Distorted-wave Born approximation calculations of the pair-production cross section for 6.0-MeV photons

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(Received 1 June 1993; revised manuscript received 19 November 1993)

We present in this communication the results of the distorted-wave Born approximation calculations of the electron-pair-production cross section by using the technique of Wright, Sud, and Kosik [Phys. Rev. C **36**, 562 (1987)] for 6.0-MeV photons on $Z=1$, 30, 50, 68, 82, and 92. The calculated cross sections are compared with the interpolated experimental data and the results obtained from the existing interpolating formulas.

PACS number(s): 32.80.−t, 25.20.Lj, 34.90.+q

I. INTRODUCTION

The pair-production cross section data are of considerable practical importance, as the pair production is a dominant mode of photon absorption in matter above a few MeV. The latest tabulations of Hubbell, Gimm, and Overbo [1] have been compiled by using the empirical Coulomb corrections to the pair-production cross section in the intermediate energy range 5–50 MeV, as the exact distorted-wave Born approximation (DWBA) results in this range were not available. The DWBA calculation of the electron-pair-production cross section in the point Coulomb field of the atomic nucleus has been performed by a number of workers [2–9]. The first DWBA calculation of Jaeger and Hulme [2] was done for only two photon energies and for a few elements. Overbo, Mork, and Olsen [3] have computed the DWBA pair-production cross sections for photon energies ranging from threshold to 5.0 MeV for a large number of atomic numbers. Overbo, Mork, and Olsen [3] obtained the analytic expression for the DWBA differential cross section by using the relativistic Coulomb wave functions for the leptons. In the DWBA formalism the radial integrals for the pair-production process can be expressed in terms of Appell's hypergeometric function F_2 (the explicit expression for the differential pair production cross section is given in Refs. [3] and [6]). Appell's hypergeometric functions F_2, which are required to compute the pair-production cross section for 5.0-MeV photons, are very slowly convergent series and their convergence further deteriorates for higher energies. This has restricted Overbo's technique for computing the pair-production cross section up to 5.0-MeV photons. The new impetus in the DWBA calculations was provided by the development of a technique [6–8] to evaluate the Dirac-Coulomb integrals. In this technique the radial integrals are obtained from the elements of the matrix Γ function. The recurrence relation satisfied by the matrix function is used to reduce the number of radial integrals required for the computation. The matrix Γ function also satisfies a differential equation in photon-energy-like parameters. The matrix differential equation is integrated to compute the accurate radial integrals for the pair-production cross section for photon energy above 5.0 MeV by using the accurate initial matrix Γ function. The initial matrix Γ function is obtained by using F_2 functions evaluated at a photon-energy parameter at which their accurate calculations are possible. We refer the reader to the work of Wright, Sud, and Soto [8] for an explicit expression for the differential pair-production cross section in terms of the elements of the matrix Γ function and for other calculational details. The technique has already been used for the computation of the pair-production cross sections at three different photon energies and two Z values (for photon energies of 10.0 and 20.0 MeV on U and of 7.5 MeV on Sn and U). We present in Sec. II the results of our DWBA calculations of pair-production cross sections for 6.0-MeV photons for a few atomic numbers ($Z=1$, 30, 50, 68, 82, and 92). We have compared the results of our calculations with the Bethe-Heitler [10], Overbo's [11], and Maximon-Gimm [12] interpolated results, and the interpolated "experimental" data.

II. RESULTS AND DISCUSSION

We present here the results of our DWBA calculations of the differential pair-production cross section in Table I by 6.0-MeV photons for different elements ($Z=1$, 30, 50, 68, 82, and 92). We are not describing here the details of the technique used by us for evaluating the Dirac-Coulomb integrals and the differential pair-production cross section, as it has been discussed in detail by Wright, Sud, and Kosik [8] and Sud and Soto Vargas [9]. However, we would like to mention that in the computational technique used by us the expression for the differential

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TABLE I. Differential pair-production cross section for different Z ($Z=1$, 30, 50, 68, 82, and 92) with 6.0-MeV photons for different positron energies (E_+); $d\sigma_{DW}/dE_+$ is the result of the present DWBA calculations and $d\sigma_{BH}/dE_+$ is the plane-wave Bethe-Heitler cross section.

