[6]Experimental values for $S(k)$ are apparently limited to values of $k \lesssim 6\,\mathrm{\AA}^{-1}$ [see D. G. Henshaw, Phys. Rev. 119, 9 (1960)]. These results give oscillations in $S(k)$ about the value unity with a minimum and second broad maximum in the region of $k=3.0$ and $4.7\,\mathrm{\AA}^{-1}$, respectively, which are 12 and 3% below and above the mean value of unity. There is a suggestion of a third shallow minumum at $k \approx 5.8\,\mathrm{\AA}^{-1}$, which is about 1% below unity.

[7]W. L. McMillan, Phys. Rev. 138, A442 (1965).

[8]An interesting pedagogical example is afforded by a one-dimensional harmonic oscillator, with classical frequency of oscillation ω_c, for which the exact solution of $S(k,t)$ can easily be written down in closed form. The series expansion of this exact $S(k,t)$ in powers of the assumed small parameter $\omega_c/(k^2/2m)$ is seen to generate the same sequence of terms as those provided by $F_1^{\,i}$, $F_2^{\,i}$, etc., for this simple model.

[9]We have verified that these two terms in Eq. (38) make negligible contributions to any condensate contribution to F_2.

[10]I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series and Products* (Academic, N. Y. 1965), p. 464.

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Streaming Instabilities in Relativistic Magnetoplasmas

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Stability of superluminous and subluminous waves propagating transverse to the direction of the external uniform magnetic field is investigated in streaming relativistic homogeneous plasmas. In the relativistic regime for $\Omega < ck$ (Ω being the electron cyclotron frequency and k the characteristic wave number), the superluminous waves remain stable as in the absence of external magnetic field; however, for the subluminous waves, the magnetic field has a tendency towards destabilization. For $\Omega \gg ck$, the superluminous waves are dynamically unstable for all streaming velocities U, which are smaller than U_c, or for magnetic fields Ω, which are greater than Ω_c, but for the waves with frequencies $\omega \ll \Omega$ ($\Omega \gg ck$), there exists a minimum streaming velocity above which the system is unstable. In the nonrelativistic regime the system is unstable if streaming is much larger than the thermal velocity but otherwise stable. The unstable region is bounded by $\Omega_{\min}$ and $\Omega_{\max}$; $\Omega_{\min}$ being kv_t (v_t being the electron thermal velocity) and $\Omega_{\max}$ being kU.

I. INTRODUCTION

The superluminous waves (waves with phase velocities exceeding the velocity of light), which are excited in a plasma by thermal fluctuations, do not exhibit any resonance effects, i.e., there is no Landau damping or growth[1,2] associated with these waves. Recently it was shown by the author that in streaming relativistic plasmas, the superluminous waves[3] propagating transverse to the direction of relative streaming U in the absence of any external magnetic field are absolutely stable, but the subluminous waves[4] in such systems are dynamically unstable for $U \geq 0.09c$. Such relativistic plasmas one encounters in nature as well as in laboratory (thermonuclear plasmas) with the difference that there is magnetic field associated with them. It would be interesting to investigate the effect of magnetic field on these waves. In nonrelativistic counterstreaming magnetoplasmas, it was shown by Lee,[5] by Tzoar and Yang,[6] and by Buti and Lakhina[7] that the magnetic field decreases the growth rate of instability of the transverse waves.

Following Buti,[3,4] here we have considered the propagation of superluminous ($\omega > ck$) as well as subluminous ($\omega < ck$) waves, in counterstreaming (relativistic or nonrelativistic streaming) relativistic plasmas in the presence of uniform magnetic field which is taken along the direction of relative streaming but transverse to the direction of wave propagation. For strong fields, namely, $\Omega \gg ck$ in the relativistic regime, i.e., $kT \ll mc^2$, the superluminous waves are found to be stable for all streaming velocities $U > U_c$, whereas the waves with frequencies $\omega \ll \Omega$ are unstable only if the streaming velocity is greater than a certain minimum velocity. For weak magnetic fields ($\Omega < ck$), however, the superluminous waves are absolutely stable but subluminous waves are dynamically unstable and the region of instability increases with the magnetic field.

In the nonrelativistic regime, i.e., $kT \gg mc^2$, these transverse waves are unstable in a region $kv_t < \Omega < kU$ provided $U \gg v_t$. In Sec. II, the general dispersion relation is derived and Secs. IV and V deal with the discussion of dispersion relation in the relativistic and nonrelativistic limit, respectively.

