PHYSICAL REVIEW A VOLUME 49, NUMBER 1 JANUARY 1994

Observation and characterization of deterministic chaos in stimulated Brillouin scattering with weak feedback

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(Received 30 August 1993)

We report the observation and characterization of chaos in stimulated Brillouin scattering (SBS) generated in a single-mode fiber in the presence of weak external feedback. We establish that the noise, which initiates this process and dominates the emission near the SBS threshold, is dramatically suppressed from the onset of strong SBS emission, giving rise to various highly deterministic dynamical features.

PACS number(s): 42.65.Es, 05.45.+b, 41.20.−q, 42.79.−e

In nonlinear optics there are many interactions that are initiated by stochastic processes, the sources of which are either thermal or quantum noise. An issue of current interest in the field of nonlinear dynamics concerns whether a noise signal can evolve to possess deterministic dynamical behavior through such nonlinear interactions and, if so, what is the influence of the noise on the dynamics. A paradigm of such interactions is stimulated Brillouin scattering (SBS) for which spontaneous emission is amplified by the nonlinear parametric coupling of a pump signal with an acoustic wave in the interacting medium. In this Rapid Communication we report the observation of deterministic chaos in SBS, which is generated under cw pump conditions in a single-mode optical fiber in the presence of weak external feedback. We establish that, upon increasing the pump signal strength, the noise that initiates this process is dramatically suppressed, giving rise to highly deterministic dynamics, the low-dimensional chaos we observe evolving from quasiperiodicity. The dynamics is found to be noise dominated only for weak pumping near the threshold for SBS. In distinguishing stochastic from deterministic chaotic behavior we have augmented standard correlation dimension analysis [1] with some of the latest measurement tools [2].

Earlier observations by the authors [3] revealed that the dynamics of SBS without feedback was strongly aperiodic, while in the presence of feedback it displayed distinct dynamical forms, e.g., limit cycles and quasiperiodicity, indicative of deterministic behavior [3–5]. Several of these features were subsequently observed in a similar experimental arrangement [6,7] and also a ring laser [8]. Most recently a characterization of the aperiodic motion provided evidence that such behavior was stochastic in nature [7]. By contrast, our earlier theoretical work established deterministic dynamics and chaos to be generic to SBS in extended media and in the absence of noise, when nonlinear refraction and its interplay with the SBS gain were accounted for [9]; omission of which resulted in only stable SBS emission. Furthermore, our analysis also showed that, in the case of weak feedback, the nonlinear refractive effects may play an important role in modifying and even significantly complicating the dynamical behavior obtained in otherwise (nonlinear) refraction-free systems. However, subsequent inclusion of noise initiation in this analysis was found to dramatically suppress the deterministic features, resulting in stochastic emission [10], as in experiments, whereas for SBS in the presence of feedback, its inclusion had little influence on the dynamics above the SBS threshold. These findings imply that feedback in conjunction with the gain for SBS are two primary control parameters in determining the nature of the dynamics. In other work the effect of frequency detuning in promoting chaos in SBS has also been recently studied theoretically [11].

The SBS emission was generated in a single-mode optical fiber pumped by cw single-mode laser emission, the experimental setup being the same as in Ref. [3]. When accurately cleaved normal to the fiber length, the natural reflectivity at the fiber ends was measured to be $\sim 3.3\%$, providing a weak boundary for the system. A stabilized cw Nd:YAG (neodymium-doped yttrium aluminum garnet) laser operating at 1.06 μm was selected as the pump source, its operation at 1.06 μm providing low scattering losses in the fiber. The laser system (SL903-1122) was suitably modified to provide a single-mode output with an intensity stability of $\pm 2\%$ and a short-term spectral bandwidth of ≤ 40 kHz. The laser output intensity was externally controlled using a standard half-wave plate and polarizer, giving a continuously variable signal strength of 0–4 W. Microscope objectives were used to couple the light into and out of the fiber, which was of length 124 m and comprised a pure SiO_2 cladding. The fiber was optically isolated from the laser using a Faraday isolator (OFR Model No. IO-5-YAG) giving an isolation factor of 35 dB. The input and transmitted pump signals together with the backscattered emission were sampled, via beam splitters, using photodiodes (Hewlett-Packard-type BPX65) with rise times of less than 0.5 nsec. A transient digitizer (LeCroy TR8828C) having a data sampling rate of 5 nsec and interfaced with a Sun WorkStation Computer was used for temporal recording and subsequent signal processing. A Fabry-Pérot interferometer with variable spacer established the Stokes emission to be first order throughout the range of pump intensities investigated.

