PHYSICAL REVIEW A VOLUME 49, NUMBER 1 JANUARY 1994

Measurement of phase differences between two partially coherent fields

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(Received 10 June 1993)

Although the operational approach to the quantum-phase problem developed earlier by us [Phys. Rev. A **45**, 424 (1992)] has now been confirmed experimentally for several different states of the electromagnetic field, so far no measurements have been reported with partially coherent light beams. This situation is remedied in the present work, where we report the results of measurements on light obeying thermal statistics with a variable degree of mutual coherence. Once again, we find agreement between experiment and our theoretical approach.

PACS number(s): 42.50.Wm, 03.65.Bz

I. INTRODUCTION

Although there exists a very large literature on the mathematical problem of identifying the phase of a quantized electromagnetic field and/or its probability density [1], relatively few phase measurements have been reported [2–6]. We have recently introduced an operational approach to the phase problem [7], based on an examination of what is typically measured, rather than on any abstract criterion for a phase operator. This leads to the conclusion that phase differences between two fields are more fundamental than the absolute phase of either one, and that different measurement schemes lead to different operators. In other words, the dynamical variables associated with the measured cosine and sine of the phase difference cannot be divorced from the detailed process of measurement.

We have tested our theoretical approach experimentally, and obtained good agreement between theory and experiment [5],[6] in all cases, which included measurements of some higher moments. We have also extended the range of average photon numbers by more than two orders of magnitude below previous measurements. Nevertheless, experimental tests of the theory have so far been limited to states that are close to pure coherent states or photon number states of the field. In the following we present experimental results for electromagnetic fields obeying thermal statistics, for which the degree of coherence between the two interferometer inputs varies between 0 and 1. Once again we obtain agreement between experiment and our theoretical approach to the phase problem.

II. THEORY OF THE MEASUREMENT

Let us consider the experimental arrangement shown in Fig. 1, in which the sine and cosine of the phase difference $\hat{S}_M, \hat{C}_M$ between two input fields $\hat{a}_1, \hat{a}_2$ is measured by the arrangement we have labeled scheme 2 [5–7]. The two inputs are derived by splitting the light from a highly stable single-mode He:Ne laser into two parts with the beam splitter BS_7 and passing these through a rotating ground glass. The two emerging light beams are then mixed in various different proportions with the help of beam splitters BS_3 and BS_5 and filters F_4, F_6, F_8, as shown. The emerging light has been shown to obey approximately thermal statistics [8–10], so that it can be described classically in terms of fluctuating Gaussian waves. It was pointed out in Ref. [7] that when phase fluctuations are accompanied by correlated intensity fluctuations, then our phase measurement technique is unable to separate the latter contribution, except in the strong-field case $\langle \hat{n}_1 \rangle, \langle \hat{n}_2 \rangle \gg 1$.

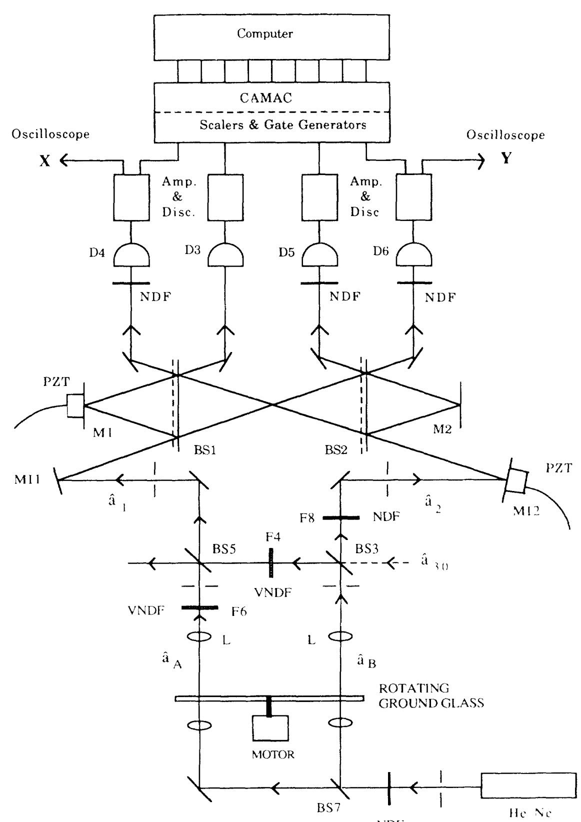

FIG. 1. Outline of the setup of the phase measurement. Above mirrors MI1,MI2 the apparatus is similar to that described in Refs. [5] and [7].