II. GENERAL THEORY

Let us consider two hot homogeneous collisionless plasmas in the presence of a uniform magnetic field $\vec{B}_0$. The electrons in these plasmas are streaming whereas the ions are immobile and provide only the neutralizing background. On using the relativistic linearized Veasov equation for the electrons, namely,

$$\frac{\partial f_1}{\partial t}+\frac{\vec{p}}{m\gamma}\cdot\frac{\partial f_1}{\partial \vec{x}}-e\left(\vec{E}_1+\frac{\vec{p}\times\vec{B}_1}{mc\gamma}\right)\cdot\frac{\partial f_0}{\partial \vec{p}}-\frac{e}{mc\gamma}(\vec{p}\times\vec{B}_0)\cdot\frac{\partial f_1}{\partial \vec{p}}=0\ , \quad (1)$$

we get the dispersion relation $|\overleftrightarrow{R}|=0$, where

$$\overleftrightarrow{R}=(c^2k^2-\omega^2)\overleftrightarrow{I}-c^2\vec{k}\vec{k}+\sum_{\alpha=1}^{2}\frac{i\omega\omega_{p\alpha}^2}{N_\alpha}\int d\vec{p}\,\vec{p}\times\int_{-\infty}^{\phi}d\phi'\frac{G}{\Omega}\left[\frac{\partial f_0^\alpha}{\partial \vec{p}'}+\frac{\vec{p}'}{m\omega\gamma}\left(\vec{k}\cdot\frac{\partial f_0^\alpha}{\partial \vec{p}'}\right)-\frac{(\vec{k}\cdot\vec{p}')}{m\omega\gamma}\frac{\partial f_0^\alpha}{\partial \vec{p}'}\right]\ , \quad (2)$$

$$\ln G=-\frac{i}{m\Omega}\left[(k_\parallel p\cos\theta-m\omega\gamma)(\phi-\phi')+k_\perp p\sin\theta(\sin\phi-\sin\phi')\right]\ . \quad (3)$$

In Eq. (1) $\gamma=(1+p^2/m^2c^2)^{1/2}$ and all the other symbols are same as in Ref. 2. The equilibrium distribution function f_0 is given by drifted Maxwellian, namely,

$$f_0^\alpha(\vec{p})=N_\alpha B_\alpha\exp[-a_\alpha(\gamma-U_\alpha\cdot\vec{p}/mc^2)]\ , \quad (4)$$

where α labels the two plasmas having the mass motion U_α and $a_\alpha=mc^2/kT_\alpha$ and

$$B_\alpha=a_\alpha[4\pi m^3c^3\gamma_\alpha^2K_\alpha(a_\alpha/\gamma_\alpha)]^{-1}\ ,$$

with K_2 as the Bessel function of second kind and $\gamma_\alpha=(1-U_\alpha^2/c^2)^{-1/2}$. In writing Eq. (2), we have taken the magnetic field $\vec{B}_0$ along the z axis and

$$\vec{p}=(p,\theta,\phi)\ ,\quad \vec{p}'=(p,\theta,\phi')\ .$$

Let us consider the wave propagation along the x axis and the streaming along z axis. Further, if we take the two plasmas to be identical and counterstreaming, i.e., $U_1=-U_2=U$, then we can show that the elements Rxz, Rzx, Ryz, and Rzy of $\overleftrightarrow{R}$ vanish and we obtain the following two modes:

$$Rzz=0\ ,\quad R_{xx}R_{yy}-R_{xy}R_{yx}=0\ .$$

Now we shall consider the former mode, which is linearly polarized, in detail.

III. LINEARLY POLARIZED WAVE

On writing $k_\perp=k$, Rzz according to Eq. (2), is given by

$$Rzz=(c^2k^2-\omega^2)+\sum_\alpha\frac{i\omega\omega_{p\alpha}^2a_\alpha}{mc^2\Omega N_\alpha}\int d\vec{p}\,pf_0(\vec{p})\cos\theta\times\int_{-\infty}^{\phi}d\phi'\left(U_\alpha-\frac{p}{m\gamma}\cos\theta-\frac{pkU_\alpha}{m\omega\gamma}\sin\theta\cos\phi'\right)\times\exp\left[-\frac{i\omega\gamma}{\Omega}(\phi-\phi')-\frac{ikp}{m\Omega}(\sin\phi-\sin\phi')\right]\ , \quad (5)$$

which, on using the transformations[8] $(\phi-\phi')/\Omega=\eta$ and $(\phi+\phi')/2=\chi$, can be rewritten in the following form:

$$Rzz=(c^2k^2-\omega^2)+\sum_\alpha\frac{i\omega\omega_{p\alpha}^2a_\alpha}{mc^2N_\alpha}\int dp\,p^3\int_0^\pi d\theta\sin\theta\cos\theta f_0(\vec{p})\int_0^\infty d\eta\int_0^{2\pi}d\chi\times\left[U_\alpha-\frac{p\cos\theta}{m\omega\gamma}-\frac{U_\alpha kp}{m\omega\gamma}\sin\theta\cos\left(\chi-\frac{\Omega\eta}{2}\right)\right]\exp\left[i\omega\gamma\eta-\frac{2ikp}{m\Omega}\sin\theta\cos\chi\sin\frac{\Omega\eta}{2}\right]\ . \quad (6)$$