E_+	$d\sigma_{DW}/dE_+$ (mb/MeV)						$d\sigma_{BH}/dE_+$
(MeV)	$Z=1$	$Z=30$	$Z=50$	$Z=68$	$Z=82$	$Z=92$	(mb/MeV)
0.52			0.6371×10^{-4}	0.1949×10^{-3}		0.2163×10^{-6}	0.02889
0.60	0.0926					0.07951	0.09301
0.62					0.0926		0.1033
0.65		0.08592					0.1173
1.00	0.2192	0.2127	0.2043	0.1917	0.1781	0.1677	0.2191
1.50	0.2856	0.2844	0.2809	0.2736	0.2608	0.2533	0.2854
2.00	0.3135	0.3130	0.3101	0.3034	0.2904	0.2853	0.3135
2.50	0.3246	0.3239	0.3213	0.3152	0.3017	0.2985	0.3247
3.00	0.3274	0.3268	0.3246	0.3194	0.3098	0.3042	0.3275
3.50	0.3246	0.3244	0.3231	0.3195	0.3079	0.3067	0.3247
4.0	0.3136	0.3145	0.3152	0.3142	0.3064	0.3058	0.3135
4.50	0.2857	0.2888	0.2935	0.2973	0.2960	0.2975	0.2854
5.0	0.2195	0.2281	0.2388	0.2518	0.2962	0.2700	0.2191
5.25	0.1558	0.1723	0.1874	0.2058	0.2230	0.2366	0.1551
5.4849						0.1784	0.01946

pair-production cross section has been expressed as a partial-wave expansion, and each term in it corresponds to a combination of lepton partial-wave quantum numbers κ_+ and κ_-. The differential pair-production cross-section expression is given as

$$\frac{d\sigma}{dE_+}=\sum_{q=1}^{\infty} T_q ,$$

where $q=|\kappa_+|+|\kappa_-|-1$ and T_q is the partial differential cross section (see Wright, Sud, and Kosik [8] for an explicit expression for T_q). We have computed T_q for $q=1-25$, 40, and 80, and intermediate values of q have been obtained by interpolating $\log(T_q)$ by spline interpolation. The contribution to $d\sigma/dE_+$, due to the noncomputed terms, is obtained by utilizing the fact that $\log(T_q)$ varies linearly with q.

We can express the differential pair-production cross section as

$$\frac{d\sigma}{dE_+}=\sum_{q=1}^{q_{\max}} T_q+R ,$$

where the remainder (R) is given as

$$R=T_{q_{\max}}\frac{e^a}{1-e^a}$$

and a is the slope of the T_q versus q curve at $q=q_{\max}$. The remainder in the present calculation is less than the calculations done by Wright, Sud, and Kosik [8] (for photon energy $w=10.0$ and 20.0 MeV) and Sud and Soto Vargas [9] ($w=7.5$ MeV). The remainder (R) is less than 1% of the sum of the terms up to $q=q_{\max}$ for calculations involving unevenly shared energy by positrons and electrons (i.e., for $E_+=0.6$, 1.0, 1.5, 4.5, 5.0, and 5.25 MeV) and less than 1.7% for other calculational points. We have estimated the uncertainty in our calculation to be less than 0.1% and have determined, by estimating the error in the remainder, by using the technique of Wright, Sud, and Kosik [8]. We have compared the results of our DWBA calculation ($Z=1$, 30, 50, 68, 82, and 92) with the plane-wave Born approximation results of Bethe and Heitler (see Table I). The Bethe-Heitler results for the differential pair-production cross section are symmetric about the evenly shared energy point (i.e., $E_+=E_-=3.0$ MeV), as expected. It can be seen from the Table I that the DWBA results for $Z=1$ are almost symmetric about the evenly shared energy point and are very close to the results obtained by the Bethe-Heitler [10] calculations. As expected, the effect of the Coulomb distortion (see columns 3–7 in Table I) is significant for higher Z values. We present in Table II the results for our total DWBA pair-production cross-section calculation, which has been obtained by numerically integrating the differential cross section by using the Gauss quadrature technique and spline interpolation for obtaining the intermediate base points. We have compared our results in Table II with

TABLE II. The total pair-production cross section for different Z values at 6.0-MeV photon energy in barns. The subscripts DWBA, expt, MG, OV, and BH correspond to the present DWBA result, interpolated experimental result, the Maximon-Gimm interpolation, the Overbo interpolation, and the Bethe-Heitler result.

