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Intensity-Dependent Propagation Characteristics of Circularly Polarized High-Power Laser Radiation in a Dense Electron Plasma*

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Circularly polarized laser radiation propagating in an electron plasma drives the electrons into circular orbits. This orbital motion induces a magnetic field which is either parallel or antiparallel to the laser beam. The generation of this magnetic field is known as the inverse Faraday effect. Because of this magnetic field and the relativistic change of the electron mass, the wave propagation is enhanced for high intensities in the sense that the critical plasma density increases with the laser-beam intensity. For the wavelength $\lambda = 1.06\ \mu$, corresponding to a neodymium-glass laser, the dependence of the various propagation characteristics on the brightness of the beam is examined in detail. Electron densities varying between 10^{20} and 6.6×10^{21} particles per cm^3 and radiation intensities in the range 10^{17}–10^{20} W/cm^2 are considered. Energy losses caused by synchrotron radiation and by electron-ion bremsstrahlung are calculated.

I. INTRODUCTION

The present paper describes certain phenomena associated with the propagation of circularly polarized high-intensity laser radiation in a dense electron plasma. Such a wave field causes the plasma electrons to gyrate in orbits whose radii depend on the radiation intensity, on the wave frequency, and on the electron density. This gyration of the plasma electrons induces a magnetic field which for left-circular polarization is parallel and for right-circular polarization is antiparallel to the direction of wave propagation. The generation of the induced magnetic field is referred to as the inverse Faraday effect. In a nonrelativistic treatment, Pomeau and Quemada[1] have considered the magnetization induced in a collisionless electron plasma by circularly polarized microwaves. V. Deschamps *et al.*[2] have observed such a magnetic field.

At laser frequencies the electron orbital motion becomes extremely rapid and the induced magnetic field theoretically so large that its effect on the electron orbital motion can not be neglected. Radiation intensities as large as 10^{17} W/cm^2, already available from high-power lasers,[3] would be capable of producing a substantial diamagnetic field as well as relativistic electrons. For radiation of such intensity the propagation is nonlinear, and circular polarization constitutes the only mode in which electromagnetic energy can propagate as pure transverse waves.[4–6] Also it should be pointed out that circularly polarized waves are the only type of wave which would avoid completely the possibility of parametric coupling to other waves by way of the relativistic variation of mass.[7] In this mode the directed electron velocity is perpendicular to the direction of wave propagation ($\vec{v}\cdot\vec{k} = 0$). For intensities exceeding 10^{18} W/cm^2 circularly polarized radiation can propagate in an electron plasma whose density is larger than the critical density

$$N_c = m\omega^2/4\pi e^2 \ . \tag{1.1}$$

Here, e and m denote, respectively, the electron charge and rest mass, and ω is the frequency of the laser radiation. This enhancement of the plasma transparency is caused by the effect upon the electron orbits of the relativistic mass increase. The induced magnetic field, not included in previous treatments of wave propagation,[4–6] further increases the wave penetrability. We present a calculation using equations that are relativistic and that are self-consistent in the sense that the equation of motion contains the induced magnetic field. In a detailed discussion of the propagation characteristics we assume a vacuum wavelength λ of 1.06×10^{-4} cm, corresponding to the radiation generated by a neodymium-glass laser. For this wavelength the circular frequency $\omega = 2\pi c/\lambda$ is 1.78×10^{15}/sec, and the electron orbital motion is pronounced relativistic.

In view of the very large directed velocity acquired by the electrons we neglect their thermal motion. For radiation intensities less than approximately 3×10^{22} W/cm^2 the ion velocity remains well below the electron velocity.[5] The ion motion has then little effect on the propagation, and we neglect it entirely.

In the optical frequency range and for laser-beam intensities above 10^{18} W/cm^2, the synchrotron radiation emitted by the gyrating electrons constitutes the principal source for the attenuation of circularly polarized waves. The bremsstrahlung losses caused by electron-ion collisions are several orders of magnitudes smaller. Because of the collective nature of the electron motions, collisions between electrons can be neglected.

We now outline the structure of our presentation.

Section II contains the derivation of the set of equations whose simultaneous solution yields the amplitudes of the laser electric and magnetic field intensities inside the plasma, the electron orbit radius, and the relativistic particle energy as functions of the local laser beam intensity, the wave frequency and the electron number density. The upshot of Sec. II is the intensity dependent dispersion relation which accounts for the nonlinear propagation characteristics. This dispersion relation sets the stage for the calculations of Sec. III, where the various propagation characteristics, such as the phase velocity, the group velocity, and the energy transport velocity, are discussed and exhibited in diagrams. In Sec. IV we consider radiative losses and rederive the dispersion relation to account for wave attenuation. In Sec. V we conclude with a few critical remarks concerning possible effects of electron drift and short laser-pulse duration.

