$$A = (\gamma_a \gamma_b)^{-1} [\gamma_a \gamma_b + 2\gamma_{ab}(p/T_1) + (p/T_1)^2] . \tag{103}$$

This increase of laser intensity with pressure comes basically from a reduction of the third-order (or saturation) term. An atom gives up energy to the radiation field and then makes a deflecting collision before it can reabsorb any radiation at the same frequency.

If there were no deflecting collisions and only phase-changing collisions [see paper I, Eq. (126)], $1/T = 0$ and A becomes

$$A = \gamma_{ab}^{-1} [\gamma_{ab} + \delta_1 p] , \tag{104}$$

where δ_1 is the broadening factor per torr from phase-changing collisions [see paper I, Eq. (144), for definition of $\delta = \delta_1 p$]. In that case the maximum intensity would have a linear variation with pressure.

Thus, if the tuning curves are measured as in Fig. 5 with $\mathfrak{N}$ constant, the existence and magnitude of the effect of deflecting collisions can easily be determined. It is not expected that the coefficient A[Eq. (103)] will be as simple as the pure velocity-changing case, but the major effects of deflecting collisions can nevertheless be discerned.

The detailed features of the tuning dip will not be discussed here. In general, the dip shows the effects of phase-changing collisions. The fine structure determined from Eq. (101) will be useful when there are only velocity-changing collisions present. This might be the case in some molecular lasers.

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Pleochroic Halos and the Constancy of Nature: A Reexamination

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A thorough reanalysis of the early measurements of pleochroic halos in light of today's standards and knowledge indicates that, contrary to widespread belief, these halos do not provide proof that the laws of radioactive decay are constant in time.

One of the great bedrocks of physics and astronomy is the belief that physical processes are invariant over cosmologically long periods of time. Despite the fundamental importance of this belief, there is extraordinarily little direct evidence of its validity.

One very familiar piece of evidence comes from the examination of the identifiable spectral lines of distant galaxies. The frequencies of these lines appear to have been the same at the time of their creation (billions of years ago) as the frequencies of equivalent lines created today in terrestrial laboratories. Observable differences between these two sets are attributed (almost certainly correctly) to the Doppler red shift caused by galactic recession.

There is another piece of evidence often cited, considerably less familiar however, which has been around for over 60 yr.[1] This is the information obtained from geological phenomena known as pleochroic halos, originally discovered by Joly and subsequently explained by him. A small grain of radioactive material, say ${}_{92}U^{238}$, at some time in the past may have become embedded in a mineral, usually biotite mica. The subsequent decay chain produces eight α particles (among other things) and each of these α's possesses a characteristic energy and, hence, range. The mica becomes chemically darkened by the α's at the end of their range, and eight concentric dark spheres are formed in the rock over a period of time.

Upon thin slicing of the mica, a pattern of eight dark circles or "halos" is evident. For ${}_{92}U^{238}$ the diameter of the concentric ring system is about 40 μ. Here then is a precise record of an event taking place hundreds of millions of years ago. And it is an event whose features and characteristics can be reproduced today for purposes of comparison. Since α energy, and hence range, is a monotonically increasing function of decay constant, a comparison of old halo radii with equivalent range values today should reveal any long-term change in α-decay half-lives.

It is the usual statement in a variety of books,[2-5] and is generally held to be true, that studies of pleochroic halos have shown that physical processes are indeed invariant over long periods of time. Two main investigators have studied these halos: Joly[1,4,6,7] and Henderson.[8-11] It is on the basis of their work that the belief in pleochroic-halo evidence is founded. The work of Henderson and his co-workers is more recent than Joly's and is considerably more accurate both theoretically and experimentally.

It is the purpose of this paper to show that a close examination of Henderson's work, in the light of modern knowledge of energy-loss phenomena, definitely does *not* support the conclusion that the α decays involved are constant in time. In fact, if his measurements were to be accepted at face value as accurate, they would tend to provide evidence that these processes *have* changed over periods of several hundred million years.

Before turning to an analysis of Henderson's work, we consider an interesting aspect of this problem which might have some bearing on its understanding: the cosmological expansion of the universe. If Hubble's constant H is taken to be the usual 100 km/sec megaparsec, then a time span of, say, 500 million years corresponds to an increase of lengths and distances of about 5%.