Let $\hat{a}_A, \hat{a}_B$ be the non-Hermitian quantum-field mode amplitudes immediately after the rotating ground glass. We shall treat $\hat{a}_A, \hat{a}_B$ as mutually incoherent and statistically independent thermal mode amplitudes, and the interferometer input mode amplitudes $\hat{a}_1, \hat{a}_2$ are related to $\hat{a}_A, \hat{a}_B$ by

$$\hat{a}_1 = r_5'[t_4(r_3\hat{a}_B + \hat{a}_{30}t_3') + \hat{a}_{40}r_4'] + t_5[t_6\hat{a}_A + r_6'\hat{a}_{60}] , \quad (1)$$

$$\hat{a}_2 = [t_8 t_3 \hat{a}_B + r_8'\hat{a}_{80} + t_8 r_3'\hat{a}_{30}] . \quad (2)$$

Here r_j, t_j and r_j', t_j' are the reflectivity and transmissivity of the beam splitter j ($j=3,5$) from one side and from the other side. t_4, t_6, and t_8 are the transmissivities of the neutral-density filters F_4, F_6, F_8, which are also modeled as beam splitters, and $\hat{a}_{30}, \hat{a}_{40}, \hat{a}_{60}, \hat{a}_{80}$ are vacuum mode amplitudes associated with BS_3, F_4, F_6, F_8, respectively. From Eqs. (1) and (2) we obtain

$$\langle \hat{n}_1 \rangle = |r_5|^2|r_3|^2|t_4|^2\langle \hat{n}_B \rangle + |t_5|^2|t_6|^2\langle \hat{n}_A \rangle , \quad (3)$$

$$\langle \hat{n}_2 \rangle = |t_3|^2|t_8|^2\langle \hat{n}_B \rangle , \quad (4)$$

$$\langle \hat{a}_1^\dagger \hat{a}_2 \rangle = r_5'^* r_3^* t_4^* t_3 t_8 \langle \hat{n}_B \rangle . \quad (5)$$

It follows that if modes 1 and 2 are allowed to come together and interfere, then the visibility ϑ of the resulting interference pattern is given by

$$\vartheta = \frac{2|\langle \hat{a}_1^\dagger \hat{a}_2 \rangle|}{\langle \hat{n}_1 \rangle + \langle \hat{n}_2 \rangle} ,$$

or, when expressed in terms of modes A and B,

$$\vartheta = \frac{2|r_5 r_3 t_3 t_4 t_8|\langle \hat{n}_B \rangle}{(|r_5|^2|r_3|^2|t_4|^2 + |t_3|^2|t_8|^2)\langle \hat{n}_B \rangle + |t_5|^2|t_6|^2\langle \hat{n}_A \rangle} . \quad (6)$$

Expressions for the expectation of the measured cosine and sine of the phase difference for different optical fields of any quantum state have been given in Ref. [7]. Because the general expressions are complicated we will focus on the special case $\langle \hat{n}_1 \rangle, \langle \hat{n}_2 \rangle \ll 1$ for simplicity, for which [7]

$$\langle \hat{C}_M \rangle = \frac{\langle \hat{a}_1^\dagger \hat{a}_2 \rangle + \text{c.c.}}{2(\langle \hat{n}_1 \rangle + \langle \hat{n}_2 \rangle)} , \quad (7)$$

$$\langle \hat{S}_M \rangle = \frac{-i(\langle \hat{a}_1^\dagger \hat{a}_2 \rangle + \text{c.c.})}{2(\langle \hat{n}_1 \rangle + \langle \hat{n}_2 \rangle)} . \quad (8)$$

