic up to 1950, and the present work contains a number of references since 1950). Figure 1 shows the potential energy curves for the electronic states of  $H_2$  and  $H_2$ <sup>+</sup> that are of interest in this work.

The molecule contains two independent emission spectra: a triplet spectrum due to the two electron spins of the molecule being aligned, and a singlet spectrum due to electronic states with opposed electron spins. Spontaneous radiative transitions between these two term schemes is strictly forbidden by the selection rule on the total electron spin of the molecule, i.e.,  $\Delta S = 0$ . An elaborate treatment of transitions in diatomic molecules is treatment of transitions in diatomic molecules is<br>given by Herzberg.<sup>11</sup> Also, a concise and usefu summary of selection rules is given by Garstang.<sup>12</sup>

In order to interpret the results of the present experiment, an understanding of the electronic states of the  $H_2$  molecule is necessary. In Appendix A we present the results of a survey of the pertinent electronic excited states of  $H_2$  and their lifetimes, and discuss which excited states have lifetimes greater than the time of flight of the  $H<sub>2</sub>$  molecule in the present experiment (0.  $29 \le t \le 0.53$   $\mu$ sec). Based on the discussion in Appendix A, we conclude that decay of all excited states, except the hydrogenlike (Rydberg) states with  $n \gtrsim 8$ , the  $F^{-1}\Sigma_g^*$  state and the  $c^{3}$ II, state, has taken place in this time interval.

First we consider the population of the hydrogenlike (Rydberg) states with  $n \ge 8$ . From measurements by Kingdon *et al.*<sup>13</sup> and Solov'ev *et al.*<sup>14</sup> we conclude that less than  $2\%$  of the H<sub>2</sub> formed by electron capture by  $H_2^*$  in Mg at 40 keV is in hydrogenlike states with  $n \ge 8$ . This is negligible compared to the observed excited state fractions of  $28\%$  at this energy.

We do not know the relative populations of the  $F^{1}\Sigma_{g}^{*}$  and  $c^{3}\Pi_{u}$  states. However, when we consider the electron-capture process, we conclude from 'Fig. 1 that the  $F^{1}\Sigma_{g}^{+}$  population should be negligible The population distribution of the vibrational levels of  $H_2^*$  ions from which the  $H_2$  beam is prepared is determined by the vibrational distribution of the H<sub>2</sub> gas and by the energy of the electrons in the ion source. Since the energy separation between the first two vibrational levels of  $H_2$  is  $\sim 0.4$  eV, essentially all of the  $H_2$  gas molecules were in the lowest vibrational level. Based on the Franck-Condon principle (i. e. , the time required for an electronic transition is small compared to the nuclear motion of the molecule, so that the initial and final states have almost the same internuclear separation), the  $H_2^*$  ions formed by electron ionization in the source will be almost exclusively in levels  $v \leq 5$ the source will be almost exclusively in levels  $v \le 5$ <br>(see Fig. 1).<sup>15</sup> When these  $H_2^+$  ions capture an electron, we see from Fig. 1 that the range of nuclear separations (not indicated in the figure) defined by the Franck-Condon principle covers most of the  $c^{3}\Pi_{u}$  state but only a small part of the  $F^{1}\Sigma_{g}^{*}$  state. We therefore expect poor overlap with the  $\overline{F}$  state in the capture process. Furthermore, from simple statistical arguments, we would expect a greater probability for capture into a triplet state relative to a singlet state. We therefore exclude the possibility of a significant population of the  $H<sub>2</sub>$  beam being in the  $F^1\Sigma_g^*$  state, and we attribute the experimental observations of excited  $H_2$  to the  $c^3\Pi_u$  state.



FIG. 2. Diagram of the experimental arrangement.

Unfortunately, there exists the nonradiative . mechanism of predissociation in the case of the  ${}^{3}\Pi_{u}$  state (see Appendix A). For the conditions of our experiment approximately one-half of the  ${}^{3}$ II. rotational-state population will be subject to allowed predissociation during their time-of-flight through the apparatus.

#### III. EXPERIMENTAL APPARATUS AND PROCEDURE

#### A. Approach

Figure <sup>2</sup> shows the experimental arrangement. ' The energetic  $H_2$  molecules were produced by electron capture when a momentum-analyzed beam of  $H_2^*$  ions traversed a neutralizing cell of Mg vapor, or  $H_2$  or  $N_2$  gas. (We have made measurements with incident  $H_2^*$ ,  $D_2^*$ , and  $HD^*$  ion beams. When the results are compared at the same velocity, they are the same within the experimental uncertainties. ) Since we are investigating the  $H_2$  molecule, the charged particles emergent from the neutralizer cell were swept out of the beam with an electric field, and the  $H_2^*$  component, detected with a Faraday cup, was used to provide a monitor of the beam intensity.

The analysis of the  $H<sub>2</sub>$  beam was performed by beam attenuation. To this end the neutral beam, consisting of ground-state and excited H and  $H_2$ , traversed a target cell containing  $H_2$  gas. We assume that the H<sub>2</sub> molecules are in either the <sup>1</sup> $\Sigma^*$ ground state or in the  ${}^{3}\Pi_{u}$  excited state (Sec. II). Since the collision cross sections of these two states are different, they attenuated differently upon traversing the target cell. By observing the emerging particles as a function of  $H_2$  gas thickness in the target cell, we were able to deduce the total-loss and ionization cross sections for the  ${}^{3}$ II<sub>u</sub> and  ${}^{1}\Sigma_{g}^{+}$ states as well as the population of these states in the beam. By varying the neutralizer conditions, i.e., the type of neutralizer used  $(Mg, H_2, \text{ or } N_2)$ and its thickness (atoms or molecules/ $\rm cm^2$ ), we were able to maximize the excited-state population or reduce it to an insignificant amount.

In order to increase the energy range of the measurements, two methods were used for particle detection. For method I (see Fig. 2) we employed particle-counting techniques: The particles were detected by CsI(Tl) crystals mounted on photomultipliers. Method I was limited to energies greater than about 10 keV/nucleon; below this energy it was difficult to resolve the pulse-height distribution of the atomic portion of the beam from the molecular portion. For method II a third gas cell (see Fig. 2), held at a constant pressure, was used to strip the fast molecules emerging from the target of an electron; the resulting  $H_2^*$  current was measured with a Faraday cup. This method yields less information than method I but is applicable to total- loss cross-section measurements below 10 keV/nucleon. Using this method we have extended the total-loss cross-section measurements to 1 ke V/nucleon.

#### B. Production of H<sub>2</sub> Molecules

The  $H_2^*$  ions were produced in a PIG (Penning ionization gauge) ion source which could also be operated as an electron-gun source. The ions were accelerated electrostatically and momentum selected by a  $90^\circ$  analyzing magnet; they then passed through a 120-cm-long drift tube before entering the experimental region shown in Fig. 2. Measurements were made with accelerating voltages between 4 and 84 kV. The lower limit was determined by the performance of the detection system and the upper limit by our accelerator.

The accelerator voltage was measured by a highimpedance divider calibrated to  $\sim 1\%$ . The uncertainty in the absolute value of the energy of the particles is estimated to be  $\sim 3\%$ , except for energies less than 2 keV/nucleon, where it is estimated to be  $\sim$  5%. This is based on the analysis of the deviation from linearity of a plot of the voltage necessary to deflect the particles versus the measured value of the accelerator voltage.

Just in front of the experimental region was an electromagnet that could be used to sweep the primary  $H_2^*$  ions out of the beam path, so that the  $H_2$ molecules produced by electron capture by  $H_2$ <sup>+</sup> from the background gas in the drift tube could be detected and subtracted as background. Under typical single-collision conditions in the neutralizer the background neutral production was  $\leq 10\%$ .

