

## Effective Hamiltonian for the radiation in a cavity with a moving mirror and a time-varying dielectric medium

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We study the quantized field in a one-dimensional electromagnetic resonant cavity. The cavity contains a linear and lossless dielectric medium with frequency-independent polarizability. The dielectric permittivity is an externally prescribed function of both the space and the time. We also allow one of the cavity's mirrors to move in a given trajectory. Unlike other previous studies on the same system, we formulate an effective Hamiltonian so that the dynamics of the cavity field can be described in the Schrödinger picture. The effective Hamiltonian is quadratic in structure, therefore two-photon generation from the vacuum state can occur. We also discuss the case of resonant behavior of the system.

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### I. INTRODUCTION

In this paper, we study the quantized field in a quite general time-dependent cavity system. Both the refractive index of the medium and the cavity size are external time-varying parameters. The quantization of the electromagnetic field in a cavity with movable perfectly reflecting boundaries was first discussed by Moore [1] two decades ago. Within the framework of Moore's approach, Dodonov and co-workers [2] recently have generalized the theory so that the effects of a time-varying refractive index of the medium inside the cavity are also included. The major interest in this kind of system is the possibility of the creation of photons [3], which can be interpreted as a nonadiabatic distortion of the electromagnetic vacuum state. It has been predicted that a moving mirror with nonuniform motion [4,5] or a sudden change of the refractive index of the medium [6,7] can produce real photons from the vacuum state. In the latter case, Yablonvitch [6] suggested that a rapidly growing plasma produced by short optical pulses could provide a large rate of change of the index of refraction with observable effects. From the point of view of quantum optics, the statistical properties of the photons are perhaps even more interesting. Since the emission is purely a quantum effect, we expect the photon statistics to carry some nonclassical features. In fact, a nonthermal distribution [8], as well as squeezing, was found in recent theoretical analyses [8,9].

The quantization of a field in a cavity with time-varying parameters has so far [1,2] been restricted to the Heisenberg picture, in which the field operators are constructed directly from the solutions of the classical wave equation, and the Hamiltonian plays no role in the theory. It is therefore not possible to know the explicit form of the state of the field. Early work [10] on formulating the dynamics from the Hamiltonian applies to a system with moving mirrors only, and a generalization of the method in Ref. [10] for a time-dependent dielectric medium seems difficult. Physically, both the moving mirror and the time-dependent dielectric medium produce

similar effects on the cavity field, so they can be treated on the same ground. In this paper, we adopt a different approach to establish an effective Hamiltonian which is consistent with the previous formalism [1,2]. This effective Hamiltonian exhibits the essential features in the physical processes and makes a Schrödinger-picture description possible. In Sec. II, we define the quantum system and derive the effective Hamiltonian. The resonance behavior of the effective Hamiltonian is discussed in Sec. III, and Sec. IV is devoted to our conclusions.

### II. FIELD QUANTIZATION AND THE EFFECTIVE HAMILTONIAN

We consider a one-dimensional cavity formed by two perfectly reflecting mirrors (see Fig. 1). One of the mirrors is fixed at the position  $x=0$  and the other is allowed to move in a prescribed trajectory  $x=q(t)$ . The space between the mirrors contains a linear, lossless, and non-dispersive dielectric medium. The relative dielectric permittivity  $\epsilon(x,t)$  of the medium is an externally prescribed real function of both space and time. For simplicity, we let the magnetic permeability  $\mu$  be a constant throughout the cavity.

The Lagrangian density of the system ( $c=1$ ,  $\mu=1$ ) is given by

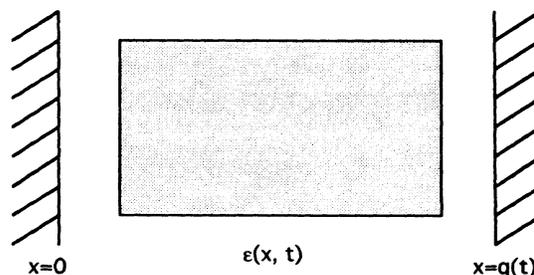


FIG. 1. The one-dimensional cavity with a moving mirror and the dielectric medium have time-varying permittivity.

$$L(x,t) = \frac{1}{2} \left[ \epsilon(x,t) \left( \frac{\partial A}{\partial t} \right)^2 - \left( \frac{\partial A}{\partial x} \right)^2 \right], \quad (2.1)$$

where  $A(x,t)$  is the vector potential. We have considered only the case of a linearly polarized field because the two polarizations are decoupled. The wave equation obtained from (2.1) has the form

$$\frac{\partial^2 A}{\partial x^2} = \frac{\partial}{\partial t} \left[ \epsilon(x,t) \frac{\partial A}{\partial t} \right] \quad (2.2)$$

and we impose the boundary conditions [1]

$$A(0,t) = A(q(t),t) = 0 \quad (2.3)$$

which guarantees that the electric fields are always zero in the rest frames of the mirrors' surfaces.