The general features of the SBS emission as a function

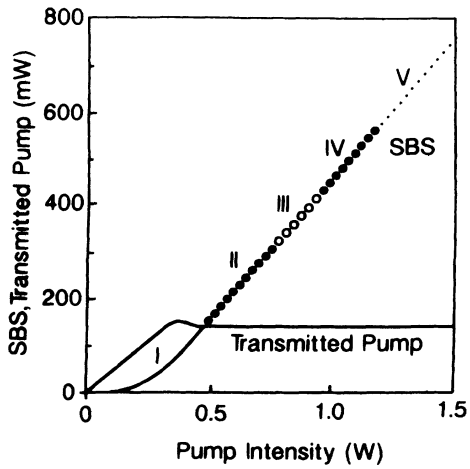

FIG. 1. Dependence of Stokes and transmitted pump intensities on input pump strength. Regions marked I–V show various forms of dynamics.

of pump strength are shown in Fig. 1. Defining a threshold for SBS as the minimum pump power for which the Stokes emission is detectable, the threshold gain was measured to be ~9, about half that for SBS without external feedback. The SBS signal strength, as seen, grows almost linearly with pump strength, with a steady increase of conversion efficiency from 5% at threshold up to 85% at a pump power of 1.5 W, the corresponding transmitted pump signal displaying a "shoulder" prior to saturation, which is attributable to the onset of strong pump depletion. These features are somewhat different from those observed for SBS without cavity feedback due to less pronounced saturation of the pump signal.

The dynamics in the vicinity of the SBS threshold (Fig. 1, region I) was erratic, exhibiting intermittent relaxation bursts with a dominant frequency component at $1/2T_r$, where T_r is the cavity trip time, the durations of the bursts increasing with the pump signal strength, typically over a range of tens to hundreds of $2T_r$ as shown in Fig. 2. The irregularity of the bursts revealed the stochastic nature of the dynamics in this region, which extended up to where the Stokes signal strength was comparable to that of the transmitted pump, a mark of the onset of strong SBS emission. These features are somewhat reminiscent of those of the laser emission near threshold. When the cavity reflectivity was significantly reduced from it optimum value of 3.3% (in effect increasing the SBS threshold), implemented by bad cleavage of the fiber ends, the dynamics became aperiodic with much higher and denser frequency content, similar to that obtained in the absence of the feedback.

The dynamics of Stokes emission above threshold was very distinct and reproduceable, exhibiting different classifiable forms in the regions marked in Fig. 1. Two windows of periodic behavior (regions II and IV) were observed, separated by a window of quasiperiodic and chaotic motion (region III). Within the first window, the dynamics exhibited sustained relaxation oscillations [see Figs. 3(a), 3(b), and 4(a) for time series, phase portrait, and power spectrum], which, on increasing the pump signal, underwent a transition to sinusoidal oscillations, the periods being equal to $2T_r$. These features are similar to those first reported by Bar-Joseph *et al.*, which were interpreted physically as arising from the interplay of the SBS with the feedback from the external cavity [12]. The amplitudes of these oscillations were quite stable over a time scale of hundreds of the cavity-round-trip time, providing a time window wide enough for characterization of the dynamics. In the second of these windows (region

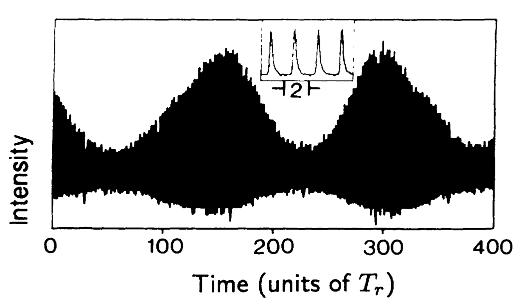

FIG. 2. Dynamics of Stokes emission in region I; the inset is an expanded window showing $2T_r$ oscillation.

