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Nonclassical states of motion in a three-dimensional ion trap by adiabatic passage

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A scheme for the preparation of nonclassical states of motion in a three-dimensional harmonic ion trap is proposed. The technique is based on adiabatic passage along dressed energy levels of the strongly coupled ion-trap system by varying the laser frequency.

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Recent experiments on laser cooling and trapping [1] have reported the observation of quantum effects related to the center-of-mass motion of ions and atoms. In ion traps, sideband cooling to the vibrational ground state of the trapping potential has been demonstrated [2], and for neutral atoms quantized motion in optical potentials has been observed in spectroscopy of optical molasses [3]. An intriguing new perspective is the possibility to prepare *nonclassical states of motion* of cold atoms and ions [4–6]. This is interesting from the point of view of high-resolution spectroscopy with cold atoms and ions. In addition this opens a new route for experiments to test fundamental questions in quantum mechanics, such as the formation and decay of macroscopic superposition states. We have proposed [5] the generation of Fock states of motion of trapped ions (energy eigenstates of the trap) by observation of quantum jumps in three-level systems placed at the node of a standing laser wave. In a similar configuration, squeezed states of motion can be prepared as a "dark state" of the atom in a multichromatic laser wave [6]. For an ion located at the node of a standing light wave there exists an interesting analogy between the ion in the trap and the Jaynes-Cummings model (JCM) of cavity quantum electrodynamics (CQED) [5,6].

In this Rapid Communication we discuss a scheme for preparation of nonclassical states of motion of trapped ions based on adiabatic passage along dressed energy levels of the strongly coupled trap-ion system. In comparison with our previous proposals [5,6] this scheme has the advantage that it can be easily extended from one to two or three dimensions. It is not restricted to the Lamb-Dicke limit (LDL); i.e., does not assume that the ion is localized in a region small compared to the optical wavelength [8], and thus it allows the preparation of superposition states larger than the wavelength of the light. Finally, the present scheme is simpler to realize experimentally and is fairly insensitive to uncertainties in experimental parameters. Adiabatic passage has been suggested in Refs. [7] to prepare nonclassical states of light in CQED.

The Hamiltonian describing the motion of a trapped ion is given by

$$H_{tp} = \sum_{j=x,y,z} \left[\frac{P_j^2}{2m} + \frac{1}{2} m \nu_j^2 R_j^2 \right] = \sum_{j=x,y,z} \hbar \nu_j \left[a_j^\dagger a_j + \frac{1}{2} \right], \quad (1)$$

where $a_j = [m\nu_j/(2\hbar)]^{(1/2)}[R_j + iP_j/(m\nu_j)]$ is the usual annihilation operator along the direction j (R and P are position and momentum operators, and m is the ion mass), and ν_j is the frequency of the ion motion in the j direction. Denoting $|n_j\rangle_j$ ($j = x, y, z$) the Fock state (eigenstate) of the harmonic oscillator with energy $\hbar\nu_j(n_j + 1/2)$, we seek to prepare (i) Fock states in three dimensions, i.e.,

$$|\Psi\rangle = |n_x, n_y, n_z\rangle \equiv |n_x\rangle_x |n_y\rangle_y |n_z\rangle_z, \quad (2)$$

and (ii) linear superpositions of these states, such as

$$|\Psi\rangle = \alpha |n_x, n_y, n_z\rangle + \beta |n_x, n_y + 1, n_z\rangle . \quad (3)$$

To prepare these states, the ion is assumed to interact with three standing waves of frequencies ω_x, ω_y, and ω_z along the directions x, y, and z, respectively. These laser beams excite an internal transition $|g\rangle \rightarrow |e\rangle$ of the ion modeled here by a two-level system with transition frequency ω_0. For the preparation of the nonclassical states (2) and (3) we propose to use a slow variation of the laser frequency in such a way that the motion of the ion is left in one of the states (2) or (3), whereas the internal atomic state is the ground state $|g\rangle$. This adiabatic passage is coherent, and we will assume that there is no spontaneous emission during the preparation. This is the case, for example, when the lasers excite an electric-dipole forbidden transition [2]. After the frequency chirp cycle the ion will be left in its internal ground state; therefore the prepared motional state persists due to the absence of spontaneous emission.