II. ANALYSIS OF INVERSE FARADAY EFFECT

We consider the electron plasma as a collection of charges rather than a material medium. We therefore concentrate on the microscopic fields $\vec{E}$ and $\vec{B}$. For circular polarization and wave propagation along the positive z axis, the electric and magnetic field intensities of the wave can be represented as

$$\vec{E}(z,t)=\mathrm{Re}[E(\hat{\epsilon}_1+i\lambda\hat{\epsilon}_2)\,e^{-i(\omega t-kz)}] \tag{2.1}$$

and

$$\vec{B}(z,t)=\mathrm{Re}[B(-i\lambda)(\hat{\epsilon}_1+i\lambda\hat{\epsilon}_2)\,e^{-i(\omega t-kz)}] \ . \tag{2.2}$$

The unit vectors $\hat{\epsilon}_1$, $\hat{\epsilon}_2$, and $\hat{\epsilon}_3$ are directed along the orthogonal coordinate axes x, y, and z, respectively. The helicity λ is $+1$ for right-circular polarization and -1 for left-circular polarization. The wave fields (2.1) and (2.2) satisfy the Maxwell equations

$$\vec{\nabla}\times\vec{B}=\frac{1}{c}\frac{\partial\vec{E}}{\partial t}+\frac{4\pi}{c}\vec{j} \ , \tag{2.3}$$

$$\vec{\nabla}\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \ . \tag{2.4}$$

If N, e, and $\vec{v}$, denote respectively, the electron number density, the electron charge, and the electron velocity, the current density is

$$\vec{j}=eN\vec{v} \ . \tag{2.5}$$

From relations (2.3)–(2.5), together with the transversality condition

$$\mathrm{div}\vec{E}=0 \ , \tag{2.6}$$

we obtain the wave equation for the electric field

$$\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}-\nabla^2\vec{E}+\frac{4\pi}{c^2}\frac{\partial}{\partial t}(eN\vec{v})=0 \ . \tag{2.7}$$

This equation can be satisfied with the expression (2.1) for the electric field and with the following expression for the electron velocity:

$$\vec{v}(z,t)=\mathrm{Re}[i(e/|e|)V(\hat{\epsilon}_1+i\lambda\hat{\epsilon}_2)e^{-i(\omega t-kz)}] \ , \tag{2.8a}$$

where

$$V=(E/4\pi|e|N\omega)(\omega^2-k^2c^2) \ . \tag{2.8b}$$

This particular solution corresponds to circular motion of the electrons. It neglects thermal motions and possible drift motions. Roberts and Buchsbaum[8] have discussed in considerable detail the various possible orbits of a single charged particle in a vacuum field consisting of a transverse circularly polarized wave propagating along a constant magnetic field. However, as mentioned in Sec. I, this electron velocity, which is normal to both $\vec{k}$ and $\vec{E}$, is required for pure circularly polarized transverse wave propagation. It should be emphasized that this electron velocity may not characterize completely the actual electron motion in a microplasma irradiated by a focused laser field, but there is every likelihood that this mode will couple to external laser fields.[5]

Each electron orbiting in accordance with the Eq. (2.8) contributes a magnetic dipole moment

$$\vec{\mu}=(1/2c)\,er^2\vec{\omega} \ , \tag{2.9}$$

where

$$r=V/\omega \tag{2.10}$$

is the orbit radius. Since the circularly polarized wave is assumed to propagate along the positive z axis, the angular velocity vector and the helicity are related by

$$\vec{\omega}=\lambda\omega\hat{\epsilon}_3 \ . \tag{2.11}$$

The magnetic induction generated by the orbital motions of the electrons, whose number density we take to be constant, is then

$$\vec{B}_{\mathrm{ind}}=4\pi N\vec{\mu}=(2\pi Ne\lambda\omega r^2/c)\,\hat{\epsilon}_3 \ . \tag{2.12}$$

By substituting the Eqs. (2.1) and (2.5) into the Maxwell equation (2.3) we obtain the amplitude relation

$$B=\left[1-\frac{r}{a}\left(\frac{\omega_p}{\omega}\right)^2\right]\frac{v_p}{c}E \ , \tag{2.13}$$

where the phase velocity

$$v_p=\omega/k \ , \tag{2.14}$$

the plasma frequency

$$\omega_p=(4\pi Ne^2/m)^{1/2} \ , \tag{2.15}$$

and the elementary orbit radius

$$a=|e|E/m\omega^2 \tag{2.16}$$

have been introduced. By utilizing the Maxwell equation (2.4) we find the further amplitude relation

$$B=(c/v_p)E \quad . \tag{2.17}$$

Equations (2.13) and (2.17) then yield the dispersion relation

$$\omega^2=k^2c^2+(r/a)\omega_p^2 \quad . \tag{2.18}$$

The intensity I of the laser radiation is given by the absolute magnitude of the Poynting vector, namely, by

$$I=|\vec{S}|=(c/4\pi)|\vec{E}\times\vec{B}| \quad . \tag{2.19}$$

The laser field amplitudes E and B can therefore be written as

$$E=(1/c)(4\pi v_p I)^{1/2} \quad , \tag{2.20}$$

$$B=(4\pi I/v_p)^{1/2} \quad , \tag{2.21}$$

where by virtue of Eq. (2.18) the phase velocity may be expressed as

$$v_p=c\left[1-\frac{r}{a}\left(\frac{\omega_p}{\omega}\right)^2\right]^{-1/2} \quad . \tag{2.22}$$

