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Possibility of producing the one-photon state in a kicked cavity with a nonlinear Kerr medium

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It is shown that the field in a cavity, periodically kicked with classical pulses and containing a nonlinear Kerr medium, can reach the one-photon quantum state if the amplitude of the kicks and the time between the kicks are appropriately chosen.

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Although number states are well known and commonly used in theoretical descriptions of quantum fields, their production in practice is by no means a trivial task. Hong and Mandel [1] have shown that an ideal one-photon state is produced in the parametric down converter if signal photons are used to gate a photodetector counting the corresponding idler photon. Such schemes of generating antibunched light have been studied theoretically by Stoler and Yurke [2]. Another possibility of preparing a highly excited Fock state is a kicked cavity into which two-level atoms are injected [3]. Recently, Brune *et al.* [4,5] have suggested a method for the preparation of a Fock state based on the quantum nondemolition (QND) measurement in which detection of the atomic phase by the Ramsey method plays the role of the QND probe, giving information on the cavity field energy. After a sequence of atomic measurements the cavity field collapses into a Fock state with an unpredictable number of photons.

In this paper, we propose a scheme in which a one-photon state can be obtained in a cavity that is periodically kicked by a sequence of classical light pulses and is filled with nonlinear Kerr medium. The evolution of the cavity field between the kicks is the quantum evolution of the field interacting with the nonlinear Kerr medium leading to the intensity-dependent "phase shift" of the field. A similar system, but one with nonlinear rather than linear kicks, has recently been discussed [6–8] in the context of a comparison between the classical and quantum dynamics in the regions of classically regular and chaotic behaviors.

The nonlinear quantum evolution of the cavity field in the Kerr medium is crucial for the preparation of a Fock state in such a system. The effectiveness of the preparation is, however, considerably diminished by the cavity losses. Nevertheless, it seems to us important that the cavity with a nonlinear Kerr medium, which is initially in a vacuum state and is pumped by a sequence of short pulses of the classical field coupled linearly to the cavity field, can reach, with a high degree of accuracy, the one-photon Fock state.

Our model is as follows. In the period of "free" evolution between the kicks the system dynamics is determined by the Hamiltonian (in the interaction picture)

$$H_{\text{int}}=\frac{\hbar\chi}{2}(a^\dagger)^2a^2 , \tag{1}$$

where χ is proportional to the third-order nonlinear susceptibility and $a,a^\dagger$ are the annihilation and creation operators of the field mode. They obey the boson commutation relation, $[a,a^\dagger]=1$.

The cavity is kicked by a train of short pulses of the electromagnetic field at the frequency of the cavity mode. The kicks are described by the Hamiltonian (in the interaction picture)

$$H_K=\hbar(\epsilon a^\dagger+\epsilon^* a)\delta_K(t) , \tag{2}$$

where

$$\delta_K(t)=\sum_{n=0}^{\infty}\delta(t-nT) , \tag{3}$$

where T is the time between the kicks. We assume here that T is sufficiently large so that $\omega\gg 2\pi/T$, where ω is the field frequency. This assumption allows us to consider the pump field as a coherent pulse of frequency ω being short enough to be modeled by a δ function. Rapid oscillations with optical frequency ω are eliminated in the interaction picture. The kick strength ϵ is associated with the complex classical field amplitude of the pumping field.

Thus the evolution operator over the kick may be written as

$$U_K=e^{-i(\epsilon a^\dagger+\epsilon^* a)} , \tag{4}$$

and the "free" nonlinear evolution operator between the kicks is given by

$$U_N=e^{-i(\chi T/2)\hat{n}(\hat{n}-1)} , \tag{5}$$

where $\hat{n}=a^\dagger a$ is the photon number operator. The evolution of the system from just before the kick up to just before the next one is governed by the unitary evolution operator

$$U=U_N U_K=e^{-i(\chi T/2)\hat{n}(\hat{n}-1)}e^{-i(\epsilon a^\dagger+\epsilon^* a)} . \tag{6}$$

The change in the field state after a kick and free evolution between the kicks (quantum map) is then given by

$$|\Psi_{k+1}\rangle=U|\Psi_k\rangle . \tag{7}$$

Since the operator U_N is diagonal in the number-state basis, and the operator U_K is in fact a displacement operator $\mathcal{D}(-i\epsilon)$ whose matrix elements in the number-state basis are known [9], the quantum map (7) can be found numerically in such a basis. Recursive application of (7) allows us to calculate the field state after k kicks for a given initial state. Knowing this state allows us in turn to calculate quantum-mechanical expectation values such as the mean number of photons, its variance, etc. We can, however, look at the evolution of the field state itself finding the amplitudes of particular Fock states. This is our goal in this paper.