The calculation involves a renormalization to correct for discarded data, as described in Ref. [7]. Hence we obtain

$$\langle \hat{C}_M \rangle = \tfrac{1}{2}\vartheta \cos(\theta_1 - \theta_2), \quad \langle \hat{S}_M \rangle = \tfrac{1}{2}\vartheta \sin(\theta_1 - \theta_2) , \quad (9)$$

where $\theta_1 - \theta_2$ is the phase difference between the two interferometer inputs, and

$$\begin{aligned}\langle (\Delta\hat{C}_M)^2 \rangle + \langle (\Delta\hat{S}_M)^2 \rangle &= \langle \hat{C}_M^2 \rangle + \langle \hat{S}_M^2 \rangle - \langle \hat{C}_M \rangle^2 - \langle \hat{S}_M \rangle^2 \\ &= 1 - \tfrac{1}{4}\vartheta^2 . \end{aligned} \quad (10)$$

A plot of $\langle \hat{C}_M \rangle / \cos(\theta_1 - \theta_2)$ versus the visibility ϑ of the interference pattern should be a straight line of slope $\frac{1}{2}$, while a plot of $\langle (\Delta\hat{C}_M)^2 \rangle + \langle (\Delta\hat{S}_M)^2 \rangle$ versus ϑ should yield a parabola (when $\langle \hat{n}_1 \rangle, \langle \hat{n}_2 \rangle \ll 1$).

The degree of mutual coherence between the two interferometer inputs is given by

$$|\gamma_{12}| = \frac{|\langle \hat{a}_1^\dagger \hat{a}_2 \rangle|}{(\langle \hat{n}_1 \rangle \langle \hat{n}_2 \rangle)^{1/2}} = \frac{|r_5 r_3 t_4 t_3 t_8|\langle \hat{n}_B \rangle}{[|r_5 r_3 t_4|^2\langle \hat{n}_B \rangle + (|t_5 t_6|^2)\langle \hat{n}_A \rangle]^{1/2}[\langle \hat{n}_B \rangle |t_3 t_8|^2]^{1/2}} . \quad (11)$$

and when $\langle \hat{n}_A \rangle = \langle \hat{n}_B \rangle$ and $|r_5|^2 = \frac{1}{2} = |r_3|^2 = |t_5|^2 = |t_3|^2 = |t_8|^2$, this reduces to

$$|\gamma_{12}| = \frac{1}{[1 + 2|t_6|^2/|t_4|^2]^{1/2}} . \quad (12)$$

The ratio $|t_6|^2/|t_4|^2$ therefore determines the degree of coherence $|\gamma_{12}|$, which can range between 0 and 1. $|\gamma_{12}|$ coincides with the visibility ϑ of the interference pattern when $\langle \hat{n}_1 \rangle = \langle \hat{n}_2 \rangle$. In practice it is convenient to make $\langle \hat{n}_1 \rangle = \langle \hat{n}_2 \rangle$, and, from Eqs. (3) and (4), this is achieved by making

$$|t_6|^2 = (1 - |t_4|^2)/2 = |r_4|^2/2 . \quad (13)$$

In that case $\langle \hat{n}_1 \rangle = \langle \hat{n}_2 \rangle = \langle \hat{n}_A \rangle / 4$, and

$$|\gamma_{12}| = |t_4| = \vartheta . \quad (14)$$

We have recently described a procedure for obtaining the probability density $p(\phi_2 - \phi_1)$ of the phase difference $\phi_2 - \phi_1$ between the two interferometer inputs [11]. For this purpose the input to channel 2 is phase shifted by some angle θ, which can be varied in small steps between $-\pi$ and π, and phase measurements are made in each case separately. As explained in Ref. [11], if $C(x|\theta)$ is the characteristic function of the phase difference $\phi_2 - \phi_1$ conditioned on the shift θ, then, by analogy with $\exp[i(\phi_2 - \phi_1)] = \cos(\phi_2 - \phi_1) + i\sin(\phi_2 - \phi_1)$, we write

$$C(x|\theta) = \langle e^{i\Theta(\{\hat{n}\})x} \rangle' , \quad (15)$$

where

$$e^{i\Theta(\{\hat{n}\})} \equiv \hat{C}_M + i\hat{S}_M = \frac{\hat{n}_4 - \hat{n}_3 + i(\hat{n}_6 - \hat{n}_5)}{[(\hat{n}_4 - \hat{n}_3)^2 + (\hat{n}_6 - \hat{n}_5)^2]^{1/2}} , \quad (16)$$