The χ integration can be immediately performed and Eq. (6) then reduces to

$$Rzz=c^2k^2-\omega^2+\sum_\alpha\frac{4i\pi\omega\omega_{p\alpha}^2a_\alpha B_\alpha}{mc^2\Omega}\int_0^\infty dp\,p^3e^{-a_\alpha\gamma}S\ , \quad (7)$$

where

$$S=\int_0^\infty dy\exp\left(\frac{2iy\gamma\omega}{\Omega}\right)\int_0^\pi d\theta\sin\theta\cos\theta\exp\frac{a_\alpha pU_\alpha}{mc^2}\times\left[\left(U_\alpha-\frac{p\cos\theta}{m\gamma}\right)J_0(b\sin\theta)+\frac{ikU_\alpha p}{m\omega\gamma}\sin\theta\cos y\,J_1(b\sin\theta)\right]\ , \quad (8)$$

with

$$b=[2kp/(m\Omega)]\sin y\ .$$

For $b<1$ (which in the relativistic case can be satisfied only if $\Omega>ck$ and in the nonrelativistic case if $\Omega>kV_t$), Eq. (7) can be further simplified to give

$$Rzz=c^2k^2-\omega^2-\sum_\alpha\frac{2\pi\omega_{p\alpha}^2B_\alpha}{U_\alpha}\int_0^\infty\frac{dp}{\gamma}p^2e^{-a_\alpha\gamma}\left\{U_\alpha\left(\cosh z-\frac{\sinh z}{z}\right)-\frac{p}{m\gamma}\left[\sinh z\left(1+\frac{2}{z^2}\right)-\frac{2\cosh z}{z^2}\right]\right.$$

$$+\frac{k^2p^3}{m^3\Omega^2\gamma}\left(\frac{\gamma^2\omega^2}{\Omega^2}-1\right)^{-1}\left[\frac{\sinh z}{z^2}\left(5+\frac{12}{z^2}\right)-\frac{\cosh}{z}\left(1+\frac{12}{z^2}\right)\right]\Bigg\}, \quad (9)$$

with $z=a_\alpha p U_\alpha/(mc^2)$. We shall now discuss this dispersion relation in the relativistic limit ($a \ll 1$) and in the nonrelativistic limit ($a \gg 1$) separately.

IV. RELATIVISTIC CASE

In the extreme relativistic case, $\gamma \approx p/mc$ and the limits of integration in Eq. (9) are $mc < p < m\Omega/2k$. Now if we take the magnetic field to be weak, namely, $ck < \Omega < \omega$ then on integration and simplifications Eq. (9) for $Rzz=0$, yields

$$\omega^2=c^2k^2-\sum_\alpha \frac{2\pi mc^2\omega_{p\alpha}^2 B_\alpha}{U_\alpha}\left[A_1+\frac{c^2k^2}{\omega^2}A_2+\frac{m^2c^4k^2\Omega^2}{\omega^4}A_3\right], \quad (10)$$

where

$$A_1=\left\{\frac{m^2c^2}{2}\left[\left(1+\frac{2}{a_\alpha\delta}\right)\frac{e^{-a_+}}{a_\alpha}-\left(1-\frac{2}{a_\alpha\delta}\right)\frac{e^{-a_-}}{a_\alpha}+\frac{1}{a_\alpha^2\delta^2}(E_1(a_+)-E_1(a_-))\right]-\frac{m^2\Omega^2}{8k^2}\left[a_\alpha\to\frac{a_\alpha\Omega}{2ck}\right]\right\}, \quad (11)$$

$$A_2=\left\{\frac{m^2c^2}{2a_\alpha\delta}\left[-E_0(a_+)-E_0(a_-)+\frac{5}{a_\alpha\delta}(E_1(a_-)-E_1(a_+))-\frac{12}{a_\alpha^2\delta^2}(a_-E_0(a_-)-a_-E_1(a_-)+a_+E_0(a_+)-a_+E_1(a_+))\right.\right.$$

$$\left.\left.+\frac{6}{a_\alpha^3\delta^3}(a_-(1-a_-)E_0(a_-)-a_+(1-a_+)E_0(a_+)+a_-^2E_1(a_-)-a_+^2E_1(a_+))\right]-\frac{m^2c\Omega}{4a_\alpha k\delta}\left[a_\alpha\to\frac{a_\alpha\Omega}{2ck}\right]\right\}, \quad (12)$$