We have performed numerical calculations of the field state after k kicks, assuming that initially the state is in the vacuum and the kick strength is small ($\epsilon < 1$). We have found that for a sufficiently small amplitude of the pump field ($\epsilon \ll 1$) and a sufficiently long time T between the kicks, the field state in the cavity evolves between the vacuum and the one-photon state without affecting noticeably the states with higher numbers of photons. This effect is shown in Fig. 1(a), where $\epsilon = \pi/50$ and $\chi T = \pi$. In all the figures we assume ϵ as being real. As ϵ increases, the states with higher n acquire noticeable population, which is seen in Fig. 1(b), where $\epsilon = \pi/10$, and the state with $n=2$ appears. This means, however, that under an appropriate choice of parameters ($\epsilon \ll 1$, $\epsilon k = \pi/2$) the cavity field is to a high degree of accuracy in the one-photon state. This is convincingly seen in Fig. 2(a), where the probabilities of the vacuum and the one-photon state are shown for $\epsilon = \pi/50$, $\chi T = \pi$, indicating clearly that after 25 kicks ($\epsilon k = \pi/2$) the field state is the almost perfect one-photon state. The preparation of the one-photon state according to this scheme becomes less perfect if the time of the "free" nonlinear evolution of the system is too short, which is seen in Fig. 2(b), where $\chi T = \pi/10$ and the state with $n=2$ becomes noticeably populated. Even in this case, however, the one-photon state preparation may be improved by taking a smaller ϵ and increasing the number of kicks k so as to have $\epsilon k = \pi/2$ again. The smaller ϵ, the better the preparation of the one-photon state. However, one has to keep in mind that, so far, we have not included losses in the system. We return to this point later on.

Our numerical calculations, indicating a possibility of preparing the cavity field in the one-photon Fock state, are supported by the following simple analytical considerations. Let us assume that initially the field state was the vacuum, then after k kicks, according to (6) and (7), the field state is given by

$$|\Psi_k\rangle = (U_N U_K)^k |0\rangle \ . \tag{8}$$

For $\epsilon \ll 1$, we can use the Taylor-series expansion of U_K, retaining the terms up to ϵ^2, which after proper or-

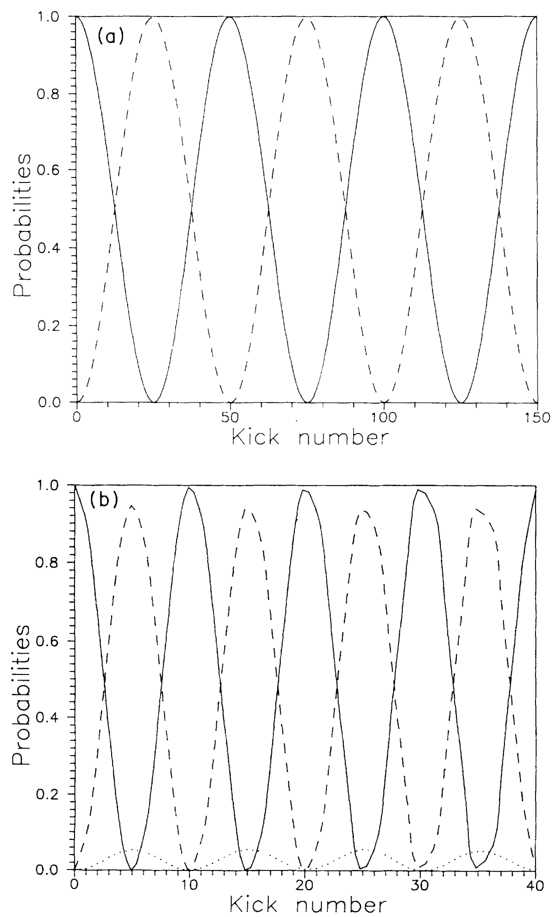


FIG. 1. Time evolution of the probabilities for the vacuum $|0\rangle$ (solid lines), one-photon $|1\rangle$ (dashed lines), and two-photon $|2\rangle$ [dotted line, (b)] states. The time $T = \pi/\chi$ and the kick strength $\epsilon = \pi/50$ (a) and $\epsilon = \pi/10$ (b).