The field quantization is achieved by constructing a field operator  $\hat{A}(x,t)$  associated with the vector potential such that it is a solution of Eqs. (2.2) and (2.3), and satisfies the following commutation relations:

$$[\hat{A}(x,t), \hat{A}(x',t)] = [\hat{\pi}(x,t), \hat{\pi}(x',t)] = 0, \quad (2.4)$$

$$[\hat{A}(x,t), \hat{\pi}(x',t)] = i\delta(x-x'), \quad (2.5)$$

where  $x$  and  $x'$  are defined in the space between the mirrors, excluding the boundaries. The operator  $\hat{\pi}(x,t)$  is the conjugate momentum obtained from the Lagrangian density:

$$\hat{\pi}(x,t) = \epsilon(x,t) \frac{\partial \hat{A}(x,t)}{\partial t}. \quad (2.6)$$

It is known that the solutions for the field operator  $\hat{A}(x,t)$  can be determined consistently through (2.2) and (2.3) regardless of the Hamiltonian. In fact, we do not attempt to formulate a quantum theory which is based on the full Hamiltonian of the system. This is because the description of the interaction between the field and the induced surface current on the moving mirror can be quite complicated. The vanishing boundary conditions on  $\hat{A}(x,t)$  are therefore used to account for these subtle interactions in a simplified way. Once the boundary conditions are assumed, it is not necessary and even not possible to find a consistent fundamental Hamiltonian [1]. However, an effective Hamiltonian does exist. As we shall see below, the Hamiltonian dynamics can be recovered in a special time-dependent mode basis.

Let us first define the "instantaneous" set of mode functions  $\{\phi_k(x;t)\}$ ,

$$\frac{\partial^2 \phi_k(x;t)}{\partial x^2} + \epsilon(x,t) \omega_k^2(t) \phi_k(x;t) = 0, \quad (2.7)$$

subjected to the boundary conditions

$$\phi_k(0;t) = \phi_k(q(t);t) = 0 \quad (2.8)$$

with the eigenvalues  $\omega_k(t)$ . The physical meaning of the mode functions  $\phi_k(x;t)$  is quite obvious. If we "freeze" the system at the instant  $t_0$ , then the dielectric permittivity and the length of the cavity are stopped at  $\epsilon(x,t_0)$  and  $q(t_0)$ , respectively. The set of mode basis functions for this system is therefore  $\{\phi_k(x;t_0)\}$ . In other words, the

role of the  $t$  in  $\phi_k(x;t)$  is a parameter. Such a set of functions is orthonormal,

$$\int_0^{q(t)} dx \epsilon(x,t) \phi_n(x;t) \phi_m(x;t) = \delta_{n,m}, \quad (2.9)$$

and is complete. Hence  $\hat{A}(x,t)$  and  $\hat{\pi}(x,t)$  can be expanded in terms of  $\phi_k(x;t)$  at any instant  $t$ :

$$\hat{A}(x,t) = \sum_k \hat{Q}_k(t) \phi_k(x;t) \quad (2.10)$$

and

$$\hat{\pi}(x,t) = \epsilon(x,t) \sum_k \hat{P}_k(t) \phi_k(x;t) \quad (2.11)$$

where  $\hat{Q}_k(t)$  and  $\hat{P}_k(t)$  are defined by

$$\hat{Q}_k(t) = \int_0^{q(t)} dx \epsilon(x,t) \hat{A}(x,t) \phi_k(x;t), \quad (2.12)$$

$$\hat{P}_k(t) = \int_0^{q(t)} dx \hat{\pi}(x;t) \phi_k(x;t). \quad (2.13)$$

The expansion (2.11) should not include the moving boundary point  $x=q(t)$ , because  $\hat{\pi}(x,t)$  is actually nonzero there (except for the case of stationary mirrors). The discrepancy at that point however does not affect the time dependence of  $\hat{P}_k(t)$  defined in (2.13).

