State preparation via quantum coherence and continuous measurement

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(Received 23 July 1993)

The theory of continuous measurement is generalized so as to be applicable to a two-mode radiation field whose properties are monitored by using a Λ system with initial coherence between two ground states. The two-mode system is shown to offer the possibility of generating a number of new quantum states. Quantum statistics of the field following continuous measurements is calculated. Results for the phase distribution, quasiprobability distribution, and the photon-number distribution of the resulting field are also presented.

PACS number(s): 42.50.Dv, 42.50.Ar, 03.65.Bz

I. INTRODUCTION

In a series of papers [1,2] Ueda and co-workers have generalized the quantum theory of continuous measurements [3] and shown how the measurement process, with either a "yes" or "no" result, affects the quantum-statistical properties of the field [4,5] on which measurements are made. In particular, they discussed the new features the *single-mode* radiation field acquires if such a field is continuously monitored by a photodetection. Some of the key elements of the continuous measurement theory are as follows.

A "yes" event, i.e., detection of a photon, changes the input field state $\rho(0)$ into

$$\rho(0^+)=\frac{\hat{a}\rho(0)\hat{a}^\dagger}{\mathrm{Tr}[\rho(0)\hat{a}^\dagger\hat{a}]}\ , \tag{1}$$

whereas a "no" event changes the input state to

$$\rho(0^+)=e^{-(1/2)g^2\tau^2\hat{a}^\dagger\hat{a}}\rho(0)e^{-(1/2)g^2\tau^2\hat{a}^\dagger\hat{a}}\ . \tag{2}$$

Equation (2) holds up to a normalization constant. Here g is the coupling constant between the single-mode field and the atom, and τ is the interaction time. It is assumed that the detector works by the absorption of a photon. Thus if the photodetection is carried out on a time interval t and if $t_1,t_2,t_3,\ldots,t_n$ are the times when the photons are detected, then the net density matrix after n detection events becomes

$$\rho(t)=\frac{e^{-g^2(\tau/2)t\hat{a}^\dagger\hat{a}}\hat{a}^n\rho(0)\hat{a}^{\dagger n}e^{-g^2(\tau/2)t\hat{a}^\dagger\hat{a}}}{\mathrm{Tr}(\rho(0)\hat{a}^{\dagger n}e^{-g^2\tau t\hat{a}^\dagger\hat{a}}\hat{a}^n)}\ . \tag{3}$$

All of the physics relevant to the present problem is contained in Eq. (3). Ueda *et al.* [1] considered the case of an input field in a squeezed coherent state and predicted, for example, the possibilities of measurement-induced oscillations in Mandel's Q parameter and the production of "CAT"-like states.

Note, however, that if the input field is either in a coherent state $|\alpha\rangle$ or in a Fock state $|N\rangle$, then the "nature" of the state does *not* change as Eq. (3) shows

$$\begin{aligned}&|\alpha\rangle\rightarrow|\alpha e^{-g^2\tau/2t}\rangle\ ,\\ &|N\rangle\rightarrow|N-n\rangle\ .\end{aligned} \tag{4}$$

Note further that if the usual photodetector is replaced by a quantum counter which operates via stimulated emission, then an equation like (3) also holds with $\hat{a}\leftrightarrow\hat{a}^\dagger$. In such a case even a coherent-state field is transformed into a nonclassical state by the process of measurement [6].

In this paper we consider the generalization of the continuous measurement theory to a radiation field consisting of *two modes,* and we demonstrate several new possibilities such as (a) production of an entangled pair of photons, (b) production of bifurcations in the phase probability distributions, and (c) transfer of coherence from one mode to the other mode. The organization of this paper is as follows. In Sec. II we introduce the scheme of continuous measurements on a two-mode system. We generalize the theory of continuous measurements to two-mode fields. In Sec. III we derive the result for the state of the two-mode system after a certain number of measurements in the long-time limit. We discuss how the coherence is transferred from one mode to the other. We show the production of an entangled state and discuss its properties. In Sec. IV we show the quasidistribution (Q function) of the resulting field. And, finally, in Sec. V we examine the phase characteristics of the field after measurements.

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II. CONTINUOUS MEASUREMENTS ON A TWO-MODE SYSTEM

Let us consider the interaction of a three-level Λ system with a two-mode radiation field in a high-Q cavity. Let us assume that the Λ system is prepared in an equal superposition of the two ground states $|g\rangle$ and $|g'\rangle$, i.e.,

$$|\psi_A(0)\rangle = \frac{1}{\sqrt{2}}[|g\rangle + e^{-i\varphi}|g'\rangle] . \tag{5}$$

This can be done, for example, by using a microwave field before the atoms enter the cavity [7,8]. We now assume that the transition $|g'\rangle \leftrightarrow |e\rangle$ is resonant with the cavity. We further assume that the transition $|g\rangle \leftrightarrow |e\rangle$ is driven by a coherent field $|\alpha\rangle$. The Hamiltonian for the interaction between the atom and fields in the interaction picture can be written as

$$H_I = g\hat{a}|e\rangle\langle g| + g\hat{b}|e\rangle\langle g'| + \text{H.c.} , \tag{6}$$

where $\hat{a}$ is the annihilation operator of the coherent field resonant with the transition $|g\rangle \leftrightarrow |e\rangle$, and $\hat{b}$ is the annihilation of the field resonant with the $|g'\rangle \leftrightarrow |e\rangle$ transition.