By utilizing Eqs. (2.16), (2.20), and (2.22) we obtain

$$|e|E^4-m\omega_p^2 rE^3=|e|(4\pi I/c)^2 \quad , \tag{2.23}$$

relating the electric field to the laser intensity I. We now determine the electron orbit radius by substituting into the relativistic equation of motion

$$\frac{d}{dt}(\gamma m\vec{v})=\frac{\partial}{\partial t}(\gamma m\vec{v})+(\vec{v}\cdot\vec{\nabla})\gamma m\vec{v}$$
$$=e[\vec{E}+(1/c)\vec{v}\times(\vec{B}+\vec{B}_{ind})] \tag{2.24}$$

the field vectors $\vec{E}$, $\vec{B}$, and $\vec{B}_{ind}$, and $\vec{v}$, where

$$\gamma=(1-|\vec{v}|^2/c^2)^{-1/2} \quad . \tag{2.25}$$

For the electron velocity (2.8) the term $(\vec{v}\cdot\vec{\nabla})\vec{v}$ vanishes and γ is a constant. We may therefore replace Eq. (2.22) by

$$\frac{\partial}{\partial t}(m\gamma\vec{v})=\gamma m\frac{\partial\vec{v}}{\partial t}=e[\vec{E}+(1/c)\vec{v}\times(\vec{B}+\vec{B}_{ind})] \quad . \tag{2.26}$$

By substituting Eq. (2.8) for the velocity and Eq. (2.12) for $\vec{B}_{ind}$ in Eq. (2.26), we obtain the force equation

$$\gamma m\omega^2 r=|e|E-(m\omega^2\omega_p^2/2c^2)r^3 \quad , \tag{2.27}$$

in which γ has the value

$$\gamma=(1-\omega^2r^2/c^2)^{-1/2} \quad . \tag{2.28}$$

In terms of the cyclotron frequency

$$\vec{\Omega}=-e\vec{B}_{ind}/\gamma mc \tag{2.29}$$

$$=-(2\pi Ne^2\lambda\omega r^2/\gamma mc^2)\hat{\epsilon}_3 \quad , \tag{2.30}$$

the orbit radius r may be expressed as

$$r=|e|E/\gamma m\omega(\omega+\Omega) \quad , \tag{2.31}$$

where $\Omega=|\vec{\Omega}|$. By substituting this expression for the orbit radius in Eq. (2.22), the phase velocity may be written

$$v_p=c\,[1-\omega_p^2/\gamma\omega(\omega+\Omega)]^{-1/2} \quad . \tag{2.32}$$

Since Ω is added to ω rather than subtracted from it, we note that the wave propagates as a so-called ordinary wave[9] irrespective of the value of the helicity. By solving Eqs. (2.23), (2.27), and (2.28) simultaneously we can now determine the electric intensity E, the orbit radius r, and the electron kinetic energy $(\gamma-1)mc^2$. The magnetic induction of the laser field is then determined by

$$B=(1-m\omega_p^2 r/|e|E)^{1/2}E \quad . \tag{2.33}$$

The fact that the three equations (2.23), (2.27), and (2.28) must be solved simultaneously is indicative of the self-consistent nature of the problem. The wave vector is intensity dependent, and neither of the field amplitudes E or B has the usual proportionality to the square root of the intensity.

We have obtained a number of solutions to these equations for the case of a neodymium-glass laser (wavelength $=1.06\times10^{-4}$ cm, $\omega=1.78\times10^{15}$/sec). In Table I we list for the electron plasma density and for several values of the radiation intensity the parameter a, the orbit radius r, the induced magnetic field B_{ind}, and the electron kinetic energy $(\gamma-1)mc^2$. For small intensities the orbit radius is close to the nonrelativistic value a, but for larger intensities the ratio r/a is decreased by almost an order of magnitude. We observe in Fig. 1 that the ratio r/a is more dependent on the radiation intensity than on the electron density. The electron kinetic energy, shown in Fig. 2, is essentially independent of the plasma density. The electric and magnetic laser field amplitudes are shown in Fig. 3. These amplitudes differ appreciably only for plasma densities near the critical density for circularly polarized waves, defined in Eq. (3.1). On the other hand the induced magnetic field, plotted in Fig. 4, is approximately proportional to the plasma density.

TABLE I. Plasma density $N=10^{21}/\text{cm}^3$.

