Born-Floquet theory of laser-assisted electron-atom collisions

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We present a non-Hermitian Born-Floquet theory of scattering of fast electrons by atoms in the presence of a strong monochromatic laser field. The interaction of the laser field with both the incident electron and the target atom is treated nonperturbatively, while the interaction of the incident electron with the target atom is treated in first Born approximation. Fluorescence is neglected. Detailed calculations are performed for the "elastic" scattering of 500 eV electrons by atomic hydrogen accompanied by the transfer of photons. The contribution of the entire spectrum of unperturbed atomic states to the dressing of the target is exactly taken into account by performing the calculations on a complex Sturmian basis set. In the nonresonant case, and for electric field strengths that are small with respect to the atomic unit, our Born-Floquet results are in agreement with those obtained using the semiperturbative approach of Byron and Joachain (in which target dressing is treated in first-order perturbation theory) even at intensities where multiphoton ionization is nonperturbative. The Born-Floquet approach is particularly useful to study resonant cases, where the laser frequency matches a transition frequency in the atom. Two such situations are analyzed.

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I. INTRODUCTION

The study of electron-atom collisions in the presence of a laser field is presently a subject of intense research activity, not only because of the importance of these processes in applied areas (such as plasma heating), but also in view of their interest in fundamental atomic collision theory. In particular, laser-assisted electron-atom collisions allow the observation of multiphoton processes at relatively moderate laser-field intensities [1] and give rise to a number of new effects with respect to field-free collisions (see, for example, [2]).

Most of the early theoretical investigations of laser-assisted electron-atom collisions were confined to the study of laser-assisted potential scattering, in which the target atom is modeled by a structureless center of force and hence does not interact with the laser field. This is the case, in particular, for the first Born calculations of Bunkin and Fedorov [3], the low frequency theory of Kroll and Watson [4] and the high-intensity, high frequency theory of Gavrila and Kaminski [5].

The analysis of electron collisions with "real" atoms (having an internal structure) in the presence of a laser field is a much more difficult problem. We have three interactions: first, the unbound electron interacts with all the constituents of the target atom (atomic nucleus and bound electrons) as in the field-free case; second, the laser field interacts with the unbound electron; third, the laser field interacts with the target atom and hence "dresses" the atomic target states.

In order to understand the role played by these three interactions in laser-assisted electron-atom collisions, Byron and Joachain [6] used a semiperturbative method, valid for fast incident electrons ($E_{k_i} \geq 100$ eV) and electric field strengths that are small with respect to the atomic unit of electric field strength $e/a_0^2 \simeq 5.1 \times 10^9$ V cm^{-1}. In this method the interaction between the fast projectile electron and the target atom is treated perturbatively by using the Born series. On the other hand, the interaction between the laser field (assumed to be described as a monochromatic, monomode and homogeneous electric field) and the projectile electron is treated exactly by using a Volkov wave function [7]. Finally, the laser-target interaction is treated by using first-order time-dependent perturbation theory, as the laser electric field is assumed to be small with respect to the Coulomb field of the nucleus experienced by the target electrons. This semiperturbative theory has been applied extensively to a variety of laser-assisted electron-atom collisions involving the transfer of several photons between the electron-atom system and the laser field: "elastic" collisions [6,8,9], inelastic (excitation) collisions [10], and $(e, 2e)$ reactions [11]. If, instead of being treated to all orders (via Volkov wave functions), the laser projectile interaction is treated to first order, the semiperturbative theory reduces to a fully perturbative treatment [12].

How can the semiperturbative theory be improved? Assuming that the projectile electron is fast (so that a perturbative treatment of the projectile target interaction is adequate), the only cause of concern is the first-order treatment of the interaction between the laser field and the target atom. The reason is twofold. First, if the electric field strength $\mathcal{E}_0$ is increased (within reasonable limits, otherwise the target atom would decay too rapidly for the laser-assisted electron scattering experiment to be performed), it is expected that higher-order terms in the laser field-target atom interaction could become significant. Second, and more importantly, a first-order treatment of the laser-field–target-atom interaction is inadequate in the immediate vicinity of a resonance, i.e., when the laser photon energy matches the excitation energy of an intermediate state; perturbation theory will exhibit a spurious divergence at the resonance. There is no divergence when the states which are resonantly coupled by the laser field are treated nonperturbatively, for example, by representing the atom by a two-state model in the rotating wave approximation [13]. Unnikrishnan [14] proposed to improve this simple two-state model, which is clearly not valid off resonance, by including the first-order corrections arising from the counter-rotating terms and by treating the coupling of the two resonant states to the other, nonresonant states by first-order perturbation theory. Unnikrishnan applied his method to study the scattering of fast electrons by atomic hydrogen in the presence of a laser-field coupling resonantly the ground state to the $2p$ state. A more general and accurate treatment of the laser-target interaction was used by Francken and Joachain [15] to analyze the resonant laser-assisted excitation of the $2\,^1S$ and $2\,^1P$ states of helium by fast electrons. Their method consists in coupling a few target states (namely, those which are near-resonantly coupled by the laser field) exactly, using the Floquet approach, while the coupling of the laser field with the remaining target states is treated perturbatively. The Floquet theory [16] is indeed a natural framework for developing nonperturbative treatments of the dressing of the target atom by the laser field [17,18].

In the Born-Floquet calculations reported in this paper we go beyond the work of Francken and Joachain [15], since in the present approach all the hydrogen atom target states are coupled exactly to the laser field and multiphoton ionization is taken into account. The interactions between the laser field and both the projectile and the target are treated in a nonperturbative way: the former by using (exact) Volkov waves and the latter by using the non-Hermitian Floquet method [19]. (The use of the non-Hermitian Floquet method distinguishes the present Born-Floquet theory from the theory proposed by Faisal [18].) The interaction between the projectile electron and the target atom is treated by using the first Born approximation. Hence, the present theory is physically meaningful for fast projectiles, provided that the laser field does not act for a long time before the collision takes place (since no allowance is made for fluorescence). It is also applicable to cases where the semiperturbative theory is questionable, e.g., at high intensity or for resonant laser fields, and is amenable to numerically converged calculations. In the present work, we apply this theory to the case of an atomic hydrogen target, for which the contribution of the entire spectrum (bound *and* continuum) to the dressing of the atomic states can be taken into account in a very concise way, by expanding the wave function of the target atom, dressed by the field and decaying by multiphoton ionization, on a discrete basis of complex Sturmian functions. It is justified to neglect exchange effects (which arise from the antisymmetry of the wave function) in the present work. Indeed, the field-free exchange effects are essentially negligible at the high impact energy—typically 500 eV or higher—we are considering [20]. Moreover, it has been shown that exchange effects are usually smaller in the presence of a laser field than in the field-free case [21]. It would not be easy to extend the Born-Floquet theory to a lower range of impact energy, where the first Born approximation breaks down (i.e., the distortion of the target by the projectile cannot be neglected) and exchange effects must be included. An R-matrix-Floquet theory in which the electron-atom interaction as well as projectile and target dressings are taken into account nonperturbatively is presently being developed to analyze laser-assisted electron-atom collisions at low energy [22].