The Hamiltonian describing the motion of the ion in the presence of the laser beams is

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$$H = H_{tp} + H_I + H_{int}, \tag{4}$$

where H_{tp} is given in (1), $H_I = \omega_0 \sigma_z/2$ ($\hbar = 1$) is the free Hamiltonian describing the two-level system, and

$$H_{int} = \sum_{j=x,y,z} \frac{\Omega_j}{2} \sin(k_j R_j + \phi_j)(\sigma_+ e^{-i\omega_j t} + \text{H.c.})$$

gives the interaction with the laser. Here, $\sigma_\pm$, σ_z are the usual spin-1/2 operators describing the two-level system, Ω_j is the peak Rabi frequency of the laser standing wave in the direction $j = x, y, z$; $k_j = \omega_j/c$, and ϕ_j depends on the relative phase between each of the lasers forming the standing wave in this direction.

In the following we will consider that at any given time only one standing wave (denoted by j) interacts with the ion. In this case, in a frame rotating with the laser frequency ω_j, the Hamiltonian is

$$H = H_{tp} - \frac{(\omega_j - \omega_0)}{2}\sigma_z + \frac{\Omega_j}{2} \sin(k_j R_j + \phi_j)(\sigma_+ + \text{H.c.}) \, . \tag{5}$$

In the absence of spontaneous emission and considering interaction with a single laser only, the motion in the three spatial dimensions decouples, which allows us to study the Hamiltonian H in one dimension. For simplicity we will suppress the subscript j in the following. In contrast to the standard theory of ion cooling [8,9], we will not assume in the following that we are in the LDL.

Below we discuss the eigenvalue spectrum and dressed states of the Hamiltonian (5) as a function of the laser frequency. In particular we are interested in the evolution of the dressed states in an adiabatic frequency sweep. Let us denote by $|n, g\rangle$ ($|n, e\rangle$) the state of the ion in which the internal two-level system is in the ground (excited) state, and with n the excitation number of the harmonic oscillator. The bare Hamiltonian ($\Omega = 0$) shows degeneracies whenever the laser detuning is a multiple of the trap frequency, $\omega - \omega_0 = k\nu$ ($k = 0, \pm 1, \ldots$); i.e., the laser is tuned to one of the "motional sidebands" corresponding to a degeneracy between $|n, g\rangle$ and $|n + k, e\rangle$. In the presence of the laser these degeneracies become avoided crossings, and for sufficiently weak laser excitation these avoided crossings will be isolated (nonoverlapping). For example, when the laser frequency is close to the two-level transition resonance ($|\omega - \omega_0| \ll \nu$) transitions changing the harmonic oscillator quantum number n are off resonance ($k = 0$) and can be neglected (arrows 1 and 3 in Fig. 1). In this case the Hamiltonian (5) can be approximated by

$$H^0 = H_{tp} - \frac{1}{2}(\omega_j - \omega_0)\sigma_z + \alpha_0 \frac{\Omega_j}{2}(\sigma_+ + \text{H.c.}). \tag{6}$$

For laser frequencies close to the lower motional sideband $|\omega - (\omega_0 - \nu)| \ll \nu$ ($k = -1$) only transitions decreasing the quantum number n by 1 are important (arrow 1 in Fig. 1). Now, H can be approximated by the Jaynes-Cummings-type Hamiltonian

$$H^{JC} = H_{tp} - \frac{(\omega_j - \omega_0)}{2}\sigma_z + \alpha_- \frac{\Omega_j}{2}(\sigma_+ a_j + \text{H.c.}). \tag{7}$$

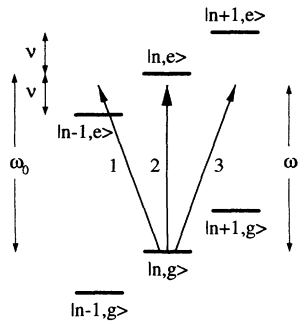

FIG. 1. Energy levels for a two-level ion moving in a harmonic potential. For $\omega \simeq \omega_0$, transitions from $|n, g\rangle \rightarrow |n, e\rangle$ are close to resonance (2), whereas transitions with $|n, g\rangle \rightarrow |n \pm 1, e\rangle$ are very far off resonance (1) and (3). For $\omega \simeq \omega_0 - \nu$, transitions from $|n, g\rangle \rightarrow |n - 1, e\rangle$ are close to resonance (1) whereas all other transitions are very far off resonance. For $\omega \simeq \omega_0 + \nu$, only transitions from $|n, g\rangle \rightarrow |n + 1, e\rangle$ are close to resonance (3).