I (W/cm^2)	a (cm)	r (cm)	B_{ind} (G)	$(\gamma-1)mc^2$ (keV)
10^{17}	6.17×10^{-6}	5.54×10^{-6}	5.48×10^{6}	30
10^{18}	1.46×10^{-5}	$1\ 02\times10^{-5}$	1.85×10^{7}	130
10^{19}	3.87×10^{-5}	1.50×10^{-5}	4.02×10^{7}	604
10^{20}	1.15×10^{-4}	1.67×10^{-5}	4.96×10^{7}	2692

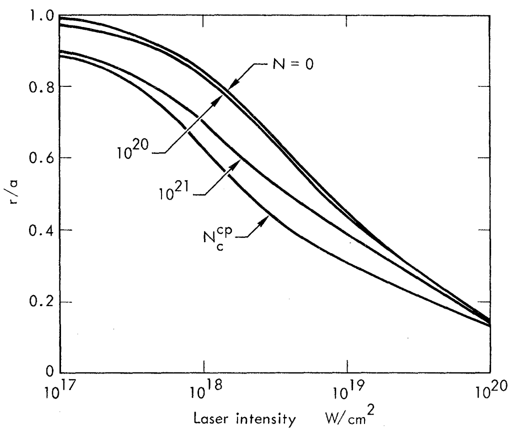

FIG. 1. Ratio r/a as a function of the radiation intensity I for various plasma densities.

III. DEPENDENCE OF PROPAGATION CHARACTERISTICS ON RADIATION INTENSITY AND PLASMA DENSITY

The dispersion relation (2.18) differs from the dispersion relation for a cold collision-free non-relativistic plasma with no magnetic field in that ω_p^2 is replaced by $(r/a)\omega_p^2$, which is an intensity-dependent term. We have seen that the ratio (r/a) diminishes from unity with increasing intensity. There are several important implications of this intensity dependence. First, the critical density of the electron plasma is no longer a function of the radiation frequency alone. By setting $k=0$ in Eq. (2.18) we obtain as the critical density for the propagation of circularly polarized light

$$N_c^{cp} = (a/r)N_c \ , \tag{3.1}$$

where N_c is given by Eq. (1.1). Second, the phase velocity, (2.22) decreases with increasing intensity. Third, while the usual definition of group

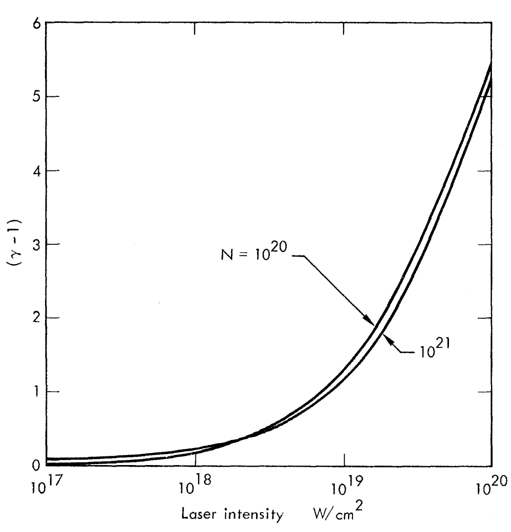

FIG. 2. Electron kinetic energy in units of mc^2.

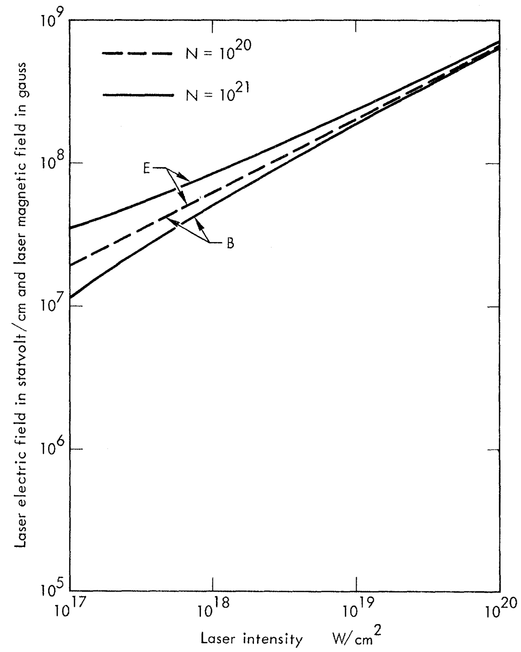

FIG. 3. Electric and magnetic fields of the wave inside the plasma. For electron densities less than $N=10^{20}$ cm^{-3} the electric intensity E and the magnetic intensity B are essentially equal, and are represented by the single broken line.

velocity may be formally written for circularly polarized waves as

$$v_g = \frac{c^2}{v_p}\left[1 - \frac{\omega_p^2}{2\omega}\frac{d}{d\omega}\left(\frac{\omega}{\gamma(\omega+\Omega)}\right)\right]^{-1} \ , \tag{3.2}$$