Let us first discuss a simple measurement scheme. The atom with the initial state, given by Eq. (5), interacts with the fields for a short time τ, which is the time of flight through the cavity. After the interaction we *measure* the atom to see if it is in the excited state $|e\rangle$. The process of measurement will reduce the state of the field in the cavity. Let $\rho(0)$ be the state of the field before the atom enters the cavity. Solving for the atom-field density operator $\rho_{a,f}$ up to second order in the coupling constant, we find

$$\rho_{a,f}(\tau) - \rho_{a,f}(0) = -i\tau[H_I, \rho_{a,f}(0)] - \frac{\tau^2}{2}[H_I,[H_I,\rho_{a,f}(0)]] , \tag{7}$$

where the initial density matrix for the combined atom-field system is given by

$$\rho_{a,f}(0) = |\psi_A(0)\rangle \rho(0) \langle \psi_A(0)| . \tag{8}$$

From Eqs. (7) and (8) we find that the state of the field immediately after the measurement (atom detected in an excited state) will be $\langle e|\rho(\tau)|e\rangle \equiv \rho_{\text{ex}}(\tau)$ up to a normalization constant, i.e.,

$$\rho_{\text{ex}}(\tau) \cong g^2\tau^2 \hat{A}\rho(0)\hat{A}^\dagger , \tag{9}$$

where

$$\hat{A} = \frac{1}{\sqrt{2}}(\hat{a} + \hat{b}e^{-i\varphi}) . \tag{10}$$

Note that $\rho_{\text{ex}}(\tau)$ is the conditional density matrix, i.e., it is the density matrix of the combined a-b system subject to the condition that the atom was detected in the excited state.

We next examine the density matrix subject to the condition that the atom is not found to be in the excited state. Note that *not* finding the atom in the excited state is also information [9] and hence the field density matrix changes. Given that the detected atom is not excited we

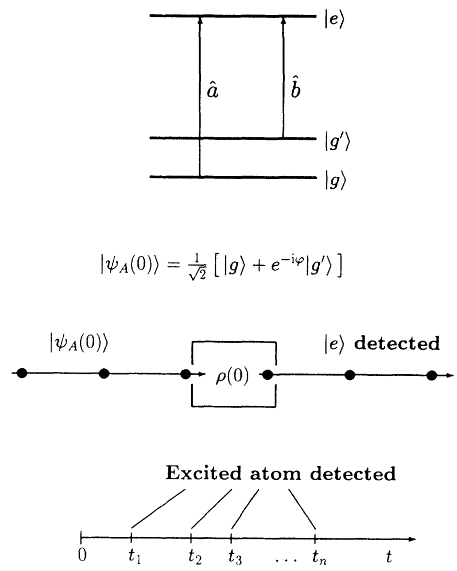

FIG. 1. Schematic diagram of the scheme proposed in this paper.

find from Eqs. (5)–(7) that the density matrix for the field is

$$\begin{aligned}\rho_{gnd}(\tau) &= \langle g|\rho_{a,f}(\tau)|g\rangle + \langle g'|\rho_{a,f}(\tau)|g'\rangle \\ &= \rho(0) - \frac{g^2\tau^2}{2}[\hat{A}^\dagger\hat{A}\rho(0) + \rho(0)\hat{A}^\dagger\hat{A}] \\ &\cong \exp(-2R\tau\hat{A}^\dagger\hat{A})\rho(0)\exp(-2R\tau\hat{A}^\dagger\hat{A}) ,\end{aligned} \tag{11}$$

where we have defined $R = g^2\tau/4$.

We now imagine the following measurement scheme (Fig. 1). A regular beam of well separated coherently prepared [Eq. (5)] atoms passes through the cavity. We assume that at any given time there is only one atom in the cavity. We continue to assume that the cavity Q is so large that the relaxation of the field inside the cavity is negligible.

Next we ask for $\rho^{(n)}(t)$, the density matrix for the field, given that n atoms are excited at times $t_1, t_2, \ldots, t_n$ during a time interval[1] 0 to t. From (10) and (11) we see that this is given by

$$\rho^{(n)}(t) = \hat{M}_n\rho(0)\hat{M}_n^\dagger / \text{Tr}[\rho(0)\hat{M}_n^\dagger\hat{M}_n] , \tag{12}$$

where

$$\begin{aligned}\hat{M}_n &= \exp[-2R(t-t_n)\hat{A}^\dagger\hat{A}]\hat{A} \\ &\times \exp[-2R(t_n - t_{n-1})\hat{A}^\dagger\hat{A}] \cdots \\ &\times \hat{A}\exp[-2R(t_2-t_1)\hat{A}^\dagger\hat{A}]\hat{A}\exp(-2Rt_1\hat{A}^\dagger\hat{A}) ,\end{aligned} \tag{13}$$

[1]Actually the interaction time of the atoms detected in the excited state is not counted, making, in this approximation ($g\tau \ll 1$), the formulas easier to read. In the same sense the atoms are assumed to have a separation equal to the cavity length.

and using the fact that $\exp(+x\hat{A}^{\dagger}\hat{A})\hat{A}\exp(-x\hat{A}^{\dagger}\hat{A}) = \hat{A}\exp(-x)$ we have

$$\rho^{(n)}(t)=\frac{\exp(-2Rt\hat{A}^{\dagger}\hat{A})\hat{A}^{n}\rho(0)\hat{A}^{\dagger n}\exp(-2Rt\hat{A}^{\dagger}\hat{A})}{\mathrm{Tr}[\rho(0)\hat{A}^{\dagger n}\exp(-4Rt\hat{A}^{\dagger}\hat{A})\hat{A}^{n}]} . \quad (14)$$

This is our key result and is the generalization of the previous results to a field with two modes and to atoms which are prepared in a coherent superposition of two ground states. In the next few sections we discuss [10] the important physical consequences of (14).

