Three proposed "quantum erasers"

Paul G. Kwiat, Aephraim M. Steinberg, and Raymond Y. Chiao
Department of Physics, University of California, Berkeley, Berkeley, California 94720

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Interaction with a measuring apparatus can cause otherwise interfering paths of a particle to become distinguishable. Under some conditions, however, it is possible to "erase" the distinguishability, thereby restoring interference. The concept of a quantum eraser is useful for understanding the role of entanglement in interference. Several attempts to investigate the phenomenon have been made using the correlated photons from spontaneous parametric down-conversion. However, none of the experiments previously performed in connection with quantum erasure has provided an optimal demonstration. We propose three improved down-conversion schemes, each of which satisfies all the criteria for a true quantum eraser. As the proposed schemes are all modifications or combinations of previously completed experiments, they are deemed to be feasible.

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I. INTRODUCTION

The complementary nature of wavelike and particlelike behavior is commonly interpreted as follows: Due to the uncertainty principle, any attempt to measure the position (particle aspect) of a quantum will lead to an uncontrollable, irreversible disturbance in its momentum, thereby washing out any interference pattern (wave aspect). The measurement provokes an irreversible "reduction of the state vector," irrevocably introducing an uncertainty in the phase. For example, if one wishes to determine which of two slits was traversed by an interfering particle by measuring the initial and final momentum of the recoiling slit mechanism, the uncertainty principle requires a sufficient uncertainty in the initial transverse position of the slits that the interference is lost [1]. This picture, however, is incomplete; no "state reduction" is necessary to destroy wavelike behavior, and measurements which do not involve reduction can be reversible in a certain sense. To fully understand this phenomenon, one must view the loss of coherence as arising from an *entanglement* of the system wave function with that of the measuring apparatus (MA); this is identical to the first step in von Neumann's measurement theory [2], but lacks the step in which the off-diagonal elements of the expanded density matrix are postulated to vanish. Through the entanglement of a quantum system to a MA (in itself a quantum system), previously interfering paths can become *distinguishable* (assuming that the final MA states are orthogonal), such that no interference is observed. This is true even though one may not actually make subsequent measurements on the MA to determine which path actually occurred, i.e., even if one does not look at the state of the MA. Whenever *welcher Weg* ("which way") information is available, in principle, about which possible path occurred, the paths are distinguishable and no interference is possible. Interference may be regained, however, if one somehow manages to "erase" the distinguishing information, by correlating the results of measurements on the interfering particle with the results of particular measurements on the MA. This is the physical content of quantum erasure [3,4].

To date, three performed experiments have been discussed in connection with the quantum eraser [5–8], all employing the correlated photon pairs produced via spontaneous parametric down-conversion [9]. For different reasons, none of these is an optimal demonstration of a quantum eraser, each lacking one or more of its desirable attributes. In fact, two are actually not quantum erasers at all in the strictest sense of the term, and the third, while incorporating the basic features of the phenomenon, is pedagogically wanting. In this paper we present three other experimental schemes for observing quantum erasure, including delayed-choice mechanisms. As these are all modifications of one of the earlier experiments [7], we believe them to be feasible.

In Sec. II we summarize the salient features of a quantum eraser. In Sec. III we discuss the three previous experiments, concentrating on the shortcomings of each, and also introducing techniques which will be relevant for the proposed experiments. These are presented in Sec. IV, where we will show that each allows a truly nonlocal, delayed-choice aspect. Conclusions are in Sec. V. A detailed calculation for one of the proposals is given in the Appendix.

II. IDEALIZED QUANTUM ERASER

In order to explain what is required for an optimal demonstration, and to understand the limitations of the experiments already performed, we first need to describe the relevant features that comprise an *ideal* quantum eraser. Such an experiment begins with an interfering particle (or particles). Envision that the particle has two processes or paths, which we label "*a*" and "*b*," leading to the same outcome (such as striking a particular point on a screen, or exiting a particular port of an interferometer), with probability amplitudes ψ_a and ψ_b, and a variable phase ϕ between them. Thus, the total amplitude for this particular outcome, in the absence of any *welcher Weg* detectors, could be written as

$$\psi_s = \psi_a + e^{i\phi}\psi_b \ . \qquad (1)$$

For instance, ψ_S might represent the value of the wave

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function at a particular point on a screen after double slits, or at one of the exit ports of a Mach-Zehnder interferometer. The squared modulus of ψ_S corresponds to the probability that this outcome would occur (i.e., that the particle would appear at the particular point on the screen, or would choose that interferometer exit port); interference arises from the cross terms $e^{i\phi}\psi_a^*\psi_b + e^{-i\phi}\psi_a\psi_b^*$. We have implicitly assumed thus far that the "a" and "b" paths are not distinguishable (e.g., that we are using a very massive, rigid slit mechanism).

Inserting a MA into one or both paths causes the system wave function to become entangled with the wave function of the MA, enlarging the relevant Hilbert space for the problem. That is, ψ_S becomes $\psi_s|\mathrm{MA}\rangle$, and we replace ψ_a by $\psi_a|A\rangle$ and ψ_b by $\psi_b|B\rangle$, where the states $|A\rangle$ and $|B\rangle$ span the Hilbert space of the MA, and are by definition orthogonal. The previous cross terms therefore vanish, and interference is lost [10]. Importantly, in principle the MA need not induce any disturbance in the center-of-mass wave function of the particle: loss of interference results from the distinguishability of the final MA states. The above description contains the essential elements of a *welcher Weg* experiment.