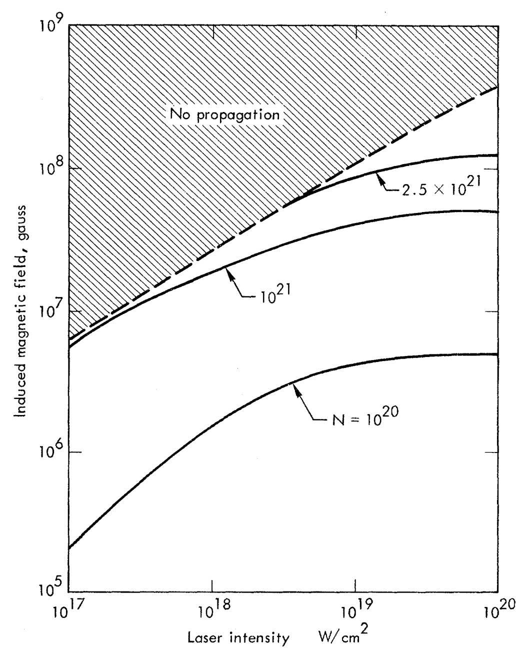

FIG. 4. Induced magnetic field.

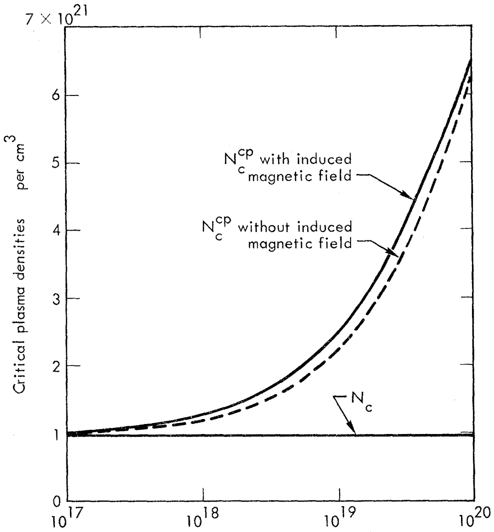

FIG. 5. Critical plasma density. The solid curve illustrates the dependence on the radiation intensity of the critical plasma density which is caused by the relativistic change of the particle mass and by the induced magnetic field. The intensity dependence of the critical plasma density neglecting the induced magnetic field is shown by the dotted curve. The horizontal line indicates the usual nonrelativistic critical plasma density.

it should be clearly recognized that v_g cannot have the physical interpretation of an energy transport velocity. We are dealing here with a nonlinear problem in wave propagation. The relativistic increase in particle mass with radiation intensity is one source of this nonlinearity. The induced magnetic field constitutes the other source. Both sources have the effect of reducing the electron orbit radius.

For a monochromatic wave, the energy transport velocity may properly be determined from the equation

$$v_{et} = I/U \quad , \tag{3.3}$$

where $I = (c/4\pi)EB$ is the magnitude of the Poynting vector, and

$$U = \frac{1}{8\pi}\left(E^2 + B^2 + B_{ind}^2\right) + mc^2(\gamma - 1)N \tag{3.4}$$

is the sum of the total electromagnetic energy density and the particle kinetic-energy density.

Figure 5 shows that the main contribution to the increase with radiation intensity of the critical plasma density stems from the relativistic change of the electron mass, whereas the induced magnetic field is responsible for a small additional increase. The propagation characteristics are rather strongly dependent on the plasma density. From Fig. 6 it is apparent that with increasing radiation intensity the phase velocity asymptotically decreases towards the speed of light, while the group and energy transport velocities both increase towards this limit. Since the phase velocity decreases with the radiation intensity, a laser pulse having a Gaussian-like intensity distribution along the transverse coordinates would be self-focusing.

IV. RADIATION LOSSES

If radiation reaction is taken into account, the electron motion must be described by the Lorentz-Dirac equation. Because of the (additional) force caused by radiative reaction, the electrons are driven into a type of orbit that is considerably more complex than circular orbits. The reaction is many orders of magnitude smaller than the force $e\vec{E}$ arising from the laser electric field. The electron acceleration continues therefore to be primarily transverse. However, it will be shown that, as a consequence of the radiative reaction, the electrons experience a longitudinal acceleration in the direction of wave propagation. The associated longitudinal electron motion, which is of the nature of a drift velocity, introduces a Doppler shift of the wave frequency which increases with time. Since these complications are beyond the scope of the present paper, the validity of our analysis is restricted to times such that the longitudinal electron velocity can be neglected, and the orbit is still approximately in a plane and circular.

For an electron moving in a circle of radius r with angular velocity ω the emitted synchrotron radiation constitutes a power loss given by[10]

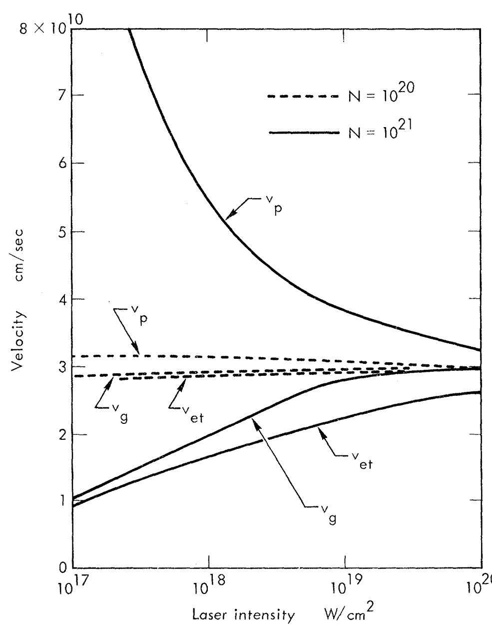

FIG. 6. Phase velocity, group velocity, and energy transport velocity as functions of the radiation intensity. For low intensities the phase velocity approaches the value $v_p = c\,[1 - (\omega_p/\omega)^2]^{-1/2}$.