III. MEASUREMENT-INDUCED CORRELATIONS IN THE LONG-TIME LIMIT

We first examine the characteristics of the density matrix (14) in the limit of long counting intervals. Clearly in the long-time limit the solution $\rho^{(n)}$ must be such that

$$\hat{A}\rho^{(n)}=0,\quad \rho^{(n)}\hat{A}^{\dagger}=0. \quad (15)$$

Note that since $\hat{A}$ is a linear combination of a and b modes, the condition (15) is not enough to determine the solution uniquely. In fact the longtime solution will also depend on the initial condition and thus this system has memory of initial correlations. We derive the long-time solution assuming that

$$\rho(0)=|\psi\rangle\langle\psi| ,\quad |\psi\rangle=\int\int d^2\alpha\, d^2\beta\psi(\alpha,\beta)|\alpha,\beta\rangle_{a,b} , \quad (16)$$

where we use the expansion of $|\psi\rangle$ in terms of the coherent states of a and b modes. On using (16) in (14) we get up to a normalization constant

$$\rho^{(n)}=|\Phi\rangle\langle\Phi| , \quad (17)$$

where

$$|\Phi\rangle\equiv\int\int d^2\alpha\, d^2\beta\,\psi(\alpha,\beta)e^{-2Rt\hat{A}^{\dagger}\hat{A}}\hat{A}^{n}|\alpha,\beta\rangle_{a,b} . \quad (18)$$

Note that

$$\hat{A}|\alpha,\beta\rangle_{a,b}=A|\alpha,\beta\rangle_{a,b},\quad A=\frac{\alpha+\beta e^{-i\varphi}}{\sqrt{2}} . \quad (19)$$

Next we have to simplify the term $\exp(-2Rt\hat{A}^{\dagger}\hat{A})|\alpha,\beta\rangle_{a,b}$. To simplify this we introduce the operator $\hat{B}$

$$\hat{B}=\frac{\hat{a}-\hat{b}e^{-i\varphi}}{\sqrt{2}} \quad (20)$$

and the coherent states $|A,B\rangle_{A,B}$ of the modes A and B. It can be shown that

$$|\alpha,\beta\rangle_{a,b}=|A,B\rangle_{A,B},\quad B=\frac{\alpha-\beta e^{-i\varphi}}{\sqrt{2}} . \quad (21)$$

Using (19)–(21) we now have

$$\begin{aligned}\exp(-2Rt\hat{A}^{\dagger}\hat{A})|\alpha,\beta\rangle_{a,b}&=\exp(-2Rt\hat{A}^{\dagger}\hat{A})|A,B\rangle_{A,B}\\&=\exp[-\tfrac{1}{2}|A|^2(1-e^{-4Rt})]|Ae^{-2Rt},B\rangle_{A,B}\xrightarrow{Rt\to\infty}e^{-(1/2)|A|^2}|0,B\rangle_{A,B}\\&=e^{-(1/2)|A|^2}\left|\frac{B}{\sqrt{2}},-\frac{B}{\sqrt{2}}e^{i\varphi}\right\rangle_{a,b} . \end{aligned} \quad (22)$$

On substituting (22) in (18) we get

$$|\Phi\rangle=\int d^2B\left|\frac{B}{\sqrt{2}},-\frac{B}{\sqrt{2}}e^{i\varphi}\right\rangle_{a,b}\Phi(B) , \quad (23)$$

where

$$\Phi(B)=\int d^2A\, e^{-(1/2)|A|^2}A^{n}\psi\left[\frac{A+B}{\sqrt{2}},\frac{A-B}{\sqrt{2}}e^{i\varphi}\right] . \quad (24)$$

Equation (23) is the key result of this section. The form (23) makes it clear that

$$\hat{A}|\Phi\rangle=0 . \quad (25)$$

The weight function Φ in (23) is determined from the initial conditions which enter through the function $\psi(\alpha,\beta)$. Note that the final state in general is an *entangled state with strong correlations.* Note further that (23) shows that the modes a and b will essentially have *identical* photon statistics irrespective of their initial state.

If initially both modes a and b are in coherent states $|\alpha\rangle$ and $|\beta\rangle$, then

$$\rho^{(n)}=|\chi\rangle\langle\chi| ;\quad |\chi\rangle=\left|\frac{\alpha-\beta e^{-i\varphi}}{2},-\frac{\alpha-\beta e^{-i\varphi}}{2}e^{i\varphi}\right\rangle_{a,b} . \quad (26)$$

In the special case of the b mode in the vacuum state we get a very interesting result:

$$|0\rangle_b\to\left|-\frac{\alpha}{2}e^{i\varphi}\right\rangle_b . \quad (27)$$

We thus have *transferred coherence from the a mode to the b mode.* Note further that the result (26) is independent [11] of the index n leading us to conclude that the results (26) and (27) are obtained even if we do not ever find the atoms in the excited states.