The advantage of adopting the Floquet approach is that it is then possible to cast the essentially time-dependent problem of laser-assisted collisions into a time-independent formalism which is more suitable for carrying out numerical computations. Clearly, reduction to a time-independent problem would not be possible without restricting the range of scattering processes that can be considered. The Floquet method is appropriate to describe experimental situations in which the duration of one optical cycle of the laser (or the inverse of the relevant atomic transition frequencies) is much shorter than (i) the temporal scale of the variation of the laser intensity experienced by the target as the laser pulse passes by; (ii) the lifetime of the target atoms in the field; and (iii) the duration of the pulses of projectiles that are directed onto the target atoms. Condition (i) is easily fulfilled since even short laser pulses encompass, in typical experiments, at least hundreds of optical cycles. However, together with condition (ii), it sets an upper limit to the pulse peak intensity that can be considered for a given frequency and pulse duration. As described in Sec. II below, we assume that the decaying atom can be characterized by a single Floquet state before the collision occurs; this would not be possible if the intensity were varying too rapidly. Moreover, representing the atom by a single (pure) state vector implies that the duration of the laser pulses we consider is limited not only by the rate of photoionization but also by the rate of fluorescence, since spontaneous decay destroys the coherence of the initial Floquet state [23]. We address here the case where the laser pulse is shorter than the fluorescent time of the atom excited by the field; in particular, we are interested in collisions in the presence of an intense field where the rate of photoionization is larger than the rate of fluorescence. As noted previously, the theory presented in this paper is not suited to the cases where the atom can decay by fluorescence before the collision occurs

or where the laser bandwidth is significant. (It is possible, experimentally, to distinguish the electrons scattered during the passage of a sub-ns laser pulse with transfer of a *nonzero* number of photons from those scattered in the absence of field, since the former and the latter have a different final energy. However, it could be difficult to measure cross sections for laser-assisted scattering *without* net transfer of photons if the pulses are very short.)

Condition (iii) amounts to impose that the energy width of the projectile wave packet is small compared to the energy of the photons and to the differences in energy of the relevant target states, so that the incident wave packet can be approximated, as in field-free time-independent scattering theory, by a monoenergetic beam of infinite duration. This makes it possible to distinguish processes in which different numbers of photons have been absorbed or emitted. Since it is not possible to define when the collision takes place if there is no uncertainty in the energy of the projectile, one should not expect the scattering cross sections to depend on the phase of the laser field when condition (iii) is fulfilled; the opposite conclusion would be reached if the process investigated were, for example, the scattering of short pulses of electrons in the presence of a microwave field of period much longer than the duration of the electron pulses. Phase-dependent cross sections for assisted scattering have been considered by Bachau and Shakeshaft [24] and by Smith and Flannery [25]. The case studied by Bachau and Shakeshaft was the scattering of protons by hydrogen in a nearly resonant laser field. The phase dependence stemmed from treating the heavy projectile as a classical particle following a well-defined trajectory with no uncertainty in position at any time, in the impact parameter formalism. The authors stressed that the measurable cross section involves an average over the phase of the field. Smith and Flannery have also used the Floquet method (including a few target states in the calculation) in a study of laser-assisted $1s$-$2s$ and $1s$-$2p$ excitation of atomic hydrogen by intermediate energy electrons, the collision dynamics being described either within the framework of the multichannel eikonal formalism or in the Born approximation. The phase-dependent cross sections considered by these authors were cross sections for transitions between *unperturbed* states of the atom in the presence of the field, not for transitions between *dressed* states of the atom. The scattering amplitudes for transitions between *dressed* states depend on the phase of the field, ϕ, in a simple way, through a phase factor which has no effect on the corresponding cross section—see Eq. (28). By contrast, the phase dependence of the scattering amplitudes for transitions between *unperturbed* states in the presence of a field is nontrivial, because unperturbed states are linear superpositions of several dressed states, each varying with ϕ in its own way.

The theory is developed in Sec. II by considering the case of laser-assisted "elastic" collisions of fast electrons by atomic hydrogen, for which detailed Floquet calculations are feasible to obtain the dressed target states. The theory can be generalized easily to the case of laser-assisted excitation of discrete states, provided their decay width can be neglected when calculating the cross section—this is briefly discussed in Sec. II, between Eqs. (25) and (26). Applications of the present Born-Floquet theory to laser-assisted excitation of atomic hydrogen will be reported in a future publication. We only give a short account of the Floquet method, which has been discussed at length in the recent literature (see, e.g., [26,27]). Numerical results are presented in Sec. III, where we discuss the nonresonant case and analyze two resonant cases.

II. THEORY

Let us consider an elastic collision between a fast (nonrelativistic) electron of mass m and charge $-e$ and a hydrogen atom, in the presence of a laser field. We take the nucleus to be infinitely massive. During this collision, a *net* number $|N|$ of photons are transferred between the electron-atom system and the field. Thus, if $\boldsymbol{k}_\mathrm{i}$ and $\boldsymbol{k}_\mathrm{f}$ denote, respectively, the wave vectors of the unbound electron before and after the collision and $E_{k_\mathrm{i}} = \hbar^2 k_\mathrm{i}^2/2m$ and $E_{k_\mathrm{f}} = \hbar^2 k_\mathrm{f}^2/2m$ are the corresponding energies, we have

$$E_{k_\mathrm{f}} = E_{k_\mathrm{i}} + N\hbar\omega, \quad N = 0, \pm 1, \pm 2, \ldots . \tag{1}$$

The laser field will be treated classically, as a spatially homogeneous, linearly polarized, monochromatic and single mode electric field:

$$\boldsymbol{\mathcal{E}}(t) = \boldsymbol{\mathcal{E}}_0 \sin(\omega t + \phi), \tag{2}$$

the corresponding vector potential being

$$\boldsymbol{A}(t) = \boldsymbol{A}_0 \cos(\omega t + \phi) \tag{3}$$

with $\boldsymbol{A}_0 = c\boldsymbol{\mathcal{E}}_0/\omega$. We describe below how we allow for spatial and temporal variations of the field that are slow on the scales set, respectively, by the range of the projectile-atom interaction and by the laser period.

We work in the dipole approximation. The nonrelativistic Hamiltonian of the electron-atomic hydrogen system in the presence of the laser field may be written in the direct arrangement channel as

$$H = H_F + H_T + V_d, \tag{4}$$

where

$$H_F = \frac{1}{2m}\left[\boldsymbol{p}_0 + \frac{e}{c}\boldsymbol{A}(t)\right]^2 \tag{5}$$

is the Hamiltonian of the free electron in the presence of the laser field,

$$H_T = \frac{1}{2m}\left[\boldsymbol{p}_1 + \frac{e}{c}\boldsymbol{A}(t)\right]^2 - \frac{e^2}{r_1} \tag{6}$$

is the Hamiltonian of the target hydrogen atom in the presence of the laser field, and

$$V_d = -\frac{e^2}{r_0} + \frac{e^2}{r_{01}}, \qquad r_{01} = |\boldsymbol{r}_0 - \boldsymbol{r}_1|, \tag{7}$$

is the projectile electron-target atom interaction in the direct arrangement channel.

The Volkov wave function

$$\chi_{\boldsymbol{k}}(\boldsymbol{r}_0,t) = (2\pi)^{-3/2} e^{i[\boldsymbol{k}\cdot\boldsymbol{r}_0 - \boldsymbol{k}\cdot\boldsymbol{\alpha}(t) - E_k t/\hbar - \zeta(t)]}, \tag{8}$$

with $\boldsymbol{\alpha}(t) = \boldsymbol{\alpha}_0 \sin(\omega t + \phi)$ and $\boldsymbol{\alpha}_0 = e\boldsymbol{\mathcal{E}}_0/m\omega^2$ is the exact solution of the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t}\chi(\boldsymbol{r}_0,t) = H_F \chi(\boldsymbol{r}_0,t), \tag{9}$$

for a free electron moving in the laser field with mean momentum $\hbar\boldsymbol{k}$; α_0 is the amplitude of oscillation that a classical electron would have in the field. In Eq. (8), $E_k = \hbar^2 k^2/2m$, $\zeta(t) = (e^2/2m\hbar c^2)\int^t A^2(t)dt$, and we have normalized $\chi_{\boldsymbol{k}}$ to a δ function in momentum.