and

$$A_3=\left\{\frac{1}{2\delta^2}\left[(1-\delta)E_0(a_-)\left(\frac{1}{a_\alpha}-\frac{3}{a_\alpha^2\delta}+\frac{1}{a_\alpha\delta}-\frac{1}{2\delta}+\frac{3}{a_\alpha^3\delta^2}-\frac{1}{a_\alpha^2\delta^2}+\frac{1}{2a_\alpha\delta^2}-\frac{1}{2\delta^2}\right)-(1+\delta)E_0(a_+)\right.\right.$$

$$\left.\left.\times\left(\frac{1}{a_\alpha}+\frac{3}{a_\alpha^2\delta}-\frac{1}{a_\alpha\delta}+\frac{1}{2\delta}+\frac{3}{a_\alpha^3\delta^2}-\frac{1}{a_\alpha^2\delta^2}+\frac{1}{2a_\alpha\delta^2}-\frac{1}{2\delta^2}\right)+\frac{(1-\delta^2)}{2\delta^2}\left(E_1(a_-)-E_1(a_+)\right)\right]-\frac{1}{2\delta^2}\left[a\to\frac{a\Omega}{2ck}\right]\right\}, \quad (13)$$

with

$$\delta=U_\alpha/c, \quad a_\pm=a_\alpha(1\pm\delta),$$

$$E_0(z)=e^{-z}/z, \quad E_1(z)=\int_1^\infty dt\, e^{-zt}/t.$$

In Eqs. (11)–(13), $[a\to a\Omega/2ck]$ means the entire expression in the preceding square bracket with (a) replaced by ($a\Omega/2ck$). Moreover, in writing Eqs. (12) and (13), we have made use of the following relation for the exponential integrals[9]:

$$E_{n+1}(z)=(z/n)[E_0(z)-E_n(z)], \quad n\ge 1. \quad (14)$$

For contrastreaming identical plasmas with $N_1=N_2=N/2$, Eq. (10) simplifies to

$$\omega^6+\omega^4(RA_1-c^2k^2)+c^2k^2A_2\omega^2+Rm^2c^4k^2\Omega^2A_3=0, \quad (15)$$

where

$$R=a\omega_p^2[2Um^2c\gamma_0^2K_2(a/\gamma_0)]^{-1},$$

with

$$\gamma_0=(1-U^2/c^2)^{-1/2}.$$

Note that Eq. (15) offers at least one negative root for ω^2 if $A_3>0$, which will ensure dynamic instability for the superluminous waves.

For $(a\Omega/2ck)\ll 1$, E_0 and E_1 in Eq. (13) can be expanded and on simplification we obtain

$$A_3=\left\{\left(\frac{a\Omega}{2ck}\right)^2\frac{1}{6\delta^3}\left[1-\frac{\delta^2}{10}-\frac{2}{5}\delta^4\left(\frac{2ck}{a\Omega}\right)\right]\right\}-\left[\frac{a\Omega}{2ck}\to a\right], \quad (16)$$

which shows that for instability for $\Omega\gg ck$, we need to satisfy the condition $U<U_c$ or $\Omega>\Omega_c$, where $U_c/c\approx(5a\Omega/4ck)^{1/4}$ and

$$\frac{\Omega_c}{2ck}=\frac{2\delta^4}{5a}\left(1-\frac{\delta^2}{10}\right)^{-1}.$$

Note that the latter condition can be satisfied only for relativistic U. For other values of $(a\Omega/ck)$, U_c is found by numerically evaluating Eq. (13) and the variation of U_c with the magnetic field is shown in Fig. 1.

Low-Frequency Waves

Equation (9), which is valid for $\Omega>ck$, in the extreme relativistic case for low frequencies ($\omega\ll\Omega$) can be integrated to give

$$\omega^2=c^2k^2-\sum_\alpha\frac{2\pi\omega_{p\alpha}^2B_\alpha mc^2}{U_\alpha}\times\left[B_1+B_2\frac{k^2}{m^2\Omega^2}+B_3\frac{k^2\omega^2}{m^4c^2\Omega^4}\right], \quad (17)$$