The idea of transfer of coherence is quite interesting and we now examine the transformation of the Fock state of the b mode by this coherence transfer. Thus we consider the initial $|\psi\rangle$ to be

$$|\psi\rangle=|\alpha\rangle_a|N\rangle_b\equiv\int d^2\beta\, e^{-(1/2)|\beta|^2}\frac{\beta^{*N}}{\sqrt{N!}}|\alpha,\beta\rangle_{a,b} , \quad (28)$$

and hence by using (22)

$$|\Phi\rangle = \int d^2\beta\, e^{-(1/2)|\beta|^2} \frac{\beta^{*N}}{\sqrt{N!}} A^n \times e^{-(1/2)|A|^2} \left| \frac{B}{\sqrt{2}}, -\frac{B}{\sqrt{2}} e^{i\varphi} \right\rangle_{a,b} = \sum_{l,m} c_{l,m} |l,m\rangle_{a,b} , \tag{29}$$

where

$$c_{l,m} = \int d^2\beta\, e^{-(1/2)|\beta|^2} \frac{\beta^{*N}}{\sqrt{N!}} A^n e^{-(1/2)|A|^2-(1/2)|B|^2} \times \frac{1}{\sqrt{l!}} \left[\frac{B}{\sqrt{2}} \right]^l \frac{1}{\sqrt{m!}} \left[-\frac{B}{\sqrt{2}} e^{i\varphi} \right]^m , \tag{30}$$

and where A and B are defined by (19) and (21). On using binomial expansions, (30) reduces to the moments of the Gaussian distribution with the result

$$c_{l,m} = e^{-(1/2)|\alpha|^2} \frac{\sqrt{N!}(-e^{i\varphi})^m(-e^{-i\varphi})^N}{\sqrt{l!m!2^n}\,2^{l+m}} \times \alpha^{n+l+m-N} \sum_k \begin{bmatrix} n \\ k \end{bmatrix} \begin{bmatrix} l+m \\ N-k \end{bmatrix} (-1)^k . \tag{31}$$

Using (17) and (29), and on restoring the normalization, the density matrix for the b mode can be written in the form

$$\rho^{(n)(b)} = \frac{\sum_{m,m',l} |m\rangle_{bb}\langle m'| c_{l,m} c^*_{l,m'}}{\sum_{l,m} |c_{l,m}|^2} . \tag{32}$$