In a quantum eraser we go one step further. By making a suitable measurement on the MA, and correlating the results to the detection of the original particle(s), one can revive the interference effect. Of course, if one post-selects only those events where either A or B was measured, then no interference fringes will be observed. But if we calculate the probability of obtaining the particular outcome of the initial system *and* finding the MA in the symmetric state $(|A\rangle+|B\rangle)/\sqrt{2}$, we find

$$\left|\left[\frac{\langle A|+\langle B|}{\sqrt{2}}\right]\psi_s|\mathrm{MA}\rangle\right|^2 = \tfrac{1}{2}|\psi_a+e^{i\phi}\psi_b|^2 = \tfrac{1}{2}(|\psi_a|^2+|\psi_b|^2+e^{i\phi}\psi_a^*\psi_b + e^{-i\phi}\psi_a\psi_b^*)\,, \quad (2)$$

a revival of the original fringes. For concreteness, we now assume that processes a and b are equally likely to lead to the outcome we are examining, and that all phase differences between ψ_a and ψ_b are included in ϕ. Then (2) is simply $(1+\cos\phi)/2$. Moreover, depending on the precise measurement made on the MA, one can actually alter the form of the interference, yielding *antifringes* $(1-\cos\phi)/2$ instead of the expected fringes. This is achieved by projecting onto the antisymmetric state $(|A\rangle-|B\rangle)/\sqrt{2}$. (If we do not correlate to the MA results, then the fringes and antifringes will cancel.) More generally, projecting along $[|A\rangle+\exp(i\theta)|B\rangle]/\sqrt{2}$ yields $[1+\cos(\phi-\theta)]/2$. (As an aside, one may note from the previous expression that the system and MA play symmetric roles, i.e., one could equally well view the system as carrying "which-state" information about the measuring apparatus.)

To emphasize the nonseparability [nonfactorizability, in the sense of Schrödinger, of two- (or more) particle wave functions] of quantum mechanics, one important aspect of an ideal quantum eraser is that there be a possible element of "delayed choice": The measurement on the MA could be made *after* the interfering particle has been detected. In fact, our decision to use the MA as a *welcher Weg* detector (by projecting along $|A\rangle$ or $|B\rangle$) or a quantum eraser [by projecting as in (2)], could also be made after detection of the interfering particle. It is only via the subsequent correlation of the results (of measurements on the original particle and the MA) that either interference or *welcher Weg* information may be recovered. In the original delayed choice discussion by Wheeler, he pointed out that the decision to display wavelike or particlelike aspects in a light beam may be delayed until after the beam has been split by the appropriate optics [11]. The situation with the quantum eraser is even more striking—the decision to measure wavelike or particlelike behavior may be delayed until after the *detection* of the quantum, an irreversible process [12].

A second desirable feature of a true quantum eraser is that it employ single particles (as opposed to coherent states, for example); the reason is that the entire discussion of "which way" information depends on the notion of the particlelike aspect of an indivisible quantum. Finally, although nowhere above did we require that the distinguishing information be carried separately from the interfering particle, this is clearly preferable from a pedagogical point of view. As we have stated before, it is the enlargement of the Hilbert space through entanglement, and subsequent reduction, which is the central feature of the quantum eraser [8]; however, the nonseparability inherent in the process becomes more apparent when a system spatially distinct from the initial interfering system serves as the measuring apparatus.

III. PAST EXPERIMENTS

All three of the experiments previously discussed in the context of quantum erasure use the simultaneously emitted photon pairs (conventionally called "signal" and "idler") produced in spontaneous parametric down-conversion [5–8]. The first experiment [6] involves an interference effect which exists only in coincidence detection. Two down-conversion crystals are pumped by coherent cw pump beams. The signal beams are mixed at a beam splitter, while the idler beams are mixed at a separate beam splitter, such that after the beam splitters there is no way of distinguishing from which crystal a given pair of photons originated. Fringes in coincidence are observed as any of the path lengths before the beam splitters are varied, while no interference is seen in the singles rates. The "delicate change" which leads both to distinguishability and to erasure in that example is the removal and reinsertion of one of the beam splitters (e.g., if the idler beam splitter is removed the idler beams then carry *which-crystal* information, and no interference occurs). In this sense, it is deficient as a quantum eraser since it is the structure of the interferometer itself, and not just the structure of the detection scheme, which determines once and for all the presence or absence of interference fringes. To put it differently, there is never interference unless a larger Hilbert space already including the idler photons is considered; removal of the idler beam splitter does not enlarge the Hilbert space.

The experiment of Ref. [7], while a remarkable demonstration of complementarity in its own right, differs fundamentally from the quantum eraser proposal, in that it is entirely a first-order (one-photon), not a second-order (two-photon), interference effect, and no delayed-choice version would be possible. Again, two nonlinear crystals, NL1 and NL2, are used [see Fig. 1(a)]; they are aligned such that the trajectories of the idler photons from each crystal overlap. A beam splitter acts to superpose the trajectories of the signal photons. The basic interference effect arises between these signal photons, as the path length from either of the crystals to the beam splitter is varied slightly. If the path lengths are adjusted correctly, and the idler beams overlap precisely, there is no way to tell, even in principle, from which crystal a photon detected at D_S originated—there results interference in the signal singles rate at D_S (and thus trivially in the coincidence rate between D_S and D_i). If the idler beam from crystal NL1 is prevented from entering crystal NL2 (or even if the two idler beams are only slightly misaligned), the interference vanishes because the presence or absence of an idler photon at D_i then "labels" the parent crystal. However, at this state the *welcher Weg* measurement is effectively irreversible. There is no way