The Mg-vapor oven mhich served as the neutralizer cell has been described in Ref. 17. The entrance and exit collimator diameters were 0. 508 and 1.27 mm, respectively. The effective length of the stainless-steel oven was 4. 4 cm, measured from the entrance collimator to the exit collimator. An operating temperature of 665 K produced a Mgvapor thickness of 1.75 $\times$ 10<sup>14</sup> atoms/cm<sup>2</sup>. A typical range of operation was from 616 to 783 K.

Before data mere taken, the oven was outgassed at a temperature higher than that needed to produce the thickest Mg-vapor neutralizer of interest. This was done to ensure both the purity of the vapor and the proper interpretation of the vapor pressure. The temperature of the oven, which must be known in order to determine the Mg vapor pressure, was determined with two chromel-alumel thermocouples (with a zero-degree centigrade reference junction) connected to a potentiometer. To minimize radiative losses, the cell was surrounded by three layers of 0. 25-mm-thick dimpled stainless steel. The calibration of the device was performed by Berkner ' ${\it et\ al.}\; , {}^{17}$  who originally used the oven as a collision cell for the measurement of electron-transfer cross sections of <sup>H</sup> and H' in Mg vapor. Data necessary for the calculations of the vapor pressure were obtained from Hultgren et  $al.$  <sup>18</sup> The assigned standard error in this Mg vapor pressure data is  $\pm 10\%.$ <sup>17</sup>

The  $H_2^*$  component emergent from the neutralizer was deflected electrostatically 20' into a Faraday cup. The signal from the cup was sent to an electrometer whose output was fed to an integrator. This accumulated charge, which is proportional (for a given neutralizer thickness) to the number of neutrals incident on the target cell, was used to monitor the neutral beam.

Since the beam entering the target cell was 35 cm from the monitor Faraday cup, strict alignment was necessary to ensure correlation between the detected particles and the monitor, which were 82 cm apart. Alignment was performed visually with the aid of a telescope. Before taking data, the beam was tuned and maximized by observing the count rate due to the neutral particles. This count rate was then compared, as a function of the tuning parameters, with the electrometer output of the monitor Faraday cup. If the two signals were well correlated to within the uncertainty of visual meter observation, i. e. , exhibited the same rise, plateau, and decrease in signal intensity as a function of the tuning parameters, data were acquired; if not, the beam was retuned. Occasionally realignment was necessary. The position of the beam was observed visually by replacing the Faraday cup with a phosphorcoated glass plate. The observed beam was much smaller than the Faraday cup diameter (2. 54 cm). Permanent magnets were used to prevent electrons from entering or leaving the cup.

The neutral beam traveled 70 cm from the oven exit to the entrance of the differentially pumped target cell (see Fig. 2). The beam entrance and exit collimators were 3. 30 and 4. 31 mm in diameter, respectively, and 1.<sup>28</sup> cm long. The effective length of the cell was 8. 8 cm. This length was measured from the midpoint of the tubular entrance collimator to the midpoint of the tubular exit collimator (changes in the target thickness owing to gas in the surrounding tank of the target cell mere inconsequential at all target pressures). The target cell was equipped with a pair of metal rods,  $\sim 0.5$ cm apart, parallel to the beam line, mhich extended over the entire length of the cell. While measuring the H<sub>2</sub> attenuation, a voltage of  $4 \text{ kV}$  was applied to one of the rods in order to produce an electric field which ensured that the exiting  $H_2$  beam was due only to attenuation, without the contribution of two-step processes, i.e.,  $H_2 \rightarrow H_2^+ \rightarrow H_2$ . While measuring the ionization cross section, the voltage was turned off so that the  $H_2^*$  component was allowed to leave the target cell and be detected. The pressure in the cell was measured by a Barocel capacitance manometer which had been calibrated numerous times over a period of several years by a McLeod gauge and an oil manometer.

#### C. Analysis of  $H_2$  Molecules

### 1. Me thod I: Single -Particle Counting

After traveling 17 cm from the target-cell exit the  $H_2^*$  component of the beam was bent  $20^\circ$  by the electric field between a set of deflection plates and traveled 24 cm further, where it was incident on a 2. 54-cm-diam Csl(Tl) scintillation crystal which was mounted on an Amperex 10-stage XP-1010 photomultiplier tube (see Fig. 2). The neutral beam traveled 20 cm after leaving the deflector plates and was intercepted by a similar arrangement. The photomultiplier tubes were not cooled since upon cooling no significant increase in energy resolution mas observed. The photomultiplier signal was amplified and shaped, discriminated, and finally scaled. A 400-channel pulse-height analyzer was used to set the upper and lower levels of the discriminators. A typical pulse-height spectrum, showing the resolution of the neutral components of the beam, is shown in Fig. 3.

Since the scintillator response (the number of photons produced per incident energetic particle) is proportional to the amount of energy transmitted



FIG. 3. Pulse-height spectrum obtained with a CsI(Tl) scintillation crystal for the neutral collision products of a 60-keV  $H_2^*$  beam.

to the crystal during the impact of the energetic particle, two atoms, each of energy  $\frac{1}{2}E$ , whose difference in impact time is considerably shorter than the characteristic decay time of the crystal  $(-0.6\times10^{-6}$  sec), would be interpreted as a single particle of energy  $\nu E$ . Hence, in the present experiment, where the maximum possible difference in the impact time of the two H atoms produced by dissociation of H<sub>2</sub> in the target cell is  $\sim 5 \times 10^{-9}$  sec, two H atoms could not be distinguished from an  $H<sub>2</sub>$ molecule.

The <sup>H</sup> atoms may, however, be spatially distinguishable by use of a low-transparency mesh. Assuming a random orientation of the  $H_2$  internuclear axis with respect to the beam line and an isotropic angular distribution for the H atoms produced by dissociation, we have calculated for the case which produces the smallest scatter in the present experiment (42 keV/nucleon and  $\sim$  2-eV dissociation energy) that, at the detector, the <sup>H</sup> atoms from dissociation in the target are uniformly distributed over a circular region with a 2. 2-mm radius. The mesh, which had a transparency of  $\sim 1\%$ , consisted of four layers of 22% transparent 2000-line/in. Cu Electromesh which had square apertures  $6.35\times10^{-3}$  mm on a side. Clearly, the grid size is significantly smaller than the majority of the H atom separations. The transmission was measured by using an H-atom beam. In order to identify the  $H_2$  molecules from the  $(H + H)$  signal, the mesh was placed in front of the neutral detector. With the mesh in the beam line the probability of obtaining a full energy pulse from two simultaneous H atoms is  $\sim 10^{-4}$ , while the probability of obtaining a full energy pulse from a  $H_2$ molecule is  $\sim 10^{-2}$ .

For each species of the beam detected  $(H_2^+, H^+,$  $H<sub>2</sub>$ , H), two sets of scalers were used to correct for random noise. By employing an incident  $H_2$ <sup>+</sup> beam, chopped at a frequency of 4. 0 Hz by applying voltage to a set of steering deflector plates in the accelerator region, one set of scalers recorded the beam plus the noise and the other set recorded the beam-off signal (i. e. , noise only), which was always much less than that of the beam-on signal. A set of timing scalers was used to determine the

beam-on and -off counting times. Subtraction of the two time-normalized counts yielded the counts produced by the beam.

For this method, the ion source was operated in the electron-gun mode to obtain low-intensity, countable beams. Typical counting rates were  $(1-4)$  $\times$ 10<sup>3</sup> counts/sec.

#### 2. Method II: Faraday Cup

Method II for analyzing the particles (refer to Fig. 2) was used to obtain low-energy ( $E \le 10 \text{ keV/nu}$ cleon) total-loss cross sections. Ionization cross sections and excited-state populations cannot be obtained by this method.