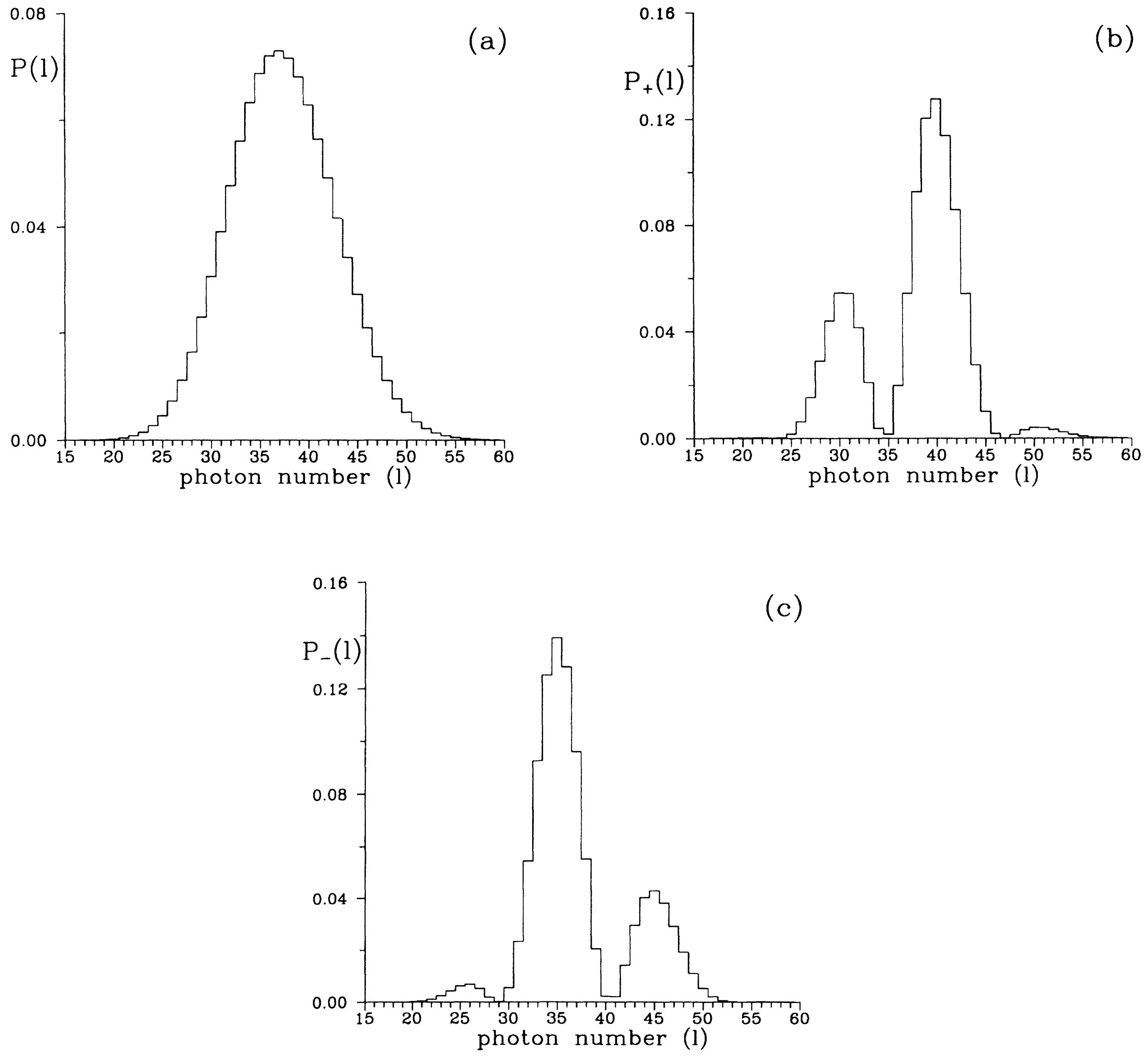

FIG. 2. (a) The number distribution $P(l)$ for both modes (it is the same) in the long-time limit with the parameters $N=40$, $n=5$; and for the eigenvectors of these modes in this limit, (b) $P_+(l)$ and (c) $P_-(l)$.

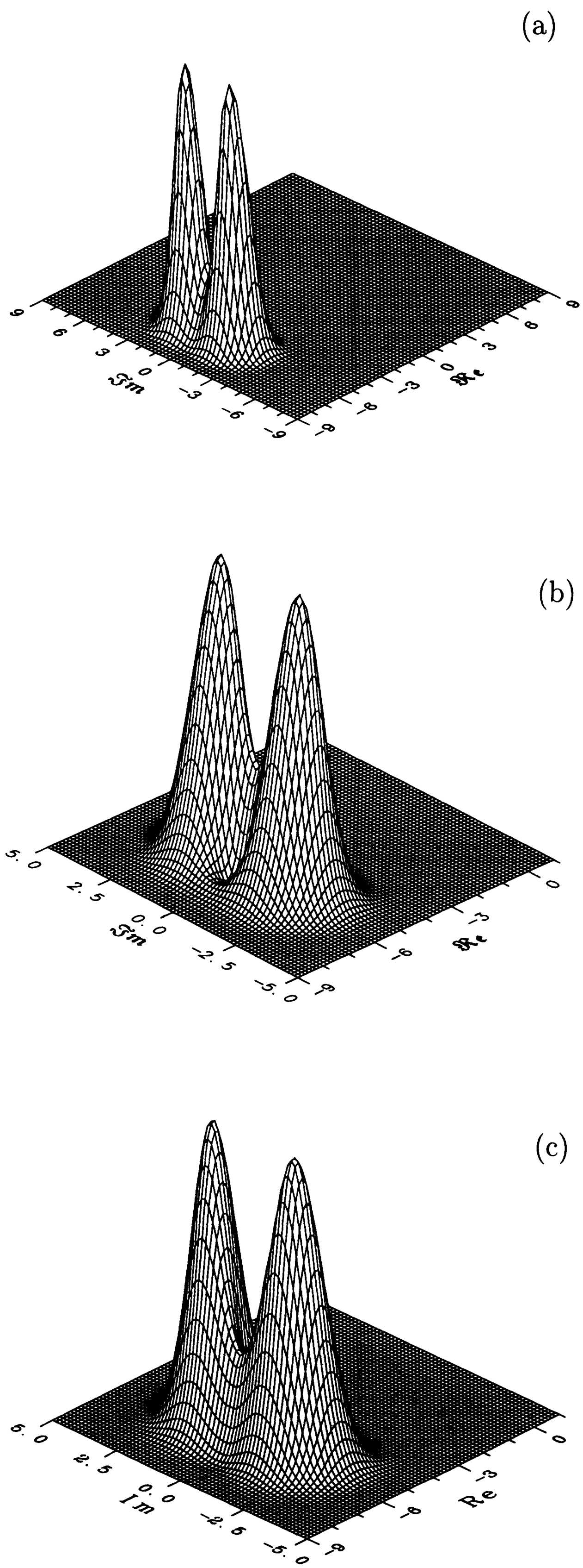

FIG. 3. (a) The Q distribution for the b mode in the long-time limit with the same parameters as in Fig. 2; and for the eigenvectors of the b mode in this limit, (b) Q_+ and (c) Q_-.

The photon distribution for the b mode is then

$$P_m^{(n)(b)} = \frac{\sum_l |c_{l,m}|^2}{\sum_{l,m} |c_{l,m}|^2} . \tag{33}$$

Using (32) the quasiprobability distribution like the Q function can also be evaluated:

$$Q_b^{(n)}(\beta,\beta^*) = \sum_{m,m'} \rho^{(n)(b)}_{m,m'} \frac{\beta^{*m}\beta^{m'}}{\sqrt{m!m'!}} e^{-|\beta|^2} . \tag{34}$$

In Figs. 2 and 3 we show the photon distribution and the Q function for certain values of the parameters. Figure 3 shows that the Q distribution is bimodal. The density matrix of the b mode can in fact be put in the diagonal form. A numerical calculation using (32) shows that

$$\rho^{(n)(b)} = \tfrac{1}{2}(|\Lambda_+\rangle\langle\Lambda_+| + |\Lambda_-\rangle\langle\Lambda_-|) , \tag{35}$$

where $|\Lambda_\pm\rangle$ are the eigenvectors of $\rho^{(n)(b)}$. We also show in Fig. 2 the distribution of photon numbers in the states $|\Lambda_\pm\rangle$ and in Fig. 3 the corresponding Q distributions, i.e., we show the quantities

$$P^{(n)(b)}_{\pm l} = |\langle l|\Lambda_\pm\rangle|^2, \quad Q_\pm = \frac{1}{\pi}|\langle\beta|\Lambda_\pm\rangle|^2 . \tag{36}$$

It should be noted that the distributions $P_\pm$ are far from Poissonian though the distribution $P(l)$ is close to Poissonian. The mean and variance of the distribution $P(l)$ are 37.37 and 29.24, respectively.

