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Two-dimensional Sisyphus cooling in a three-beam laser configuration

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Calculations on the steady-state distribution of atoms cooled in three coplanar laser beams are presented. For this three-beam configuration, the achievements of Sisyphus cooling are independent of fluctuations in the relative time phases between the beams. The atomic motion is quantized and the steady-state solution of the density-matrix equations is found within a secular approximation. We obtain a minimum width of the cooled momentum distributions on the order of six photon momenta.

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Laser cooling has been the subject of much work during the last 20 years [1,2] in both experiments and theory. In "optical molasses" experiments, atoms with kinetic energies equivalent to temperatures of a few microkelvin have been observed [3,4], corresponding to rms momenta on the order of the photon momentum $\hbar k$. Theoretically, such low temperatures have been accounted for in one-dimensional laser configurations within both semiclassical [5] and quantum treatments of the atomic motion [6], and recently two-dimensional laser cooling has also been studied by full quantum calculations [7,8] similar to those of [6]. Extensive numerical calculations on two-dimensional (2D) and 3D cooling have been performed within a standard semiclassical description [9,10], but in the case of optimum cooling this approach is not necessarily justified.

In this Rapid Communication, quantum calculations on polarization gradient cooling in a three-beam, two-dimensional laser configuration are presented. We use the secular approximation described in [6], which is useful for the type of polarization gradient cooling named Sisyphus cooling. The three-beam configuration is experimentally attractive since in this case no control over the time phases between the beams is needed [11], as long as phase fluctuations are slow on the time scale of the atomic motion. Changes in the two relative phases correspond to a translation in the two-dimensional space.

The simplest transition that exhibits Sisyphus cooling is one of angular momenta $J_g = \frac{1}{2} \to J_e = \frac{3}{2}$. When the oscillating electric field is confined to a plane, there is no coherence between the two ground-state Zeeman sublevels of this system. This property is independent of simplifications in the pattern of spontaneous emission, as demonstrated in Ref. [8].

As in the 2D experiments of Ref. [11], the laser field studied is composed of three traveling waves of equal strength and with coplanar wave vectors which are rotated 120° with respect to each other. Each component is linearly polarized in the plane. Since the relative phases are irrelevant, the positive frequency part of the electric field may be represented by the expression

$$\vec{E}^{(+)}(\vec{r}) = E_0\left[\vec{e}_y e^{i\vec{k}_1\cdot\vec{r}} + \left(\frac{\sqrt{3}}{2}\vec{e}_x - \frac{1}{2}\vec{e}_y\right)e^{i\vec{k}_2\cdot\vec{r}} + \left(-\frac{\sqrt{3}}{2}\vec{e}_x - \frac{1}{2}\vec{e}_y\right)e^{i\vec{k}_3\cdot\vec{r}}\right], \quad (1)$$

with the wave vectors

$$\vec{k}_1 = k\vec{e}_x, \quad \vec{k}_2 = -\frac{k}{2}\vec{e}_x - \frac{\sqrt{3}k}{2}\vec{e}_y, \quad \vec{k}_3 = -\frac{k}{2}\vec{e}_x + \frac{\sqrt{3}k}{2}\vec{e}_y.$$

We study the case of a weak laser field, i.e., the saturation parameter

$$s_0 = \frac{\Omega^2/2}{\delta^2 + \Gamma^2/4}$$

is small, $s_0 \ll 1$. Here, δ denotes the frequency detuning, $\delta = \omega_{\text{laser}} - \omega_{\text{atom}}$, Γ the linewidth of the atomic transition, and Ω the Rabi frequency corresponding to one traveling wave and a transition with maximum value, d, for the dipole moment, $\Omega = 2dE_0/\hbar$. In this limit, it is appropriate to perform an adiabatic elimination of the excited states and work with an atomic density matrix in the basis of ground-state sublevels only [12,13]. Then the ground-state density matrix solves the equation