This method required a third gas region (the stripper), which contained  $H_2$  gas held at a constant pressure, to ionize the  $H_2$  molecules transstant pressure, to folling the  $r_2$  molecules trans-<br>mitted through the target. The resulting  $H_2^*$  bean exiting from the stripper was proportional to the  $H_2$ beam transmitted through the target and was deflected to a Faraday cup. The resulting signal was fed to an electrometer whose output was sent to an integrator.

For this method the ion source was operated in the PIG mode to obtain higher beam intensities, measurable by a Faraday cup. Typical  $H_2^+$  currents measured by the final Faraday cup ranged from  $10^{-13}$ to  $10^{-10}$  A.

#### IV. DATA ANALYSIS

A. Total-Loss Cross Sections and Excited Fractions (Method I)

For this measurement an electric field of about 8 kV/cm was applied transverse to the beam within the target cell in order to deflect the charged collision products from the beam as they formed. We neglect excitation and deexcitation collisions (see Appendix B). The equations that describe the surviving  $H_2$  molecules are

$$
F(\pi_t) = [1 - f(\pi_n)] e^{-\sigma_t \pi_t} , \qquad (1a)
$$

$$
F^{\ast}(\pi_t) = f(\pi_n) e^{-\sigma \hat{t} \pi_t} \quad , \tag{1b}
$$

where  $F(\pi_t)$  is the number of ground-state  $\text{H}_{2}\, ({}^1\Sigma_g^{\texttt{+}}\,$ molecules in the beam, after traversing a target thickness  $\pi_t$ , per H<sub>2</sub> molecule incident on the target cell;  $F^*(\pi_t)$  is the number of excited  $H_2(^{3}\Pi_u)$  molecules in the beam, after traversing a target thickness  $\pi_t$ , per H<sub>2</sub> molecule incident on the target cell;  $f(\pi_n)$  is the fraction of the incident H<sub>2</sub> molecules in the  ${}^{3}$ II<sub>u</sub> state, produced by electron capture by  $H_2$ <sup>+</sup> ions in a neutralizer of thickness  $\pi_n$ ; and  $\sigma_t$ ,  $\sigma_t^*$  are the total-loss cross sections for the  ${}^{1}\Sigma_{g}^{*}$  (ground) and  ${}^3\Pi_u$  states.

The detector system is not capable of distinguishing between the two classes of molecules; therefore we measured the sum  $y(\pi_t)$  =  $F(\pi_t)$  +  $F^*(\pi_t)$  for several target thicknesses  $\pi_t$ . Figure 4 shows the quantity  $y(\pi_t)$  versus target thickness for a 50-keV



FIG. 4. Observed attenuation of a beam of  $D_2$  molecules in an  $H_2$  target by method I. The molecules were produced in a Mg neutralizer (thin target) by electron capture by  $50 - keV D_2^*$ . The solid line is the result of a least-squares fit of the data to the sum of two exponentials. Curve A is an extrapolation of the thick-target asymptote; curve B is the difference between the data and curve A. Solid circles are experimental points; open circles are. derived points.

 $D<sub>2</sub>$  beam produced in a thin-target Mg neutralizer (i. e. , single-collision conditions applied) by electron capture by  $D_2^+$  ions. The solid line is the result of a least-squares fit of the data to  $y(\pi_t)$ . The nonlinear portion is due to the presence of  $H_2(^3\Pi)$ molecules which attenuate faster than ground-state molecules  $(\sigma_t^* > \sigma_t)$ . As the target thickness increases the attenuation becomes linear on the semilogarithmic plot, reflecting the loss of essentially all  ${}^{3}$ II<sub>u</sub> molecules in the target cell and the transmission of only  ${}^{1}\Sigma_{g}^{+}$  molecules. By subtracting the extrapolated portion (curve A} of the linear region from the data we obtain the lower straight line (curve B) which is the  $H_2(^{3}II_u)$  attenuation. The straight lines are represented by the equations

$$
\ln F(\pi_t) = -\sigma_t \pi_t + \ln[1 - f(\pi_n)] \text{ (curve A)}, \quad (2a)
$$

$$
\ln F^*(\pi_t) = -\sigma_t^* \pi_t + \ln f(\pi_n) \qquad \text{(curve B)} \qquad (2b)
$$

The cross sections are therefore easily obtained from the slopes as

$$
\sigma_t = -\Delta \ln F(\pi_t) / \Delta \pi_t \tag{3a}
$$

$$
\sigma_t^* = -\Delta \ln F^*(\pi_t) / \Delta \pi_t \tag{3b}
$$

Also, the intercept of curve B yields the fraction of the beam in the  ${}^{3}\Pi_u$  state, since from Eq. (1b) we have

$$
F^*(0) = f(\pi_n) \tag{4}
$$

When the thickness of the Mg-vapor neutralizer

was increased to large values  $(\pi_n \gtrapprox 1 \times 10^{16} \text{ atoms})$ cm<sup>2</sup>) or when  $H_2$  or  $N_2$  gas was used as a neutral izer, only a simple exponential attenuation was observed, i.e.,  $F^*(0) = 0$ . The decrease in the <sup>3</sup>II. component with increasing Mg-vapor- neutralizer thickness reflects its larger total-loss cross section. In the case of the  $H_2$  and  $N_2$  gas neutralizers, the absence of detectable  ${}^{3}$ II<sub>u</sub> molecules is due to the large ground-state electron-capture process.

When the  $H_2$  beam contained no  ${}^{3}H_2$  component, the resulting simple-attenuation curve had the same slope as curve A of Fig. 4, as expected. This enabled us to get an independent measurement of  $\sigma_t$ .

### B. Total-Loss Cross Sections (Method II)

The H<sub>2</sub> beam transmitted through the target cell was passed through the stripper where a fraction of it was ionized  $(H_2 + H_2^*)$ . The  $H_2^*$  current was integrated for a preset accumulation of charge at the monitor Faraday cup. The collected charge  $Q$  can be expressed as

expressed as  
\n
$$
Q(\pi_t) = \alpha_1 \left[1 - f(\pi_n)\right] e^{-\sigma_t \pi_t} + \alpha_2 f(\pi_n) e^{-\sigma_t^* \pi_t} , \qquad (5)
$$

where  $\alpha_1$  and  $\alpha_2$  are constants for a given stripper thickness and reflect the efficiency of producing  $H_2$ <sup>+</sup> ions from  $H_2({}^1\Sigma_g^*)$  and  $H_2({}^3\Pi_u)$  molecules. The measurement was performed by observing the attenuation of  $Q(\pi_t)$  for several target cell thicknesses  $\pi_t$ . Figure 5 shows the quantity  $Q(\pi_t)$  (in arbitrary units vs  $H_2$ -target thickness for a  $D_2$ <sup>+</sup> beam produced in a thin Mg neutralizer  $(8.5 \times 10^{13} \text{ atoms/cm}^2)$  and transmitted through the  $H_2$  target cell. The nonlinearity of the curve in Fig. 5 is larger than the nonlinearity produced by direct observation of the  $D_2$  molecules (method I) under similar conditions (cf. Fig. 4). This is due to the fact that the cross section for  $D_2^*$  production from  $D_2(^3\Pi_u)$  is larger than that from  $D_2({}^1\Sigma_g^*)$ , i.e.,  $\alpha_2 > \alpha_1$ . Without a knowledge of  $\alpha_1$  and  $\alpha_2$ , we cannot determine  $f(\pi_n)$ , but we still are able to determine  $\sigma_t$  and  $\sigma_t^*$ . The analysis of the data to yield values for  $\sigma_t$  and  $\sigma_t^*$  is identical to method I.

#### C. Ionization Cross Sections

With our experimental arrangement we were able to study one of the processes that contribute to the total loss of  $H_2$  molecules: collisional ionization,  $H_2 + H_2^*$ . For this measurement the transverse electric field in the target cell was turned off so that the  $H_2^*$  component in the beam owing to the ionization of  $H_2$  could be measured. Detection of the emerging  $H_2^*$  ions and  $H_2$  molecules was performed by single-particle counting techniques (method I).