IV. PHOTON STATISTICS OF THE b MODE FOR AN ARBITRARY COUNTING INTERVAL

In this section we examine (14) for arbitrary counting intervals. We assume the initial state (28), which on substituting in (14) yields

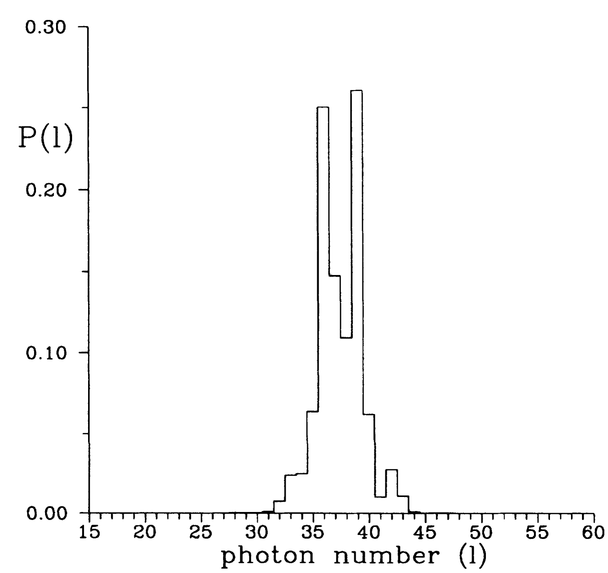

FIG. 4. The photon number distribution $P^{(n)(b)}(l)$ of the b mode for $N=40$, $n=5$ at an intermediate $\Gamma=3$.

$$
\begin{aligned}
|\Phi\rangle &= \int d^2\beta e^{-(1/2)|\beta|^2} \frac{\beta^{*N}}{\sqrt{N!}} A^n e^{-2Rt\hat{A}^\dagger\hat{A}} |\alpha,\beta\rangle_{a,b} , \\
&= \int d^2\beta e^{-(1/2)|\beta|^2} \frac{\beta^{*N}}{\sqrt{N!}} A^n e^{-2Rt\hat{A}^\dagger\hat{A}} | A,B\rangle_{A,B} , \\
&= \int d^2\beta e^{-(1/2)|\beta|^2} \frac{\beta^{*N}}{\sqrt{N!}} A^n e^{-(1/2)|A|^2(1-e^{-4Rt})} | Ae^{-2Rt},B\rangle_{A,B} , \\
&= \int d^2\beta e^{-(1/2)|\beta|^2} \frac{\beta^{*N}}{\sqrt{N!}} A^n e^{-(1/2)|A|^2(1-e^{-4Rt})} |\tilde{\alpha}(t),\tilde{\beta}(t)\rangle_{a,b} ,
\end{aligned}
\tag{37}
$$

where

$$
\tilde{\alpha}(t) = \frac{Ae^{-2Rt}+B}{\sqrt{2}}, \quad \tilde{\beta}(t) = \frac{Ae^{-2Rt}-B}{\sqrt{2}} e^{i\varphi} , \tag{38}
$$

and where A and B are defined by (19) and (21). The result (37) can again be expressed as

$$
|\Phi\rangle = \sum_{l,m} c_{l,m}(t) |l,m\rangle , \tag{39}
$$

where

$$
c_{l,m}(t) = \int d^2\beta\, e^{-(1/2)|\beta|^2} \frac{\beta^{*N}}{\sqrt{N!}} A^n e^{-(1/2)|A|^2(1-e^{-4Rt})} e^{-(1/2)|\tilde{\alpha}(t)|^2-(1/2)|\tilde{\beta}(t)|^2} \frac{[\tilde{\alpha}(t)]^l}{\sqrt{l!}} \frac{[\tilde{\beta}(t)]^m}{\sqrt{m!}} . \tag{40}
$$