$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H,\rho] + \mathcal{L}_{\text{relax}}[\rho] \quad (2)$$

with operators for an atom of mass M

$$H = \frac{\vec{P}^2}{2M} + U, \quad (3a)$$

$$U = \frac{1}{2}\hbar\delta s_0 \mathcal{V}^{(-)}\mathcal{V}^{(+)}, \quad (3b)$$

$$\mathcal{L}_{\text{relax}}[\rho] = -\frac{1}{4}\Gamma s_0\left\{\mathcal{V}^{(-)}\mathcal{V}^{(+)},\rho\right\} + \frac{1}{2}\Gamma s_0\frac{3}{8\pi}\int d\Omega_{\vec{n}}\sum_{\vec{\epsilon}\perp\vec{n}} W_{\vec{\epsilon}}(\vec{n})\rho W_{\vec{\epsilon}}(\vec{n})^{\dagger}, \quad (3c)$$

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$$W_{\vec{\epsilon}}(\vec{n}) = \left(\vec{\Delta}^{(+)} \cdot \vec{\epsilon}\right)^{\dagger} e^{-ik\vec{n}\cdot\vec{R}} \mathcal{V}^{(+)}. \tag{3d}$$

The operators of atomic position and momentum have been denoted $\vec{R}$ and $\vec{P}$. The dipole operator $\vec{D}$ has been decomposed in a raising and a lowering part and expressed in the form $\vec{D} = d(\vec{\Delta}^{(+)} + \vec{\Delta}^{(-)})$, and the dimensionless operators $\mathcal{V}^{(+)}$ and $\mathcal{V}^{(-)}$ represent the corresponding terms in the atom-laser interaction potential $V = -\vec{D} \cdot \vec{E}$, in the rotating-wave approximation,

$$V = \frac{\hbar\Omega}{2}\left[\mathcal{V}^{(+)} + \mathcal{V}^{(-)}\right]. \tag{4}$$

In the basis of standard polarizations, $\vec{e}_q, q = 0, \pm 1$, the components of $\vec{\Delta}^{(+)}$ read

$$\Delta_q^{(+)} = \sum_m \mathcal{C}_m^q \, |e, m+q\rangle\langle g, m|. \tag{5}$$

Here, the states correspond to a well defined angular momentum along the z axis, and the quantities $\mathcal{C}_m^q$ denote the Clebsch-Gordan coefficients of the atomic transition. The summation in (3c) extends over two mutually orthogonal polarization vectors for each value of the unit vector $\vec{n}$, indicating the direction of spontaneous emission of a photon within the solid angle $d\Omega_{\vec{n}}$.

With the electric field of (1) and the simple atomic transition $J_g = \frac{1}{2} \to J_e = \frac{3}{2}$, we find that the optical potential U is diagonal in the basis of ground states $|g,m\rangle \equiv |g_{\pm}\rangle$ with diagonal elements

$$\begin{aligned} U_{\pm}(\vec{R}) = -U_0 \Bigg\{ & \left[1 + \sin^2\left(\frac{\sqrt{3}}{2}kY\right) - \cos\left(\frac{3}{2}kX\right)\cos\left(\frac{\sqrt{3}}{2}kY\right)\right] \\ & \pm \frac{\sqrt{3}}{2}\left[\cos\left(\frac{3}{2}kX\right)\sin\left(\frac{\sqrt{3}}{2}kY\right) - \sin\left(\frac{\sqrt{3}}{2}kY\right)\cos\left(\frac{\sqrt{3}}{2}kY\right)\right]\Bigg\}, \end{aligned} \tag{6a}$$

$$U_0 = \frac{2}{3}\hbar|\delta|s_0. \tag{6b}$$

Here, we have chosen a negative detuning, δ, to see cooling. We note from (6) that U_+ and U_- differ by a translation in space only.

Due to the periodicity of the optical potential, the eigenstates of the Hamiltonian, H, may be characterized by a band index n and a Bloch vector $\vec{q}$, in addition to the internal state $g_{\pm}$. We find the eigenstates in, e.g., U_+ as the direct product of $|g_+\rangle$ and a spatial eigenfunction of Bloch type $|n, \vec{q}\rangle$

$$\langle \vec{R}|n,\vec{q}\rangle = e^{i\vec{q}\cdot\vec{R}}\phi_{n,\vec{q}}(\vec{R}), \tag{7}$$

where $\phi_{n,\vec{q}}(\vec{R})$ has the periodicity of $U_+(\vec{R})$. The vector $\vec{q}$ is chosen within the first Brillouin zone which is a hexagon of outer radius k with two corners at $\pm k\vec{e}_x$.