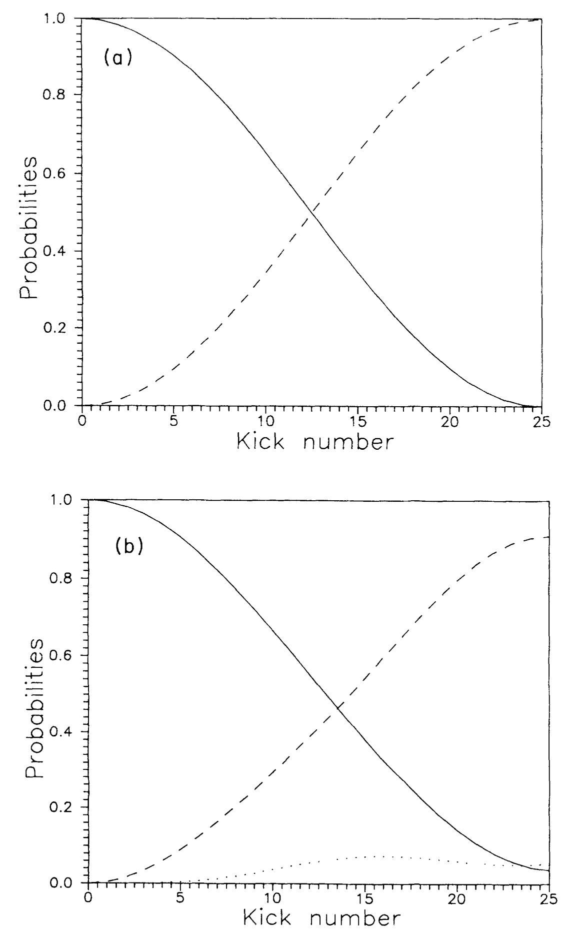


FIG. 2. Same as for Fig. 1, but for the time $T = \pi/\chi$ (a) and $T = \pi/10\chi$ (b). The kick strength $\epsilon = \pi/50$.

dering of the terms gives us

$$|\Psi_k\rangle=(1-\tfrac{1}{2}k^2\epsilon^2)|0\rangle-ik\epsilon|1\rangle$$
$$-\tfrac{1}{2}\{A\ldots(A(A(A+3)+5)+7)+\cdots$$
$$+(2k-1)\}\sqrt{2}\epsilon^2|2\rangle\ , \tag{9}$$

where the quantity A is a phase factor generated by the "free" evolution of the system, and is given by

$$A=e^{-i\chi T}\ . \tag{10}$$

From Eq. (9) it is obvious that the amplitude of the state $|2\rangle$ depends on the time T between the kicks, when the evolution of the system is due to the nonlinear Hamiltonian (1). It is seen that the nonlinear coupling between two consecutive kicks acts as a "phase shifter" and couples incoherently the state $|2\rangle$. Obviously, this coupling is much slower than the coherent excitation process between the vacuum $|0\rangle$ and the one-photon $|1\rangle$ state. The expression in braces in Eq. (9) can be written as

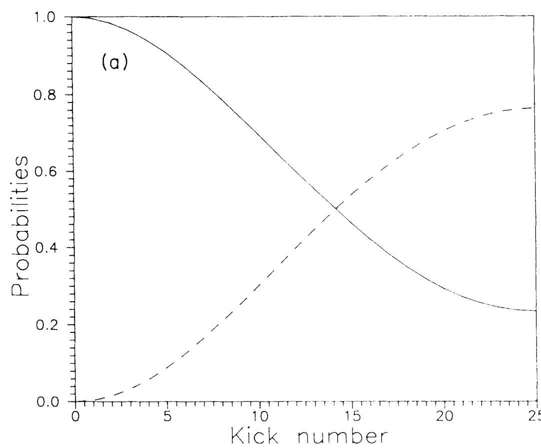
$$\frac{1}{2}\sum_{l=0}^{k-1}(2l+1)A^{k-l-1}=\frac{2k(1-A)-(1+A)(1-A^k)}{2(1-A)^2}\ , \tag{11}$$