Let us now turn to the Schrödinger equation describing the target atom dressed by the laser field,

$$i\hbar \frac{\partial}{\partial t}\Phi(\boldsymbol{r}_1,t) = H_T \Phi(\boldsymbol{r}_1,t). \tag{10}$$

Under the assumption that the intensity and the frequency are constant or vary adiabatically, this equation can be transformed into a time-independent problem by making the Floquet ansatz, namely, by writing the wave function $\Phi(\boldsymbol{r}_1,t)$ as

$$\Phi(\boldsymbol{r}_1,t) = e^{-iEt/\hbar - i\zeta(t)} \sum_{M=-\infty}^{+\infty} e^{-iM\omega t} \mathcal{F}_M(\boldsymbol{r}_1). \tag{11}$$

The harmonic components $\mathcal{F}_M(\boldsymbol{r})$ are solutions of the system of equations

$$\begin{aligned}(E + M\hbar\omega - H_0)\mathcal{F}_M = {} & e^{-i(\phi-\pi/2)} V_+ \mathcal{F}_{M-1} \\ & + e^{i(\phi-\pi/2)} V_- \mathcal{F}_{M+1},\end{aligned} \tag{12}$$

where $H_0 = p_1^2/2m - e^2/r_1$ is the target Hamiltonian in the absence of the laser field, and

$$V_\pm = \mp \frac{e\hbar}{2mc} \boldsymbol{A}_0 \cdot \boldsymbol{\nabla}_1. \tag{13}$$

It is worth noting that the dependence on the phase ϕ can be eliminated easily from the system (12), by introducing the functions

$$\mathcal{G}_M(\boldsymbol{r}_1) = e^{iM(\phi-\pi/2)} \mathcal{F}_M(\boldsymbol{r}_1), \tag{14}$$

which satisfy the system

$$(E + M\hbar\omega - H_0)\mathcal{G}_M = V_+ \mathcal{G}_{M-1} + V_- \mathcal{G}_{M+1}. \tag{15}$$

Since the atom is initially in a bound state, its physical wave function must have a pure outgoing (+) wave behavior in the open channels. Accordingly, the harmonic components $\mathcal{F}_M(\boldsymbol{r})$ must be regular at the origin and satisfy the Siegert boundary conditions

$$\mathcal{F}_M(\boldsymbol{r}_1) \underset{r\to\infty}{\sim} \sum_{M'} f_{M'M}(\hat{\boldsymbol{r}}_1) r_1^{i\gamma_{M'}} \frac{e^{ik_{M'}r_1}}{r_1} \tag{16}$$

with $\gamma_{M'} = me^2/\hbar^2 k_{M'}$ and

$$k_{M'} = \left[\frac{2m}{\hbar^2}(E + M'\hbar\omega)\right]^{1/2}. \tag{17}$$

The branch of the square-root function in Eq. (17) is chosen such that the exponential function is decreasing at large distance in the closed channels, and increasing in the open channels, with an outgoing wave behavior. The quasienergy E is, therefore, a complex eigenvalue of the system of Eqs. (12):

$$E = E_i + \Delta - i\Gamma/2, \tag{18}$$

E_i being the eigenenergy of the initial (unperturbed) state of the target, Δ its dynamical Stark shift, and $\Gamma/\hbar$ its total rate of multiphoton ionization.

The boundary conditions are implemented implicitly in our calculations, by expanding the harmonic components on a discrete basis set consisting of spherical harmonics $Y_{\ell m}(\hat{\boldsymbol{r}}_1)$ and complex radial Sturmian functions $S^\kappa_{n\ell}(r_1)$ [29–31]. The latter are given by

$$\begin{aligned}S^\kappa_{n\ell}(r_1) = {} & \mathcal{N}^\kappa_{n\ell}\, (-2i\kappa r)^{\ell+1}\, e^{i\kappa r} \\ & \times {}_1F_1(\ell + 1 - n; 2\ell + 2; -2i\kappa r),\end{aligned} \tag{19}$$

where $\mathcal{N}^\kappa_{n\ell}$ is a normalization constant. The confluent hypergeometric function ${}_1F_1$ can be reduced to an associated Laguerre polynomial. The complex parameter κ is chosen to lie in the first (upper right) quadrant of the complex plane; hence the Sturmian functions oscillate and decrease exponentially at large distance. We choose the axis of quantization of the angular momentum along the polarization direction. Then only the spherical harmonics with the same value of the magnetic quantum number m_i as the initial state need to be retained in the expansion (the quasienergy depends on the value of $|m_i|$), and we write

$$\begin{aligned}\Phi^{(+)}(\boldsymbol{r}_1,t) = {} & e^{-iEt/\hbar - i\zeta(t)} \sum_{M=-\infty}^{+\infty} e^{-iM\omega t} e^{-iM(\phi-\pi/2)} \\ & \times \sum_{n\ell} c^{(M)}_{n\ell} r_1^{-1} S^\kappa_{n\ell}(r_1) Y_{\ell m_i}(\hat{\boldsymbol{r}}_1).\end{aligned} \tag{20}$$

In numerical calculations, the expansion on M is truncated to a finite number of terms, and the coefficients $c^{(M)}_{n\ell}$ are obtained by solving the linear eigenvalue system representing the coupled Eqs. (12) on the basis [28,27]. The relevant eigensolution of the system of Eqs. (12) is the one whose harmonic component with photon index $M = 0$ reduces either *adiabatically* or *diabatically* to the initial unperturbed state in the zero field limit (e.g., [32]):

$$\begin{aligned}\Phi^{(+)}(\boldsymbol{r}_1,t) \;&\sim\; e^{-iE_i t/\hbar} \Phi^{(0)}_{n_i \ell_i m_i} \\ &=\; e^{-iE_i t/\hbar} R_{n_i \ell_i}(r_1) Y_{\ell_i m_i}(\hat{\boldsymbol{r}}_1).\end{aligned} \tag{21}$$

In writing this last equation we should exclude the case where the target is a hydrogenic system initially in a state other than $1s$, $2s$, $2p$, or $3p$, for otherwise the Floquet state would reduce to a *superposition* of unperturbed states belonging to the same n_i manifold and of the same

parity but with different values of ℓ. We assume also that the atom stays, in good approximation, in a *single* Floquet state as the intensity varies during the passage of the laser pulse. Either the intensity increases slowly and the atom remains on the adiabatic quasienergy curve which develops from the unperturbed energy, or the intensity increases rapidly enough (on the time scale set by the largest relevant Rabi frequency) that the atom remains on a single diabatic quasienergy curve. It is not easy to extend the theory to cases where the atom is in a superposition of several Floquet states, as a result of very fast intensity variations or incomplete population transfer at avoided crossings, because of the difficulty to calculate the weight of each state in the superposition for such inherently time-dependent processes. We do not consider here the case where the atom is in an incoherent mixture of different states, as could be produced by spontaneous decay.