and $\langle\ \rangle'$ denotes the quantum expectation in the phase-shifted state. The $\hat{n}_3,\hat{n}_4,\hat{n}_5,\hat{n}_6$ refer to the photon numbers counted by the four photodetectors and these are expressible in terms of the input modes to the interferometer [5–7]. Because experimental outcomes with $n_4=n_3$ and $n_6=n_5$ do not give meaningful values of $\hat{C}_M$ or $\hat{S}_M$ we discard these outcomes and renormalize the results. This procedure has been criticized recently [12], but the suggested alternatives can lead to even more meaningless consequences [13]. By Fourier inversion of $C(x|\theta)$ we obtain the probability density $p(\phi_2-\phi_1|\theta)$ conditioned on θ,

$$p(\phi_2-\phi_1|\theta)=\frac{1}{2\pi}\int_{-\infty}^{\infty}C(x|\theta)e^{-ix(\phi_2-\phi_1-\theta)}dx\ , \qquad (17)$$

whose average over all θ yields the corrected probability distribution $p_c(\phi_2-\phi_1)$,

$$p_c(\phi_2-\phi_1)=\frac{1}{2\pi}\int_{-\pi}^{\pi}d\theta\, p(\phi_2-\phi_1|\theta)\ . \qquad (18)$$

We now apply the formalism to the experiment illustrated in Fig. 1. As the light emerging from the rotating ground glass has been shown to obey Gaussian statistics [8–10], the input modes 1 and 2 must be Gaussian also. On using the procedure given in Ref. [7] for expressing expectations of the form $\langle f(\hat{n}_3,\hat{n}_4,\hat{n}_5,\hat{n}_6)\rangle$ in terms of normally ordered operators, and then using the so-called optical equivalence theorem [14,15,16] for calculating the normally ordered expectation as a c-number average, we obtain for the phase-shifted characteristic function $C(x|\theta)$

$$C(x|\theta)=\sum_{\{n\}}\left\{\frac{(n_4-n_3)+i(n_6-n_5)}{[(n_4-n_3)^2+(n_6-n_5)^2]^{1/2}}\right\}^x \times\frac{\langle |v_1-v_2e^{i\theta}|^{2n_3}|v_1+v_2e^{i\theta}|^{2n_4}|-iv_1+v_2e^{i\theta}|^{2n_5}|-v_1+iv_2e^{i\theta}|^{2n_6}e^{-|v_1|^2-|v_2|^2}\rangle}{4^{n_3+n_4+n_5+n_6}n_3!n_4!n_5!n_6!}\ . \qquad (19)$$

Here v_1,v_2 are c-numbers corresponding to $\hat{a}_1,\hat{a}_2$, and the expectation $\langle\ \rangle$ is to be calculated by taking the phase-space density of v_1,v_2 to be a bivariate Gaussian distribution with $\langle |v_1|^2\rangle=\langle n_1\rangle=\langle |v_2|^2\rangle$ and $\langle v_1^*v_2\rangle=\gamma_{12}\langle n_1\rangle$. From Eq. (19) the probability density $p_c(\phi_2-\phi_1)$ then follows with the help of Eqs. (17) and (18). These equations are applicable to the experiment so long as the measurement or photon counting time T is much shorter than the coherence time T_c, so that $\phi_2-\phi_1$ has no time to change during the measurement.

In practice it is often convenient to increment θ in steps of equal width B and to present the results of the phase measurement as a histogram with bins of width $\Delta(\phi_2-\phi_1)=B$. Then the theoretically expected values of the probability distribution P_N $(N=0,\pm1,\pm2,\ldots,\pm\pi/B)$ are given by [11]

$$P_N=\int_{N-B/2}^{N+B/2}p_c(\phi_2-\phi_1)d(\phi_2-\phi_1)\ . \qquad (20)$$

The averages in Eq. (19) have been evaluated numerically for two Gaussian variates v_1,v_2 with $\langle |v_1|^2\rangle=\langle |v_2|^2\rangle$ and normalized cross correlation γ_{12}.