Now we let  $\hat{Q}_k(t)$  and  $\hat{P}_j(t)$  obey the commutation relations

$$[\hat{Q}_k(t), \hat{Q}_j(t)] = [\hat{P}_k(t), \hat{P}_j(t)] = 0, \quad (2.14)$$

$$[\hat{Q}_k(t), \hat{P}_j(t)] = i\delta_{kj}$$

which guarantee the commutation rules (2.4) and (2.5) among the field operators. Therefore we can interpret from (2.14) that  $\hat{Q}_k(t)$  and  $\hat{P}_j(t)$  are the natural generalized position and momentum operators, respectively, for the field.

By taking the time derivative of (2.12) and (2.13), and using the relations (2.6), (2.7), and (2.9), we obtain the equations of motion for  $\hat{Q}_k(t)$  and  $\hat{P}_k(t)$

$$\frac{d\hat{Q}_k}{dt} = \hat{P}_k + \sum_j G_{k,j}(t) \hat{Q}_j(t), \quad (2.15)$$

$$\frac{d\hat{P}_k}{dt} = -\omega_k^2(t) \hat{Q}_k - \sum_j G_{j,k}(t) \hat{P}_j(t), \quad (2.16)$$

where the time-dependent coefficient  $G_{k,j}(t)$  is defined by

$$G_{k,j}(t) = - \int_0^{q(t)} dx \epsilon(x,t) \phi_k(x;t) \frac{\partial \phi_j(x;t)}{\partial t}. \quad (2.17)$$

Considering Eqs. (2.15) and (2.16) as the Heisenberg equations of motion,  $\hat{O} = i[H_{\text{eff}}, \hat{O}]$ , we can construct the effective Hamiltonian up to an arbitrary constant,

$$H_{\text{eff}} = \frac{1}{2} \sum_k \{ \hat{P}_k^2 + \omega_k^2(t) \hat{Q}_k^2 + G_{k,k}(t) (\hat{P}_k \hat{Q}_k + \hat{Q}_k \hat{P}_k) \} \\ + \sum_{\substack{j,k \\ j \neq k}} G_{k,j}(t) \hat{P}_k \hat{Q}_j, \quad (2.18)$$

which generates the equations of motion (2.15) and (2.16). In order to describe the system in Fock space, we now introduce the "instantaneous" creation and annihilation

operators:

$$a_k^\dagger = \frac{1}{\sqrt{2\omega_k(t)}} [\omega_k(t)\hat{Q}_k - i\hat{P}_k], \quad (2.19)$$

$$a_k = \frac{1}{\sqrt{2\omega_k(t)}} [\omega_k(t)\hat{Q}_k + i\hat{P}_k]. \quad (2.20)$$

Once again we take the time derivative of (2.19) and (2.20) to obtain the equations of motion. Because of the explicit time dependence of  $\omega_k(t)$  in the definitions, the effective Hamiltonian that governs the motion of  $a_k$  and  $a_k^\dagger$  has some extra terms. The final form of the effective Hamiltonian is given by

$$\begin{aligned} \tilde{H}_{\text{eff}} = & \sum_k \omega_k(t) a_k^\dagger a_k + i \sum_k \xi_k(t) (a_k^{\dagger 2} - a_k^2) \\ & + \frac{i}{2} \sum_{\substack{j,k \\ j \neq k}} \mu_{k,j}(t) (a_k^\dagger a_j^\dagger + a_k^\dagger a_j - a_j a_k - a_j^\dagger a_k), \end{aligned} \quad (2.21)$$

where we have used the abbreviation

$$\xi_k(t) = \frac{G_{k,k}(t)}{2} + \frac{1}{4\omega_k(t)} \frac{d\omega_k(t)}{dt} \quad (2.22)$$

and

$$\mu_{k,j}(t) = \left[ \frac{\omega_k(t)}{\omega_j(t)} \right]^{1/2} G_{k,j}(t). \quad (2.23)$$

It is worth noting that if the mirrors are fixed in positions and the dielectric permittivity is spatially homogeneous  $\epsilon(x,t) = \epsilon(t)$ , then all coefficients  $G_{k,j}$  are zero [but  $\xi_k(t) \neq 0$ ]. In this special case, different modes are decoupled from each other and the effective Hamiltonian describes a system of decoupled oscillators with time-dependent frequencies [11].