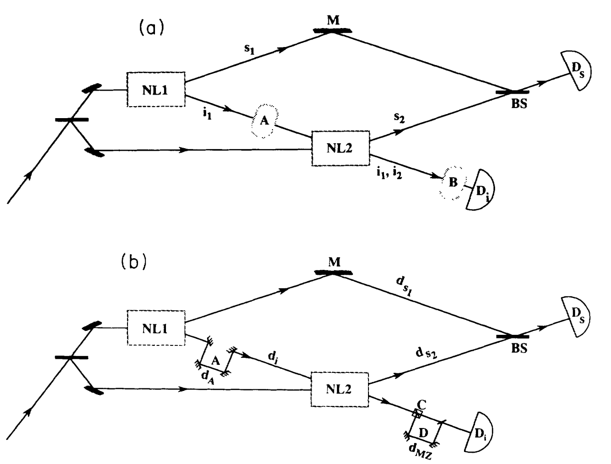

FIG. 1. Schematic of setup used in Ref. [7], with the possible inclusion of additional elements to make it suitable for a quantum eraser. (a) A half-wave plate at A distinguishes the idler photons; the which-crystal information may be erased with a polarizer at B. (b) Two other versions add a delay of length d_A (much greater than the coherence length of the idler photons) between the two crystals. Erasure is performed after crystal NL2 using an unbalanced Mach-Zehnder interferometer, whose path-length difference d_{MZ} is essentially equal to d_A. Fringes in the correlated detection events between D_S and D_i will have 50% or 100% visibility, depending on the detector time resolution (see text). In the second version of this type, where the visibility is always 100%, a half-wave plate at A rotates the NL1 idler polarization so that it is orthogonal to the NL2 idler polarization. In addition, the first beam splitter (at C) of the Mach-Zehnder interferometer is a polarizing beam splitter, oriented so that idler photons from NL1 (NL2) take the short (long) path. Finally, a second wave plate in one of the arms (e.g., at D) makes identical the idlers' polarizations.

at all in practice to "erase" the distinguishability by any transformation on the idler state alone, and certainly no possibility of a delayed choice, even in principle [13]. Because detection events (of the signal photons) are never compared with measurements on the "measuring apparatus" (the idler photons, here), there is no way to recover fringes or antifringes.

The most recent experiment [8] also employs a nonclassical second-order interference effect, so that coincident detection of the two photons is required, but uses only a single crystal. The signal and idler photons are directed to opposite sides of a 50%-50% beam splitter. If the lengths of the signal and idler paths are different, then the photons act independently at the beam splitter, and coincidences are observed half of the time between detectors at the output ports of the beam splitter. On the other hand, if both photons reach the beam splitter simultaneously, they always exit the same port. There results a dip in the coincidence rate as one of the path lengths is varied slightly [14]. The interference stems from the indistinguishability of the transmission-transmission and reflection-reflection processes which could lead to coincidences [15]. A half-wave plate may be used to change the polarization of one of the photons inside the interferometer (prior to the wave plate, the photons are horizontally polarized), so that the two photons have orthogonal polarizations. The Hilbert space is enlarged to include the polarizations of the photons, as well as their directions. This serves to label the two interfering processes, and the interference consequently disappears, despite the fact that the final detectors do not explicitly "measure" the photons' polarizations.

However, by using polarizers before the two detectors it is possible to erase the distinguishing information, and recover the interference, even though the photons have already left the interferometer. Put differently, to correctly understand the results, one must adhere to Bohr's dictum to consider the *entire* experimental system, including the polarizers after the interferometer. This experiment is pedagogically superior as a quantum eraser to the obvious "classical" analog [16] (the canonical two slit-experiment with a classical light field, a polarization-rotator in front of one slit, and a variable polarizer-analyzer in front of the detection screen), because its use of single-particle states permits the notion of *welcher Weg* information [17]. However, it is not pedagogically optimal because the *welcher Weg* information is carried by the interfering particles themselves, not stored in some external measuring system. As a result, performing a delayed choice version of the experiment would be very difficult, requiring quantum nondemolition detection of the photons before a subsequent measurement on the polarization part of their wave function. It would be preferable if the entanglement were to an external system, not to an internal degree of freedom of the interfering particles [18].

IV. PROPOSED QUANTUM ERASERS

The basic setup is the same as that of the second experiment discussed above [7]. The difference is the inclusion of the additional elements A and B [see Fig. 1(a)], which we shall discuss presently. Note that for the signal beams to interfere, it is crucial that the idler photons be indistinguishable after crystal NL2, even though we need not detect them to observe the interference at the signal detector D_S. In particular, the idler photons from the two crystals must have the same polarization and color, and must arrive at the idler detector D_i at essentially the same time. (If this were not the case, then one could *in principle* determine which crystal emitted the detected *signal* photon by making a careful measurement at D_i of the polarization or energy or arrival time of the conjugate *idler* photon.) We shall purposely violate these constraints to distinguish the idler photons from NL1 and NL2, removing the first-order interference at D_S. This scheme is superior to the previous experiment [8] because the *welcher Weg* information is not carried by the interfering particles, the signal photons. Correlating the counts at D_S with subsequent measurements on the idler photons, we can implement a quantum eraser.