Similarly, for $|\omega - (\omega_0 + \nu)| \ll \nu$, only transitions increasing the quantum number n by 1 ($k = +1$) contribute to the Hamiltonian H_{int} (arrow 3 in Fig. 1). In this case, H can be approximated by the *anti*-Jaynes-Cummings Hamiltonian

$$H^{AJC} = H_{tp} - \frac{(\omega_j - \omega_0)}{2}\sigma_z + \alpha_+ \frac{\Omega_j}{2}(\sigma_+ a_j^\dagger + \text{H.c.}). \tag{8}$$

In Eqs. (6)–(8) the α's are numerical factors depending on the dimensionless Lamb-Dicke parameter $\eta = [\hbar k^2/(2m\nu)]^{(1/2)}$ and the phase ϕ_j. For an isolated avoided crossing the dressed states are readily obtained by diagonalizing the 2×2 matrix of degenerate states.

Figure 2 is a numerical example of a dressed energy spectrum. In this figure we have plotted the exact eigenvalues of H (calculated numerically) as a function of the laser frequency ω, for $\eta = 0.5$ [10], $\Omega = 0.3\nu$, and $\phi = \pi/4$. The dotted line represents the energies of the "bare" states. For the laser frequencies far below the two-level resonance (point C in Fig. 2) no transitions between states $|g\rangle$ and $|e\rangle$ take place, since they are very far

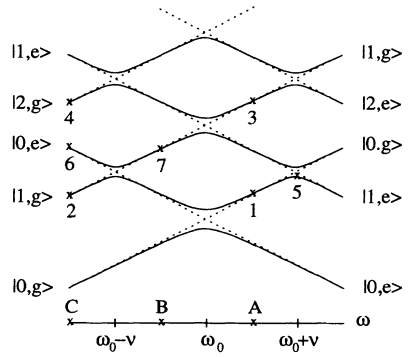

FIG. 2. Energy level diagram for a two-level ion interacting with laser light, as a function of the laser frequency ω. Dotted lines correspond to the bare states.

off resonance. Hence, the eigenstates of H are those of the free part $H_{tp}+H_I$, i.e., $|n,g\rangle$ and $|n,e\rangle$. For increasing values of the laser frequency, states $|n,g\rangle$ couple to states $|n-1,e\rangle$ via the Hamiltonian H^{JC}, and therefore the eigenstates (or dressed states) are linear combinations of these two states. In particular, for $\omega=\omega_0-\nu$ it can be easily shown that the eigenstates are

$$|\Psi_{\pm}^n\rangle=\frac{1}{\sqrt{2}}(|n+1,g\rangle\pm|n,e\rangle)\ \ (n=0,1,\ldots),$$
$$|\Psi_0\rangle=|0,g\rangle, \tag{9}$$

which corresponds to the first level anticrossing of Fig. 2. For increasing values of the laser frequency, but still far from resonance (point B), states $|n,g\rangle$ and $|m,e\rangle$ are again decoupled. If the laser frequency is increased to the two-level resonance, states $|n,g\rangle$ couple to states $|n,e\rangle$ via Hamiltonian H^0, and for $\omega=\omega_0$ the eigenstates are

$$|\Psi_{\pm}^n\rangle=\frac{1}{\sqrt{2}}(|n,g\rangle\pm|n,e\rangle), \tag{10}$$

which corresponds to the second level anticrossing of Fig. 2. Similarly, for increasing values of the laser frequency one can easily identify the eigenstates of H by considering transitions between states $|n,g\rangle$ and $|n+1,e\rangle$ via the Hamiltonian H^{AJC}. In particular, for $\omega=\omega_0+\nu$, the eigenstates are

$$|\Psi_{\pm}^n\rangle=\frac{1}{\sqrt{2}}(|n,g\rangle\pm|n+1,e\rangle)\ \ (n=0,1,\ldots),$$
$$|\Psi_0\rangle=|0,e\rangle. \tag{11}$$

The ability to change adiabatically the laser frequency offers the possibility to generate Fock states of the atomic motion by adiabatically following one of the dressed states. To illustrate this, let us assume that the initial state of the ion is in $|0,g\rangle$. This is the vibrational ground state in which the ion is left after sideband cooling. The laser frequency is initially set to a certain value between ω_0 and $\omega_0+\nu$ (denoted by A in Fig. 2), so that all transitions are off resonance. The corresponding energy level is denoted by 1 in the energy diagram of Fig. 2. When the laser frequency is decreased adiabatically, the state of the ion changes according to the dressed energy diagram of Fig. 2. Once the frequency ω reaches C, the state of the ion has evolved to $|1,g\rangle$ (point 2 in the figure). Now the laser is switched off and on again, but with the original frequency A. Thus, the state of the ion corresponds to the point 3 in the figure. By decreasing adiabatically the laser frequency again the ion ends up in the state $|2,g\rangle$ (point 4 in the figure). By repeating this cycle n times, the state of the ion becomes $|n,g\rangle$; i.e., a Fock state with n quanta is prepared. Note that after any cycle, the internal structure of the ion is left in its ground state $|g\rangle$, and therefore the Fock state is not modified due to spontaneous emission from the electric-dipole forbidden transition.