Under thin-target conditions the collision relation governing the  $H_2^+$  component  $F^+$  in the beam, owing to the ionization of H<sub>2</sub> ( ${}^{1}\Sigma_{g}^{+}$ ) and H<sub>2</sub> ( ${}^{3}\Pi_{u}$ ) in the target cell, may be written as



FIG. 5. Observed attenuation of a beam of  $D<sub>2</sub>$  molecules in an  $H_2$  target by method II. The figure is a plot of the collected  $D_2^*$  charge Q measured by the Faraday cup after the stripper vs H<sub>2</sub> target gas thickness  $\pi_t$ . The  $D_2$ <sup>+</sup> ions were produced by ionization of  $D_2$  in the H<sub>2</sub> target (stripper)  $\sim 2 \times 10^{14}$  molecules/cm<sup>2</sup> thick. The D<sub>2</sub> molecules were produced by electron capture of 62-keV  $D_2^*$  ions in a Mg-vapor target (neutralizer) of thickness  $8.5 \times 10^{13}$  atoms/cm<sup>2</sup>. Curve A is an extrapolation of the thick-target asymptote; curve B is the difference between the data and curve A. Solid circles are experimental points; open circles are derived points.

$$
\frac{d}{d\pi_t} F^*(\pi_t) = (\sigma_{H_2^*}) F(\pi_t) + (\sigma_{H_2^*}^*) F^*(\pi_t) ,
$$
 (6)

where  $\sigma_{H_2^+}$  and  $\sigma_{H_2^+}^*$  are the cross sections for ionwhere  $\sigma_{H_2^+}$  and  $\sigma_{H_2^+}^*$  are the cross sections for<br>ization  $(H_2 \rightarrow H_2^+$  and  $H_2^* \rightarrow H_2^*)$ . In terms of the fraction of excited  $H_2$  molecules, f, this equation yields

$$
F^{\star}(\pi_t) = \{ (\sigma_{H_2^*}) [1 - f(\pi_n)] + (\sigma_{H_2^*}^*) f(\pi_n) \} \pi_t .
$$
 (7)

Figure 6 is a plot of  $F^*(\pi_t)$  vs  $\pi_t$ .

When the incoming  $H_2$  molecules contained no observable fraction of  ${}^{3}$  $\Pi_u$  states, the slope of the linear relation between  $\ddot{F^*}(\pi_t)$  and  $\pi_t$  yielded  $\sigma_{\rm H_{2^*}}$ (curve B). When the  $H_2$  beam did contain an observable fraction of  ${}^{3}\Pi_u$  molecules (thin Mg neutralizer), the same technique yielded the composite cross section

$$
(\sigma_{\mathrm{H}_2} \ast) \left[1 - f(\pi_n)\right] + (\sigma_{\mathrm{H}_2}^{\ast} \ast) f(\pi_n)
$$

[curve A: see Eq. (7)]. Using the value of  $f(\pi_n)$ from independent attenuation measurements (see Sec. IVA) for the same neutralizer conditions, the cross section for ionization of the  ${}^{3}$ II<sub>u</sub> state,  $\sigma_{\mathrm{H_2^+}}^*$ , can be determined.

#### D. Particle Yields

The present section describes the measurement of the particle yields due to energetic  $H_2$ <sup>+</sup> ion collisions with Mg vapor. For this measurement the Mg oven was moved to the target cell position. This was done in order to ensure complete collection of the breakup products. (No attempt was made to distinguish between ground-state and excited  $H_2$ .) The exit aperture of the oven was enlarged until there was no observable change in the measured beam components for our thickest target. The final oven collimation was 0. 33-mm entrance diam and 2. 8-mm exit diam. The diameter of the particle detectors (25. 4 mm) was larger than the maximum possible beam spreading which is defined by the oven collimation. For this measurement the Mg-vapor thickness was varied from  $2\times10^{14}$  to  $1\times10^{16}$  atoms/cm<sup>2</sup>.

The measurements were performed by passing  $H_2^*$  ions through the oven for several Mg-vapor thicknesses and counting the exiting  $H_2^*$ ,  $H^*$ ,  $H$ , and  $H<sub>2</sub>$  components of the beam. This enabled us to obtain the yields of the reactions

$$
H_2^* + Mg \rightarrow H_2 - e + \cdots, \qquad (8)
$$

$$
H_2^* + Mg \rightarrow H + H - e + \cdots , \qquad (9)
$$

$$
H_2^+ + Mg \rightarrow H + H^+ + \cdots, \qquad (10)
$$

$$
{H_2}^* + Mg \to H^* + H^* + e + \cdots \qquad (11)
$$

(The H<sup>-</sup> yield was measured at 20 keV/nucleon and was found to be negligible. We expect the H<sup>-</sup> yield to peak at energies less than those used in the present work. )

In order to separate the  $H_2$  and  $H + H$  contributions, the neutral particles had to be detected once with the mesh in and once with it out. From these measurements we were able to account for the total



FIG. 6. Production of  $D_2^+$  ions from the ionization of 60-keV  $D_2$  molecules in collision with a H<sub>2</sub> gas target. The  $D_2$  molecules were produced by electron capture by  $D_2^*$  ions in a Mg-vapor neutralizer. Line A connects the experimental data using a thin Mg-vapor neutralizer  $(\pi_n < 1 \times 10^{14} \text{ atoms/cm}^2)$ ; line B connects the experimental data using a thick Mg-vapor neutralizer  $(\pi_n \geq 1)$  $\times 10^{16}$  atoms/cm<sup>2</sup>).



FIG. 7. The observed dependence on the Mg-vaportarget thickness  $\pi_n$  of the particle yields from the collisional breakup of  $D_2^*$ . The beam energy was 10 keV/ nucleon.  $\blacksquare$ ,  $D_2^*$  + Mg  $\rightarrow$  D<sub>2</sub> – e +  $\cdot \cdot$ , D + D – e + **0**,  $D_2^* + Mg \rightarrow D_2 - e + \cdots$ ;  $\triangleright$ ,  $D_2^* + Mg \rightarrow D + D^* + \cdots$ ;  $\blacklozenge$ ,  $D_2^*$ +Mg  $\rightarrow$  D<sup>+</sup>+D+e+ $\cdot \cdot \cdot$ . The lines drawn through our points are to guide the eye and have no other significance.

beam and calculate the individual beam-component fractions.

### E. Error Analysis

For all experimental results of this work, the assignment of uncertainty is based on the reproducibility of the data occurring in a number of runs taken months apart, on the standard deviation of the least-squares fit of the data, and on systematic experimental errors. The long-term reproducibility of the ground-state measurements was  $\pm 5\%$  with a standard deviation of the least-squares fit of the data of  $\pm 3\%$ . This was true in both methods I and II, although for  $E \ge 20$  keV/nucleon the results using the stripper-Faraday-cup combination for detection (method II) were consistently  $9\%$  lower than the results using single-particle counting techniques (method I). A possible explanation for the difference in the results is that the vibrational population distribution of the ions was not the same in the two cases because of the different modes of ion-source operation.

For the excited-state measurements, the longterm reproducibility was  $\pm 17\%$ , and the standard deviation of the least-squares fit of the data varied from  $\pm 5$  to  $\pm 20\%$ . Statistical errors in counting were always less than  $4\%$  and in all but a few cases less than  $1\%$ . Possible systematic experimental uncertainties resulting from pressure and targetlength measurements are estimated to total about  $+ 7\%$ .

The effect of collimator interceptions and gas background were determined to be negligible since the fraction of the  $H<sub>2</sub>$  beam surviving with no target gas was always  $>0.998$ . Other possible sources of error which are determined to be inconsequential in this work are target and source gas contamination and detector signals of unknown origin that were modulated with the beam.