III. EXPERIMENT AND RESULTS

An outline of the experiment is shown in Fig. 1. The light beam from a stable, single-mode He:Ne laser is divided into two equal parts by the 50%:50% beam splitter BS_7 and the two resulting beams pass through a ground glass plate rotating at 10 revolutions/sec at a distance of 6 cm from the wheel center. The rotating plane imposes an approximately Gaussian random modulation on the two light beams, which emerge uncorrelated and with near thermal statistics [8–10] with a coherence time T_c determined by the ratio of grain size to grain velocity, which is of order 3×10^{-6} sec. By making the rotating ground glass the focal plane of the lenses L shown in Fig. 1, we ensure that each emerging beam is close to being spatially coherent. Beam splitters BS_3 and BS_5 and neutral density filters F_4 and F_6 mix the two thermal beams, so that the fields $\hat{a}_1,\hat{a}_2$ at the two interferometer inputs are partially coherent, with controllable degree of coherence $|\gamma_{12}|$. The 50% attenuator F_8 placed in the $\hat{a}_2$ input arm helps to balance the two inputs $\langle \hat{n}_1\rangle,\langle \hat{n}_2\rangle$. In practice the parameters $|t_4|,|t_6|$ are adjusted so as to satisfy Eq. (13) and keep the two average photon numbers $\langle \hat{n}_1\rangle,\langle \hat{n}_2\rangle$ equal as $|\gamma_{12}|$ is varied. However, the degree of coherence determined from the visibility of the interference pattern turns out to be about 10% smaller than that given by Eqs. (12) or (14), possibly because of imperfections in the overlap and alignment of the light emerging from the ground glass. This explanation is reinforced by the fact that there are also variations of a few percent in the visibility seen by the four different photodetectors.

From the mirrors MI1,MI2 on, the interferometer is essentially identical to that described previously [5],[6]. Four output beams fall on four photon counting photodetectors D_3,D_4,D_5,D_6, whose quantum efficiencies are carefully balanced, and the numbers of photons n_3,n_4,n_5,n_6 registered in each counting interval T by each detector are recorded. From a large number of data sets n_3,n_4,n_5,n_6, the average measured cosine and sine of the phase difference, and the corresponding dispersions are determined by the procedure described previously [5–7]. The visibility or the degree of coherence is extracted from the maximum and minimum values of

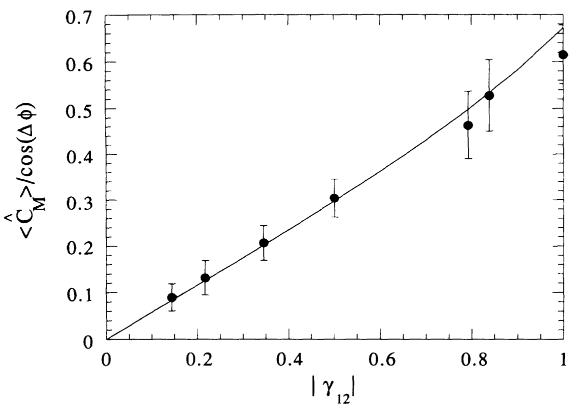

FIG. 2. Results of measurements of $\langle \hat{C}_M \rangle$ as function of the degree of coherence $|\gamma_{12}|$. Each data point is the average of 24 different, equally spaced values of the phase difference. The error bars correspond to one standard deviation. The full curve is theoretical.

$\langle \hat{C}_M \rangle$ as the optical path difference is varied by piezoelectric displacement of one of the interferometer mirrors. We chose the counting interval T to be 0.5 μsec, which makes $\langle n_1 \rangle \approx 1 \approx \langle n_2 \rangle$, and also ensures that T is several times shorter than T_c.

Figures 2 and 3 show experimental results for $\langle \hat{C}_M \rangle / \cos(\theta_2 - \theta_1)$ and $\langle (\Delta \hat{C}_M)^2 \rangle + \langle (\Delta \hat{S}_M)^2 \rangle$ for various values of the visibility ϑ or the degree of coherence $|\gamma_{12}|$ and of $\theta_2 - \theta_1$. The full lines correspond to the theoretical predictions given by the theory of Ref. [7], which is based on Eq. (16), when $\langle \hat{n}_1 \rangle = 1 = \langle \hat{n}_2 \rangle$, but they have the approximate general form given by Eqs. (9) and (10) for the case $\langle \hat{n}_1 \rangle, \langle \hat{n}_2 \rangle \ll 1$.