To make the problem tractable, we replaced the spontaneous emission pattern in (3c) by a distribution which contains photons emitted along the z axis and along the directions of the three laser beams only. This distribution preserves the triangular symmetry of the problem in the xy plane. The last term in the relaxation operator $\mathcal{L}_{\text{relax}}$ then becomes

$$\frac{1}{2}\Gamma s_0 \sum_{\vec{n}} \alpha_{\vec{n}} \sum_{\vec{\epsilon}\perp\vec{n}} W_{\vec{\epsilon}}(\vec{n})\rho W_{\vec{\epsilon}}(\vec{n})^{\dagger}, \tag{8}$$

where $\alpha_{\vec{n}}$ are normalization factors of $\frac{1}{4}$ for $\vec{n} = \pm\vec{e}_z$ and of $\frac{1}{6}$ for $\vec{n} = \pm\frac{\vec{k}_j}{k}, j = 1,2,3$. With this emission pattern, only three Bloch vectors are coupled and we have used the triple

$$\vec{q}_1 = \vec{0}, \quad \vec{q}_2 = k\vec{e}_x, \quad \vec{q}_3 = \frac{k}{2}\vec{e}_x + \frac{\sqrt{3}k}{2}\vec{e}_y.$$

When diagonalizing the Hamiltonian in order to find the periodic functions $\phi_{n,\vec{q}}$, we use a Fourier expansion in momentum states and introduce a cutoff at momenta $|p_x|, |p_y| = \mathcal{N}\hbar k$. In the results to be presented below, the parameter $\mathcal{N}$ ranges between 16.5 and 25.5.

As discussed in the papers [6–8], the secular approximation is applicable when the ratio between a typical splitting between eigenenergies in the optical potential and a typical linewidth of a transition between them is large. In two-dimensional systems, these two quantities are on the order of the recoil energy of the atom associated with the emission of a photon, $E_R = \hbar^2k^2/2M$, and $\hbar\Gamma s_0$, such that the validity condition reads

$$\frac{E_R}{\hbar\Gamma s_0} = \frac{2}{3}\frac{|\delta|}{\Gamma}\frac{E_R}{U_0} \gg 1. \tag{9}$$

Within this region, the equation of evolution in (2) can be reduced to a set of rate equations between diagonal elements of the density matrix $\Pi_i = \langle i|\rho|i\rangle$ in the basis of eigenstates $|i\rangle = |n, \vec{q}, \pm\rangle$ of the Hamiltonian (3a),

$$\frac{d\Pi_i}{dt} = -\sum_j \gamma_{i\to j}\Pi_i + \sum_j \gamma_{j\to i}\Pi_j, \tag{10}$$

as also described in [6]. The rates $\gamma_{j\to i}$ are obtained from the last term in (3c) with the emission pattern of (8), and one finds the expression

$$\gamma_{j\to i} = \frac{1}{2}\Gamma s_0 \sum_{\vec{n}} \alpha_{\vec{n}} \sum_{\vec{\epsilon}\perp\vec{n}} |\langle i|(\vec{\Delta}^{(+)} \cdot \vec{\epsilon})^{\dagger} e^{-ik\vec{n}\cdot\vec{R}} \mathcal{V}^{(+)}|j\rangle|^2. \tag{11}$$

Apart from the common factor of Γs_0, the rates depend upon the parameters in the problem only through the ratio $U_0/E_R = \frac{2}{3}\hbar|\delta|s_0/E_R$ that appears in the matrix elements. Consequently, the steady-state populations Π_i depend on U_0/E_R only.