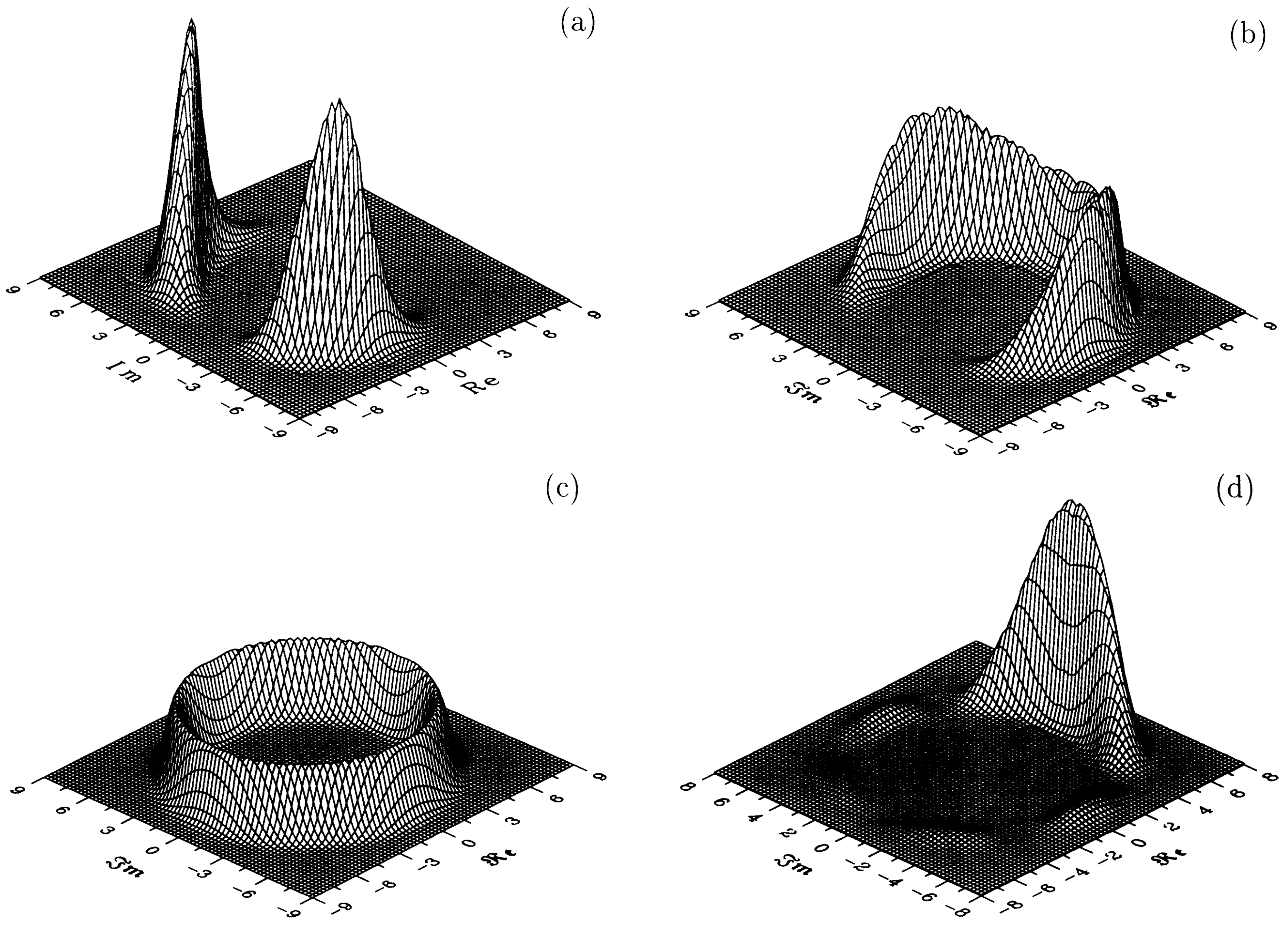

FIG. 5. The Q distribution of the b mode for the same situation as in Fig. 4, with (a) $\Gamma=3$, and (b) $\Gamma=1$. These are to be compared with the Q distribution f or (c) a number state $|40\rangle$, and (d) the state $\sum_{n=35}^{40}|n\rangle$.

On carrying out the binomial expansions and the Gaussian integrals we find that

$$c_{l,m}(t)=e^{-(1/2)|\alpha|^2}\frac{\sqrt{N}!(-e^{i\varphi})^m(-e^{-i\varphi})^N}{\sqrt{l!m!2^n2^{l+m}}}\alpha^{n+l+m-N}\times\sum_k\begin{bmatrix}n\\k\end{bmatrix}(-1)^k\sum_q\begin{bmatrix}l\\q\end{bmatrix}\begin{bmatrix}m\\N-k-q\end{bmatrix}(1-e^{-2Rt})^{m+k-N+2q}(1+e^{-2Rt})^{N+l-k-2q}\,, \tag{41}$$

which in the limit $Rt \ll 1$ reduces to

$$c_{l,m}(t)=e^{-(1/2)|\alpha|^2}\frac{\sqrt{N}!(-e^{i\varphi})^m(-e^{-i\varphi})^N}{\sqrt{l!m!2^n}}\alpha^{n+l+m-N}\sum_k\begin{bmatrix}n\\k\end{bmatrix}\sum_q\begin{bmatrix}l\\q\end{bmatrix}\begin{bmatrix}m\\N-k-q\end{bmatrix}(-1)^k(Rt)^{m+k-N+2q}\,. \tag{42}$$

The time-dependent density matrix for the b mode is then obtained by using (41) in (32).

For numerical work we choose the initial excitation in the two fields to be equal $N=|\alpha|^2$. We also introduce the parameter

$$\Gamma=Rt|\alpha|^2\,, \tag{43}$$

which is equal to the excitation rate (in the field α) multiplied by the counting interval. The characteristics of the resulting field depend on the parameter Γ and the number of "yes" measurements. In Fig. 4 we display the distribution of photons in the b mode starting from an initial Fock state with the number of photons equal to 40. Note that the variance of this distribution is quite small relative to its mean. The numerical calculation gives mean equal to 37.45; variance equal to 4.07. The variance generally increases with an increase in the parameter Γ. This can be understood from the fact that as the counting interval increases, a Fock state evolves into a superposition of more and more Fock states. In Fig. 5 we show the Q distribution of the field for two different counting times. For comparison we also show the Q distribution for the input field as well as the Q distribution for a field which is the uniform superposition of $|n\rangle$

$$|\psi\rangle=\sum_{n=35}^{40}|n\rangle\,. \tag{44}$$

As Wigner distributions show a rather oscillatory behavior for relatively high number states and the evolving field distributions, we show in Fig. 6 for completeness this distribution only for a small initial number of photons ($N=10$) where a bifurcation is still recognizable.