Although cosmological red shifts of distant galactic lines force a consistent belief that all space is expanding on a very small scale,[12] this does not necessarily imply that physical objects such as rocks are also expanding. If rocks do *not* expand cosmologically, then the interpretation of ancient-modern α-range consistencies (or inconsistencies) is straightforward.

Still the possibility that rocks expand cosmologically cannot be ruled out and should this be the case, the interpretation of range-consistency measurements is more involved. It might appear at first thought that Henderson's pleochroic halos from the Precambrian period (> 500 million years ago) should be at least 5% shorter than equivalent ranges today, providing, of course, that rocks do expand cosmologically. (Some of his uranium halos are Devonian and 350–400 million years old. They are less accurately measured.) In this case pleochroic halos would be a possible way in which the cosmological expansion of the universe might be verified.

However, the Precambrian halos were made in specific samples of mica which were more dense (because their dimensions were, say, 5% less) than they are today. For a given energy α, its range is inversely proportional to the stopping material's density. Hence even assuming that the decay α energies were the same 500 million years ago, the density of the mica sample was not, and it is an incorrect oversimplification to look for a simple 5% lengthening in the halo radius over a 500 million year span.

The problem becomes further entangled because it is difficult to conceive of the solid crystalline biotite as expanding in time without the individual molecules and atoms comprising it also expanding. This follows since the same electrostatic forces and laws of quantum mechanics are at work in the solid-state rock crystal as in the atoms. Perhaps the laws of nature are such that bound electrostatic macrosystems can expand while the molecules and atoms that comprise them do not, but it does not seem plausible. We emphasize again that rocks may not expand cosmologically, in which case, of course, atoms and molecules do not expand either.

We can get some insight into how these atomic expansions might affect the range of α particles in time by examining the standard expression for α ranges. Except for small corrections the relevant formula for the range of an α of energy E in a material of atomic charge Z with atomic density N is[13,14]

$$R(E)=\frac{1}{128\pi}\;\frac{m_\alpha}{m_e}\,I^2\,\frac{1}{NZe^4}\,f\left(4\frac{m_e}{m_\alpha}\frac{E}{I}\right), \qquad (1)$$

where $f(x)=\int_0^x (x/\ln x)\,dx$. In Eq. (1) I is an ionization energy specific to the material involved. If the expression in Eq. (1) increases in time as Ht (for $Ht \ll 1$), then ranges preserved from long ago will agree *exactly* in length with present day ranges. This is obvious since the stretching of the rock by Ht will be exactly matched by the fact that present day ranges (used for comparison) are Ht greater than ancient ranges. Of course, agreement is also obtained if $R(E)$ is independent of time and rocks do not expand cosmologically.

It is of interest, therefore, to determine under what conditions we can expect $R(E)$ to grow in time as Ht. Any deviation of pleochroic-halo ranges from modern values would then indicate a violation of one of these conditions.

We make the following general assumptions: (i) The Bohr radius a_0 and the atomic nucleus both expand in time as Ht for $Ht \ll 1$; (ii) the frequency of atomic spectral lines is constant in time as is the speed of light[15]; (iii) the uncertainty principle holds at all times.

From $E_\nu = h\nu$ for spectral lines and (ii) above we conclude that E_ν must vary in time in the same way that h does. Since I is a spectral energy it varies as E_ν does. On the other hand the magnitude of E is obtained from the uncertainty relation for an α confined to an atomic nucleus. Using (i) and (iii) we have that the momentum p_α varies as h/t and hence E varies as $h^2/m_\alpha t^2$. But we have

$$\frac{h^2}{m_\alpha t^2} = \frac{m_e}{m_\alpha}\frac{m_e e^4}{h^2}\frac{h^4}{m_e^2 e^4}\frac{1}{t^2}, \tag{2}$$

which varies as

$$(m_e/m_\alpha)\, I\, a_0^2\, t^{-2} \quad ,$$

and we conclude that (providing m_e and m_α vary in the same way in time) E and I have identical time variations. Examining Eq. (1) and using the fact that the ratio I^2/e^4 must vary as E_ν^2/e^4 which is t^{-2}, we conclude that $R(E)$ varies as t since N varies as t^{-3}.

The above arguments are not foolproof (even assuming rocks expand cosmologically) but they are important because they imply that even the existence of expanding rock probably would *not* lead to a deviation of pleochroic-range values from present day ranges. Exact consistency of ranges over hundreds of millions of years is perfectly compatible with expanding rocks, but this would also appear to require that some or all of the constants m, e, and h be time dependent.