$$P_{syn} = (2e^2/3c^3)\gamma^4\omega^4 r^2 \quad . \tag{4.1}$$

The corresponding radiative reaction force per particle is[11]

$$\vec{F} = -(2e^2/3c^3)\omega^2\gamma^4\vec{v} \quad . \tag{4.2}$$

This frictional force modifies the equation of motion (2.26) to

$$\gamma m \frac{\partial \vec{v}}{\partial t} = e\vec{E} + (e/c)\vec{v}\times(\vec{B} + \vec{B}_{ind}) - (2e^2/3c^3)\omega^2\gamma^4\vec{v} \quad . \tag{4.3}$$

In order to satisfy Eq. (4.3), the velocity $\vec{v}$ originally given by Eq. (2.8) must now be shifted slightly forward in time phase according to the equation

$$\vec{v} = \mathrm{Re}[i(e/|e|)V(\hat{\epsilon}_1 + i\lambda\hat{\epsilon}_2)e^{-i\alpha}e^{-i(\omega t - kz)}] \quad . \tag{4.4}$$

The phase shift α directs a component of the force $e\vec{E}$ along the velocity $\vec{v}$, which compensates exactly for the radiative reaction and thereby maintains constant the electron energy for steady-state propagation. The required shift in phase

$$\alpha = \sin^{-1}(2e^2/3c^3)\omega^3\gamma^4 r/|e|E$$
$$\simeq (2e^2/3c^3)\omega^3\gamma^4 r/|e|E \tag{4.5}$$

can be obtained by substituting Eq. (4.4) into Eq. (4.3). As a consequence of this phase shift, there arises a force $(e/c)\vec{v}\times\vec{B} = (|e|/c)|\vec{v}|\,|\vec{B}|(\sin\alpha)\hat{\epsilon}_3$. This force is the source of the above-mentioned longitudinal acceleration,

$$\frac{dV_3}{dt} = (1/\gamma m)(|e|/c)VB\sin\alpha$$
$$= (1/\gamma m)^2(4\pi e^2/c^2)I(\sin\alpha)/(\omega+\Omega)$$
$$= \frac{8\pi e^4\gamma\omega^2}{3m^3c^5(\omega+\Omega)^2}I \quad , \tag{4.6}$$

where in the second equation the function $\sin\alpha$ has been replaced by its argument α. The validity of the Eqs. (4.1)–(4.4) hinges on the assumption $V_3 \ll c$. In this case the acceleration (dV_3/dt) is approximately time independent, and this criterion is therefore satisfied for time intervals

$$\tau \ll \frac{C}{dV_3/dt} \quad . \tag{4.7}$$

In Table II we list values of

$$\tau = 0.01\,\frac{c}{dV_3/dt} \tag{4.8}$$

TABLE II. Plasma density $N = 10^{21}/\mathrm{cm}^3$.

I (W/cm^2)	α	Ω (sec^{-1})	dV_3/dt (cm/sec^2)	τ (sec)
10^{17}	1.25×10^{-8}	0.91×10^{14}	1.12×10^{16}	2.68×10^{-8}
10^{18}	1.91×10^{-8}	2.61×10^{14}	0.90×10^{17}	3.33×10^{-9}
10^{19}	0.98×10^{-7}	3.24×10^{14}	0.84×10^{18}	3.57×10^{-10}
10^{20}	2.54×10^{-6}	1.39×10^{14}	1.04×10^{19}	2.88×10^{-11}

that are representative of laser-pulse durations for which the present considerations are valid.

Equation (4.6) constitutes a generalization of the result obtained by Sanderson[12] and by Kibble[13] for the longitudinal electron acceleration associated with radiation friction; their result corresponds to Eq. (4.6) with $\gamma = 1$ and $\Omega = 0$. If expression (4.4) for the modified velocity is substituted into the wave equation (2.7), the dispersion relation including radiative damping

$$\omega^2 = k^2c^2 + e^{-i\alpha}(r/a)\omega_p^2 \tag{4.9}$$

is obtained. The wave number k is therefore complex, and in polar form

$$k = |k|e^{i\theta} \quad , \tag{4.10}$$

where

$$|k| = (1/c)[\omega^4 - 2(r/a)\omega^2\omega_p^2\cos\alpha + (r/a)^2\omega_p^4]^{1/4} \quad , \tag{4.11}$$

and

$$\theta = \tfrac{1}{2}\tan^{-1}\{(r/a)\omega_p^2(\sin\alpha)/[\omega^2 - (r/a)\omega_p^2\cos\alpha]\} \quad . \tag{4.12}$$

Since for a complex wave number the intensity becomes z dependent, and since the wave number itself is a function of the local intensity, the wave number itself becomes a function of z:

$$k(z) = k_r(z) + ik_i(z) \quad . \tag{4.13}$$

The factor $e^{-i(\omega t - kz)}$ appearing frequently in our previous equations must be replaced by the factor $\exp[-i(\omega t - \int_{z_0}^{z} k(\eta)d\eta)]$, where the wave is assumed to have a known intensity at the initial point z_0.