To experimentally investigate the possibility of the presence of unknown systematic errors, we measured the well-known single-electron-capture cross section for energetic protons in collision with  $H<sub>2</sub>$  gas in the (10-30)-keV energy range. Our measurements agree to within  $\pm 5\%$  with those of Stier and Barnett.<sup>19</sup> We also measured the total-loss cross section for energetic hydrogen atoms passing through H, gas. These results also agreed to within  $± 5\%$  with those of Ref. 19.

The estimated absolute uncertainty in the present results ranges from  $\pm 18$  to  $\pm 25\%$  for the excitedstate cross sections and populations (those that survive for  $t \stackrel{\geq}{\sim} 0.5$   $\mu$ sec), is  $\pm 10\%$  for the ground-state cross sections, and ranges from  $\pm 10$  to  $\pm 15\%$  for the particle-yield measurements. The confidence level associated with these uncertainties is estimated to be 60%. The assigned absolute uncertainty for each data point may be found with the tabulated data in Sec. V.

#### V. RESULTS AND DISCUSSION

#### A. Beam Populations

### 1. YieLds of Atoms and Molecules from Collisions of  $H_2$ <sup>+</sup> Ions in Mg Vapor

The data plotted in Fig. 7 show the variation with Mg-vapor thickness of the experimental yields of reaction products due to 10-keV/nucleon  $D_2^*$  ion collisions with Mg vapor. We see that at  $10 \text{ keV}/$ nucleon, with  $\pi_n \sim 2 \times 10^{15}$  atoms/cm<sup>2</sup>, approximately  $90\%$  of the incident  $H_2^*$  ions have been converted to neutrals, and the reaction  $H_2^+$  + Mg  $\rightarrow$  H + H –  $e$  +  $\cdots$ is the dominant inelastic process.

In Fig. 8 we illustrate the variation, with Mg-vapor thickness and with  $H_2^+$  particle energy, of the experimental  $H_2$  yields  $\mathfrak F$  from  $H_2^*$  ion collisions with Mg vapor. The maximum  $H_2$  yields occur around  $10^{15}$ atoms/ $\rm cm^2$  for all energies measured and increase with decreasing energy. Also shown in the figure are the results of Kingdon et al. at 20 keV/nucleon.<sup>20</sup>

No  $H_2$  or  $D_2$  ions were observed. Analysis of the yields allows us to assign an upper bound of  $10^{-21}$  $\text{cm}^2/\text{atom}$  to the cross section for double electron capture into  $H_2^-$  states with lifetimes longer than  $\sim$  10<sup>-7</sup> sec. The H<sup>-</sup> yield at 20 keV/nucleon was observed to be less than 1% for  $\pi_n \stackrel{<}{\scriptstyle \sim} 10^{15}$  atoms/cm<sup>2</sup>.

### 2. Yields of  ${}^{3}\Pi_u$  Molecules from Collisions of  $H_2^+$ Ions in Mg Vapor

The results shown in Fig. 9 illustrate the variation with Mg-vapor thickness and with  $H_2$ <sup>+</sup> particle



FIG. 8. Fraction of incident  $H_2^*$  ions converted to  $H_2$  molecules vs Mg-vapor thickness  $\pi_n$ . Solid symbols, present work; open symbols, Ref. 20;  $\times$ , 7.5 keV/ nucleon;  $\bullet$ , 10 keV/nucleon; +, 15 keV/nucleon;  $\triangle$ ,  $\blacktriangle$ , 20 keV/nucleon;  $\bullet$ , 25 keV/nucleon;  $\blacksquare$ , 30 keV/nucleon. (For  $E < 20$  keV/nucleon the incident projectile was  $D_2^*$ .) The lines through the points are drawn in to guide the eye and have no other significance.

energy of the measured fraction  $f$  of the H<sub>2</sub> molecules formed in the  ${}^{3}$ II<sub>u</sub> state by electron capture by  $H_2^*$  ions in Mg vapor. We see from Fig. 9 that, for  $\pi_n \leq 2 \times 10^{14}$  atoms/cm<sup>2</sup>, f is independent of target thickness, i. e. , single-collision conditions exist, and it is a slowly decreasing function of energy.

From calculations for electron capture into the  $n=2$  level by protons from Mg, <sup>7</sup> we expect up to 50% of the electron capture by  $H_2^*$  from Mg to be into the  $n = 2$  H<sub>2</sub>(<sup>3</sup>II<sub>u</sub>) state at the lower end of our energy range. For the broad rotational population distribution anticipated for our ion source, we also estimate that approximately one-half of the  ${}^{3}$ II<sub>u</sub> molecules formed are susceptible to predissocia-

tion (Sec. II). Our thin-target results for  $f$  (which vary from 0.36 at 11.2 keV/nucleon to 0.23 at 35 keV/nucleon) are consistent with these estimates.

When the results for  $f$  were compared at equal  $H_2^*$ ,  $D_2^*$ , and HD<sup>\*</sup> velocities (at 15 and 20 keV/ nucleon), no differences were observed, within experimental uncertainties. Since the nuclear symmetry associated with  $H<sub>2</sub>$  and  $D<sub>2</sub>$  breaks down for the heteronuclear isotopic molecule HD, the equivalence of the results at equal velocities seems to indicate that the interaction between the electronic and rotational motion is small. In this case the electronic dipole selection rules for homonuclear molecules are very good approximations for heteronuclear molecules.  $11,12$ 

We also looked for a  ${}^{3}\Pi_u$  component in an H<sub>2</sub> beam prepared by  $H_3^+$  dissociation in thin-target Mg vapor. The  $H_2$  molecules were formed via the reactions

$$
{H_3}^* + Mg \rightarrow \begin{cases} H_2 + H - e + \cdots \\ H_2 + H^* + \cdots \end{cases}
$$

This measurement was performed at 7.5 keV/nucleon for two different Mg-vapor thicknesses, 0. 9  $\times10^{14}$  and  $2\times10^{14}$  atoms/cm<sup>2</sup>. No  $\text{H}_{2}(^{3}\text{II}_{\mu})$  molecules were observed.

#### **B.** Cross Sections

# 1. Total-Loss Cross Sections of  ${}^{1}\Sigma_{\epsilon}^{+}$  and  ${}^{3}\Pi_{u}$  H<sub>z</sub> Molecules in Collisions with  $H<sub>2</sub>$  Gas

The total-loss cross sections for the excited state  $\sigma_t^*$  and for the ground state<sup>21</sup>  $\sigma_t$  are given in Table I and in Fig.  $10.$  McClure<sup>22</sup> has obtained the ground-state total-loss cross section at  $5 \text{ keV}$ nucleon by a very different technique. His result is in excellent agreement with the present results (see Fig. 10).

We are not aware of any theoretical calculations

FIG. 9. Fraction of  $H_2$  molecules that are in the  $c^3\Pi_u$  state vs Mg-vapor thickness  $\pi_n$ .  $\bullet$ , 11.2 keV/nucleon; +, 15 keV/nucleon; ▲, 20 keV/nucleon; ◆,<br>25 keV/nucleon; ■, 30 keV/nucleon; ×, 35 keV/nucleon. (For  $E < 20$  keV/nucleon the incident projectile was  $D_2^*$ .) The lines through the points are drawn in to guide the eye and have no other significance.



TABLE I. Total collisional-loss cross sections (in units of  $10^{-16}$  cm<sup>2</sup>/molecule) for H<sub>2</sub>(<sup>1</sup> $\Sigma_e$ ),  $\sigma_t$ , and H<sub>2</sub>(<sup>3</sup> $\Pi_u$ ),  $\sigma_t^*$ , in H<sub>2</sub> gas. The assigned absolute uncertainties indicated are based on an estimated 60% confidence level. (For  $E < 20$  keV/ nucleon the incident projectile was  $D_2^*$ .)