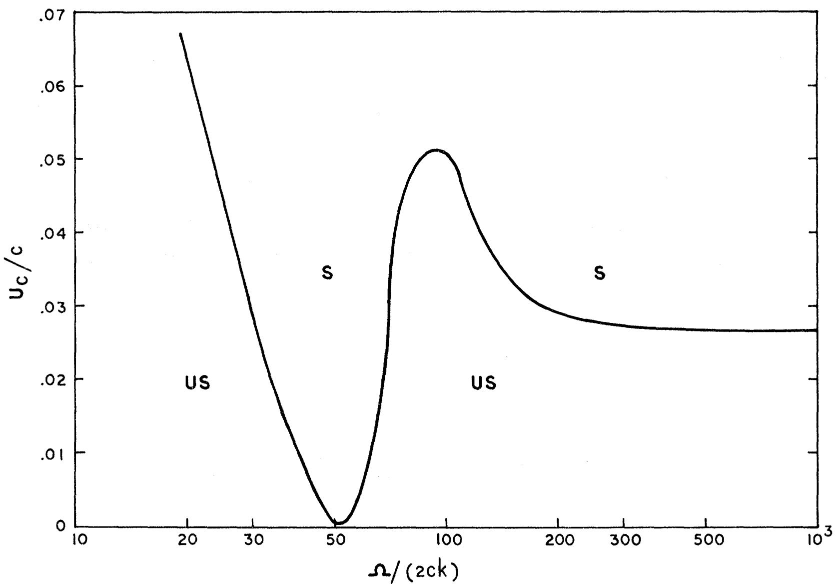

FIG. 1. Variation of U_c with magnetic field for $a = 0.01$.

where

$$B_1 = \frac{m^2c^2}{a_\alpha^2\delta}\left[\left(1+\frac{a_\alpha\delta}{2}\right)e^{-a_+} + \left(1-\frac{a_\alpha\delta}{2}\right)e^{-a_-} + \frac{1}{\delta}\left[E_1(a_+)-E_1(a_-)\right]\right], \quad (18)$$

$$B_2 = \frac{m^4c^4}{2a_\alpha\delta}\left\{\left[1+\frac{12}{a_\alpha^2\delta^2}+\frac{2}{a_-}+\frac{2}{a_-^2}-\frac{5}{a_\alpha\delta}\left(1+\frac{1}{a_-}\right)\right]E_0(a_-) + \left[1+\frac{12}{a_\alpha^2\delta^2}+\frac{1}{a_+}\left(1+\frac{1}{a_+}\right)\left(2+\frac{5}{\delta}\right)\right]E_0(a_+) + \frac{12}{a_\alpha^3\delta^3}\left[E_1(a_+)-E_1(a_-)\right]\right\}, \quad (19)$$

and

$$B_3 = \frac{m^6c^6}{2a_\alpha\delta}\left[E_0(a_-)\left\{1-\frac{4}{a_-}\left(1-\frac{3}{a_-}\right)-\frac{5}{a_\alpha\delta}\left(1+\frac{3}{a_-}\right) + \frac{12}{a_\alpha^2\delta^2}+\left(1+\frac{1}{a_-}\right)\left(\frac{24}{a_-^3}-\frac{30}{a_\alpha a_-^2\delta}+\frac{24}{a_\alpha^2 a_-\delta^2}-\frac{12}{a_\alpha^3\delta^3}\right)\right\} + E_0(a_+)\{\delta\to-\delta\}\right]. \quad (20)$$

Once again $\{\delta \to -\delta\}$ stands for the expression in the preceding curly bracket with δ replaced by $(-\delta)$. We may point out that in deriving Eqs. (18)–(20), the upper limit of integration, which should be $(mc\Omega/\omega)$, was replaced by ∞; this is justified because $\Omega \gg \omega$ and large values of p in Eq. (9) contribute negligibly anyway. The rate of growth of unstable waves represented by Eq. (17) is given by

$$\left(\frac{\mathrm{Im}\,\omega}{\omega_p}\right)^2 = \left(1+\frac{a}{2\epsilon}\xi^2\Lambda C_3\right)^{-1}\left[\epsilon + \Lambda\left(C_1 + \tfrac{1}{2}a\xi C_2\right)\right], \quad (21)$$

where

$$\epsilon = c^2k^2/\omega_p^2\,,\quad \xi = c^2k^2/\Omega^2\,,\quad C_1 = B_1 a^2 U/m^2c^3\,,$$

$$C_2 = B_2(2a\delta/m^4c^4)\,,\qquad C_3 = B_3(2a\delta/m^6c^6)\,,$$

$$\Lambda = (1-\delta^2)\left[2a\delta^2 K_2(a_+^{1/2}a_-^{1/2})\right]^{-1}.$$

Equation (21) is valid for identical counterstreaming plasmas and this guarantees the existence of instability only if the right-hand side of Eq. (21) is positive which as shown in Tables I and II, occurs if U exceeds a certain minimum velocity $U_{\min}$. It is also evident from these tables that the growth rate increases by increasing ϵ and Ω and it decreases by increasing U and v_t, i.e., the temperature of the plasma and the streaming velocity have a stabilizing effect, whereas the magnetic field has the destabilizing effect on these low-frequency waves. Large values of ϵ are not taken because these transverse instabilities[10] are important for $\epsilon \lesssim 1$.