Besides the Floquet wave function with outgoing wave behavior, we also need the wave function with ingoing wave behavior, $\Phi^{(-)}$. Using these Floquet wave functions, the S-matrix element for direct "elastic" scattering in the presence of the laser field is given in first Born approximation by

$$S_{\rm el}^{\rm B1,F} = -\frac{i}{\hbar}\int_{-\infty}^{+\infty} dt \langle \chi_{\boldsymbol{k}_{\rm f}}(\boldsymbol{r}_0,t)\Phi^{(-)}(\boldsymbol{r}_1,t)|V_d \times|\chi_{\boldsymbol{k}_{\rm i}}(\boldsymbol{r}_0,t)\Phi^{(+)}(\boldsymbol{r}_1,t)\rangle, \tag{22}$$

where the initial and final states of the system (distorted by the electromagnetic field) are coupled to first order by the projectile-target interaction. Once complex conjugated, the harmonic components $\mathcal{G}_M$ corresponding to a quasienergy E satisfy the Floquet Eq. (14) with a quasienergy E^*. Their asymptotic behavior is to decrease exponentially in the closed channels and to increase exponentially in the open channels with an ingoing wave behavior. Therefore, the Floquet wave function is simply given by

$$\Phi^{(-)}(\boldsymbol{r}_1,t) = e^{-iE^*t/\hbar - i\zeta(t)} \sum_{M=-\infty}^{+\infty} e^{-iM\omega t} e^{-iM(\phi-\pi/2)} \times \sum_{n\ell} c_{n\ell}^{(M)*} r_1^{-1}[S_{n\ell}^{\kappa}(r_1)]^* Y_{\ell m_{\rm i}}(\widehat{\boldsymbol{r}}_1). \tag{23}$$

The spherical harmonics are not complex conjugated, so that $\Phi^{(-)}(\boldsymbol{r}_1,t)$ has the same zero-field limit (21) as $\Phi^{(+)}(\boldsymbol{r}_1,t)$—provided the radial wave function $R_{n_{\rm i}\ell_{\rm i}}(r_1)$ is real; we are free to not complex conjugate the spherical harmonics, since their azimuthal phase factor can be factored out of Eq. (12) in the case considered. The physical interpretation of the asymptotic behavior is the following. Near the nucleus, $|\Phi^{(-)}|^2$ increases exponentially in time with a rate $\Gamma/\hbar$. Therefore, the flux of electron density in the direction of the nucleus, at a fixed radius r, must also increase exponentially in time. This is possible only if at any given fixed time the electron density increases exponentially in r in the open channels, since the mean velocity of an electron flowing towards the nucleus is constant at large distance. The radial integral over r_1 in Eq. (22) must be defined as the analytic continuation of a divergent Riemann integral, owing to the exponential increase of the harmonic components at large distance. This is achieved by making use of the expansions (20) and (23), to express the matrix element of V_d as a double sum of well-defined integrals (e.g., [29]).

It is interesting to note that the product $\Phi^{(-)*}(\boldsymbol{r}_1,t)\,\Phi^{(+)}(\boldsymbol{r}_1,t)$ is a periodic function of t, and, in particular, that the cycle average of $\langle\Phi^{(-)}|\Phi^{(+)}\rangle$,

$$\frac{\omega}{2\pi}\int_0^{2\pi/\omega} dt\,\langle\Phi^{(-)}|\Phi^{(+)}\rangle = \sum_{M=-\infty}^{+\infty}\sum_{nn'\ell} c_{n'\ell}^{(M)} c_{n\ell}^{(M)} \int_0^{\infty} dr\; S_{n'\ell}^{\kappa}(r) S_{n\ell}^{\kappa}(r) \tag{24}$$

is constant in time. Therefore, by normalizing the Floquet wave function according to

$$\frac{\omega}{2\pi}\int_0^{2\pi/\omega} dt\langle\Phi^{(-)}|\Phi^{(+)}\rangle = \int_0^{\infty} dr\, r^2 R_{n_{\rm i}\ell_{\rm i}}^2(r) = 1, \tag{25}$$

we ensure that the norm of the initial field-free state is preserved at any intensity—recall that the atom is assumed to stay in a single Floquet state. The Floquet state with ingoing wave behavior and the normalization of Floquet states are also discussed by Pont *et al.* [33], within the time-reversal operator formalism.

The time integration in Eq. (22) is readily performed and leads to an energy-conservation δ function, even though the atomic energy level is broadened by the field. The case of "nonelastic" collisions is different in this respect, since only for "elastic" collisions the exponential increase in time of the ingoing Floquet wave function of the final state and the exponential decrease in time of the outgoing Floquet wave function of the initial state exactly compensate one another in the integrand of Eq. (22). In the case of "nonelastic" scattering an energy-conservation δ function is still obtained if the difference between the energy widths of the initial state and of the final state can be neglected. (This simplification can be made when their lifetimes in the field are longer than the duration of the projectile electron wave packets.)

Working from now on in atomic units (a.u.), and making use of the formula

$$\exp[-i\boldsymbol{K}\cdot\boldsymbol{\alpha}_0\sin(\omega t+\phi)] = \sum_{p=-\infty}^{+\infty} e^{-ip(\omega t+\phi)} J_p(\boldsymbol{K}\cdot\boldsymbol{\alpha}_0), \tag{26}$$

we have

$$S_{\rm el}^{\rm B1,F} = \frac{i}{2\pi}\sum_{N=-\infty}^{+\infty}\delta(E_{\boldsymbol{k}_{\rm f}} - E_{\boldsymbol{k}_{\rm i}} - N\omega) f_{{\rm el},N}^{\rm B1,F}, \tag{27}$$

where $f_{{\rm el},N}^{\rm B1,F}$, the first Born approximation to the "elastic" scattering amplitude with the transfer of $|N|$ photons, is given by

$$f_{{\rm el},N}^{\rm B1,F} = e^{-iN\phi}\sum_{M,M'=-\infty}^{+\infty} i^{M-M'} J_{N+M'-M}(\boldsymbol{K}\cdot\boldsymbol{\alpha}_0) \times \sum_{n'\ell'}\sum_{n\ell} c_{n'\ell'}^{(M')} c_{n\ell}^{(M)} f_{n'\ell' m_{\rm i}, n\ell m_{\rm i}}^{\rm B1}. \tag{28}$$

In the above equation, $\boldsymbol{K} = \boldsymbol{k}_\mathrm{i} - \boldsymbol{k}_\mathrm{f}$ is the momentum transfer and

$$f^{\mathrm{B1}}_{n'\ell'm_\mathrm{i},n\ell m_\mathrm{i}} = -\frac{2}{K^2}\int d\boldsymbol{r}\, r^{-2} S^{\kappa}_{n'\ell'}(r) Y^{*}_{\ell' m_\mathrm{i}}(\hat{\boldsymbol{r}}) \times[\exp(i\boldsymbol{K}\cdot\boldsymbol{r}) - 1] S^{\kappa}_{n\ell}(r) Y_{\ell m_\mathrm{i}}(\hat{\boldsymbol{r}}). \tag{29}$$

We remark that since we are dealing with an elastic transition and because of our choice of quantization axis parallel to the polarization direction of the field the magnetic quantum number m_i of the target state must be unchanged.

The angular part of the integral (29) is performed by expanding $\exp(i\boldsymbol{K}\cdot\boldsymbol{r})$ in partial waves. This gives rise to radial integrals of a product of two associated Laguerre polynomials, an exponential, an integral power of r, and a spherical Bessel function $j_\lambda(Kr)$, which are delicate to evaluate. They can be calculated by expanding the spherical Bessel function either in a power series in Kr (when the momentum transfer is small, roughly $K < 0.5$ a.u.) or in a power series in $(Kr)^{-1}$, and using recurrence relations for the resulting integrals [34].

As expected from the discussion in the Introduction, the Born-Floquet differential cross section,

$$\frac{d\sigma^{\mathrm{B1,F}}}{d\Omega_{\mathrm{el},N}}(\boldsymbol{K},\boldsymbol{\alpha}_0) = \frac{k_\mathrm{f}}{k_\mathrm{i}} |f^{\mathrm{B1,F}}_{\mathrm{el},N}|^2, \tag{30}$$

does not depend on the phase ϕ of the laser field. The ponderomotive acceleration the projectile experiences when penetrating and leaving the laser beam should be taken into account when relating this differential cross section to experimental data [35], as well as the spatio-temporal distributions of laser intensity, of nonionized target atoms and of ions created through multiphoton ionization. Ponderomotive acceleration tends to deflect the electrons away from the regions where the intensity is large, which may make it difficult to measure accurately cross sections for scattering in very intense fields, unless the pulses are very short. This classical effect originates from the macroscopic spatial variations of $|\boldsymbol{A}_0|$, and is distinct from the quantum microscopic problem we focus on.