Having obtained the general effective Hamiltonian (2.21), the time evolution of the system is determined by the Schrödinger equation

$$\tilde{H}_{\text{eff}}|\Psi\rangle = i \frac{\partial |\Psi\rangle}{\partial t}, \quad (2.24)$$

where  $|\Psi\rangle$  is the state vector represented in the Fock space. We emphasize that the Fock space here is dynamical in nature because it is based on the set of time-dependent mode basis functions  $\{\phi_k(x;t)\}$ . As we change the system's parameters in time, the vacuum state changes accordingly. Therefore, the bosons associated with the "instantaneous" creation and annihilation operators (2.19) and (2.20) may not be regarded as real photons unless we can specify a measurement process to detect them. Nevertheless, these bosons become real photons once  $\epsilon(x,t)$  and  $q(t)$  stop changing with time, since the ordinary Fock space is recovered when the mode functions become stationary [12].

There are basically two kinds of nonadiabatic processes in the system. The first kind is the zero-photon process characterized by the  $a_k^\dagger a_j$  terms in the Hamiltonian. Photons are scattered from one mode to another without changing the total number of photons. The second kind

is the two-photon processes characterized by the terms  $a_k^\dagger a_j^\dagger$  and  $a_k^{\dagger 2}$ , so that photon pairs can be created from the vacuum state. This two-photon character of the field is related to the squeezing phenomena that were recently found [8,9]. It must be noted that the effects of these two processes are determined by the time dependence of the  $\xi_k(t)$  and  $\mu_{k,j}(t)$ . Nonadiabatic behavior happens only if  $\xi_k(t)$  and  $\mu_{k,j}(t)$  change appreciably in the typical time scales of the system. For the scattering process the time scale is given by the inverse of the frequency difference between the two scattering modes, whereas the time scale for the two-photon process is the inverse of the frequency sum of the two modes concerned. These two time scales can be different from each other by many orders of magnitude. As an example, take a one-meter-long cavity. If we consider only the optical field, it requires that  $\xi_k(t)$  and  $\mu_{k,j}(t)$  have frequency components in the optical domain in order to create photons. On the other hand, the scattering between neighboring modes requires a much slower motion of  $\xi_k(t)$  and  $\mu_{k,j}(t)$ , which is in the microwave frequency region.

### III. RESONANCES IN THE WEAKLY PERTURBED REGIME

When the cavity field is weakly perturbed periodically by some appropriate choices of the motion in the cavity's parameters, resonances [9] would occur and cavity modes can be selectively excited. To facilitate our discussions, let us concentrate on the moving-mirror system in the absence of the dielectric medium, i.e.,  $\epsilon(x,t) = 1$ . The mode functions are given by

$$\phi_k(x;t) = \left[ \frac{2}{q(t)} \right]^{1/2} \sin \frac{k\pi x}{q(t)} \quad (3.1)$$

and the eigenvalues are  $\omega_k(t) = k\pi/q(t)$ . Hence, the effective Hamiltonian reads

$$\begin{aligned} \tilde{H}_{\text{eff}} = & \sum_k \omega_k(t) a_k^\dagger a_k + i \sum_k \frac{\dot{q}(t)}{4q(t)} (a_k^{\dagger 2} - a_k^2) \\ & + \frac{i}{2} \sum_{\substack{j,k \\ j \neq k}} (-1)^{j+k} \frac{kj}{j^2 - k^2} \left[ \frac{k}{j} \right]^{1/2} \frac{\dot{q}(t)}{q(t)} \\ & \times (a_k^\dagger a_j^\dagger + a_k^\dagger a_j - a_j a_k - a_j^\dagger a_k), \end{aligned} \quad (3.2)$$

where  $\dot{q}(t) = dq(t)/dt$ . Expression (3.2) is not the same as the one obtained in a different approach [10]. The discrepancy is due to the different definition of the field operators.

A convenient choice of the mirror's trajectory  $q(t)$  is to make  $\dot{q}(t)/q(t)$  purely sinusoidal:

$$q(t) = L \exp \left[ \frac{q_0 \cos \Omega t}{L} \right]. \quad (3.3)$$

The  $q(t)$  itself is actually very close to a simple harmonic motion because we will let  $q_0 \ll L$ , in order to keep the system in the weak perturbation regime, where the cavity frequencies are well defined by  $\omega_k \approx k\pi/L$ .