There are several ways to proceed. The simplest is to insert a half-wave plate between the two crystals [at A, in Fig. 1(a)], rotating the polarization of the idler from NL1 by 90°. Since the idlers are now distinguishable, the interference of the possible signal paths disappears. Just as above [8], however, we can use a polarizer before D_i (at B, in Fig. 1) to erase the distinguishing information, and correlate the counts at D_S and D_i. If the polarizer is aligned along either of the idler-polarization directions, no interference will be seen in singles or coincidence. If aligned between the two (i.e., at $\pm 45°$), we will obtain either fringes or antifringes in coincidence (but not in singles). An experiment closely related to this is currently in progress [19]. The delayed-choice feature is that we could, in principle, decide on the polarizer setting and, hence, whether we would see fringes, antifringes, or no fringes, *after* the signal photon was already detected. Experimentally, this could be accomplished by having an optical delay before the polarizer, and using a Pockels cell to effectively rotate the polarizer very quickly.

A second way of making the idler photons distinguishable is to increase the path length between the two crystals, by adding a delay of length d_A (greater than the photon's coherence length) at position A [see Fig. 1(b)]. To subsequently make the idler photons indistinguishable again, we essentially need to add two delay lines after the second crystal, the difference in whose path lengths is equal to d_A. In practice, we can achieve this using an unbalanced Mach-Zehnder interferometer before detector D_i. The choice of whether to detect fringes or antifringes is set by the detailed phase difference in the Mach-Zehnder interferometer, which could in principle be chosen after the signal photons had been detected.

We now present a simplified calculation demonstrating the above claims. (A detailed formal calculation is given in the Appendix.) For clarity we assume that the down-converted photons are degenerate at frequency ω_0. Let $\tau_s = \tau_{s_1} - \tau_{s_2}$ be the extra time required by a signal photon from crystal 1 (to reach the beam splitter before D_S) relative to a signal photon from crystal 2; similarly, let τ_i be the idler photon propagation time from crystal 1 to crystal 2, *when no delay line is present.* (These times are related to the distances labeled in the figure by $\tau = d/c$.) Furthermore, assume that with no delay line, fringes are observable in the signal singles rate, i.e., the difference in these two times is much less than τ_c, the two-photon correlation time of the down-converted photons. Adding an extra delay τ_A ($\gg \tau_c$) to τ_i then makes the two interfering processes distinguishable and singles fringes are not observed. Finally, let τ_{MZ} be the difference between the propagation times through the long and short paths in the Mach-Zehnder interferometer. There are four possible ways for the detectors D_S and D_i to register photons: (1) photons from NL1, idler takes short path in the Mach-Zehnder interferometer (denoted $s_1 i_{1,S}$); (2) photons from NL1, idler takes long path ($s_1 i_{1,L}$); (3) photons from NL2, idler takes short path ($s_2 i_{2,S}$); and (4) photons from NL2, idler takes long path ($s_2 i_{2,L}$). For arbitrary values of τ_A and τ_{MZ} these four possibilities are, in general, distinguishable. However, if we consider $|\tau_A - \tau_{\mathrm{MZ}}| \ll \tau_c$, then processes $s_1 i_{1,S}$ and $s_2 i_{2,L}$ are indistinguishable. To calculate the probability of coincidence, we sum the amplitudes of these indistinguishable processes, take the absolute square, and add the absolute square of each of the distinguishable processes,

$$P_c \propto \left| e^{i\omega_0\tau_s} \frac{i}{\sqrt{2}} e^{i\omega_0(\tau_i+\tau_A)} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \frac{i}{\sqrt{2}} e^{i\omega_0\tau_{\mathrm{MZ}}} \frac{i}{\sqrt{2}} \right|^2 + \left| e^{i\omega_0\tau_s} \frac{i}{\sqrt{2}} e^{i\omega_0(\tau_i+\tau_A)} \frac{i}{\sqrt{2}} e^{i\omega_0\tau_{\mathrm{MZ}}} \frac{i}{\sqrt{2}} \right|^2 + \left| \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \right|^2$$

$$= \tfrac{1}{2} + \tfrac{1}{4}\sin[\omega_0(\tau_s + \tau_i + \tau_A - \tau_{\mathrm{MZ}})] , \qquad (3)$$

where we have included coefficients of $1/\sqrt{2}$ ($i/\sqrt{2}$) for transmission (reflection) at each of the beam splitters. Implicit in (3) is the additional condition that our detection scheme could not exclude the contributions of noninterfering counts arising from the in-principle distinguishable $s_1 i_{1,L}$ and $s_2 i_{2,S}$ processes. This will be the case if we have slow detectors, and the visibility of the coincidence fringes will be limited to 50%. However, if the electronic resolution time ΔT of the coincidence equipment is less than τ_{MZ}, the visibility can be as high as 100% (see the Appendix). We recently observed this same sort of detector-dependent effect in connection with a Franson experiment to violate a Bell inequality based on energy and time [20].

As an aside, yet another way to restore indistinguishability to the idler photons in this delay-line configuration is simply to use a narrow-band interference filter (instead of a Mach-Zehnder) before D_i. If the resulting coherence length is greater than d_A, then interference will be restored in the coincidence rate. Correlating the signal photons with the idler photons transmitted through the interference filter will recover the fringes; correlating with the idlers *reflected* from the filter (assuming a nonabsorbing filter) will yield antifringes. The same technique was used by us in a previous experiment to demonstrate the single-photon Berry's phase [21].