The power radiated by electron-ion bremsstrahlung may be determined from the semiclassical formula[14]

$$p_{brems} = (16e^2N_eN_iZ^2\omega_b r_0^2/3\beta)$$
$$\times\int_{\omega_{min}/\omega_b}^{\omega_{max}/\omega_b}\ln(\omega_b/\omega)d(\omega/\omega_b) \quad , \tag{4.14}$$

in which $\omega_b = \beta^2c^2m\gamma^2/\hbar$. The power loss given by Eq. (4.14) is quite insensitive to the lower limit of integration, and we take for this limit the plasma cutoff frequency $\omega_{min} = \omega_p$; for the upper limit we take the energetic limit $\omega_{max} = (\gamma - 1)mc^2/\hbar$. On performing the integration we obtain

$$p_{brems} = \frac{16e^2N_eN_iZ^2\omega_b r_0^2}{3\beta}\left[\frac{\omega_{max}}{\omega_b}\left(1 + \ln\frac{\omega_b}{\omega_{max}}\right) - \frac{\omega_{min}}{\omega_b}\left(1 + \ln\frac{\omega_b}{\omega_{min}}\right)\right] \quad . \tag{4.15}$$

The power densities associated with electron synchrotron radiation and electron-ion bremsstrahlung are compared in Fig. 7 for the case of a hydrogenic plasma ($Z = 1$). For low-Z plasma

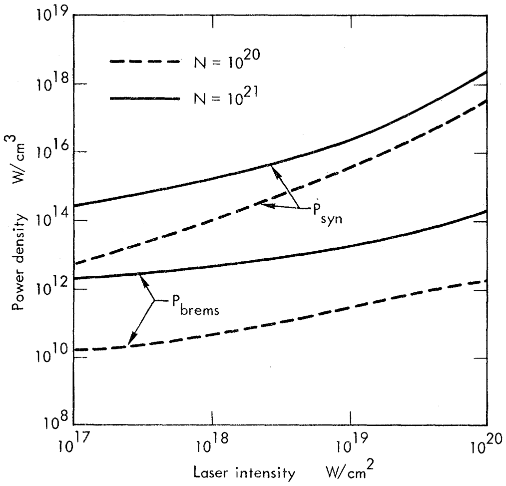

FIG. 7. Radiative losses caused by electron synchrotron radiation and by electron-ion bremsstrahlung. The energy radiated per unit plasma volume per unit time is plotted.

the bremsstrahlung loss is much smaller than the loss by synchrotron radiation and it may be neglected in the determination of α and k_i. At the maximum synchrotron power density shown in Fig. 7 the phase shift α is still quite small, namely, 2.5×10^{-6}. The variation of k_i with radiation intensity is illustrated in Fig. 8.

V. CONCLUSIONS AND CRITICAL COMMENTS

Extremely large intensities of laser radiation are realizable only in a small focal volume. Our assumption of a plane wave is justified even for a focal volume as small as 10^{-6} cm^3, for which the linear dimension would still be 100 wavelengths.

A more serious question concerns the amplitude of the induced magnetic field. Laser radiation is capable of penetrating into a dense plasma because the frequency is so large. However, a Fourier analysis in time of the induced magnetic field associated with a short laser pulse would contain components much lower in frequency than the laser frequency itself and would therefore lead to a relatively large diamagnetic response of the plasma. It must be realized that in a plasma with dimensions larger than the width of the laser beam, boundary effects may play a vital role in determining the induced magnetic field. A full analysis of this problem is beyond the scope of our paper, but should lead to a more accurate calculation of the amplitude of the induced magnetic field.

Even though the magnetic field generated by the inverse Faraday effect may theoretically attain values in the MG range for the laser-beam intensities we have considered, the increase of the critical plasma density is predominantly the result of the relativistic change of the electron mass. The relativistic enhancement of the wave propagation is by no means a unique feature of circular polarization. It is, however, more pronounced for this particular state than for any other state of polarization. Furthermore, circular polarization constitutes the only situation for which an intensity-dependent dispersion relation can be derived without encountering the problem of a particle mass varying with time.