	$\sigma_t (\pm 10\%)$		$\sigma_t^*$	
Energy (keV/nucleon)	Particle counting (method I)	Faraday cup (method II)	Particle counting (method I)	Faraday cup (method II)
1.0 1.7 3,0		1.70 1.85 2.10		
5.0		2.50		13.3 $\begin{cases} +2.6 \\ -2.3 \end{cases}$
7.5		2.70		
8.7		2.80		13.4 $\begin{cases} +2.3 \\ -2.4 \end{cases}$
11.2		3.10		$14.0 \pm 2.5$
12.2	3.25		13. 0 $\begin{cases} +2.5 \\ -2.3 \end{cases}$	
15.0	3.35	3.20	12.3 $\begin{cases} +2.5 \\ -2.1 \end{cases}$	11.7 $\begin{cases} +2.4 \\ -2.3 \end{cases}$
20.0	3.52		13.5 $\begin{cases} +2.2 \\ -2.6 \end{cases}$	
22.5		3.10		9.3 $\begin{cases} +1.6 \\ -1.5 \end{cases}$
25.0	3.50	3.18	10.4 $\begin{cases} +1.9 \\ -1.8 \end{cases}$	
30.0	3.55	3.15	$9.0 \pm 2.2$	8.2 $\left\{\begin{matrix}+1.8\\-1.4\end{matrix}\right.$
35.0	3.50		8.1 $\begin{cases} +1.8 \\ -1.4 \end{cases}$	
42.0	3.50	3.15		7.6 $\left\{\begin{matrix}+1.8\\-1.6\end{matrix}\right\}$

to compare with the present results.

### 2. Ionization Cross Sections of  ${}^{1}\Sigma_{g}^{+}$  and  ${}^{3}\Pi_{u}$  H<sub>2</sub> Molecules in Collisions with  $H_2$  Gas

One of the cross sections contributing to the total-loss cross section, that for ionization  $(H_2 - H_2^+),$ has also been determined for both the  ${}^{1}\Sigma_{g}^{*}$  state  $\sigma_{H_{2}+}$ and the  ${}^3\Pi_u$  state  $\sigma_{\text{H}_{2+}}^*$  in  $H_2$  gas. The results are presented in Table II and in Fig. 11. For the ground-state ionization cross section we see from Fig. 11 that the experimental results of McClure<sup>22</sup> are in excellent agreement with the present results over the entire energy range investigated. Again, we know of no theoretical calculations to compare with these results.

### VI. SUMMARY AND CONCLUSIONS

The cross sections for the total loss and ionization of  $H_2({}^1\Sigma_g^*)$   $[1 \leq E$  (keV/nucleon)  $\leq 42$ ] and  $H_2({}^3\Pi_u)$ molecules  $[5 \leq E$  (keV/nucleon)  $\leq 42$ ] in collisions with  $H<sub>2</sub>$  gas have been measured. The result of  $\rm{McClure}^{22}$  at 5 keV/nucleon for the total-loss cross section of the  ${}^{1}\Sigma_{s}^{+}$  state is in excellent agreement with the present measurements. The data of Mc-

Clure for the ionization cross section of the  ${}^{1}\Sigma_{\epsilon}^{+}$ state are also in excellent agreement. We have not found any data on collisional cross sections for the  ${}^{3}$ II<sub>u</sub> state to compare with present results.

The four cross sections measured in the pres-



FIG. 10. Total-loss cross sections for collisions of energetic H<sub>2</sub>( $\Sigma_g^*$ ),  $\sigma_f$ , and H<sub>2</sub>( ${}^3$ II<sub>u</sub>),  $\sigma_f^*$  with H<sub>2</sub> gas. Present results: triangles,  $\sigma_t^*$ ; circles,  $\sigma_t$ . Solid symbols, particle counting technique (method I); open symbols, Faraday cup technique (method II). (For  $E < 20$  keV/ nucleon the incident projectile was  $D_2^*$ .) The result of McClure (Ref. 22) at 5 keV/nucleon for  $\sigma_{\boldsymbol{t}}$  is indicate  $by +$ .

ent work  $(\sigma_{H_2^*}, \ \sigma_{H_2}^*, \ \sigma_t, \ \sigma_t^*)$  vary gradually over<br>the energy range of the measurements. The excited-state cross sections range from two to five times larger than the ground-state cross sections. The contribution of the ionization cross section to the total-loss cross section, both for the ground and excited states, increases from  $\sim$  30 to  $\sim$  60% as the energy increases over the present range.

The data agreed, within experimental uncertainties, using  $H_2^*$ ,  $D_2^*$ , and HD<sup>+</sup> ions of equal velocity. This suggests that for the  ${}^{3}\Pi_u$  molecular state strong coupling between the nuclear rotation and the electronic motion does not exist.

We have measured the  $H_2(^{3}\Pi_u)$  yields due to collisions of  $H_2^+$  ions in Mg vapor. From the results it can be shown that 6.5% of the incident  $H_2$ <sup>+</sup> beam can be converted to  $H_2(^3\Pi_u)$  molecules at 11.2 keV/nucleon using an optimized Mg-vapor-target thickness  $\sim$  7 $\times$ 10<sup>14</sup> atoms/cm<sup>2</sup>. This yield is a steeply decreasing function of energy (decreasing to 1. 8% at 20 keV/nucleon) owing to the rapid decrease in the total production of  $H_2$  molecules rather than to a significant change in the fraction of the  $H_2$  molecules in the  ${}^{3}$ H<sub>u</sub> state. The fraction of the H<sub>2</sub> molecules in the  ${}^{3}$ II<sub>u</sub> state varies slowly over the present energy range from  $36\%$  at 11.2 keV/nucleon to  $23\%$  at 35.0 keV/nucleon for Mg-vapor thicknesses less than  $\sim 1 \times 10^{14}$  atoms/cm<sup>2</sup>. We have found no significant  ${}^{3}$ II<sub>u</sub> yields for Mg-vapor-neutralizer thicknesses greater than  $\sim 1 \times 10^{16}$  atoms/cm<sup>2</sup>, nor for  $H_2$  or  $N_2$  neutralizers. These results are consistent with estimates by Hiskes. '

The presence of  $H_2$  ( ${}^{3}H_u$ ) molecules from the dissociation of  $H_3$ <sup>+</sup> ions in Mg vapor has not been observed. Also, no  $H_2^-$  ions were observed from the passage of  $H_2^*$  ions through Mg vapor. This ob-



FIG. 11. Cross sections for the ionization of energetic  $H_2({}^1\Sigma_g^*)$ ,  $\sigma_{H_2}^*$ , and  $H_2({}^3H_u)$ ,  $\sigma_{H_2}^*$ , in collision with  $H_2$  gas. Present results: solid squares,  $\sigma_{\text{H}_2}^*$ , open squares,  $\sigma_{\text{H}_2}$ . (For  $E < 20$  keV/nucleon the incident projectile was  $D_2^*$ .) The results of McClure (Ref. 22) for  $\sigma_{H2}$ + are indicated by solid circles.

TABLE II. Collision cross sections for the ionization of  $H_2({}^1\Sigma_g^*)$ ,  $\sigma_{H_2}^*$ , and  $H_2({}^3\Pi_u)$ ,  $\sigma_{H_2}^{**}$ , in  $H_2$  gas. The assigned absolute uncertainties are based on an estimated 60% confidence level. (For  $E < 20$  keV/nucleon the incident projectile was  $D_2^*$ .) The cross sections, in units of  $10^{-16}$  cm<sup>2</sup>/ molecule, were obtained by method I.