Weak Magnetic Field ($\Omega < ck$)

In this case, it is better to derive the dispersion relation from Eq. (5) rather than Eq. (7). In the limit $\Omega/ck \to 0$, if we use the relation[2]

$$\int_{-\infty}^{\phi} d\phi' \frac{G}{\Omega}\Psi(\phi') = -\frac{im\Psi(\phi)}{(\vec{k}\cdot\vec{p}-m\omega\gamma)} + \frac{m\Omega}{(\vec{k}\cdot\vec{p}-m\omega\gamma)}\frac{\partial}{\partial\phi}\left[\frac{m\Psi(\phi)}{(\vec{k}\cdot\vec{p}-m\omega\gamma)}\right] + \frac{im\Omega^2}{(\vec{k}\cdot\vec{p}-m\omega\gamma)}\frac{\partial}{\partial\phi}\left\{\frac{m}{(\vec{k}\cdot\vec{p}-m\omega\gamma)}\frac{\partial}{\partial\phi}\left[\frac{m\Psi(\phi)}{(\vec{k}\cdot\vec{p}-m\omega\gamma)}\right]\right\} + \cdots, \quad (22)$$

TABLE I. Variation of growth rate with $a=mc^2/KT$, $\xi=c^2k^2/\Omega^2$, and U/c for $\epsilon=c^2k^2/\omega_p^2=0.01$. Dots indicate the absence of instability.

	$a=1.0\times10^{-4}$			$a=0.05$		
U/c \ ξ	0.001	0.01	0.05	0.001	0.01	0.05
1.0×10^{-4}	...	...	...	2.3410×10^{3}	2.3408×10^{2}	4.6817×10^{1}
1.0×10^{-3}	...	...	...	2.4777×10^{1}	2.4641×10^{0}	4.9279×10^{-1}
1.0×10^{-2}	1.3291×10^{-9}	1.3217×10^{-10}	2.6420×10^{-11}	...	...	...
0.1	3.1654×10^{-9}	3.1586×10^{-10}	6.3160×10^{-11}	8.7245×10^{-4}	8.0287×10^{-5}	1.5802×10^{-5}
0.2	2.7362×10^{-9}	2.7311×10^{-10}	5.4612×10^{-11}	7.7993×10^{-4}	6.9785×10^{-5}	1.3722×10^{-5}
0.3	2.2250×10^{-9}	2.2216×10^{-10}	4.4425×10^{-11}	6.5371×10^{-4}	5.6808×10^{-5}	1.1159×10^{-6}
0.4	1.7317×10^{-9}	1.7296×10^{-10}	3.4589×10^{-11}	5.1852×10^{-4}	4.4235×10^{-5}	8.6876×10^{-6}
0.5	1.2849×10^{-9}	1.2838×10^{-10}	2.5674×10^{-11}	3.8585×10^{-4}	3.2798×10^{-5}	6.4469×10^{-6}
0.6	8.8859×10^{-10}	8.8803×10^{-11}	1.7760×10^{-11}	2.6324×10^{-4}	2.2631×10^{-5}	4.4572×10^{-6}
0.7	5.4564×10^{-10}	5.4541×10^{-11}	1.0908×10^{-11}	1.5715×10^{-4}	1.3847×10^{-5}	2.7355×10^{-6}
0.8	2.6710×10^{-10}	2.6704×10^{-11}	5.3407×10^{-12}	7.3884×10^{-5}	6.7477×10^{-6}	1.3380×10^{-6}
0.9	7.4185×10^{-11}	7.4177×10^{-12}	1.4835×10^{-12}	1.9537×10^{-5}	1.8644×10^{-6}	3.7128×10^{-7}
0.95	1.9621×10^{-11}	1.9621×10^{-12}	3.9241×10^{-13}	5.0338×10^{-6}	4.9180×10^{-7}	9.8154×10^{-8}

then Eq. (7) to order Ω^2 reduces to

$$Rzz=c^2k^2-\omega^2+\sum_\alpha \frac{\omega\omega_{p\alpha}^2 a_\alpha}{N_\alpha c^2}\int d\vec{p}\,\frac{p\cos\theta f_0(\vec{p})}{(\vec{k}\cdot\vec{p}-m\omega\gamma)}\left\{\left[U_\alpha-\frac{p\cos\theta}{m\gamma}-\frac{kpU_\alpha\sin\theta\cos\phi}{m\omega\gamma}\right]-\frac{ikpU_\alpha\sin\theta}{\omega\gamma}\right.$$
$$\left.\times\frac{\partial}{\partial\phi}\left[\frac{\cos\phi}{(\vec{k}\cdot\vec{p}-m\omega\gamma)}\right]+\frac{m\Omega^2kpU_\alpha\sin\theta}{\omega\gamma}\frac{\partial}{\partial\phi}\left[(\vec{k}\cdot\vec{p}-m\omega\gamma)^{-1}\frac{\partial}{\partial\phi}\left(\frac{\cos\phi}{\vec{k}\cdot\vec{p}-m\omega\gamma}\right)\right]\right\}. \tag{23}$$