To locate the resonance conditions, we notice that each

Heisenberg operator  $a_k^{\dagger 2}$  carries a zeroth-order time dependence  $\exp[i(2k\pi/L)t]$ . This fast oscillatory phase factor has to be canceled by  $\dot{q}(t)/q(t)$  for resonance to occur. Therefore if  $\Omega=2m\pi/L$ , where  $m$  is an integer, the  $k=m$  mode will be resonantly excited. A similar argument also applies for the operators  $a_k^{\dagger}a_j^{\dagger}$ , when we have resonance for  $\Omega=(m+n)\pi/L$ . In this case the  $k=m$  and  $j=n$  modes are excited simultaneously. For the scattering case  $a_k^{\dagger}a_j$ , the  $k=m$  and  $j=n$  modes are resonant when  $\Omega=(m-n)\pi/L$ .

It is not difficult to see that among all the terms in expression (3.2), there are only a few of them that are on resonance, when a specific choice of  $\Omega$  is given. The effective Hamiltonian can therefore be greatly simplified by keeping only those resonant terms. This is the so-called rotating-wave approximation. In the following, we write down the approximate form of the resonant effective Hamiltonian for the first three resonances of the system:

(i)  $\Omega=\pi/L$ :

$$\tilde{H}_{\text{eff}}^{(i)} \approx \frac{q_0\pi}{4L^2} \sum_k f_1(k)(a_{k+1}^{\dagger}a_k + a_k^{\dagger}a_{k+1}); \quad (3.4)$$

(ii)  $\Omega=2\pi/L$ :

$$\begin{aligned} \tilde{H}_{\text{eff}}^{(ii)} \approx & \frac{q_0\pi}{4L^2} (a_1^2 + a_1^{\dagger 2}) \\ & + \frac{q_0\pi}{4L^2} \sum_k f_2(k)(a_{k+2}^{\dagger}a_k + a_k^{\dagger}a_{k+2}); \end{aligned} \quad (3.5)$$

(iii)  $\Omega=3\pi/L$ :

$$\begin{aligned} \tilde{H}_{\text{eff}}^{(iii)} \approx & \frac{q_0\pi}{2\sqrt{2}L^2} (a_1a_2 + a_1^{\dagger}a_2^{\dagger}) \\ & + \frac{q_0\pi}{4L^2} \sum_k f_3(k)(a_{k+3}^{\dagger}a_k + a_k^{\dagger}a_{k+3}); \end{aligned} \quad (3.6)$$

where the function  $f_{\alpha}(k)$  ( $\alpha=1,2,3$ ) is given by

$$f_{\alpha}(k) = \frac{k(k+\alpha)}{2k+\alpha} \left[ \left( \frac{k+\alpha}{k} \right)^{1/2} + \left( \frac{k}{k+\alpha} \right)^{1/2} \right]. \quad (3.7)$$

In deriving the effective Hamiltonians (3.4), (3.5), and (3.6), we have made rotating-wave approximations and neglected the correction in the cavity frequencies due to the modulation of  $q(t)$ . There is no time dependence in the expressions because they are represented in rotating frames.

The first resonant Hamiltonian (3.4) describes the scattering interactions between neighboring modes, and the total photon number is conserved. In the second case, a degenerate parametric oscillator associated with the lowest mode  $k=1$  appears in (3.5), and the total photon number is not conserved. The third resonance is similar, but with a nondegenerate parametric oscillator associated with the modes  $k=1$  and  $k=2$ . In all cases, the complicated form of the scattering terms forbids us from finding the analytic solutions, and a perturbative ap-

proach could be useful. If the initial state of the system is the vacuum, the evolution of the system in the short-time domain, to first order in  $t$ , is given by

$$|\Psi(t)\rangle \approx |\text{vac}\rangle - i \frac{q_0\pi}{4L^2} t a_1^{\dagger 2} |\text{vac}\rangle \quad \text{for } \Omega=2\pi/L \quad (3.8)$$

and

$$|\Psi(t)\rangle \approx |\text{vac}\rangle - i \frac{q_0\pi}{2\sqrt{2}L^2} t a_1^{\dagger} a_2^{\dagger} |\text{vac}\rangle \quad \text{for } \Omega=3\pi/L \quad (3.9)$$

where the case of the first resonance is not considered. It is clear that the dynamics of the system is dominated by the parametric oscillator for small  $t$ , and scattering processes will appear only as higher-order effects. When the time increases, the scattering terms act as an oscillator bath and cause damping in the motion of the parametric oscillator.