The conditions for the adiabatic following can be easily estimated for a laser frequency varying linearly with time. In this case one finds $\bar{\Omega}T\gg\nu/\bar{\Omega}$, where T is the time duration of each cycle, and $\bar{\Omega}$ is the effective Rabi frequency for transitions $|n,g\rangle\rightarrow|n-1,e\rangle,|n,e\rangle$, which for the sake of simplicity we assume to be similar. For example, for a trap frequency of $(2\pi)1$ MHz, and an effective Rabi frequency of $(2\pi)50$ kHz, the adiabaticity condition implies interaction times $T\gg 60$ μs. These numbers are within reach of current experiments with trapped ions. It is worth emphasizing the fact that under low-excitation conditions [11] and when the adiabaticity condition is satisfied, this procedure for preparing Fock states is insensitive to the specific values of the parameters involved in the process. Furthermore, it can be easily generalized to other laser configurations, such as traveling wave excitations.

General linear combinations of neighboring Fock states can be prepared following the above procedure, with the initial state of the ion in the superposition state $(\alpha|g\rangle+\beta|e\rangle)|0\rangle$. As can be deduced following the level diagram of Fig. 2, after one cycle the state of the ion becomes $(\alpha|1\rangle+\beta|0\rangle)|g\rangle$. After n cycles, the state of the ion will be $(\alpha|n\rangle+\beta|n-1\rangle)|g\rangle$.

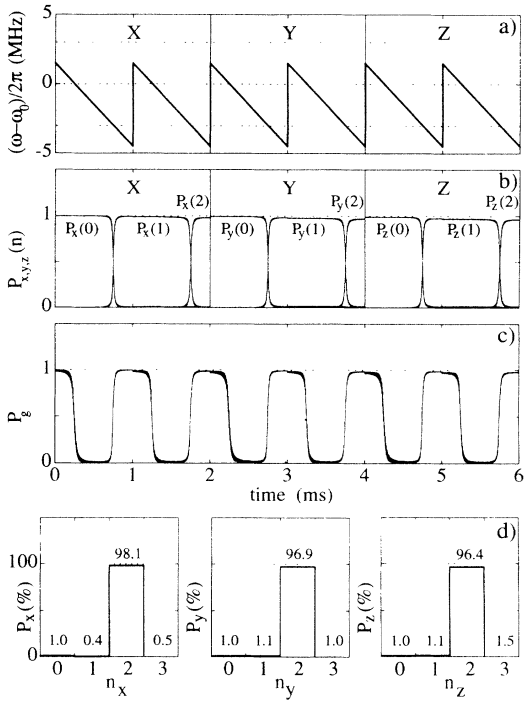

FIG. 3. Preparation of the 3D Fock state $|2\rangle_x|2\rangle_y|2\rangle_z$ by adiabatic variation of the laser detuning. (a) Variation of the laser frequency as a function of time (X, Y, and Z indicate the direction of propagation of the laser that is on during each time interval; these time intervals are separated by vertical lines in the figure). (b) Populations of the states $|n\rangle_x$, $|n\rangle_y$, and $|n\rangle_z$ ($n=0,1,2$) vs time. Note that we have plotted the populations only during the time intervals in which they change due to the presence of the laser. (c) Population of the internal ground state $|g\rangle$ vs time. (d) Occupation probabilities P_x, P_y, and P_z of the Fock states of the harmonic oscillator along the directions x, y, and z, respectively, at the end of the preparation (time=6 ms). Parameters are $\nu/(2\pi)=3$ MHz, $\Omega/(2\pi)=300$ kHz, $\phi=\pi/4$, and $\eta=0.5$. Note that under these conditions we are not in the Lamb-Dicke limit.

Fock states in two and three dimensions may be prepared as follows. We assume that the ion has been cooled via sideband cooling to its lowest state along the three directions; i.e., the initial state of the ion is $|0,0,0,g\rangle$. Applying lasers along one direction, for example, x, and with the procedure described above, the ion ends up in the state $|n_x,0,0,g\rangle$. Next, the lasers along x are switched off, and those along y switched on. By adiabatic change of the laser frequency, the state $|n_x,n_y,0,g\rangle$ is prepared. Finally, proceeding in the same manner with the z direction, one can prepare state (2). Obviously, by combining this procedure with that described in the previous paragraph one is able to generate state (3).