Before coming to the description of our numerical results, let us comment briefly on two points. First, our Born-Floquet scattering amplitude (28) is gauge invariant. Under a gauge transformation of the electromagnetic field, the wave functions $\chi_{\boldsymbol{k}_\mathrm{i}}(\boldsymbol{r}_0,t)$ and $\chi_{\boldsymbol{k}_\mathrm{f}}(\boldsymbol{r}_0,t)$ are simply multiplied by a phase factor $\exp[i\gamma(\boldsymbol{r}_0,t)]$, and the Floquet wave funtions $\Phi^{(+)}(\boldsymbol{r}_1,t)$ and $\Phi^{(-)}(\boldsymbol{r}_1,t)$ by a phase factor $\exp[i\gamma(\boldsymbol{r}_1,t)]$; the phases in the bra vector and those in the ket vector of Eq. (22) cancel each other. However, the choice of gauge is important as far as computations are concerned: the velocity gauge is more appropriate for performing Floquet calculations on a basis of Sturmian functions [27]. There is no gauge invariance if approximations are made in the Floquet wave functions, for example, if too few harmonic components or partial waves are included in the numerical calculations. [The energy scale may shift under a gauge transformation. In particular, the quasienergy is larger by a quantity $E_\mathrm{pond} = e^2\mathcal{E}_0^2/4m\omega^2$ when calculated in the $\boldsymbol{\mathcal{E}}\cdot\boldsymbol{r}$ gauge rather than by solving the system (12); $E_\mathrm{pond}t$ is the secular component of $\zeta(t)$. Of course, this energy shift has no effect on the cross sections.]

The second point is that the Born-Floquet scattering amplitude for "elastic" scattering from the ground state does correctly reduce for electric field strengths $\mathcal{E}_0 \ll e/a_0^2$ to the semiperturbative scattering amplitude of Francken and Joachain [9] (that is, the scattering amplitude in Born approximation when the target is dressed only within the first-order in perturbation theory, in the $\boldsymbol{E}\cdot\boldsymbol{r}$ gauge). Upon solving the system (12) in first-order perturbation theory, the scattering amplitude (28) becomes, for $\phi = 0$,

$$f^{\mathrm{B1,1PT}}_{\mathrm{el},N} = J_N(\boldsymbol{K}\cdot\boldsymbol{\alpha}_0) f^{\mathrm{B1}}_{1s,1s} + \frac{i}{2c}\sum_n \left[\frac{J_{N-1}(\boldsymbol{K}\cdot\boldsymbol{\alpha}_0)}{\omega_{np,1s}-\omega} + \frac{J_{N+1}(\boldsymbol{K}\cdot\boldsymbol{\alpha}_0)}{\omega_{np,1s}+\omega}\right] f^{\mathrm{B1}}_{1s,np}\langle np|\boldsymbol{A}_0\cdot\boldsymbol{\nabla}|1s\rangle - \frac{i}{2c}\sum_n \left[\frac{J_{N+1}(\boldsymbol{K}\cdot\boldsymbol{\alpha}_0)}{\omega_{np,1s}-\omega} + \frac{J_{N-1}(\boldsymbol{K}\cdot\boldsymbol{\alpha}_0)}{\omega_{np,1s}+\omega}\right] \langle np|\boldsymbol{A}_0\cdot\boldsymbol{\nabla}|1s\rangle^*\, f^{\mathrm{B1}}_{np,1s}, \tag{31}$$

where the sums extend over all the spectrum of states of p symmetry (including the continuum), and the scattering amplitudes $f^{\mathrm{B1}}_{1s,1s}$, $f^{\mathrm{B1}}_{1s,np}$, and $f^{\mathrm{B1}}_{np,1s}$ are the field-free first Born amplitudes for the $1s \to 1s$, $1s \to np$, and $np \to 1s$ transitions, and $\omega_{np,1s} = E_{np} - E_{1s}$. (The symbol n denotes the principal quantum number of the unperturbed atomic states that act as intermediate states, in this paragraph.) The terms with $|M| + |M'| > 1$ are neglected when defining $f^{\mathrm{B1,1PT}}_{\mathrm{el},N}$, since they are of higher order in $\mathcal{E}_0$. By using the relations

$$\langle np|\boldsymbol{A}_0\cdot\boldsymbol{\nabla}|1s\rangle^* f^{\mathrm{B1}}_{np,1s} = \langle np|\boldsymbol{A}_0\cdot\boldsymbol{\nabla}|1s\rangle f^{\mathrm{B1}}_{1s,np} \tag{32}$$

and

$$\langle np|\boldsymbol{A}_0\cdot\boldsymbol{\nabla}|1s\rangle = -\frac{c}{\omega}\,\omega_{np,1s}\,\langle np|\boldsymbol{\mathcal{E}}_0\cdot\boldsymbol{r}|1s\rangle, \tag{33}$$

Eq. (31) can be reduced to Eq. (9) of Ref. [9] :

$$f^{\mathrm{B1,1PT}}_{\mathrm{el},N} = J_N(\boldsymbol{K}\cdot\boldsymbol{\alpha}_0) f^{\mathrm{B1}}_{1s,1s} - 2iJ'_N(\boldsymbol{K}\cdot\boldsymbol{\alpha}_0)\sum_n \frac{\omega_{np,1s}}{\omega^2_{np,1s}-\omega^2} \times f^{\mathrm{B1}}_{1s,np}\langle np|\boldsymbol{\mathcal{E}}_0\cdot\boldsymbol{r}|1s\rangle. \tag{34}$$

Byron and Joachain [6] introduced a low frequency approximation to $f^{\mathrm{B1,1PT}}_{\mathrm{el},N}$, namely,

$$f^{\mathrm{B1,BJ}}_{\mathrm{el},N} = J_N(\boldsymbol{K}\cdot\boldsymbol{\alpha}_0) f^{\mathrm{B1}}_{1s,1s} - 2iJ'_N(\boldsymbol{K}\cdot\boldsymbol{\alpha}_0)\sum_n \frac{1}{\omega_{np,1s}} f^{\mathrm{B1}}_{1s,np}\langle np|\boldsymbol{\mathcal{E}}_0\cdot\boldsymbol{r}|1s\rangle. \tag{35}$$

The scattering amplitude takes on the particularly simple form

$$f_{\mathrm{el},N}^{\mathrm{B1,ND}} = J_N(\boldsymbol{K}\cdot\boldsymbol{\alpha}_0) f_{1s,1s}^{\mathrm{B1}} \tag{36}$$

when the dressing of the target is completely neglected; $f_{\mathrm{el},N}^{\mathrm{B1,ND}}$ is the Bunkin-Fedorov [3] amplitude for scattering by the static potential of the target.

The Sturmian method is convenient to calculate the semiperturbative amplitude $f_{\mathrm{el},N}^{\mathrm{B1,1PT}}$ to arbitrary accuracy, directly from Eq. (28) with the sums over M and M' truncated as explained above, without need to perform the cumbersome sum over unperturbed eigenstates on the right-hand side of Eq. (34). In perturbation theory, $\mathcal{F}_0(\boldsymbol{r}_1)$ is identical to the unperturbed wave function of the initial state $\Phi^{(0)}_{n_i\ell_i m_i}(\boldsymbol{r}_1)$, and $\mathcal{F}_1(\boldsymbol{r}_1)$ and $\mathcal{F}_{-1}(\boldsymbol{r}_1)$ are obtained by solving the equations

$$(E_i \pm \hbar\omega - H_0)\mathcal{F}_{\pm 1} = e^{\mp i(\phi-\pi/2)} V_{\pm} \Phi^{(0)}_{n_i\ell_i m_i} \tag{37}$$

on the basis set.