We have performed exact numerical calculations on the time evolution of the photon number in a few lowest modes of the cavity. Figure 2(a) corresponds to the resonance with  $\Omega=2\pi/L$ . We see that the fundamental mode  $k=1$  is resonantly excited. The  $k=2,4$  modes are basically empty because the value of  $\Omega$  permits resonant

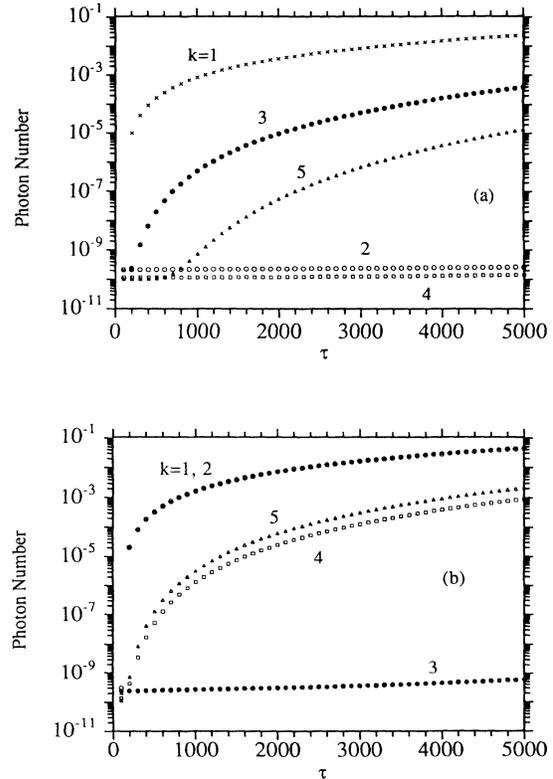


FIG. 2. The time evolution of the photon number in the lowest five modes of the cavity. Initially, the field is in the vacuum state. The time axis  $\tau=t/T$  is dimensionless, where  $T=2L/c$  is the round-trip time. The parameters are  $L=0.3$  m,  $q_0=10^{-5}L$ . Each of the points in the figure was taken for the times at every  $100T$ , when the system is instantaneously at rest. (a)  $\Omega=2\pi/L$ , (b)  $\Omega=3\pi/L$ .

scattering from the fundamental mode to odd  $k$  modes only. In Fig. 2(b), the frequency is  $\Omega=3\pi/L$ , so both  $k=1$  and  $k=2$  modes are excited at the same time. The characteristic of two-photon emission is quite apparent because the two curves for these two modes almost coincide. There is a relatively small amount of photons in the modes  $k=4$  and  $k=5$ , which is created by the scattering from the modes  $k=1$  and  $k=2$ , respectively. In both figures, we have found good agreement between the numerical solutions and the perturbative results on the photon number in the resonant modes, as described by the wave functions in (3.8) and (3.9).

#### IV. CONCLUSION

In conclusion, we have derived the effective Hamiltonian of the field in a one-dimensional cavity with a moving mirror and a dielectric medium with time-varying index of refraction. The effective Hamiltonian is found in a quadratic form. We have discussed the time scales that are associated with the nonadiabatic processes. In the case of resonance, the resonant mode can be regarded as

a parametric oscillator. We have demonstrated numerically the growth of the photon number in the regime of the first two parametric resonances of the moving mirror system. Finally, we hope that the effective Hamiltonian can provide a convenient way for further study of the atom-field interaction. The response of atoms to the field in this type of cavity should be quite unusual and may provide us with indirect ways to probe nonadiabatic changes of a vacuum field. It is because the environment that the atoms experience is modified with time. Virtual transitions and the self-dressing processes of atoms would become important, a fact that may lead to emission of photons even if the atoms are in the ground state [13].

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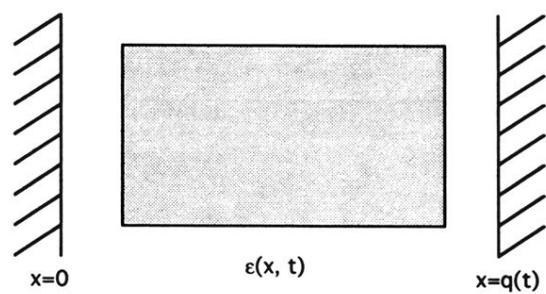


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