We now turn to an analysis of Henderson's work. What follows is independent of the preceding, the purpose of which was to give us some idea of what might be expected as to deviations. In particular, we should be looking for changes in α ranges of the order of 3% in Devonian rocks and perhaps a few percent more in Precambrian. These values are only indicative and must not be taken too literally. It is important to bear in mind that at the time Henderson did his work the age of Devonian and Precambrian rocks was reasonably well known but the age of the universe was thought to be about two billion years. Hence inconsistency in that framework meant variations of 15% to 25% not 3% to 5%. Any apparent consistency to a few percent was thought at that time to be strong evidence for the invariance of nature. We must be considerably more exacting today.

In Henderson's and Joly's work the radii of halos were measured and then compared, not to present day ranges in mica, *but to equivalent present day ranges in air*. This procedure raises severe theoretical and instrumental problems which were not realized at that time and which have only recently become apparent.

In any case, Henderson, who used a specially designed photometer to measure rings, claimed an accuracy of air-equivalent range determinations for the halos of no worse than 3%. This error is due to finite size of the radioactive central speck, systematic equipment errors, uncertainty in the theoretical air-mica conversion factor, and other small effects. Normalizing one mica range to agree with the equivalent air range, i.e., empirically determining the air-mica conversion factor once, Henderson claimed a general agreement between old and modern ranges of 2% to 3% for thorium and uranium, although the latter had one deviation of 5.5%. One thorium range had a possible deviation of 12%. Note that he would have masked any true deviation effect near the normalization point unless the empirical conversion factor could be independently checked.[16]

If we were to stop here it would be obvious that in light of current expectations the agreement shown by Henderson does not substantiate the claim of invariance of α decay over periods of 300 million to 500 million years.[16]

However, the actual situation is more interesting because Henderson based his results on the apparently inaccurate results of Briggs,[17] which showed only a slight change (about $1\frac{1}{2}$%) in the mica-air conversion factor for the spread of relevant energies. (Henderson ignored these differences, in any case, specifically indicating they were of no particular interest.)

Recent theoretical and experimental work[14,18] has shown that this conversion factor varies over the range of interest (about 4–9 MeV) considerably more than this. The reason for this is readily apparent from Eq. (1). Suppose we take the ratio of two specific α-energy ranges and do this in air and in mica. We have, on Henderson's assumption, that

$$f\left(4\frac{m_e}{m_\alpha}\frac{E_1}{I_A}\right)\Big/ f\left(4\frac{m_e}{m_\alpha}\frac{E_2}{I_A}\right) = f\left(4\frac{m_e}{m_\alpha}\frac{E_1}{I_M}\right)\Big/\left(4\frac{m_e}{m_\alpha}\frac{E_2}{I_M}\right). \tag{3}$$

However, this equality holds only if both sides are essentially independent of I.[19] Taking $E_1 = 4.5$ MeV and $E_2 = 7.5$ MeV, which does not even fully span the actual α-energy range, we obtain[14] for the left-hand side a ratio of 0.442 and for the right-hand side a ratio of 0.474. The mica ratio is more than 7% higher than the air ratio. This effect was totally ignored by Henderson, who assumed a constant ratio throughout the energy range. In checking his experimental air-mica conversion factor by doing a theoretical calculation of it, he used Bragg's rule which has recently been found to be in error by several percent when used for compounds rather than mixtures.[20] The point here is that Henderson's theoretical check was possibly no check at all. A solely empirical conversion factor would wash out any real deviation effect and, of course, would cause close agreement for ranges near to the normalization point as mentioned above.

If one chooses to accept Henderson's work at face value as establishing that old mica ranges and present day air ranges do agree to within a few percent, using a constant conversion factor, then his results clearly imply that a modern reevaluation using the correct energy-dependent conversion factor will produce deviations of several percent between old and new ranges. The implication is then that something in the physical realm *has* indeed changed in the last several hundred million years by about the amount expected from the present value of Hubble's constant.

In light of Henderson's large deviations for some radii and his inadequate theoretical handling of the air-mica equivalency difficulty, his results should be considered extremely doubtful.

It is clear that in light of today's theoretical understanding and with modern instrumentation, the measurement of pleochroic halos should be undertaken again. It appears quite within the realm of possibility to measure consistency to within 2% for rock in excess of 500 million years. It might even be feasible to measure the range of present day machine-produced α's of appropriate energy in the very same samples of mica containing the halos. This would allow probably an even more direct consistency check of even sharper accuracy (perhaps 1%). Long term deviations of 5% or more would be readily apparent.