The third quantum eraser method we propose is in some sense a hybrid of the other two. Once again, a half-wave plate is inserted at position A between the two crystals, in addition to the optical delay line of length d_A [see Fig. 1(b)]. The polarization of the idler beam from NL1 is rotated so as to be orthogonal to the polarization of the idler beam from NL2. Next, a polarizing beam splitter is used in the unbalanced Mach-Zehnder interferometer. The polarizing beam splitter is oriented so that idler photons from NL1, with the extra optical delay d_A, take the short Mach-Zehnder path, while idler photons from LN2 take the long Mach-Zehnder path. Finally, a second half-wave plate inserted at position D in the long path undoes the polarization rotation from the wave plate at A. Thus, polarization no longer labels the parent crystal. As before, if the difference in the Mach-Zehnder path lengths is equal to d_A, then the parent crystal of a given idler photon is unknowable, and interference will be observed in coincidence. The advantage of this scheme over the previous one is that the visibility of the interference fringes can be 100%, *regardless of the speed of the idler detector,* because the use of polarization permits us to force the idler photons to take the appropriate paths in the Mach-Zehnder interferometer, eliminating the noninterfering processes.

V. CONCLUSION

The quantum eraser offers an important perspective on interference and loss of quantum coherence in terms of (in)distinguishability of paths. The loss of wavelike behavior is not necessarily due to the uncertainty principle, but may merely be due to an entanglement of the interfering system with a measuring apparatus (or the entire "environment"). If the coherence of the measuring apparatus is maintained, then interference may be recovered by correlating results of measurements on the original system with results of particular measurements on the measuring apparatus. The state involved in interference is the *total* physical state, which in addition to photon spatial wave functions may include photon polarization, or even distant photons or atoms.

Fairly simple experiments are possible using correlated down-conversion photons, and several have been performed. As reported, however, none of these possesses all the attributes of a true quantum eraser. This deficiency is remedied by modification of one of the setups. The simplest extension uses polarization to provide a distinguishable label on the contributing paths; a suitable polarization measurement on the idler photons can then serve to yield *welcher Weg* information or interference fringes, *after* the interfering signal photons have been detected. Instead we may make use of the simultaneity of the down-converted twin photons by introducing a time delay into one of the idler arms. An unbalanced Mach-Zehnder interferometer can then be used to reduce the enlarged Hilbert space by projecting onto various linear combinations of the measuring apparatus states (here, the states of the idler photon). If the polarization and delay-line techniques are combined, one may achieve 100% visibility even with slow detectors.

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APPENDIX

In the calculations that follow, we assume a monochromatic pump beam at frequency $\omega_p = 2\omega_0$, and use a single spatial-mode treatment for each of the down-converted photons: the signal (idler) modes from crystal 1 and crystal 2 are labeled s_1 (i_1) and s_2 (i_2), respectively. Furthermore, we assume that the modes of the idler photons from the two nonlinear crystals are *spatially* indistinguishable ($i_1 = i_2 = i$). then we may write the wave function after the two crystals as

$$|\psi\rangle = \int_{-\infty}^{\infty} d\omega_1 A(\omega_1)\frac{1}{\sqrt{2}}[|\omega_0+\omega_1,\omega_0-\omega_1\rangle_{s_2,i} + e^{i(\omega_0-\omega_1)\tau_{iA}}|\omega_0+\omega_1,\omega_0-\omega_1\rangle_{s_1,i}] , \qquad (A1)$$

where we have omitted the vacuum term (for the predominant, but uninteresting, case in which neither crystal down-converts), and higher-order terms (for the very unlikely case in which more than one pump photon down-converts). The state (A1) describes a signal-idler photon pair, satisfying energy conservation (enforced by the effectively infinite interaction time), and originating with equal probability in crystal 1 (first term) or crystal 2 (second term). $A(\omega)$, the probability amplitude, which includes the down-conversion efficiency as well as the pump field strength, is determined ultimately by phase-matching constraints, but is limited in practice by filters and irises before the detectors. $\tau_{iA} \equiv \tau_i + \tau_A$, where $d_i = c\tau_i$ is the initial separation between the two crystals (along the idler direction), and $d_A = c\tau_A$ is the extra delay inserted in our modified scheme [see Fig. 1(b)].

The operator $E_S^{(+)}(t)$ for the positive-frequency part of electric field at the signal detector D_S may be expanded in terms of single-mode photon-annihilation operators,

$$E_s^{(+)}(t) = \int_0^{\infty} d\omega_s e^{-i\omega_s t}\left[\frac{i}{\sqrt{2}}e^{i\omega_s\tau_s}\hat{a}_{s_1}(\omega_s) + \frac{1}{\sqrt{2}}\hat{a}_{s_2}(\omega_s)\right] , \qquad (A2)$$

where we have omitted normalization constants, and assumed beam splitter transmission and reflection coefficients of $1/\sqrt{2}$ and $i/\sqrt{2}$, respectively. $d_s = d_{s_1} - d_{s_2} = c\tau_s$ is the additional distance travelled by the signal field originating in crystal 1 relative to that originating in crystal 2. According to the standard Glauber theory for photodetection, the probability per unit time of detecting a count with a detector D_S during a duration T (assumed to be much longer than any other time scale in the problem, but short enough that we can neglect multiple pairs of photons) is proportional to

$$R_S = \frac{1}{T}\int_{-T/2}^{T/2} dt\, P_S(t) ,$$
$$P_S(t) = \langle \psi | E_s^{(-)}(t) E_s^{(+)}(t) | \psi \rangle . \tag{A3}$$