Figure 4 gives experimental results for the derived probability distribution of the phase difference with bins of width $B = 15°$ for three different values of the degree of coherence $|\gamma_{12}|$ determined from Eq. (14). Also shown are the theoretically computed values of $p_c(\phi_2 - \phi_1)$ from Eqs. (15)–(20). The probability distribution is strongly peaked for two mutually coherent light beams, but becomes flat over the range $-\pi$ as π for two mutually incoherent beams. There appears to be a small discrepancy between the experimental data and the theory that may be concerned with departures from Gaussian statistics of the light scattered from the rotating ground glass. Such departures have been observed before [8–10], and we have some evidence that the rotating scatterer imposes

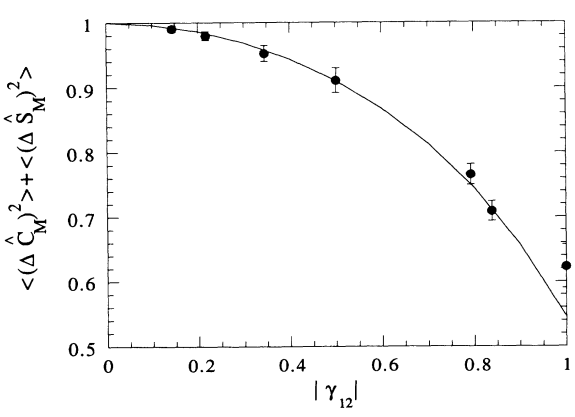

FIG. 3. Results of measurements of $\langle (\Delta \hat{C}_M)^2 \rangle + \langle (\Delta \hat{S}_M)^2 \rangle$ as a function of the degree of coherence $|\gamma_{12}|$. Each data point is the average of 24 different, equally spaced values of the phase difference. The error bars correspond to one standard deviation. The full curve is theoretical.