It is clear that the early work of Joly and Henderson, though superior in its day, can no longer pass muster by today's standards. Contrary to established thinking, pleochroic halos at the moment do *not* establish that radioactive α decay is constant over geological periods of time. If anything, they imply the opposite.

The author wishes to thank Dr. Richard Ward and Dr. Cyrus Moazed for helpful discussions.

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PHYSICAL REVIEW A VOLUME 5, NUMBER 3 MARCH 1972

Structures and Energies of Grandjean-Cano Liquid-Crystal Disclinations*

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Closed analytical expressions for the orientation of the director and the total distortional elastic energy are given for de Gennes's model of a Grandjean-Cano disclination. The director expression is shown to be equivalent to de Gennes's infinite-sum solution. This model is generalized and similar expressions for single disclinations on one of the parallel surfaces and for double disclinations between the boundaries are derived. Approximate energies are computed for these singularities in the case of nonparallel surfaces, and comparisons are made to predict the conditions for their stability. A more realistic model where many equally spaced disclinations occur within wedge-shaped boundaries is proposed and elastic distortion energies are calculated for this case.

I. INTRODUCTION

Many kinds of disclinations are known to occur in cholesteric liquid crystals. The most commonly observed is the Grandjean-Cano single disclination produced by a discontinuous change of one half-turn of the cholesteric helix.[1] Recently, the Orsay Liquid Crystal Group[2,3] has observed double disclinations which, under certain circumstances, are more stable than single disclinations and appear as heavier lines under the microscope. They are produced by the abrupt change of one full turn in the helicoidal structure.

The relative stability of these singularities can be predicted by comparing their total energies calculated on the basis of an appropriate model. Two models have been proposed. The topological models of Klêman and Friedel[4] do not seem suitable for quantitative treatment. We choose to use de Gennes's[5,6] mathematical model for a single disclination in the center of the layer. This model can easily be generalized to higher-order singularities occurring in the middle or on the surface. de Gennes gives a solution for the director in the form of an infinite series, but does not calculate elastic energies.

Kassubek uses a different procedure to obtain a solution for the director.[7] His method is general and covers higher-order singularities, and he points out that an elastic term not considered in de Gennes's treatment may be important in computing the energies of these disclinations. The results of his approximate energy calculations, however, are not precise enough to predict the relative stability of the various disclinations.

A closed analytical solution is developed for the director in the generalized de Gennes disclination model and we are able to give analytical expressions for the total elastic deformation energies. These calculations are extended to a wedge geometry where there is a series of equally spaced disclinations. The energies of the various singularities can then be precisely compared to predict their stabilities.

II. ELASTIC-ENERGY EXPRESSION

The second-order phenomenological elastic theory of Nehring and Saupe[8] contains two additional elastic constants that were left out in the earlier work of Frank.[9] We begin by assuming a planar cholesteric structure with D_∞ symmetry. The free-energy density g for such a structure having an intrinsic pitch of $2\pi/q_0$ is

$$g=\tfrac{1}{2}k_{11}(\nabla\cdot\vec{L})^2+\tfrac{1}{2}k_{22}(q_0+\vec{L}\cdot\nabla\times\vec{L})^2+\tfrac{1}{2}k_{33}[(\vec{L}\cdot\nabla)\vec{L}]^2+k_{13}\nabla\cdot(\vec{L}\nabla\cdot\vec{L}) . \quad (1)$$

The first three terms are, respectively, pure splay, twist, and bend deformation-energy densities. The elastic constants k_{11}, k_{22}, and k_{33} can be experimentally measured.[10] The value of the k_{13} elastic constant has not yet been determined. A crude theoretical model that neglects all short-range effects gives $k_{11}:k_{22}:k_{33}:k_{13}=5:11:5:-6$.[11]

The z direction is defined to be perpendicular to the unit director vector $\vec{L}$. A two-dimensional problem is made by requiring that there be no variation in $\vec{L}$ along the y direction. Let φ be the angle $\vec{L}$ makes with the x axis. $\vec{L}$ in the Cartesian coordinate system (x,y,z) is then

$$\vec{L}=(\cos\varphi(x,z), \sin\varphi(x,z), 0) . \quad (2)$$