which for $A=1$, i.e., $\chi T=2\pi n$, $(n=0,1,\ldots)$, gives $k^2/2$, and for $A\neq 1$ it has a term linear in k. This means that for $\epsilon\sim k^{-1}$ the state $|2\rangle$ amplitude is of the order of k^{-1} and is negligible for large k. In the optimal case $A=-1$, which we have used in our numerical calculations, this amplitude takes the value

$$-\frac{(k\epsilon)^2}{2}\frac{\sqrt{2}}{k}\ . \tag{12}$$

By increasing the number of kicks k and simultaneously decreasing the kick strength ϵ so as to keep $k\epsilon$ constant, the amplitude of the state $|2\rangle$ can be suppressed. In this case, the field evolution takes place between the states $|0\rangle$ and $|1\rangle$ only. One can also notice that the amplitudes of the states $|0\rangle$ and $|1\rangle$ are the first terms of the Taylor-series expansions of the cosine and sine functions of $k\epsilon$, respectively. Our numerical results confirm this observation and show that, for $k\epsilon=\pi/2$ with k sufficiently large, the system evolves into the one-photon state. It is noticeable from our discussion that our system behaves as a two-level system undergoing the Rabi oscillations, with the Rabi frequency determined by the strength of the external classical field.

In real physical situations we cannot avoid dissipation, so we cannot take ϵ too small in order to avoid complete damping of the field during the evolution between the subsequent pumping kicks. Moreover, the dissipation in the system will cause mixing of the quantum states, and the clear, pure state picture of the field evolution presented above will be obscured. Nevertheless, if the damping is weak, it is still possible to get the field in a cavity being very close to the one-photon state. Since, for the case of the Kerr medium with linear damping, the corresponding master equation can be solved exactly [10], and the solution was used by Milburn and Holmes [7] in their discussion of quantum and classical dynamics of a pulsed parametric oscillator with a Kerr nonlinearity, we simply take advantage of this solution and apply it here to take into account the dissipation in the system. The results are shown in Fig. 3, where the probabilities of the vacuum and the one-photon state are plotted for $\epsilon=\pi/50$, $\chi T=\pi$, and $\gamma/\chi=0.01$ (a); $\gamma/\chi=0.1$ (b), where γ is the linear damping constant. It is seen that for $\gamma/\chi=0.01$ it is still over 75% of the population that is found in the state $|1\rangle$, while for $\gamma/\chi=0.1$ it is already less than 15%. Thus, the dissipation in the system drastically lowers the effectiveness of producing the one-photon state.



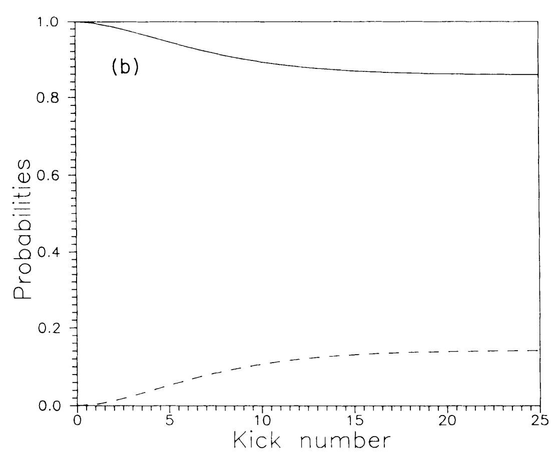


FIG. 3. Probabilities for the vacuum $|0\rangle$ (solid lines) and one-photon $|1\rangle$ (dashed lines) states. The damping constant $\gamma=0.01\chi$ (a) and $\gamma=0.1\chi$ (b), the kick strength $\epsilon=\pi/50$, and the time $T=\pi/\chi$.

Of course, our considerations have a rather model character, and many technical questions should be solved to make this method of producing the one-photon state feasible. It would require a high Q cavity to store the field for a long enough time to ensure sufficiently large nonlinear "phase shifts," and, simultaneously, the nonlinear Kerr medium should have very low linear damping to have $\gamma/\chi\ll 1$. These are very strong requirements that will not be easy to satisfy. However, recent experiments [11,12], in which the very subtle effect of "vacuum Rabi splitting" was measured, give us some hope for making our scheme of producing the one-photon state feasible.

Concluding, we have found one more interesting feature of the quantum evolution associated with the nonlinear Kerr Hamiltonian (1): it can lead to the one-photon Fock state in a cavity that is pumped by a periodic sequence of short pulses of small intensity, provided the system damping is low enough.

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