In Fig. 1, we show the populations of the lowest ten energy bands found by solution of (10) in the steady state, for three different values of the cutoff parameter $\mathcal{N}$. These populations vary smoothly with U_0/E_R, as in the one-dimensional case of [6]. This is in contrast to the results of [7], where tunneling through a potential barrier between a shallow and a deep potential well led to

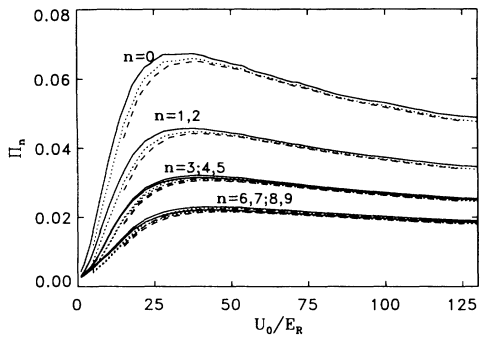

FIG. 1. The steady-state populations of the lowest ten bound energy bands shown for different values of the cutoff in momentum space, $|p_x|, |p_y| \leq \mathcal{N}\hbar k$. The solid curves give the results for $\mathcal{N} = 16.5$, the dotted curves for $\mathcal{N} = 21$, and the dashed lines for $\mathcal{N} = 25.5$.

resonances, especially for the population of the ground state, Π_0. The values of, e.g., Π_0 are smaller than those found in 1D due to the higher density of states resulting from the higher dimensionality, as also discussed in [7]. For values of the potential depth parameter U_0/E_R larger than or on the order of 50 and for $\mathcal{N}$ larger than around 17, the populations do not depend on $\mathcal{N}$, and we may rely upon our predictions for the populations.

In Fig. 2, the momentum distributions for a relatively small and a larger value of the potential depth are shown. For the shallower potential, clear asymmetries are observed, with wings in the directions of the laser beams, $\pm\frac{\vec{k}_j}{k}$. The same directions appear as lines of escape in semiclassical calculations, similar to those presented in [8]. Here, the trajectory of an atom is determined from a Langevin type of equation with a potential which alternates between U_+ and U_- at the known transition rates between the states $|g_+\rangle$ and $|g_-\rangle$. Only Doppler effects that were not included in our calculations eventually slow down these "escaping" atoms.

As we have discussed in [8], the existence of lines of escape in the semiclassical picture of Sisyphus cooling indicates that quantities such as the mean squared momentum, may not be well defined. This is illustrated by the lack of convergence of $\langle\vec{p}^2\rangle$ with increasing value of the cutoff parameter, as shown in Fig. 3(a). On the other hand, as indicated in part (b) of the figure, it is meaningful to evaluate a kinetic energy parameter p_e^2/M where p_e is the radius of the circle in momentum space contain-

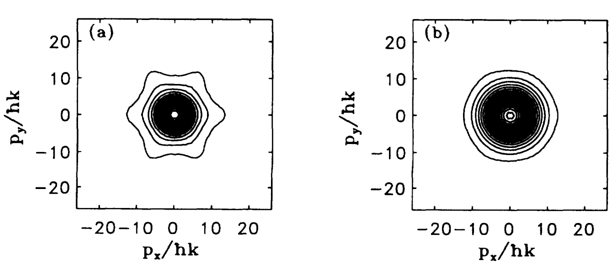

FIG. 2. Contour plots of the momentum distributions for (a) $U_0/E_R = 20$ and (b) $U_0/E_R = 50$. Note the pronounced asymmetry along the directions of the laser beams. The calculations were performed with a momentum cutoff at $\mathcal{N} = 25.5$.