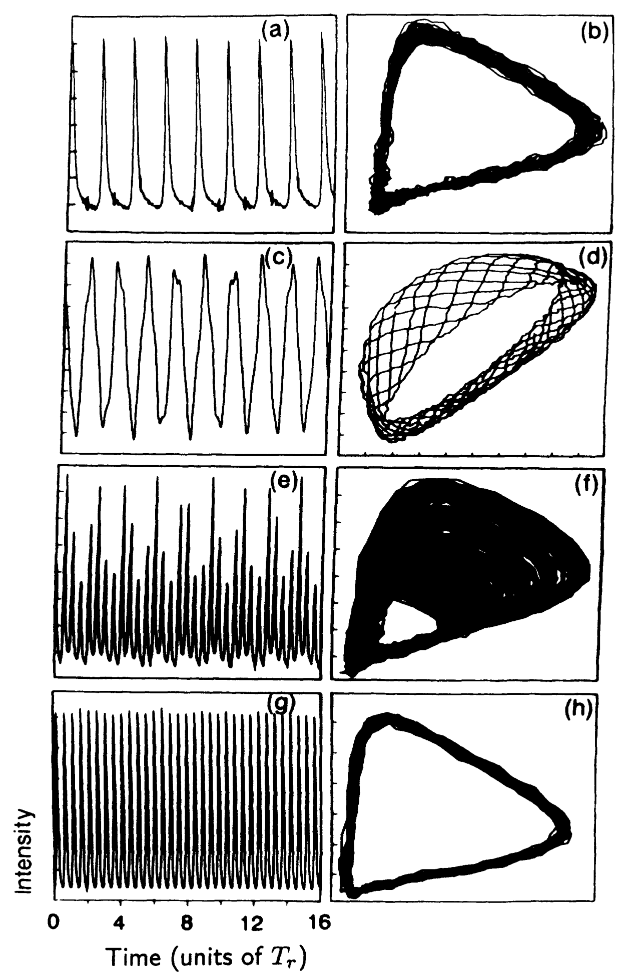

FIG. 3. Time series (left column) shows periodic [traces (a) and (g)], quasiperiodic [trace (c)], and chaotic [trace (e)] emission obtained in the regions II–IV of Fig. 1. The corresponding phase portraits are displayed in the right column.

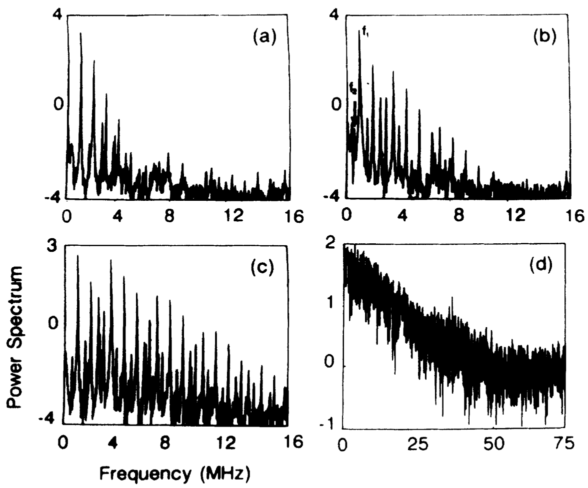

FIG. 4. Power spectra of the Stokes emission for (a) periodic relaxation oscillation, (b) quasiperiodic emission, and (c) chaotic motion; and trace (d) is that for noise-dominated emission in the absence of feedback.