III. RESULTS AND DISCUSSION

The differential cross section for "elastic" scattering of electrons from hydrogen assisted by a laser field of 620 nm wavelength is displayed in Fig. 1 as a function of the scattering angle, for a fixed laser intensity of 1.4×10^{13} W/cm^2 corresponding to a field strength $\mathcal{E}_0 = 0.02$ a.u. These results have been obtained in linear polarization and for an incident electron energy of 500 eV, like all other results presented in this paper; a first-order treatment of the projectile-target potential V_d is expected to be adequate at this large energy. At 620 nm, the cross sections obtained by using the Byron and Joachain scattering amplitude of Eq. (35) are very close to those obtained by using the full semiperturbative scattering amplitude (34) (they are within less than 1%). Taking only decay through multiphoton ionization into account, the half life of the atom is 0.8 ns at 1.4×10^{13} W/cm^2, the corresponding field-induced width of the ground-state energy being 2.2×10^{-8} a.u. The rate of multiphoton ionization departs significantly from perturbation theory (the induced width is 3.6×10^{-6} a.u. at this intensity, when calculated in perturbation theory).

Results for two different geometries are shown in Fig. 1. Let us begin by the geometry in which the polarization direction is kept parallel to the momentum transfer, and thus changes with N and with the scattering angle. As is well known, both projectile dressing and target dressing are important in this case, and this is confirmed by the results of Fig. 1. The most interesting conclusion that can be drawn from the figure, however, is that the nonperturbative Born-Floquet cross sections (F, solid curves) are very close to the semiperturbative results (1PT, dashed curves). The same is true for $N = 3$, 4, and 5 also. This is in marked contrast with the nonperturbative character that other multiphoton processes, such as multiphoton ionization, have at this intensity. The physical reason behind the success of the semiperturbative theory for the case of Fig. 1 seems clear, intuitively: while dressing is important to all orders in the field for the projectile electron, dressing for the target electron can be taken as a perturbation in the present case, since the laser electric field is much weaker than the Coulomb binding field for a laser that is not superintense. The harmonic components of the target wave function which contribute most to the differential cross sections are, therefore, those with photon number M close to zero and are also the ones that are best described by perturbation theory, even at large intensity. The harmonic components with large photon number, which contribute to the rate for multiphoton ionization rate, are more sensitive to nonperturbative effects.

Table I reveals, however, that this simple physical argument does not explain the *close* agreement between the Born-Floquet results and the semiperturbative results.

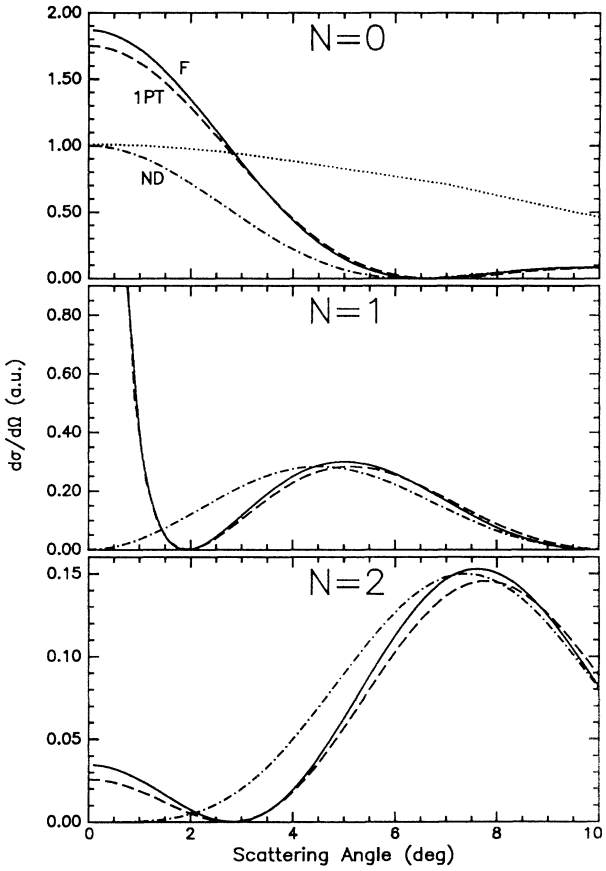

FIG. 1. The differential cross section (in atomic units) for elastic electron-H(1s) scattering in the presence of a linearly polarized laser field as a function of the scattering angle (in deg). The laser photon energy is 2 eV, the electric field strength is 0.02 a.u., the incident energy is 500 eV, and the polarization vector is parallel to the momentum transfer. N is the net number of photons absorbed by the projectile during the collision. Results are shown for $N = 0$, 1, and 2. Full curve (F): results when the target is dressed nonperturbatively in the Floquet approach [Eq. (28)]; broken curve (1PT): results when the target is dressed in first-order perturbation theory [Eq. (34)]; dot dashed curve (ND): results obtained by neglecting the dressing of the target [Eq. (36)]. The dotted curve represents the results with Floquet target dressing for the case where the laser beam is polarized parallel to the incident electron beam.

TABLE I. Differential cross section (in atomic units) for $\hbar\omega = 2$ eV, $\mathcal{E}_0 = 0.02$ a.u., $E_{k_i} = 500$ eV, and two different scattering angles. The polarization is parallel to the momentum transfer. μ is the maximum value of $|M| + |M'|$ (see text); the letter u indicates that the summation over M and M' is unrestricted. F: results calculated with a nonperturbative target wave function obtained by solving the Floquet system (12) truncated to $-5 \leq M \leq 12$ and $0 \leq \ell \leq 7$; F(0,1): results calculated with the same Floquet wave function, but ℓ and ℓ' were restricted to be 0 or 1 in Eq. (28); PT: results calculated with a target wave function obtained in first-order perturbation theory; PT(n): PT results normalized as is explained in the text. The numbers in brackets indicate powers of ten.

N	μ	$\theta = 0.5°$				$\theta = 8.0°$			
		PT	PT(n)	F(0,1)	F	PT	PT(n)	F(0,1)	F
0	0	9.79[−1]	9.04[−1]	7.97[−1]	6.91[−1]	5.54[−2]	5.12[−2]	4.65[−2]	4.22[−2]
	1	1.77	1.64	1.17	1.02	4.02[−2]	3.71[−2]	3.87[−2]	3.48[−2]
	2				1.35				4.58[−2]
	3				1.74				3.93[−2]
	4				1.79				4.24[−2]
	u				1.83				4.26[−2]
1	0	9.98[−3]	9.22[−3]	8.13[−3]	7.04[−3]	5.13[−2]	4.74[−2]	4.32[−2]	3.91[−2]
	1	2.50	2.31	6.97[−1]	6.35[−1]	7.50[−2]	6.95[−2]	5.44[−2]	4.99[−2]
	2				6.11[−1]				6.29[−2]
	3				2.39				7.38[−2]
	4				2.38				7.55[−2]
	u				2.59				7.61[−2]
2	0	3.35[−5]	3.09[−5]	2.72[−5]	2.36[−5]	1.45[−1]	1.34[−1]	1.22[−1]	1.11[−1]
	1	2.70[−2]	2.50[−2]	7.82[−3]	7.11[−3]	1.47[−1]	1.36[−1]	1.23[−1]	1.12[−1]
	2				6.59[−3]				1.44[−1]
	3				2.43[−2]				1.45[−1]
	4				2.74[−2]				1.50[−2]
	u				3.20[−2]				1.50[−2]
3	0	7.79[−8]	7.20[−8]	6.34[−8]	5.50[−8]	6.84[−2]	6.32[−2]	5.75[−2]	5.21[−2]
	1	9.71[−5]	8.97[−5]	2.84[−5]	2.58[−5]	5.83[−2]	5.39[−2]	5.24[−2]	4.73[−2]
	2				2.27[−5]				6.22[−2]
	3				9.50[−4]				5.79[−2]
	4				1.02[−3]				5.90[−2]
	u				4.12[−5]				5.87[−2]