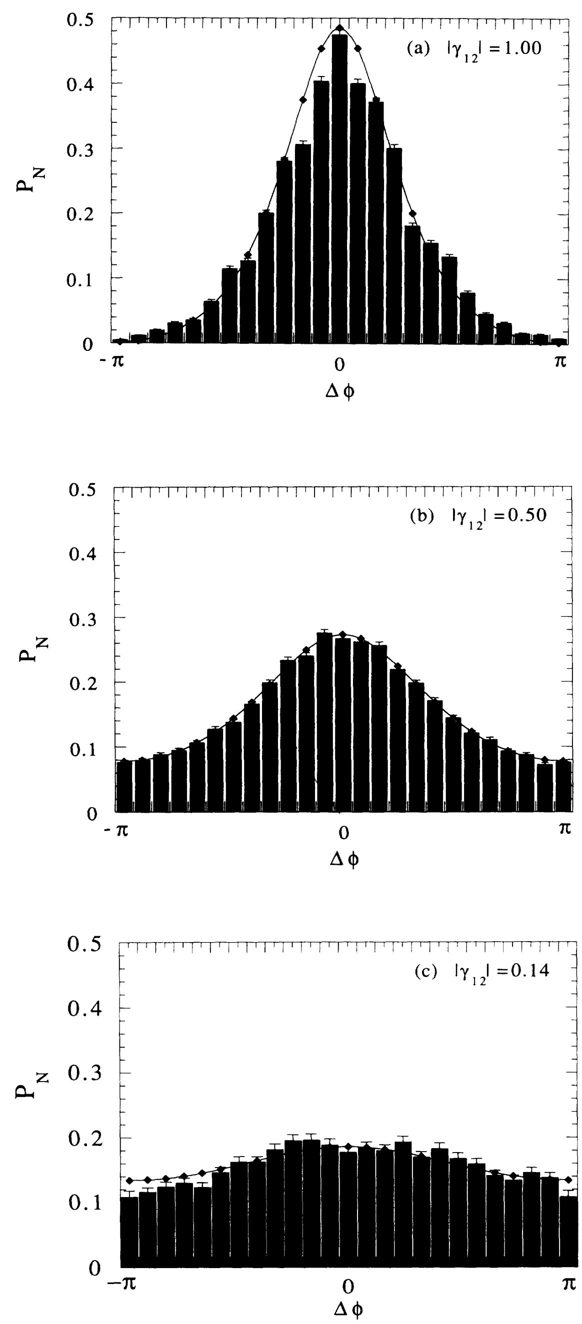

FIG. 4. Histogram of the experimentally derived probability distribution of the phase difference with bins of width 15° for several different values of the degree of coherence $|\gamma_{12}|$. The "T" above each rectangle represents one statistical standard deviation. The diamonds give the corresponding theoretical values for (a) $|\gamma_{12}| = 1$; (b) $|\gamma_{12}| = 0.5$; (c) $|\gamma_{12}| = 0.14$.

some vibration and a small periodic modulation on the light. In the absence of more detailed information on the statistics of the scattered field we cannot improve the fit, but the general form of $p(\phi_2-\phi_1)$ and its variation with $|\gamma_{12}|$ are well confirmed. It should be noted that two quite different mechanisms are responsible for the spread of the observed phase difference. When $|\gamma_{12}|=1$ and the two inputs are mutually coherent, the spread of $\Delta\phi$ is connected only with the quantum-mechanical uncertainties of the phase measurement. When $|\gamma_{12}|$ falls below zero, there is an additional phase spread due to lack of coherence between the two sources.

The results of these measurements of the phase difference under conditions of partial coherence, generally lend additional credence to our approach to the problem of identifying the measured quantum phase operator.

ACKNOWLEDGMENTS

This work was supported by the National Science Foundation and by the U.S. Office of Naval Research.

[1] See Refs. [6] and [7] for extensive references to various approaches to the phase problem.

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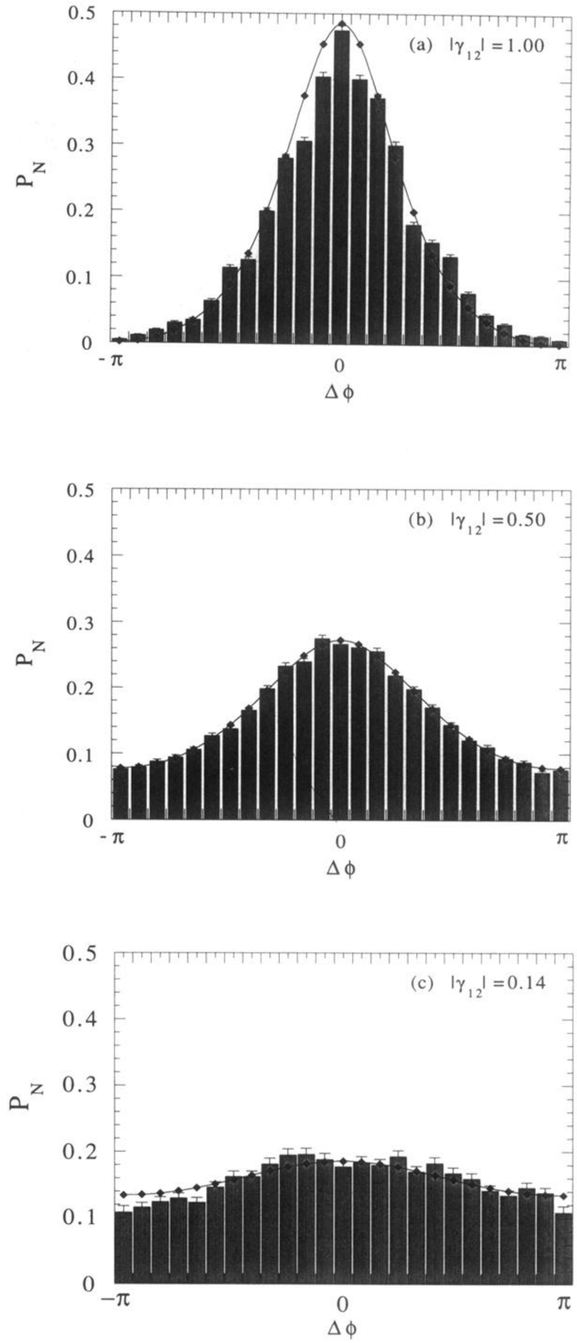

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