IV), periodic behavior was found to occur with frequencies at various harmonics of the characteristic frequency $1/2T_r$, the relative strength of which changed with the pump strength. On increasing the pump signal from region II to III (see Fig. 1), a new frequency emerged [see f_2 in Fig. 4(b)], the irrationality of this with the characteristic frequency $1/2T_r$ (f_1 as marked) resulting in quasiperiodic motion in the time series [Fig. 3(c)] and manifested as a torus in the corresponding phase portrait [Fig. 3(d)]. The dynamics was found in this case to change dramatically with a change of its frequency components, occasionally reverting to limit cycles when the frequencies locked. Within this region, loss of stability of the quasiperiodicity led to weak chaotic emission as measured below and shown in Figs. 3(e) and 3(f) and the corresponding power spectrum Fig. 4(c). Also shown in Fig. 4(d) for comparison is the spectrum of a noise-dominated emission for operation in the absence of feedback. We note that chaos was found to prevail in this region only when the system was optimized to give a good launch of the laser beam into the fiber and an accurate cleavage of the fiber normal to its axis. The dynamics was otherwise limited to periodic or quasiperiodic motion. We also note that for the emergence of chaos, low scattering loss in the fiber may also be essential, since such dynamics were not previously reported [5–7] when using laser sources of shorter wavelength (higher loss in fiber) in otherwise the same experimental arrangements. Approaching region V, the SBS was characterized by slow modulations on the periodic emission, typically on a time scale of milliseconds, the modulation depth of which increased with pump signal strength in this region, eventually breaking the sustained dynamics to form intermittent bursting action, dispersed within increasingly long regions of temporally stable emission. These dynamical features are similar to those reported earlier [4,5], and interestingly have been recently stabilized using a positive feedback loop [13].

A most remarkable feature of our observation is the suppression of stochastic fluctuations by the deterministic dynamics from the onset of strong SBS emission, the almost perfect periodicity of the Stokes signal revealing how clean this dynamics and, by inference, the chaotic dynamics are, in spite of the influence of noise. Towards a quantitative characterization of the chaos, standard correlation dimension analysis [1] has been applied to and compared with those of the time difference and phase randomized (surrogate) sets of the chaotic time series [2]. By way of example, a convergent correlation dimension with the increase of embedding dimension was obtained at $D_2 \sim 2.1$, as shown in Fig. 5(a), using 16 000 data points from the time series as in Fig. 3(e). As seen in the same figure, the measured values for the time difference data show little deviation from that for the original time series, whereas a significant deviation is displayed in the values extracted from the phase-randomized data. These findings strongly suggest that the time series is of low-dimensional chaos. Here we note that the correlation dimensions for different surrogate data sets (using different random phase series) showed little change in their values. By contrast, the results obtained, using similar measurements of the aperiodic motion in the vicinity of the threshold showed a lack of a definable correlation dimension, indicative of stochasticity, an example of which is shown in Fig. 5(b).

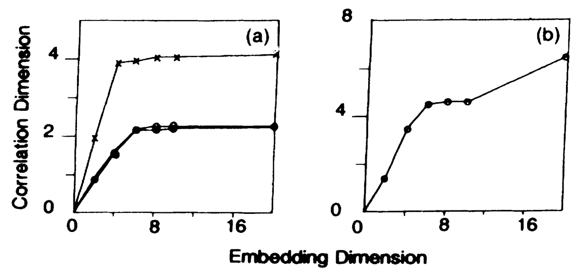

FIG. 5. Correlation dimension as a function of embedding dimension for (a) the chaotic time series of Fig. 3(e), and (b) a typical aperiodic motion in the vicinity of the SBS threshold. The open circles are for the original data, while the closed circles and crosses are for the time difference and surrogate data, respectively.

In summary we have reported experimental observations along with data analysis of the dynamics of SBS in the presence of feedback, and established that, from the onset of strong SBS oscillation, stochastic noise, which initiates this process, is dramatically suppressed, giving rise to highly deterministic dynamics, including low-dimensional chaos evolving from quasiperiodicity. The effect of the noise on the dynamics is found to be strongly felt only in the vicinity of threshold where the emission exhibits erratic relaxation bursts or stochastic-type fluctuations. These findings may have a relevance to the dynamical behavior of other noise-driven nonlinear systems.

The authors wish to thank Dejin Yu for his stimulating discussions. This work was jointly supported by the Science and Engineering Research Council (SERC, UK) and the U.S. Air Force Office of Scientific Research (AFOSR).

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