$H_2$ energy (keV/nucleon)	$\sigma_{\text{H}_{2}}^{\text{+}}$ (± 10%)	$\sigma_{\text{H}_2}^*$ .
8, 7	1.12	
11.2	1.26	
12.2	1.37	6.2 $\brace{-1.1}^{+1.2}$
15.0	1.48	6.2 $\begin{cases} +1.5 \\ -1.3 \end{cases}$
20.0	1.70	6.5 $\begin{cases} +1.2 \\ -1.5 \end{cases}$
25.0	2.0	$5.5\begin{cases} +0.9 \\ -1.3 \end{cases}$
30.0	2.1	$4.8 + 1.2$
35.0	2.1	$5.1\ \{-1.1\}$
42.0	2.0	

servation has allowed us to place an upper bound of  $10^{-21}$  cm<sup>2</sup>/atom on the cross section for double electron capture into  $H_2^-$  states with lifetimes longer than  $\sim 10^{-7}$  sec by 7- to 20-keV/nucleon  $H_2^+$ ions in collision with Mg vapor.

As shown in the present work, measurements using  $H_2$  beams that have been prepared from  $H_2$ <sup>+</sup> collisions in Mg vapor are sensitive to the vapor pressure, because this determines the  ${}^{3}$ II<sub>*u*</sub> fraction</sub> in the beam. Such beams must be treated in terms of two states, each of which has a different attenuation cross section. For example, Solov'ev et al.<sup>14</sup> obtained  $H_2^+$  electron-capture cross sections in Mg vapor by a process that involved the conversion of  $H_2$  molecules to  $H_2$ <sup>+</sup> ions in a He stripper cell. Without any information about the existence of  ${}^{3}$ II<sub>n</sub> molecules in the beam, they analyzed their data by using only ground-state-ionization cross sections. Estimates based on the results of the present experiment show that the  $H_2^+$  electron-capture cross sections obtained in Ref. 14 could be too large by as much as a factor of 2.

We expect that  $H_2$  beams, prepared by passage of  $H_2^+$  ions through other low-ionization-potential vapors such as Li, Cs, Na, and K, would contain large fractions of  $H_2(^3\Pi_u)$  molecules.

More detailed results for the yields of  $H<sub>2</sub>$ , H, and  $H^*$  from collisions of  $H_2^*$  ions in Mg vapor and the use of  ${}^{3}$ II<sub>u</sub> beams in neutral-injection controlledfusion experiments will be reported elsewhere.

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### APPENDIX A: EXCITED STATES OF  $H_2$  MOLECULE

In this appendix we survey the pertinent electronic excited states of  $H<sub>2</sub>$  to determine which excited states have lifetimes greater than  $\sim 10^{-6}$  sec. For the present discussion we can divide the excited states into three groups: (i) highly excited states  $(n\geq8)$ , (ii) intermediate-excited states  $(n<sup>5</sup>8)$ , and (iii) low-lying states having excitation energies less than  $\sim$  12. 5 eV.

First, we address ourselves to groups (i) and (ii). Since the excited electrons in H and  $H_2$  move in approximately the same potential, they will have approximately the same radiative lifetimes. From the theoretical work of Hiskes, Tarter, and Moody,  $^{23}$  we find that the atomic hydrogen states with  $n \geq 8$ , group (i), have lifetimes which are greater than  $\sim 10^{-6}$  sec. For group (ii) it can be shown that these intermediate states decay in times shown that these intermediate states decay in tin<br>shorter than ~  $10^{-6}$  sec to lower states,  $^{23}$  and further, that no anomalies resulting in longer than<br>normal lifetimes are expected.<sup>11</sup> normal lifetimes are expected.<sup>11</sup>

We now turn our attention to group (iii) (the  $H_2$ ) excited states shown in Fig. 1 comprise this group) Lichten $^{24}$  first verified the existence of metastable molecular hydrogen by use of the atomic-beam magnetic-resonance technique. Specifically, he found that the lowest vibrational level  $(v = 0)$  of the  $c^{3}\Pi_{u}$  state of H<sub>2</sub> is metastable, i.e., does not radiatively decay by electric dipole transitions. Recently, Brooks et al.<sup>25</sup> have found experimental evidence which leads them to conclude that there is at least one additional long-lived level of the  $c^{3}$ II<sub>n</sub> state other than the  $v=0$  level. They conclude that the level is most likely, but not conclusively, the  $v = 1$  level. The Kronig selection rules for an allowed radiative transition in a diatomic system<sup>26</sup> show that the  $c \, {}^3\Pi_u$  (v = 0 level), although not able to decay via electric dipole transitions, can decay radiatively by magnetic dipole, electric quadrupole, or higher moment emissions to the  $b$   $^3\Sigma_u^*$  state [the  $b$   $^3\Sigma_u^*$  state is repulsive and dissociates into two ground-state H atoms in  $\sim 10^{-14}$ sec (Ref. 24)]. The radiative lifetime for these transitions has been estimated by Freis and Hiskes<sup>27</sup> to be  $10^{-3}$  sec for  $H_2$ , and recently Johnson<sup>28</sup> has obtained experimental values of  $\sim 10^{-3}$  sec for  $H_2$ ,  $D_2$ , and HD. Freis and Hiskes<sup>27</sup> have also calculated the electric-dipole radiative lifetimes for the

 $c \nvert^3 \Pi_u$ ,  $v > 0$ , levels decaying to the only allowable final state,  $a^3 \Sigma_g^*$  [this state has been calculated to decay to the  $b^3 \Sigma_g^*$  state in ~10<sup>-8</sup> sec (Ref. 29)]. Their calculated times are about  $10^{-4}$  sec or longer for all vibrational levels. Thus we conclude that all vibrational states of  $c \, {}^{3} \Pi_u$  have radiative lifetimes greater than  $10^{-4}$  sec.

We now turn our attention to predissociation, the nonradiative transition process in diatomic molecules.<sup>30</sup> The  $c^3\Pi_u$  state is susceptible to perturbations from the repulsive  $b^3\Sigma_u^*$  state when the potential energy functions of these states come close enough together so that an overlapping of wave functions exists and causes a nonzero matrix element for predissociation. There are two types of predissociation of the  $c \, {}^3\Pi_u$  state: allowed, induced by rotational-electronic perturbations; and forbidden, induced by spin-orbit and spin-spin couplings. The selection rules governing predissocia- $\text{tion}^{\{1\}}$  indicate that, although there are many states close to the  $c \, {}^{3}\Pi_u$  state, predissociation can take place only via the  $b^3\Sigma_u^*$  state.

As has been pointed out by Kronig,  $31$  under very strong coupling conditions allowed predissociation<br>lifetimes could be as short as  $10^{-11}$  sec. Lichter lifetimes could be as short as  $10^{-11}$  sec.  $\,$  Lichten in order to explain his experimental observations, has concluded that for the  $c \, {}^3\Pi_u$  state allowed predissociation lifetimes are  $\sim 10^{-9}$  sec. Herzberg, 33 based on experiments dealing with the absorption spectrum in the visible region of  $H_2$  excited by a flash discharge, has estimated an upper bound of  $\tau$ = 0.3  $\mu$ sec on the lifetime for allowed predissociation of the  $v=2$  level of  $c \, {}^3\Pi_u$ . Also, owing to the change in proximity of the  $b^3\Sigma_d^+$  and  $c^3\Pi_u$  potential energy states with changes in internuclear separation, we expect the predissociation lifetime to decrease with increasing vibrational excitation of the  $c^{3}$ II<sub>u</sub> state (see Fig. 1). Herzberg<sup>33</sup> has experimentally verified this behavior. Therefore, based on this available information, we bound the allowed predissociation lifetime as follows: nanoseconds  $\leq \tau$  stenths of microseconds.

A survey of the selection rules for allowed predissociation shows that half of the rotational states of the  $c \, {}^3\Pi_u$  molecule are susceptible to this loss mechanism.  $^{11}$ 

rinally, Bottcher<sup>34</sup> and Chiu<sup>35</sup> have performe theoretical calculations on the forbidden predissociation lifetimes and concluded that they are of the order of  $10^{-3}$  sec, while Lichten has experimentally measured lifetimes ranging from  $0.1 \times 10^{-3}$  to 0.5  $\times 10^{-3}$  sec.  $^{32}$ 

Hence, we conclude that all the levels of the  $c^3\Pi_u$ state have radiative lifetimes longer than  $\sim 10^{-6}$  sec. but half of their rotational states are likely to undergo allowed predissociation within this time.