Z	σ_{DWBA}	σ_{expt}	σ_{MG}	σ_{OV}	σ_{BH}
1	0.001 358		0.001 349	0.001 349	0.001 349
30	1.218		1.213	1.214	1.214
50	3.354	3.376 ±0.010	3.362	3.367	3.372
68	6.170		6.174	6.180	6.237
82	8.764	8.749 ±0.009	8.854	8.862	9.070
92	11.000	10.98 ±0.01	10.95	10.96	11.42

the interpolated "experimental" data, Bethe-Heitler [10] results, and the results obtained from the formula of Overbo [11] and Maximon and Gimm [12]. We elucidate here the details of how the experimental data has been obtained. The experimental attenuation coefficient for 6.0-MeV photon energy for $Z=50$, 82, and 92 has been obtained by spline interpolation from the available experimental data. We have used the experimental attenuation coefficient data at energies 5.435, 6.405, 7.725, and 10.833 MeV (Barlett and Donahue [13]), 6.13 MeV (Paul [14]), and 6.418 MeV (Moreh and Wand [15]) for $Z=50$; at energies 4.508, 4.945, 5.278, and 5.548 MeV (Henry and Kennet [16], 5.435 and 6.405 MeV (Barlett and Donahue [13]), and 7.646 MeV (Moreh, Saltzmann, and Wand [17]) for $Z=82$; and at energies 4.508 and 4.945 MeV (Henry and Kennet [16]), 5.30 MeV (Rosenblum, Snrader, and Warner [18]), 6.13 MeV (Paul [14]), 6.418 MeV (Moreh and Wand [15]), and 7.279 and 7.646 MeV (Moreh, Saltzmann, and Wand [17]) for $Z=92$. The uncertainty in the interpolated experimental data at 6.0 MeV (for Sn 0.3%, Pb 0.1%, and U 0.1%) has been estimated by taking into account the uncertainty in the experimental data as well as the uncertainty in the interpolation. For $Z=30$ and 68 the experimental data are not available in the energy region of interest. The experimental pair-production cross section has been obtained by subtracting the sum of the atomic cross sections (Rayleigh scattering σ_R, photoelectric $\sigma_{\rm ph}$, Compton σ_c, and triplet production σ_t cross sections) and the photonuclear cross section ($\sigma_{\rm ph.n}$) from the $\sigma_{\rm tot}$, and is given as

$$\sigma=\sigma_{\rm tot}-(\sigma_R+\sigma_{\rm ph}+\sigma_c+\sigma_t)-\sigma_{\rm ph.n} \; .$$

The atomic cross sections in parenthesis have been tabulated by Hubbell, Gimm, and Overbo [1], and we have evaluated it by using the computer program XCOM of Berger and Hubbell [19]. We present in Table III the atomic cross sections (σ_R, $\sigma_{\rm ph}$, σ_c, and σ_t) for 6.0-MeV photons and $Z=1$, 30, 50, 68, 82, and 92. The total photonuclear absorption cross section ($\sigma_{\rm ph.n}$) is also included in Table III. The photonuclear absorption cross section can be represented by one or two Lorentz-shaped resonance lines. The parameters [20] (σ_0, the absorption cross section at the peak energy E_0; and Γ, the peak width at half maxima) of the Lorentz-shaped resonance line, which is given as

$$\sigma(E)=\sigma_0\frac{E^2\Gamma^2}{(E_0^2-E^2)^2+E^2\Gamma^2} \; ,$$

have been obtained by fitting the measured photoneutron cross sections. For the nucleus, Zn, the peak absorption cross section has been increased by 66% to take into account the photoproton cross section [21]. Hubbell, Gimm, and Overbo [1] have claimed that the uncertainty in the atomic cross sections is 0.5% or better. It may be noted that the major source of uncertainty in the tabulated atomic cross sections is in the pair-production cross section (we discuss it in detail in the following paragraph), and we are using only the non-pair-production atomic cross section to estimate the experimental pair-production cross section. The uncertainty in the non-pair-production component of the atomic cross sections [1,22,23] is about 0.2% at the 10.0-MeV energy range. The uncertainty in the non-pair-production component of the atomic cross section (the major part of which is the incoherent scattering cross section) at 6.0 MeV is expected to be better than 0.2%. We have however reported the uncertainty in the interpolated "experimental" pair production (in Table II) to be the same as the uncertainty in the total experimental attenuation cross section. To compare the experimental data with our theoretical result $\sigma_{\rm DWBA}$, we obtain $\sigma_{\rm expt}$ by dividing σ by $f(1-R)$ and by using the following expression:

$$\sigma=f(1-R)\sigma_{\rm expt} \; ,$$

where f is the radiative correction and $(1-R)$ is the screening factor. We have used the radiative correction to the pair production obtained by using the Mork-Olsen [22] formula along with arbitrary sine-function low-energy cutoff and screening factor from the table of Hubbell, Gimm, and Overbo [1]. The factors $f(1-R)$ are 0.977 (U), 0.980 (Pb), and 0.987 (Sn). The discrepancy between our computed $\sigma_{\rm DWBA}$ and $\sigma_{\rm expt}$ is attributed to the uncertainty in the factor $f(1-R)$.

The pair-production cross section obtained from the two interpolating formulas (Maximon and Gimm [12] and Overbo [11]) differ from each other. The difference is minimum for $Z=1$, maximum for $Z=68$, and 0.0% and 0.25% of the Bethe-Heitler result, respectively. The results obtained from Overbo's [11] formula are higher than those obtained by using the Maximon-Gimm [12] formula. The DWBA cross section for $Z=30$ is higher than the interpolated value of Maximon and Gimm [12] by 0.4% and lower than that of Overbo [11] by 0.015%, whereas for $Z=50$, the present result is lower than the result obtained by Overbo [11] as well as that of Maximon and Gimm [12] by 0.2% and 0.4%, respectively.

TABLE III. Rayleigh scattering (σ_R), photoelectric ($\sigma_{\rm ph}$), Compton (σ_c), triplet production (σ_t), and photonuclear ($\sigma_{\rm ph.n}$) cross sections for 6.0-MeV photon-atom interaction for different Z values in barns.

Z	$\sigma_{\rm R}$	$\sigma_{\rm ph}$	$\sigma_{\rm c}$	$\sigma_{\rm t}$	$\sigma_{\rm ph.n}$
1	1.285×10^{-7}	1.508×10^{-10}	7.343×10^{-3}	5.042×10^{-4}	
30	1.690×10^{-3}	3.898×10^{-3}	2.202	1.511×10^{-2}	1.470×10^{-2}
50	6.940×10^{-3}	3.846×10^{-2}	3.670	2.510×10^{-2}	6.200×10^{-3}
68	1.667×10^{-2}	1.495×10^{-1}	4.991	3.405×10^{-2}	5.298×10^{-2}
82	2.939×10^{-2}	3.404×10^{-1}	6.017	4.098×10^{-2}	1.678×10^{-2}
92	4.234×10^{-2}	5.671×10^{-1}	6.750	4.590×10^{-2}	1.235×10^{-1}

For $Z=68$ the difference between the present DWBA result and the one obtained from the interpolating formulas is only 0.17% for Overbo [11] and 0.1% for Maximon and Gimm [12]. Our results are lower by slightly more than 1.0% and higher by 0.4% than the results obtained by Maximon and Gimm [12] as well as that of Overbo [11] for $Z=82$ and 92, respectively.

In conclusion we mention that the discrepancy between our DWBA calculation and the σ_{expt} is larger than the experimental uncertainty, and thus the present calculation further supports the views expressed by Sud and Soto Vargas [9] about the need for reinvestigation of the screening effects in the energy region 5.0–10.0 MeV. The discrepancy between our DWBA results and the results obtained from the two interpolating formulas of Overbo [11] and Maximon and Gimm [12] varies with Z and has no specific pattern, so the tabulations of the total photon absorption cross section based on them need to be modified. The present calculations provide accurate data needed for developing a modified interpolating formula for evaluating the pair-production cross sections in the energy range 5–10 MeV.

ACKNOWLEDGMENTS

We would like to thank Dr. J. H. Hubbell for helpful correspondence and Dr. E. Hayward for providing us the photonuclear cross sections.

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