All the entries in Table I have been calculated by using Eq. (28), the sum over the photon numbers M and M' being either unrestricted or restricted to the terms such that $|M| + |M'| \leq \mu$. Four different sets of coefficients $c^{(M)}_{n\ell}$ were used, as explained in the table caption. The PT results for $\mu = 1$ are nothing else than the semiperturbative (1PT) cross sections; they are based on a target wave function calculated in first-order perturbation theory. The PT(n) correspond to a set of coefficients $c^{(M)}_{n\ell \mathrm{PT(n)}}$ obtained by normalizing this first-order perturbative wave function following the same prescription as for the Floquet wave function, namely,

$$\sum_{M=-1}^{+1} \sum_{nn'\ell} c^{(M)}_{n'\ell \mathrm{PT(n)}} c^{(M)}_{n\ell \mathrm{PT(n)}} \int_0^\infty dr\, S^\kappa_{n'\ell}(r) S^\kappa_{n\ell}(r) = 1. \tag{38}$$

Because of the normalization, the PT(n) results are smaller than the PT cross section, by a constant factor 0.92 . It is particularly interesting to compare the PT(n) results and the F(0,1) results. The wave functions they are based on are expected to be very similar since $\mathcal{E}_0$ is much smaller than one atomic unit. Their similarity can be measured by evaluating the distance $\mathcal{D}$ between the corresponding sets of Sturmian coefficients $c^{(M)}_{n\ell}$, as follows:

$$\mathcal{D} = \frac{\sum_n |c^{(M)}_{n\ell \mathrm{PT(n)}} - c^{(M)}_{n\ell \mathrm{F}(0,1)}|}{\sum_n |c^{(M)}_{n\ell \mathrm{F}(0,1)}|}. \tag{39}$$

This measure is meaningful only if the same set of Sturmian functions are used to represent both wave functions. The difference between the (normalized) perturbative wave function and the nonperturbative Floquet wave function is indeed very small: $\mathcal{D}$ is only 1% for $(M = 0, \ell = 0)$, and 0.8% for $(M = \pm 1, \ell = 1)$. Considering that these wave functions are so close, it is surprising that the corresponding cross sections sometimes differ appreciably, as happens for example at $\theta = 0.5°$. A more detailed analysis shows that the small differences in the wave functions are amplified by complicated cancellations into much larger differences in the cross sections. The full Born-Floquet results (F) for $\mu = 0$ and $\mu = 1$ are not identical to the F(0,1) cross sections, since ℓ is not restricted to 0 or 1 in the full Born-Floquet calcula-

tions. However, as more and more harmonic components are taken into account, i.e., as μ increases, the F cross sections converge towards a limit (the F results of Fig. 1) which happens to be rather close, in the present case, to the semiperturbative results. We conclude, therefore, that the general agreement between the Born-Floquet cross sections and the semiperturbative ones for $\mathcal{E}_0 \ll 1$ a.u. can be understood by the simple argument developed above, although a detailed understanding of this agreement requires the consideration of the normalization of the wave functions and of cancellations occurring in the contributions of harmonic components with $|M| > 1$ which are not included in the semiperturbative theory.

In Fig. 1 is also displayed the cross section for transfer of zero photons ($N = 0$) for the geometry in which the polarization direction is kept along $\boldsymbol{k}_i$. In contrast with the previous case, this cross section (shown as a dotted line) cannot be distinguished from the field-free cross section on the scale of the figure. The argument of the Bessel functions of Eq. (28) is nearly zero in the present geometry, since the momentum transfer is small and the polarization direction is nearly perpendicular to $\boldsymbol{K}$ for small scattering angles. Therefore, the values of the Bessel functions are either very close to 0 (when $M \neq M'$) or very close to 1 (when $M = M'$), and projectile dressing has no effect on the "elastic" cross section. If one performs an analysis similar to the one of Table I one can see that the Born-Floquet cross section is dominated by the contribution of the s wave of the harmonic component with $M = 0$, although the other partial waves and the harmonic components with $M \neq 0$ play a non-negligible role in the remarkable agreement with the field-free cross section. For example, the differential cross section which is obtained without taking them into account is 18% smaller, for scattering at 0.5°, than the full Born-Floquet result. A likely explanation of the near absence of any physical effect arising from the dressing of the target is that the laser field does not modify much the distribution of momentum of the atomic electron in any direction normal to the electric field vector (we are far from any resonance), so that transfer of momentum to the projectile perpendicularly to the polarization direction occurs as in the absence of field—at least in Born approximation. It is not difficult to show that $f_{\mathrm{el},N}^{\mathrm{B1,F}}$ has the same large K limit as the field-free elastic scattering amplitude, for $N = 0$ with $\boldsymbol{K} \perp \boldsymbol{\mathcal{E}}_0$, when the Floquet wave function is normalized according to Eq. (25).

The differential cross section for scattering at 0.5° is shown in Fig. 2 for a field of 620 nm wavelength and varying strength. At a field strength of 0.0377 a.u., corresponding to an intensity of 5.0×10^{13} W/cm^2, multiphoton ionization is fully nonperturbative [36]. The half life of the atom is 3.2 ps. Despite these extreme conditions, the differential cross section calculated with nonperturbative Floquet target dressing, on the one hand, and the cross section calculated with target dressing taken into account to first order, on the other hand, are in good agreement. This illustrates, once again, how different the importance of certain harmonic components is in the scattering case compared to the multiphoton ionization case. Actually, one can neglect in Eq. (11) the harmonic components with photon index M such that $\mathrm{Re}(E + M\hbar\omega) > 0$, without any significant effect on the cross sections of Fig. 2, although the multiphoton ionization rate would vanish in this approximation. It is clear that the open-channel part of the Floquet wave function does not contribute much to the cross section, at least in the absence of resonances, even in cases where the total multiphoton ionization rate is large. In other words, the projectile is scattered essentially as if the atom were not decaying and the process could be described by a Hermitian Floquet theory. By contrast, it is important to retain a sufficiently large number of angular momentum components in the wave function; in the present case values of l up to 7 must be included in order to obtain cross sections converged to within 1%. It is also worth noting that the contribution of the *continuum* states of the field-free atom to the *closed*-channel part of the Floquet wave function is considerable. For example, in the case of Fig. 2 and for $\mathcal{E}_0 = 0.0377$ a.u., the semiperturbative differential cross sections for $N = 1$ or $N = 2$ are reduced by 35% when the continuum states are neglected in the sum over intermediate states of Eq. (34).

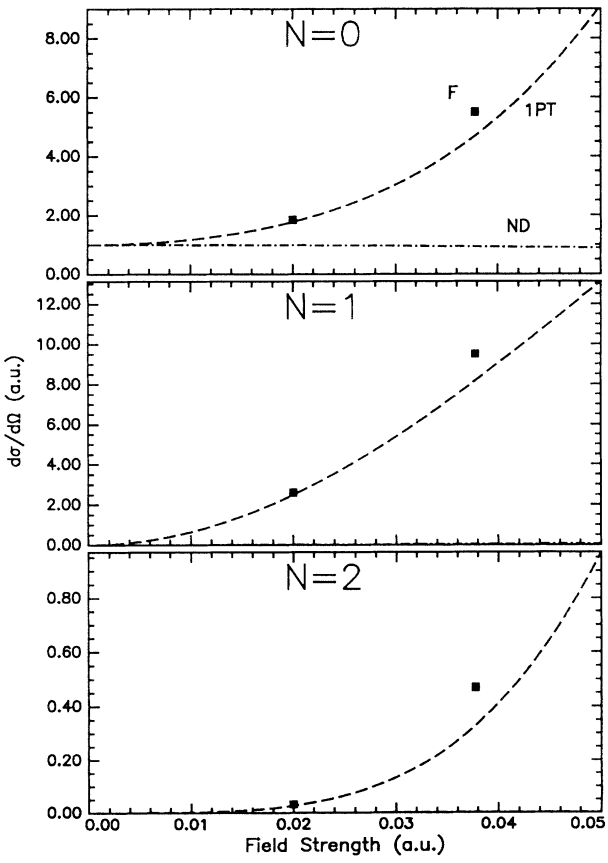

FIG. 2. Same as in Fig. 1, but for a fixed scattering angle (0.5°) and varying electric field strength (in atomic units). The results with Floquet dressing are represented by solid squares. The broken curve corresponds to results obtained when the target is dressed by using first-order perturbation theory (1PT). The dot dashed curve (ND) refers to no target dressing.