We now consider the remaining low-lying excited states of H<sub>2</sub> (see Fig. 1). The  $a^3\Sigma_g^*$  state has been

 $\stackrel{\sim}{{\mathsf{S}}}$  shown to decay to the  $b\ ^3\Sigma_u^*$  state in ~10  $^8$  sec.  $^{29}$ Also, the  $B^1\Sigma_u^+$  and the  $C^1\Pi_u$  states have both been shown to decay by allowed transitions in  $(8 \pm 2) \times 10^{-10}$ shown to decay by anowed transitions in  $(6 \pm 2) \times 10^{-10}$  sec, respectively.<sup>36</sup> This is not the case, however, for the  $n=2 \frac{1}{2} \sum_{\ell}^{\infty}$  state. Davidson<sup>37</sup> has studied this state and computed the potential curve. He finds the potential function to contain two minima: the  $E^{1}\Sigma_{g}^{+}$  state at  $R = 1.0 \text{ Å}$ , contain two infirmally the E  $Z_g$  state at  $R = 1.6$  K,<br>and the  $F^{1}\Sigma_g^*$  state at  $R = 2.27$  Å. Wolniewicz<sup>38</sup> has calculated the transition probabilities for the  $F^{1}\Sigma_{\sigma}^{+}$  $B^{1}\Sigma_{u}^{+}$  transition and finds approximately half the transitions of the  $F$ - $B$  band to have lifetimes greater than  $10^{-5}$  sec. For comparable transitions of the  $E$ -B band we conclude, based on Wolniewicz's calculation of the band strengths, that the lifetimes  $\alpha$  are  $\leq 10^{-8}$  sec.

In summary, we conclude that the  $c \, {}^3\Pi_u$  and  $F \, {}^1\Sigma^*_e$ states and the  $n \geq 8$  states have lifetimes long enough to allow them to traverse typical apparatus lengths and must be considered in the interpretation of our experimental results.

#### APPENDIX B: EFFECTS OF COLLISIONAL EXCITATION AND DEEXCITATION OF THE PRESENT MEASUREMENTS

The analysis of the  $H_2$  attenuation curves in Sec. IV A is based on the assumption that the loss processes remove  $H_2$  molecules from the beam. This, of course, is not true for collisions which shuffle the populations of the ground and  ${}^{3}$  $\Pi_u$  state, i.e., excitation from  ${}^{1}\Sigma_{g}^{*}$  to  ${}^{3}\Pi_{u}$  or deexcitation from  ${}^{3}\Pi_{u}$ to  ${}^{1}\Sigma_{r}^{*}$ . Both of these processes require a change in spin, which can be achieved only in an exchange collision in which one electron is ejected and another captured. This is an unlikely process in our energy range.

The fact that excitation and deexcitation collisions do not affect the attenuation analysis in the case of triplet excited states has been demonstrated by Gilbody et al.<sup>39</sup> for He  $(2<sup>3</sup>S)$ . Since the effect of the above processes should be a function of the ionization potential of the target gas used,

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the measured value of the excited-state population should also depend on the target. Gilbody et al.  $39$ used a wide variety of gases with no observable change in the metastable excited-state populations. They concluded that over their energy range  $(7.5 87.5 \text{ keV/nucleon}$  collisional deexcitation of He  $(2<sup>3</sup>S)$  was not important.

Although we did not carry out a systematic study of the  $c \, {}^3\Pi_u$  fraction in the  $H_2$  beam as a function of target gas, one other target was used so as to experimentally convince ourselves of the above arguments. The vapor of  $C_6H_6$  was chosen as the target, because its ionization potential is less than the excitation energy of the  $H_2(^{3}\Pi_n)$  state (9.6 and 11.9) eV, respectively), thus permitting Penning Ionization to occur. That is, the reaction  $H_2(^3\Pi_u)+C_6H_6$  $-H_2({}^1\Sigma_g^*)$  +  $(C_6H_6)^*$  + e is energetically possible; this greatly increases the available final-state phase space and so enhances the deexcitation probability.

If we are unable to observe a change in the measured value of the  ${}^{3}\Pi_{u}$  fraction by using  $C_{6}H_{6}$ , we may assume that within the accuracy of the measurements there are no effects occurring which our detectors are not capable of "seeing. " No change in the  ${}^{3}$ II<sub>u</sub> fraction was observed at 22.5 keV/nucleon.

As part of this investigation we have also measured the total-loss cross sections in  $C_6H_6$ . We obtained  $\sigma_t^* = (3.50 \pm 0.90) \times 10^{-14}$  and  $\sigma_t = (1.55 \pm 0.25) \times 10^{-15}$  cm<sup>2</sup>/molecule.  $\times 10^{-15}~\mathrm{cm}^2/\mathrm{molecular}$ 

We note two other possibilities for collisional deexcitation: collisional mixing of the  $c \, {}^{3} \Pi_u$  state and its neighboring  $a^3\Sigma_g^*$  state, with subsequent decay in  $\sim 10^{-8}$  sec to the  $b^3\Sigma_u^*$  state, <sup>29</sup> or collisional mixing causing the transition of  $c^{3} \Pi_{u} - b^{3} \Sigma_{u}^{+}$  directly. But the  $b^3\Sigma_u^*$  state is repulsive and dissociates to two ground state atoms in  $\sim 10^{-14}$ sec,  $^{24}$  thereby causing the loss of the H<sub>2</sub> molecule from the beam. This loss process is included in the total-loss cross section that is measured from the attenuation data and, therefore, does not affect the analysis in Sec. IV.

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## Onsager Symmetry Relations and the Spectral Distribution of Scattered Light

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The Onsager symmetry relations, used in conjunction with appropriate linear transformations of the hydrodynamic variables, allow one to analyze the distribution of the eigenvalues of the hydrodynamic matrix and to establish symmetry characteristics of its diagonalizing matrices. The implications of these results for the analysis of the spectral distribution of the polarized component of scattered light are pointed out.

#### I. INTRODUCTION

In recent years there has been considerable activity, both experimentally as well as theoretically, in the study of the spectral distribution of scattered light.<sup>1</sup> The most commonly used approach to calculate the spectrum is based on Onsager's assumption concerning the regression of fluctuations.<sup>2,3</sup> The number of peaks in the spectrum can be established by determining the number of complex eigenvalues of the hydrodynamic matrix. For a more detailed analysis one may undertake a normal-mode decomposition of the spectral distribution. It is the purpose of this paper to show that the Onsager symmetry relations used in conjunction with appropriate linear transformations of hydrodynamic variables allow one to determine the maximum number of

complex roots of the hydrodynamic matrix and to establish symmetry characteristics of the diagonalizing matrices. The distribution of eigenvalues establishes that the spectrum of light scattered from an ordinary fluid can have at most three peaks. The symmetry of the diagonalizing matrices of the hydrodynamic matrix results in a simplification in the normal-mode decomposition.

### II. SPECTRUM OF SCATTERED LIGHT

As is well known,  $<sup>4</sup>$  the spectral-intensity distribu-</sup> tion of the polarized component of scattered light  $I(\vec{k}, \Omega + \omega)$  is proportional to the scattering function  $S_{\epsilon}(\vec{k}, \omega)$ , where  $S_{\epsilon}(\vec{k}, \omega)$  is the spectral density of  $\Delta \epsilon(\vec{k}, t)$  the k<sup>th</sup> spatial Fourier component of the fluctuation in the local dielectric constant. Here  $\vec{k}$ is the change in wave vector,  $\omega$  the change in fre-