Using (A1) and (A2) in (A3) gives

$$R_S = \frac{1}{T}\int_{-T/2}^{T/2} dt \int d\omega_1' A^*(\omega_1') \frac{1}{\sqrt{2}} [{}_{s_1,i}\langle \omega_0+\omega_1', \omega_0-\omega_1' | e^{-i(\omega_0-\omega_1')\tau_{iA}} + {}_{s_2,i}\langle \omega_0+\omega_1', \omega_0-\omega_1' |]$$
$$\times \int d\omega_s' \int d\omega_s e^{i\omega_s' t} e^{-i\omega_s' t} \frac{1}{2} [-ie^{-i\omega_s'\tau_s} \hat{a}_{s_1}^\dagger(\omega_s') + \hat{a}_{s_2}^\dagger(\omega_s')][ie^{i\omega_s\tau_s}\hat{a}_{s_1}(\omega_s) + \hat{a}_{s_2}(\omega_s)]$$
$$\times \int d\omega_1 A(\omega_1) \frac{1}{\sqrt{2}} [|\omega_0+\omega_1, \omega_0-\omega_1\rangle_{s_1,i} e^{i(\omega_0-\omega_1)\tau_{iA}} + |\omega_0+\omega_1, \omega_0-\omega_1\rangle_{s_2,i}] . \tag{A4}$$

The canonical commutation relations yield the delta functions $\delta(\omega_s' - (\omega_0+\omega_1'))$ and $\delta(\omega_s - (\omega_0+\omega_1))$, while the integral over t enforces an effective delta function $\delta(\omega_s' - \omega_s)$. Combining these gives the additional constraint $\delta(\omega_1' - \omega_1)$, and (A4) reduces to a single integral,

$$R_S = K \int d\omega_1 A^*(\omega_1) A(\omega_1) [-ie^{-i(\omega_0-\omega_1)\tau_{iA}} e^{-i(\omega_0+\omega_1)\tau_s} + 1][ie^{i(\omega_0-\omega_1)\tau_{iA}} e^{i(\omega_0+\omega_1)\tau_s} + 1]$$
$$= K \int d\omega_1 | A(\omega_1)|^2 \{2 - 2\sin[\omega_0(\tau_A+\tau_i+\tau_s) - \omega_1(\tau_A+\tau_i-\tau_s)]\} . \tag{A5}$$

Here and below, K represents numerical constants, incidental to the calculation.

If $\tau_A = 0$ (no extra delay between the crystals), then (A5) will yield fringes of 100% visibility as long as τ_s and τ_i differ by no more than some incremental amount $\Delta\tau$ (much less than the coherence time of the signal photons). Henceforth, we assume that we start in such a balanced condition: $\tau_i = \tau_s + \Delta\tau$. Under this condition, the fringes will vanish for $\tau_A \gg 1/\sigma$, where σ is the characteristic bandwidth set by the filter function $A(\omega)$. Physically, $\tau_A \gg 1/\sigma$ implies that one could in principle determine the parent crystal of a given down-converted pair by making precise timing measurements. In other words, the inclusion of the extra delay τ_A serves to distinguish the previously interfering signal photons, yielding *welcher Weg* information by the timing of the idler photons; consequently, no interference is observed.

We now examine the effect of adding an unbalanced Mach-Zehnder interferometer (with path-length imbalance $d_{\mathrm{MZ}} = c\tau_{\mathrm{MZ}}$) before the idler detector D_i. The idler field at D_i may be expressed as follows:

$$E_i^{(+)}(t) = \int_0^\infty d\omega_i e^{-i\omega_i t} \frac{1}{\sqrt{2}} (1 - e^{i\omega_i\tau_{\mathrm{MZ}}}) \hat{a}_i(\omega_i) . \tag{A6}$$

Clearly, the above calculation of signal singles is not altered by this, since it did not depend on $E_i^{(+)}(t)$. The probability of coincidences, i.e., correlations between detectors D_i and D_s, can show a revival of interference, however. The appropriate fourth-order correlation function is given by $P_C(t_1,t_2) = \langle \psi | E_s^{(-)}(t_1) E_i^{(-)}(t_2) E_i^{(+)}(t_2) E_s^{(+)}(t_1) | \psi \rangle$. In practice, one must include the coincidence time resolution of the detection system, set, for instance, by an electronic gate window of width ΔT. (Due to the extra delay τ_A, we have to adjust the timing of the coincidence gate by this amount.) The probability of a coincidence detection within a duration T is then

$$R_C = \frac{1}{T}\int_{-T/2}^{T/2} dt_1 \int_{t_1+\tau_A-\Delta T/2}^{t_1+\tau_A+\Delta T/2} dt_2 P_C(t_1,t_2)$$
$$= \frac{1}{T}\int_{-T/2}^{T/2} dt_1 \int_{t_1+\tau_A-\Delta T/2}^{t_1+\tau_A+\Delta T/2} dt_2 \int d\omega_1' A^*(\omega_1') \frac{1}{\sqrt{2}} [{}_{s_1,i}\langle \omega_0+\omega_1', \omega_0-\omega_1' | e^{-i(\omega_0-\omega_1')\tau_{iA}} + {}_{s_2,i}\langle \omega_0+\omega_1', \omega_0-\omega_1' |]$$
$$\times \int d\omega_s' \int d\omega_s e^{i\omega_s' t_1} e^{-i\omega_s t_1} \frac{1}{2} [-ie^{-i\omega_s'\tau_s} \hat{a}_{s_1}^\dagger(\omega_s') + \hat{a}_{s_2}^\dagger(\omega_s')][ie^{i\omega_s\tau_s}\hat{a}_{s_1}(\omega_s) + \hat{a}_{s_2}(\omega_s)]$$
$$\times \int d\omega_i' \int d\omega_i e^{i\omega_i' t_2} e^{-i\omega_i t_2} \frac{1}{2} [1 - e^{-i\omega_i'\tau_{\mathrm{MZ}}}] \hat{a}_i^\dagger(\omega_i') [1 - e^{i\omega_i\tau_{\mathrm{MZ}}}] \hat{a}_i(\omega_i)$$
$$\times \int d\omega_1 A(\omega_1) \frac{1}{\sqrt{2}} [|\omega_0+\omega_1, \omega_0-\omega_1\rangle_{s_1,i} e^{i(\omega_0-\omega_1)\tau_{iA}} + |\omega_0+\omega_1, \omega_0-\omega_1\rangle_{s_2,i}] . \tag{A7}$$

