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Theory of resonances and bound-state management

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Complicated potential deformations performing a desired changing of the decay rates of quasistable states (resonance widths) or deleting chosen bound states are explained surprisingly simply. Exotic stalactitelike potential deformations "carrying" the chosen states through potential barriers are elementary constituents of the general theory of resonance management.

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Recently how to manage the bound-state parameters has been clarified [1–4]. But achieving the same was also desirable in the second part of quantum mechanics: in the scattering theory. Now we have succeeded in elaborating the qualitative algorithm of resonance management. It appears that for this it was essential to understand the mechanism of removing the chosen bound states.

It is often necessary to change the spectrum and resonance parameters in a desired way. For instance, this is needed when comparing experimental and theoretical spectroscopic and scattering data. This is also of principal interest for deeper insight into the peculiarities of the quantum world.

Let us consider at first the deleting of the chosen bound states. On the one hand, it is equivalent to shifting the *infinite* number of energy levels: each in place of its upper neighbor. But it is impossible to imagine the result of the addition of the corresponding potential perturbations [1–4]. The essence of the necessary potential transformation $[\overset{\circ}{V}(x) \to V(x)]$ was understood after a careful investigation of the quantum pictures such as those in Fig. 1 (for the deleted second, third, or fourth levels) which were drawn using the exactly solvable model [5–7]

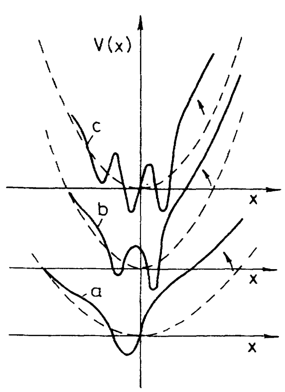

FIG. 1. The deformations (solid lines) of the oscillator potential wells (dashed lines) by deleting (a) the second, (b) the third, and (c) the fourth energy levels.

$$V(x) = \overset{\circ}{V}(x) - 2\frac{d}{dx}\left(\frac{\overset{\circ}{M}{}_{\mu}^{2}\,\overset{\circ}{F}{}_{\mu}^{2}(x)}{1-\overset{\circ}{M}{}_{\mu}^{2}\int_{-\infty}^{x}\overset{\circ}{F}{}_{\mu}^{2}(y)dy}\right). \qquad (1)$$

Here $\overset{\circ}{M}_{\mu}$ is the normalizing parameter corresponding, in the case of the initial oscillator potential, to the μth eigenfunction $\overset{\circ}{F}_{\mu}$ with the asymptotic behavior $\overset{\circ}{F}_{\mu}(x) \propto \overset{\circ}{M}_{\mu}\, x^{\mu}\exp(-x^2/2)$ far to the left (at large x). The following clear qualitative explanation of these potential transformations was found. We have to remove one knot in each of an infinite number of states above the deleted one; that is, to make them a half-wave shorter. This is achieved by narrowing the potential well above the chosen level, e.g., by shifting the right potential wall as in Fig. 1. But in a narrower well, the energy levels below the deleted state must go up. To push them down to their previous positions, the lower part of the perturbed well has to be changed according to the rule described in [2–4]. The levels are shifted down with so many additional partial wells as there are maxima of the module of the state under the deleted one.

There were indeed some hesitations about the unexpected symmetry violation of the present potential. The original potential was symmetrical as was the modulus of the deleted state. We have constructed the present potential according to the requirement of level positions, and the shape of the symmetrical potential has been determined solely by energy level positions in it. Why then does the asymmetry appear? It was introduced unintentionally into the kernel of the inverse problem equation (see [2]) when the norming constants fix the left-hand-side asymptotic behavior of the eigenfunctions as in the original well. A trivial illustration of removing only the ground state from the equidistant spectrum of the special original (oscillator) potential was published by Sukumar [7]. In this case, the potential is simply shifted up by the energy interval between the levels. But at that time, there were no general algorithms of the qualitative prediction without mathematical manipulations of how the shape of the arbitrary initial potential must be changed to remove arbitrary energy levels as has been done here.