The orbital radius and the orbital velocity of an electron drifting either parallel or antiparallel to the direction of wave propagation may differ substantially from the values predicted by the calculations of Sec. II, unless the drift velocity is negligible in comparison with light velocity. There are three reasons for these differences: (i) The drift velocity produces a Dopper shift in the angular frequency of the electric field experienced by the particle, (ii) the interaction between the particle and the laser magnetic field modifies the total force driving the particle in its orbit; and (iii) the drift velocity contributes directly to the relativistic mass increase. All of these effects must be taken into account when the electrons are assumed to have a distribution of drift velocities along the direction of propagation. In the case of particle drift normal to the direction of wave propagation, there is also Dopper shift of the frequency. The analysis of the electron motion then becomes even more complex.

In our analysis the electron density is treated as a constant, $n(\vec{x}, t) = N$. In order to account for density variations which may be caused, for example,

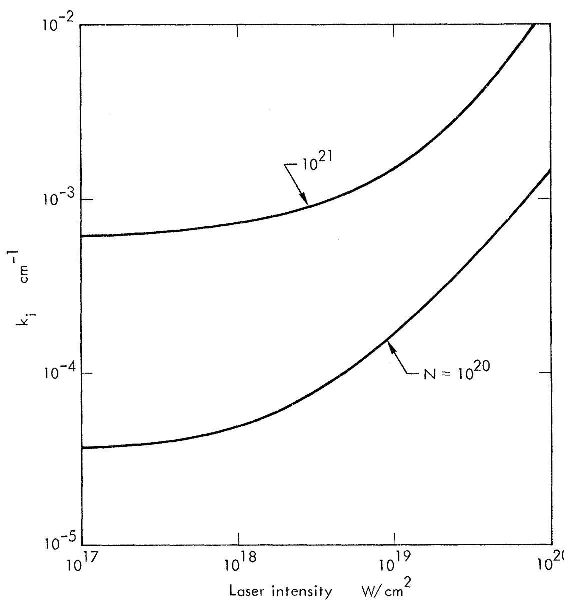

FIG. 8. Imaginary part of the wave number k as a function of intensity. The real part of the wave number is essentially unchanged by radiative losses.

by radiation pressure, finite laser-beam width, longitudinal modes, etc., it is necessary to adjoin the relativistic continuity equation

$$\frac{\partial}{\partial t}(\gamma n)+\mathrm{div}(\gamma n\vec{v})=0$$

to the set of simultaneous equations (2.23), (2.24), and (2.28). In a more general treatment the fluid approach would be replaced by a kinetic description of the plasma.

The electron temperature of dense plasmas created by laser irradiation of solid materials is on the order of several hundred eV,[15] whereas the electron kinetic energy associated with the directed orbital motion varies for the radiation intensities considered here between tens of keV and several MeV. The essential aspects of our problem should therefore be adequately represented in a fluid approximation.

Finally, it should be remarked that for a finite pulse length there exists a longitudinal component of the electron velocity independent of radiative reaction.[12,13,16–19]

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[1]Yves Pomeau and Daniel Quemada, Compt. Rend. 264, 517 (1967).

[2]J. Deschamps, M. Fitaire, and M. Lagoutte, Phys. Rev. Letters 25, 1330 (1970).

[3]M. Cox *et. al.*, Laser Focus 3 (No. 23), 21 (1967).

[4]A. I. Akhiezer and R. V. Polovin, Zh. Eksperim. i Teor. Fiz. 30, 915 (1956) [Sov. Phys. JETP 3, 696 (1956)].

[5]P. Kaw and J. Dawson, Phys. Fluids 13, 472 (1970).

[6]W. Lünow, Plasma Phys. 10, 973 (1968).

[7]N. L. Tsintsadze, Zh. Eksperim. i Teor. Fiz. 59, 1251 (1970) [Sov. Phys. JETP 32, 684 (1971)].

[8]Charles S. Roberts and S. J. Buchsbaum, Phys. Rev. 135, A381 (1964).

[9]P. A. Sturrock, in *Proceedings of the International School of Physics, Enrico Fermi, Course* 39 (Academic, New York, 1967).

[10]A. A. Sokolov and I. M. Ternov, *Synchrotron Radiation* (Permagon, New York, 1968), p. 12, Eq. (1.6).

[11]Expression (4.2) can be derived from Eqs. (7.1), (7.6), (7.11), and (7.11a) in Chap. I of Ref. 10, together with the equation $(d^2\vec{v}/dt^2)=-\omega^2\vec{v}$, which is satisfied by our expression (2.8a) for the orbital electron velocity.

[12]J. J. Sanderson, Phys. Letters 18, 114 (1965).

[13]T. W. B. Kibble, Phys. Letters 20, 627 (1966).

[14]J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962), p. 515.

[15]H. Puell, Z. Naturforsch. 25a, 1807 (1970).

[16]T. W. B. Kibble, Phys. Rev. 150, 1060 (1966).

[17]Jospeh H. Eberly and Arthur Sleeper, Phys. Rev. 176, 1570 (1968).

[18]Joseph H. Eberly, in *Progress in Optics*, edited by E. Wolf (North-Holland, Amsterdam, 1969), Vol. 7.

[19]E. S. Sarachik and G. T. Schappert, Phys. Rev. D 1, 2738 (1970).