As a numerical example, we have plotted in Fig. 3 the time evolution for a pulse sequence that leads to the preparation $|0,0,0\rangle \rightarrow |2,2,2\rangle$ by an adiabatic variation of the laser detuning. These curves were calculated by solving the full time-dependent Schrödinger equation using a truncated basis of oscillator eigenstates. The frequencies of the lasers along the three directions as a function of time are plotted in the upper part of Fig. 3(a). Figure 3(b) shows the populations $|n\rangle_x$, $|n\rangle_y$, and $|n\rangle_z$ $(n=0,1,2)$ of the trap states and Fig. 3(c) is a plot of the occupation of the internal ground state $|g\rangle$ as a function of time. As indicated in the population histogram in Fig. 3(d) a three-dimensional (3D) Fock state $|2,2,2\rangle$ has been prepared after the completion of the three adiabatic cycles.

Finally, entangled states $|\Psi\rangle = \alpha|0,1,0\rangle + \beta|1,0,0\rangle$ can also be prepared. For example, to prepare this state with $\alpha=\beta=1/\sqrt{2}$ one can proceed as follows. In a first step with the ion initially in its ground state, the frequency of the laser along the x direction is adiabatically increased from A in Fig. 2 up to $\omega_0+\nu_x$. This corresponds to moving from points 1 to 5 in Fig. 2. According to (11), the state of the ion in this first step becomes $(|0,0,0,g\rangle + |1,0,0,e\rangle)/\sqrt{2}$. In a second step the frequency of the laser in the direction y is increased from C to B, which corresponds to moving the second term of the wave function from 6 to 7 (the first part of the wave function remains the same, as can be deduced from the figure). Again, the internal state of the ion factorizes in the final state, and therefore it is left in its ground state.

In summary, we have proposed a scheme for the preparation of a variety of three-dimensional nonclassical states of motion in a trap. This technique is based on the excitation of an electric-dipole forbidden transition by different lasers in an optical-molasses configuration, in such a way that during the preparation no spontaneous emission takes place. Nonclassical states can be produced by adiabatically varying the frequency of the lasers. The scheme presented here is valid beyond the Lamb-Dicke limit, and is not sensitively dependent on the specific values of the laser parameters, trap frequencies, etc. Furthermore, after the state of motion is prepared, the internal structure of the ion is left in its ground state, and therefore the nonclassical state persists. The parameters of this scheme are well within the reach of current experiments with single trapped ions, such as Ba^+, Ca^+, or Hg^+.

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[1] Special issue of J. Opt. Soc. Am. B **6** (11) (1989), edited by S. Chu and C. Wieman; S. L. Gilbert and C. E. Wieman, Opt. Phonics News **4**, 8 (1993).

[2] F. Diedrich *et al.*, Phys. Rev. Lett. **62**, 403 (1989).

[3] P. Verkerk *et al.*, Phys. Rev. Lett. **68**, 3861 (1992); P. S. Jessen *et al.*, *ibid.* **69**, 49 (1992); A. Hemmerich and T. W. Hansch, *ibid.* **70**, 410 (1993); G. Grynberg *et al.*, *ibid.* **70**, 2249 (1993).

[4] D. J. Heinzen and D. J. Wineland, Phys. Rev. A **42**, 2977 (1990).

[5] J. I. Cirac *et al.*, Phys. Rev. Lett. **70**, 762 (1993).

[6] J. I. Cirac *et al.*, Phys. Rev. Lett. **70**, 556 (1993).

[7] J. M. Raimond *et al.*, in *Laser Spectroscopy IX*, edited by M. S. Feld, J. E. Thomas, and A. Mooradian (Academic, New York, 1989), p. 140; A. S. Parkins, Phys. Rev. Lett. **71**, 3095 (1993).

[8] See, for example, S. Stenholm, Rev. Mod. Phys. **58**, 699 (1986).

[9] J. I. Cirac *et al.*, Phys. Rev. A **46**, 2668 (1992).

[10] For excitations considered in Fig. 3 the parameter $\eta=0.5$ is outside the validity of the LDL, and our results differ significantly from those derived with the assumption of a lowest-order expansion in η.

[11] By low excitation we mean that transitions only take place under close-to-resonance excitation. For transitions $|n\rangle \rightarrow |n\rangle, |n\pm1\rangle$ this is the case for $\Omega\sin(\phi), \Omega\eta\cos(\phi) \ll \nu$.