The canonical commutation relations can be used to remove four of the integrals, leaving

$$\frac{K}{T}\int_{-T/2}^{T/2}dt_1\int_{t_1+\tau_A-\Delta T/2}^{t_1+\tau_A+\Delta T/2}dt_2\int d\omega_1'\int d\omega_1 A^*(\omega_1')A(\omega_1)e^{i(\omega_0+\omega_1')t_1}e^{-i(\omega_0+\omega_1)t_1}e^{i(\omega_0-\omega_1')t_2}e^{-i(\omega_0-\omega_1)/t_2}$$
$$\times(-ie^{-i(\omega_0-\omega_1')\tau_{iA}}e^{-i(\omega_0+\omega_1')\tau_s}+1)(ie^{i(\omega_0-\omega_1)\tau_{iA}}e^{i(\omega_0+\omega_1)\tau_s}+1)$$
$$\times(1-e^{-i(\omega_0-\omega_1')\tau_{\mathrm{MZ}}})(1-e^{i(\omega_0-\omega_1)\tau_{\mathrm{MZ}}}) \ . \tag{A8}$$

Next, we evaluate the time integrals, first the integral over t_2,

$$\int_{t_1+\tau_A-\Delta T/2}^{t_1+\tau_A+\Delta T/2}dt_2 e^{i(\omega_0+\omega_1')t_1}e^{-i(\omega_0+\omega_1)t_1}e^{i(\omega_0-\omega_1')t_2}e^{-i(\omega_0\omega_1)t_2}$$
$$=e^{i(\omega_1'-\omega_1)t_1}\int_{t_1+\tau_A-\Delta T/2}^{t_1+\tau_A+\Delta T/2}dt_2 e^{-i(\omega_1'-\omega_1)t_2}=\frac{\sin[(\omega_1-\omega_1')\Delta T/2]}{(\omega_1-\omega_1')/2}e^{i(\omega_1-\omega_1')\tau_A} \ . \tag{A9}$$

Since the above expression no longer depends on t_1, integration over t_1 simply gives a multiplicative factor of T. From (A9) we are led to make a change of variables to $\delta\omega=\omega_1'-\omega_1$ and $\Omega=\omega_1'+\omega_1$. Also, we again make the substitution $\tau_i=\tau_s+\Delta\tau$ (i.e., a balanced initial setup). Finally, for concreteness we choose a definite form for our filter function,

$$A^*(\omega_1)=A(\omega_1)=\frac{1}{\sqrt{2\pi\sigma}}e^{-(\omega_1)^2/2\sigma^2} \ . \tag{A10}$$

With the above variable substitution, this yields

$$A^*(\omega_1')A(\omega_1)=\frac{1}{2\pi\sigma}e^{-(\Omega/2\sigma)^2}e^{-(\delta\omega/2\sigma)^2} \ . \tag{A11}$$

Modulo overall constants, which we subsume into K, the coincidence probability (A8) is then

$$R_C=K\int d(\delta\omega)\int d\Omega e^{-(\Omega^2+\delta\omega^2)/(2\sigma)^2}\frac{\sin(\delta\omega\Delta T/2)}{\delta\omega}e^{-i\delta\omega\tau_A}$$
$$\times[1+e^{i\delta\omega(\Delta\tau+\tau_A)}+ie^{i\omega_0(\tau_{iA}+\tau_s)}e^{-i(\Omega-\delta\omega)(\Delta\tau+\tau_A)/2}-ie^{-i\omega_0(\tau_{iA}+\tau_s)}e^{i(\Omega+\delta\omega)(\Delta\tau+\tau_A)/2}]$$
$$\times[1+e^{i\delta\omega\tau_{\mathrm{MZ}}}-e^{-i\omega_0\tau_{\mathrm{MZ}}}e^{i(\Omega+\delta\omega)\tau_{\mathrm{MZ}}/2}-e^{i\omega_0\tau_{\mathrm{MZ}}}e^{-i(\Omega-\delta\omega)\tau_{\mathrm{MZ}}/2}] \ . \tag{A12}$$

After integration over Ω using the general result

$$\int_{-\infty}^{\infty}d\Omega e^{-(\Omega/2\sigma)^2+iX\Omega}=2\sigma\sqrt{\pi}e^{-(\sigma X)^2} \ , \tag{A13}$$

many of the terms are proportional to $\exp[-(\sigma\tau_A)^2]$ or $\exp[-(\sigma\tau_{\mathrm{MZ}})^2]$, and are thus negligible. (We assume here the case of interest that $\tau_A, \tau_{\mathrm{MZ}} \gg 1/\sigma$, so that there are no signal singles fringes.) Equation (A12) reduces to

$$R_C=K\int d(\delta\omega)e^{-(\delta\omega/2\sigma)^2}\frac{\sin(\delta\omega\Delta T/2)}{\delta\omega}$$
$$\times\{e^{-i\delta\omega\tau_A}+e^{i\delta\omega\Delta\tau}+e^{-i\delta\omega(\tau_A-\tau_{\mathrm{MZ}})}+e^{i\delta\omega(\Delta\tau+\tau_{\mathrm{MZ}})}$$
$$+2\sin[\omega_0(\Delta\tau+\tau_A-\tau_{\mathrm{MZ}}+2\tau_s)]e^{i\delta\omega(\Delta\tau-\tau_A+\tau_{\mathrm{MZ}})/2}e^{-[\sigma(\Delta\tau+\tau_A-\tau_{\mathrm{MZ}})/2]^2}\} \ . \tag{A14}$$