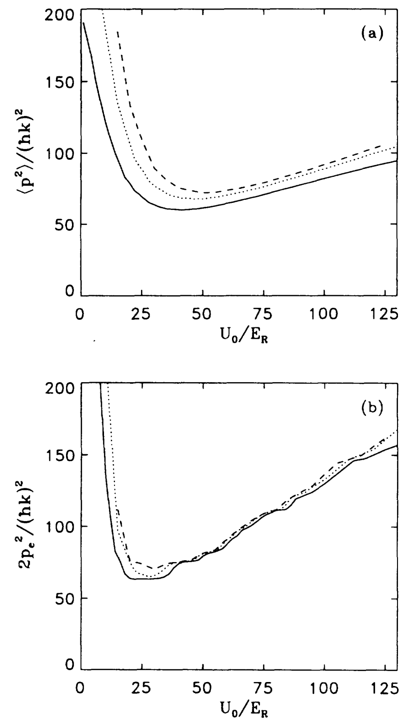

FIG. 3. (a) The variation of $\langle\vec{p}^2\rangle$ as a function of the potential depth parameter U_0/E_R, shown for different values of the cutoff parameter $\mathcal{N}$. Solid line, $\mathcal{N} = 16.5$; dotted line, $\mathcal{N} = 21$; and dashed line, $\mathcal{N} = 25.5$. No convergence is observed with increasing cutoff, which is a consequence of the existence of escape lines as seen in the momentum distributions in Fig. 2. In (b) we show the corresponding variation of the quantity, $2p_e^2$, which is the 2D kinetic-energy parameter that may be compared with the mean squared momentum. Here convergence is reached, for U_0/E_R larger than around 40.

ing a fraction of $(1 - \frac{1}{\sqrt{e}})$ of the atoms. For a symmetric Gaussian, p_e corresponds to the rms momentum along an axis.

In experiments on two-dimensional Sisyphus cooling, the resulting atomic distribution has been named an "optical crystal" [14]. In Fig. 4, we show a position distribution of all atoms where high densities appear around the six potential minima present within the area displayed

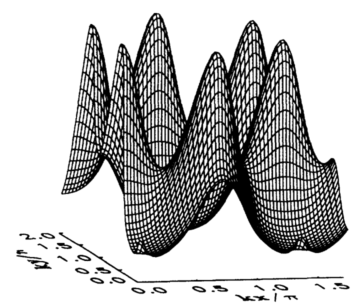

FIG. 4. Position distribution for all atoms, for $U_0/E_R = 41$ with a cutoff in momentum space at $\mathcal{N} = 21$. Six potential minima, three for each of U_+ and U_-, are present within the area displayed, as reflected in the six positions of maximum density.

in the figure. These six minima appear as the corners of a hexagon and the variation in atomic density along the perimeter of the hexagon amounts to a little more than a factor of 2, whereas the difference in density between global potential maxima and minima is larger, on the order of 9. From the results in [7] and in this Rapid Communication, one indeed observes such significant modulation of the position distributions in these optical potentials that one expects a two-dimensional lattice of cooled atoms if the density becomes of the order of an atom per potential minimum. In the experiments, however, such high densities have not yet been attained [11].

With the four-beam laser configuration studied previously [7,8], resonances appear in the populations of the lowest lying eigenstates in the optical potential as functions of potential strength, due to tunneling between two types of potential minima. The situation studied here leads to only one type of minimum and the populations of all eigenstates vary smoothly with U_0/E_R. We note that resonances might appear with the three-beam configuration for an atomic transition of higher angular momentum since they have been found in 1D Sisyphus cooling on a $J_g = 4$ to $J_e = 5$ transition [15].

In a comparison of experimental results for the temperature in 1D and 3D Sisyphus cooling, on atoms of transition with higher angular momenta, $J_g = 3 \rightarrow J_e = 4$ (Rb) and $J_g = 4 \rightarrow J_e = 5$ (Cs), the curvature of the optical potentials emerged as a relevant parameter [16]. This indicates that the steady-state distribution is determined mainly by the atoms bound around the bottom of the potential well: The lowest bound eigenstates and their energy bands can, to a good approximation, be determined from a harmonic potential, and these eigenstates are more sensitive to local surroundings, i.e., the potential curvature, than global surroundings, e.g., the maximum potential depth. In our 2D calculations, we have compared the variation of $2p_e^2$ with the values of the laser parameters for three different laser setups. The minimum width of $p_e \sim 5.9\hbar k$ of the cooled momentum distributions in the three-beam configuration studied in this paper exceeds the minimum width of $p_e \sim 3.5\hbar k$ for the four-beam configuration studied in [8], with the optimum choice for the relative time phases. With a different choice of relative phases in the four-beam configuration, we have found a minimum width on the order of 5.5 $\hbar k$. The three minimum values are obtained with approximately the same curvature of the potential well, but if we consider also the overall variation of $2p_e^2$, it is not clear whether the curvature or the depth of the potential is the appropriate parameter to choose.

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