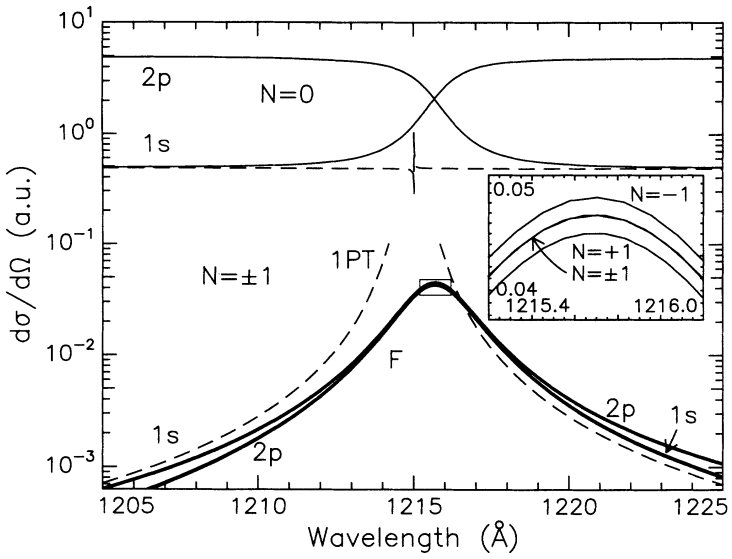

FIG. 3. The differential cross section (in atomic units) at a scattering angle of 10° for elastic electron-hydrogen scattering in the presence of a linearly polarized laser field of intensity 10^{10} W/cm^2, vs wavelength (in Å), in the vicinity of the one-photon $1s$-$2p$ resonance. The results with Floquet dressing are shown for scattering from the (adiabatic) $1s$ or $2p$ dressed states and for $N = 0, 1$, or -1; the results with perturbative dressing are shown only for scattering from the $1s$ state and $N = 0$ or 1. The inset is a magnification of the region delimited by the box.

The nonperturbative Floquet approach adopted in this work is particularly relevant when the field brings the initial state into resonance with another state. This is illustrated by Fig. 3. In the weak field limit, the $1s$ and the $2p$ states are in resonance through a one-photon dipole coupling at a wavelength of 121.5 nm. While the scattering cross section diverges at this wavelength when calculated in first-order perturbation theory, it remains finite when the coupling of these two states is taken into account nonperturbatively. Off resonance, at 122.5 nm the ionization widths of the $1s$ and $2p$ states are, respectively, 4.36×10^{-10} a.u. and 7.66×10^{-8} a.u. The ionization width of the $1s$ state is 2.8×10^{-8} a.u. at 121.5 nm. This last number should be compared to the natural width of the unperturbed $2p$ level, 1.5×10^{-8} a.u. We see that loss of coherence is not a cause of concern in this particular case, since photoionization is faster than spontaneous decay. Keeping the intensity constant and increasing the wavelength adiabatically, the Floquet state corresponding to the dressed $1s$ state below 121.5 nm loses its character as the resonance is passed, and takes on a $2p$ character; conversely, the dressed $2p$ state acquires the character of a dressed $1s$ state above 121.5 nm. The character interchange of the two states, which is associated with an avoided crossing in the real part of their quasienergies [37], manifests itself clearly in the cross section for elastic scattering without a net exchange of photons. (The crossing of the curves occurs actually at a slightly longer wavelength than in the zero-field limit, because of the shift and width of the dressed states.) On the other hand, the cross sections for scattering from the dressed $1s$ or from the dressed $2p$ state with a net exchange of 1 photon are also very close near resonance. The $|N| = 1$ Born-Floquet results actually consist of four curves, namely, two curves for $N = 1$ (one for each dressed state) and two curves for $N = -1$. The results for $N = \pm1$ are very close. At the crossing, two of the four curves osculate: this is why we see only three curves in the inset of the figure. The difference between the cases $N = 0$ and $|N| = 1$ is not difficult to understand once it is noted (i) that the cross section is dominated by the term with $M' - M = N$ in Eq. (28) at the very weak intensity and small momentum transfer considered; and (ii) that the $1s$ and $2p$ Floquet wave functions are essentially similar superpositions of the $1s$ and $2p$ unperturbed states near the resonance (though with quasienergies differing by $\hbar\omega$, and different relative phases between the harmonic components). As expected, and for both $N = 0$ and $|N| = 1$, it is only in the immediate vicinity of the resonance that there is a substantial difference between the results obtained with target dressing treated perturbatively and those obtained with nonperturbative target dressing.

Finally, differential cross sections in the neighborhood

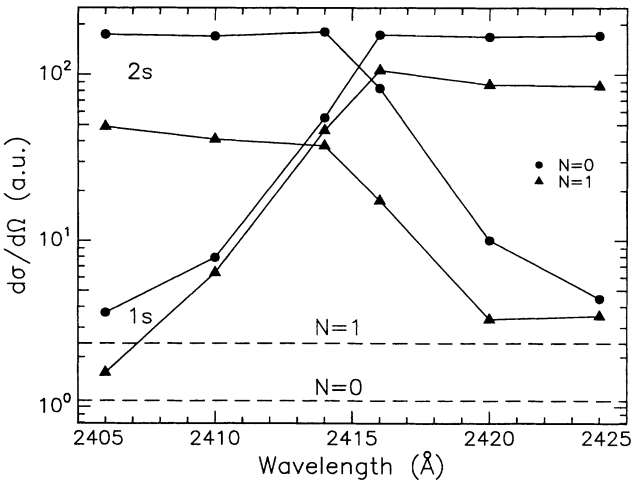

FIG. 4. The differential cross section (in atomic units) at a scattering angle of 0.5° for elastic electron-hydrogen scattering in the presence of a linearly polarized laser field of intensity 10^{13} W/cm^2, vs wavelength (in Å), in the vicinity of the two-photon $1s$-$2s$ resonance. The results with Floquet dressing are shown for scattering from the (adiabatic) $1s$ or $2s$ dressed states and for $N = 0$ (solid circles) or $N = 1$ (solid triangles). The solid lines are a guide for the eyes; the results with dressing in first-order perturbation theory are shown only for scattering from the $1s$ state.

of a two-photon resonance (between the $1s$ and the $2s$ states) are presented in Fig. 4. At an intensity of 10^{13} W/cm^2 and a wavelength of 242.4 nm, the widths of the $1s$ and $2s$ states are, respectively, 1.75×10^{-4} a.u. and 1.65×10^{-3} a.u. The structure of the Floquet wave functions of the two states is now more complicated than in the case of Fig. 3: there are significant differences, for $N = 1$ as well as for $N = 0$, between the results for scattering from the ground state and those for scattering from the resonant state. The results for $N = -1$ (not shown) are again very close to the ones for $N = +1$. Of course, no resonance structure is found when the atom is not dressed beyond first order in perturbation theory.

The variation of the cross section with the wavelength near a three-photon resonance between the initial $1s$ state and the $2p$ state has been described elsewhere [38]. In contrast with the previous cases, the crossing between the two quasienergy curves is a true crossing for this three-photon resonance—that is, the real parts of the quasienergies intersect. Nevertheless, the Born-Floquet cross section was found to be strongly enhanced at the resonance.

ACKNOWLEDGMENTS

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