For $\Omega=0$, Eq. (23) goes back to the result obtained in Ref. 3. Note that Ω term is odd in ϕ and hence vanishes on ϕ-integration, and Ω^2 term for superluminous waves for $\gamma\approx p/mc$, is of the order of Ω^2/ω^2 (which is much less than unity) compared to first term in the integral and hence can only add a real correction term to the dispersion relation, which one would get in the absence of external magnetic field.[3] Thus superluminous waves in the presence of weak magnetic field remain stable.

For subluminous waves, however, Eq. (23) for $Rzz=0$, for contrastreaming identical plasmas can be simplified to give

$$1+\eta^2=(a\omega_p^2/c^2k^2)(\delta^2-F), \tag{24}$$

where

$$\eta=(\mathrm{Im}\,\omega/ck)$$

with

$$|\eta|<1$$

and

TABLE II. Variation of growth rate $(\mathrm{Im}\omega/\omega p)^2$, with $a=mc^2/KT$, $\xi=c^2k^2/\Omega^2$, and U/c for $\epsilon=c^2k^2/\omega_p^2=1.0$. Dots indicate the absence of instability.

	$a=1.0\times10^{-4}$			$a=0.05$		
U/c \ ξ	0.001	0.01	0.05	0.001	0.01	0.05
1.0×10^{-4}	...	...	...	2.3532×10^{5}	2.3409×10^{4}	4.6817×10^{3}
1.0×10^{-3}	...	...	...	5.5138×10^{3}	2.4776×10^{2}	4.9290×10^{1}
1.0×10^{-2}	2.1533×10^{-7}	1.4041×10^{-8}	2.6750×10^{-9}	5.1465×10^{-1}	...	...
0.1	3.9136×10^{-7}	3.2334×10^{-8}	6.3460×10^{-9}	9.0446×10^{-1}	9.2880×10^{-2}	5.3054×10^{-3}
0.2	3.3065×10^{-7}	2.7881×10^{-8}	5.4840×10^{-9}	8.7944×10^{-1}	7.2994×10^{-2}	4.2107×10^{-3}
0.3	2.6064×10^{-7}	2.2597×10^{-8}	4.4578×10^{-9}	8.3150×10^{-1}	5.0888×10^{-2}	3.0163×10^{-3}
0.4	1.9624×10^{-7}	1.7527×10^{-8}	3.4681×10^{-9}	7.5050×10^{-1}	3.2302×10^{-2}	2.0191×10^{-3}
0.5	1.4115×10^{-7}	1.2965×10^{-8}	2.5725×10^{-9}	6.2393×10^{-1}	1.8793×10^{-2}	1.2764×10^{-3}
0.6	9.5003×10^{-8}	8.9418×10^{-9}	1.7784×10^{-9}	4.4734×10^{-1}	9.8637×10^{-3}	7.5256×10^{-4}
0.7	5.7034×10^{-8}	5.4788×10^{-9}	1.0918×10^{-9}	2.4734×10^{-1}	4.4578×10^{-3}	3.9698×10^{-4}
0.8	2.7413×10^{-8}	2.6774×10^{-9}	5.3435×10^{-10}	8.7423×10^{-2}	1.5512×10^{-3}	1.6819×10^{-4}
0.9	7.5034×10^{-9}	7.4262×10^{-10}	1.4839×10^{-10}	1.2424×10^{-2}	2.9244×10^{-4}	4.1369×10^{-5}
0.95	1.9726×10^{-9}	1.9631×10^{-10}	3.9245×10^{-11}	1.8071×10^{-3}	6.2241×10^{-5}	1.0338×10^{-5}

$$F=\frac{\eta}{\gamma_0^4}\int_0^{\sin^{-1}\eta}\frac{d\theta\sin\theta\cos^2\theta}{(1-\delta^2\cos^2\theta)(\eta^2+\sin^2\theta)^{1/2}}\times\left[\frac{(1+3\delta^2\cos^2\theta)}{(1-\delta^2\cos^2\theta)^2}-\frac{3a^2\Omega^2\sin^2\theta}{16c^2k^2}(4\eta^2-\sin^2\theta)\right]. \quad (25)$$