There are two regimes to consider, depending on the value of $|\tau_A-\tau_{\mathrm{MZ}}|$; each has two cases, depending on the time resolution ΔT. First we consider the situation in which the Mach-Zehnder path imbalance is quite different from the extra delay added between the crystals; quantitatively, $|\tau_A-\tau_{\mathrm{MZ}}|\gg 1/\sigma$. Then the last term in (A14) is negligible. If the resolution time is greater than any of the path lengths ($\Delta T \gg \tau_A, \tau_{\mathrm{MZ}}$), the sinc function behaves like a δ function, and the first four terms in the curly bracket all have a value of 1. If the resolution time is less than τ_A and τ_{MZ}, then integration over $\delta\omega$ will wash out all but the second term. For no value of ΔT are fringes observed.

Henceforth, we restrict ourselves to the case $\tau_A=\tau_{\mathrm{MZ}}+\Delta\tau_{\mathrm{MZ}}$, with $\Delta\tau_{\mathrm{MZ}}\ll 1/\sigma$. As before, a large value of ΔT causes an effective delta-function $\delta(\delta\omega)$. Under these conditions, (A14) predicts fringes of 50% visibility,

$$R_C \propto 4+2\sin[\omega_0(\Delta\tau+\Delta\tau_{\mathrm{MZ}}+2\tau_s)] \ . \tag{A15}$$

Finally, if the coincidence time resolution is less than the delay times τ_A and τ_{MZ}, the first and fourth terms of (A14) average out after integration over $\delta\omega$, and we recover 100% visibility fringes,

$$R_C \propto 2+2\sin[\omega_0(\Delta\tau+\Delta\tau_{\mathrm{MZ}}+2\tau_s)] \ . \tag{A16}$$

Physically, the background terms which appear in (A15) [but not in (A16)] are due to contributions from the following processes: (1) photons originate in crystal 1, but the idler takes the long path in the Mach-Zehnder; and (2) photons originate in crystal 2, but the idler takes the short path in the Mach-Zehnder. With a sufficiently short-time resolution, these noninterfering contributions may be removed, yielding (A16).

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[9] Several proposals have been made for demonstrating quantum erasers by using atoms or neutrons [M. O. Scully, R. Shea, and J. D. McCullen, Phys. Rep. **13**, 485 (1978); B.-G. Englert, J. Schwinger, and M. O. Scully, Found. Phys. **18**, 1045 (1988); J. Schwinger, M. O. Scully, and B.-G. Englert, Z. Phys. D **10**, 135 (1988); B.-G. Englert, H. Walther, and M. O. Scully, Appl. Phys. B **54**, 366 (1992)], but all of these are at best very difficult in practice. To date, the most promising of the proposed experiments on massive particles involve the interference manifested in the quantum-beat phenomenon [A. Zajonc, Phys. Lett. **96A**, 61 (1983); M. O. Scully and H. Walther, Phys. Rev. A **39**, 5229 (1989)]. However, in addition to also being rather difficult, though possibly feasible, these experiments suffer the conceptual disadvantage that there are not actually spatially separated paths as in the double-slit versions.

[10] This problem is usually addressed using the density-matrix formalism. The prescription for describing a measurement on one system is to trace over the states of the (possibly unobserved) measuring system (e.g., the MA or the environment; see, for instance the discussion of "reduction" by Zurek [W. H. Zurek, Phys. Today **44**, 36 (1991)]). This effectively removes the coherences between the formerly intefering states, although the full density matrix has undergone only unitary evolution and retains the original coherences.

[11] J. A. Wheeler, in *Problems in the Formulation of Physics,* edited by G. T. diFrancia (North-Holland, Amsterdam, 1979).

[12] There is as yet no clear answer to the question of exactly *when* or even *how* the detection becomes an irreversible event. Presumably, once the detector has coupled to some large number of degrees of freedom, the process will be effectively irreversible. In practice, we can confidently claim that by the time the event has attained the level of a macroscopic pulse out of the detector, it is thermodynamically irreversible.

[13] One might imagine recombining the two idler beams at a second beam splitter after the second crystal. But this is then topologically equivalent to the first pseudoeraser discussed above [6].

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[15] It has also been observed that this indistinguishability, and the energy entanglement of the down-converted photon pairs, lead to a cancellation of the effects of linear group velocity dispersion [A. M. Steinberg, P. G. Kwiat, and R. Y. Chiao, Phys. Rev. Lett. **68**, 2421 (1992)]. This allows for high-resolution time-of-flight measurements [A. M. Steinberg, P. G. Kwiat, and R. Y. Chiao, Phys. Rev. Lett. **71**, 708 (1993)].

[16] We use the term "classical" to indicate that the effect has an explanation using a classical field model. In such a theory there is no meaning to the concept of "which-path" informaton, since a wave can follow two paths simultaneously; even the quantum-mechanical description of such an experiment has many particles, some of which follow each route, and which are indistinguishable from one another.

[17] The nonclassical nature of the scheme is apparent from the fact that the same setup has been used in tests of Bell's inequalities [Z. Y. Ou and L. Mandel, Phys. Rev. Lett. **61**, 50 (1988); Y. H. Shih and C. O. Alley, *ibid.* **61**, 2921 (1988)].

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