The annihilation of a level can be understood as a limit of the continuous decreasing of the corresponding norming constant (this means some kind of connection be-

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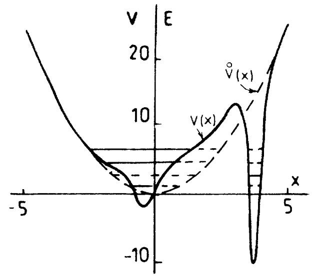

FIG. 2. The deformation of the oscillator potential by suppressing the tail (on the left) of the wave function for the second energy level. This is the intermediate stage for the subtraction of the chosen level when the narrow additional stalactite-type well "carries away" to the right infinity the localization domain of the wave function of the state to be annihilated.

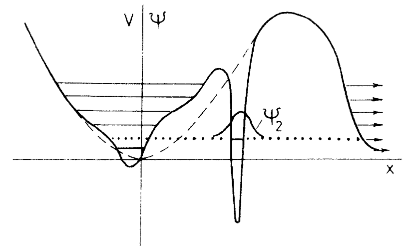

FIG. 3. The potential differing from that in Fig. 2 by bending the infinite right wall so that bound states become decaying. The stalactitelike narrow potential well containing the main part of the chosen decaying state can carry it through the barrier. Thus, one can, at will, increase the decay rate by bringing this well (and the wave function in it) to the outer boundary of the barrier.

tween independent spectral parameters E_ν and M_ν). The typical picture for the small norming constant is shown in Fig. 2.

A typical "stalactite" appears in the perturbed potential whose shape favors the retaining of the standing half-wave of the state chosen for the annihilation. The remaining part of this eigenfunction becomes more and more *self-suppressed inside the left well* when its main part is carried away by the stalactite. Unlike this, for all other states there is *self-suppressing inside the stalactite* and the concentration of the wave inside the big well. The further the additional well moves away, the higher the repulsive "lap" grows, which makes the main well narrower. The first knots from the right (one for each of these states) are posed inside the intermediate barrier. In the limit, these first knots are moving away to infinity.

In the process of removing *several* bound states there appear a corresponding number of stalactites. Their positions depend on the relation of the values of the proper norming constants. Particularly, the stalactites can move to the left (e.g., as a mirror reflection of Fig. 2, symmetrically with respect to the center of the original potential).

The connection of level annihilation with variations of normalizing constants elucidates in an alternative way the behavior of the potential perturbations by the variation of reduced widths [1,2].

The peculiarities of the present quantum pictures are so clearly understood that it is possible to give qualitative predictions without computers or analytical manipulations. For example, for the creation of the new level above the third one the well must be made *wider* with the upside-down shape of the perturbed bottom (with respect to Fig. 1). It is also easy to predict the shape of deformed potentials on the half axis in Gelfand-Levitan's and Marchenko's approaches. All these predictions were confirmed by exact calculations.

What is more important is that the mechanism considered above elucidates a possibility to also manage the decay rates of quasibound states. Indeed, let us bend the right wall of the initial potential $\mathring{V}(x)$ to transform it into a finite potential barrier vanishing at large x values (see Fig. 3). Then, the bound states become the quasistable states which decay through the barrier, and it is clear that the higher the level of the decaying state lies the bigger the corresponding decay rate. Let us choose some quasibound state. We can require that its decay width will increase, whereas the other states will remain unchanged. It is attained with the special auxiliary stalactitelike potential well carrying to the outer boundary of the barrier the chosen quasibound state while all other states remain at their previous positions. So, for the chosen state it becomes easier to decay through the narrower barrier.

This mechanism has a general character. Arbitrary decay rates for all quasibound states can be obtained by the combination of stalactites: one for each level. The discovered phenomenon can be investigated within the exactly solvable models (see references in [8] and [2]). Now we have the clear intuitive notion about the algorithms of spectral management: the rules of arbitrary variation of level positions, the corresponding norming constants [2,4], and also the recipes of deleting and creating levels. Now it becomes clear that even by the weak variation of normalizing constants there always appear embryos of the stalactites. See, for example, [2] and the perturbations of the Coulomb potential considered in [9]. In the case of infinite vertical potential walls the wave functions are confined inside the finite interval. In this case, the deleted state is "pressed into the wall."

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