Equation (24) can be satisfied only if $(a\omega_p^2/c^2k^2)>1$ and $\delta^2>F$. The latter inequality puts a lower limit on U, say U^* above which these waves are unstable. From Eq. (25), it is apparent that $F(\Omega\neq 0)<F(\Omega=0)$ and hence the instability criterion $(\delta^2>F)$ can be satisfied by smaller streaming velocities, i.e., $U^*(\Omega\neq 0)<U^*(\Omega=0)$. Thus the magnetic field $(\Omega<ck)$ once again has a destabilizing tendency for the subluminous waves.

V. NONRELATIVISTIC CASE

In this case $a\gg 1$ and $\gamma\approx(1+p^2/2m^2c^2)$; on making these substitutions, Eq. (9) which is valid for $\Omega>kv_t$ for $Rzz=0$ can be rewritten as

$$\omega^2=c^2k^2+\psi_1+\frac{\psi_2}{(\omega^2-\Omega^2)}, \quad (26)$$

where

$$\psi_1=-\sum_\alpha\frac{2\pi\omega_{p\alpha}^2B_\alpha e^{-a_\alpha}}{U_\alpha}\int_0^\infty dp\,p^2\exp\left(-\frac{a_\alpha p^2}{2m^2c^2}\right)\times\left\{U_\alpha\left(\cosh z-\frac{\sinh z}{z}\right)-\frac{p}{m}\left[\sinh z\left(1+\frac{2}{z^2}\right)-\frac{2\cosh z}{z}\right]\right\} \quad (27)$$

and

$$\psi_2=\sum_\alpha\frac{2\pi\omega_{p\alpha}^2k^2B_\alpha}{m^3}\int_0^\infty dp\,p^5\exp\left(-\frac{a_\alpha p^2}{2m^2c^2}\right)\times\left[\frac{\cosh z}{z}\left(1+\frac{12}{z^2}\right)-\frac{\sinh z}{z^2}\left(5+\frac{12}{z^2}\right)\right]. \quad (28)$$

Equations (27) and (28) can be readily integrated to yield (for contrastreaming plasmas)

$$\psi_1=(2\pi mkT)^{3/2}\omega_p^2B\exp(-a+mU^2/2KT) \quad (29)$$

and

$$\psi_2=\psi_1k^2v_t^2(1+mU^2/KT), \quad (30)$$

which show that both ψ_1 and ψ_2 are positive quantities. Equation (27) can have pure imaginary root only if $\Omega^2<\Omega^{*2}$, where

$$\frac{\Omega^{*2}}{k^2v_t^2}=\left(1+\frac{\psi_1}{c^2k^2}\right)^{-1}\frac{\psi_1}{c^2k^2}\left(1+\frac{aU^2}{c^2}\right). \quad (31)$$

Numerical evaluation of ψ_1 shows that for relativistic streaming $a\psi_1<c^2k^2$, i.e., $\Omega^{*2}<k^2v_t^2$. So we cannot have an instability because the inequality $k^2v_t^2<\Omega^2<\Omega^{*2}$ cannot be satisfied. For nonrelativistic streaming, however, $\psi_1\gtrsim c^2k^2$, so that

$$\frac{\Omega^{*2}}{k^2v_t^2}\approx\left(1+\frac{U^2}{v_t^2}\right),$$

which shows that instability can occur if $U^2\gg v_t^2$. The same conclusions we had drawn in our earlier paper.[11]

For weak magnetic fields, i.e., $\Omega^2<k^2v_t^2$, if we use Eq. (22), we recover the dispersion relation given in Ref. 11. We shall not discuss this case here.

VI. CONCLUSIONS

The superluminous waves propagating transverse to the external magnetic field in counterstreaming relativistic plasmas are stable if the field is weak, i.e., if $\Omega<ck$, otherwise they are unstable for $U<U_c$ or $\Omega>\Omega_c$ for frequencies $\omega>\Omega$ but for low frequencies $(\omega\ll\Omega)$, they are unstable for $U^*>U>U_{\min}$. Both U^* and $U_{\min}$, however, happen to be nonrelativistic. For $U>U_{\min}$ the streaming as well as the temperature of the plasma reduces the growth rate of instability whereas the magnetic field increases the growth rate. The subluminous waves, which are dynamically unstable in the absence of magnetic field, are further destabilized by the field.

In the nonrelativistic plasmas, these waves are unstable for $k^2v_t^2<\Omega^2<k^2U^2$ provided U is nonrelativistic but much greater than the thermal velocity.

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