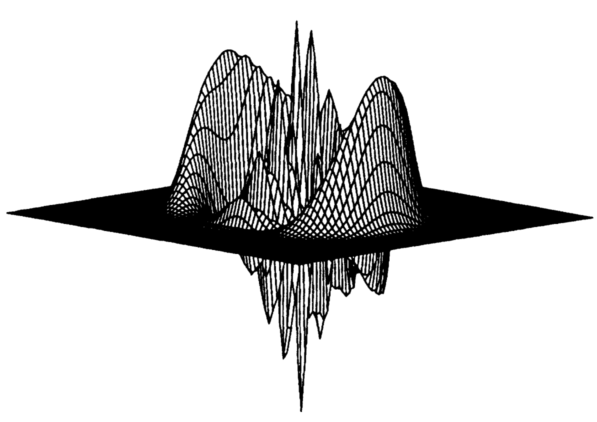

FIG. 6. The Wigner distribution of the b mode for the parameters $N=10$, $n=5$, and $\Gamma=1.5$.

V. PHASE DISTRIBUTION OF THE FIELD FOLLOWING CONTINUOUS MEASUREMENTS

In the literature considerable attention has been paid to the phase properties [12] of the radiation field. The b mode of the field has been taken to be in a Fock state. This mode has a uniform phase distribution, i.e.,

$$P(\theta)=\frac{1}{2\pi},\quad P(\theta)=\frac{1}{2\pi}\langle\theta|\rho|\theta\rangle,\quad |\theta\rangle=\sum e^{in\theta}|n\rangle\,. \tag{45}$$

We have already seen in the earlier section that in the present model there is transfer of coherence from the a mode to the b mode. We would thus expect the phase distribution of the b mode to undergo remarkable changes. The phase distribution can be calculated using (32) and (41) in (45). Some typical results are shown in Fig. 7. We find that the phase distribution first narrows and then bifurcates [13]. One can give an elementary ar-

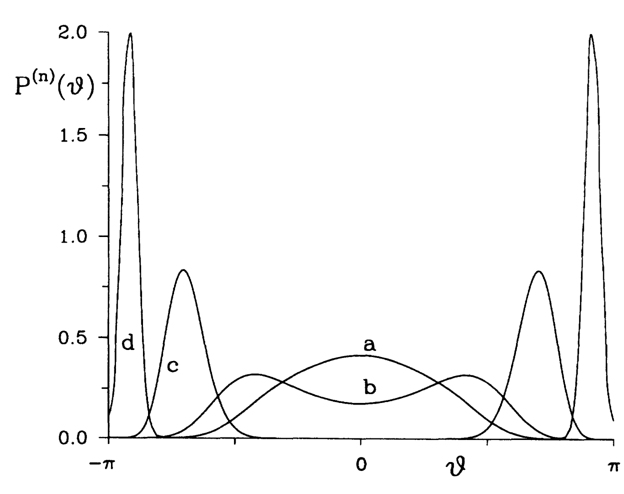

FIG. 7. The phase distribution of the b mode for the parameters $N=40$, $n=5$ and the various Γ values: (a) $\Gamma=0.5$, (b) $\Gamma=1$, (c) $\Gamma=3$, and (d) $\Gamma\to\infty$.

gument to understand how the changes in the phase distribution come about. Consider the excitation of the atom by the two-mode field in the state $|\alpha,N\rangle$. One has two pathways of excitation—the photon can be absorbed either from the mode a or from the mode b. These two pathways change the state of the field to

$$\begin{aligned} |\alpha,N\rangle_{a,b} &\rightarrow \alpha|\alpha,N\rangle_{a,b} , \\ |\alpha,N\rangle_{a,b} &\rightarrow \sqrt{N}\,|\alpha,N-1\rangle_{a,b} . \end{aligned} \tag{46}$$

Now one also has the probability of interference between these two pathways since the atom is initially prepared in a coherent superposition of two ground states. Thus after the absorption the state of the b mode is

$$|N\rangle_b \rightarrow (\alpha|N\rangle_b + e^{-i\varphi}\sqrt{N}\,|N-1\rangle_b) . \tag{47}$$

One has a superposition of two Fock states. Further absorption events produce a superposition of more and more Fock states. However "no" absorption events degrade such a superposition and a combined effect of both "yes" and "no" events results in a distribution like the one shown in Fig. 7. This is also clear from our comparisons of Figs. 5(d), 5(a), and 5(b). It is interesting to observe that each peak of the bimodal distribution narrows as the counting time increases.

In conclusion we have shown how the continuous measurements on a two-mode radiation field have potential for the production of a variety of quantum states of the field.

ACKNOWLEDGMENTS

This work was partially supported by the Office of Naval Research. One of us (M.O.) would like to thank Fondecyt and Fundacion Andes for partial support.

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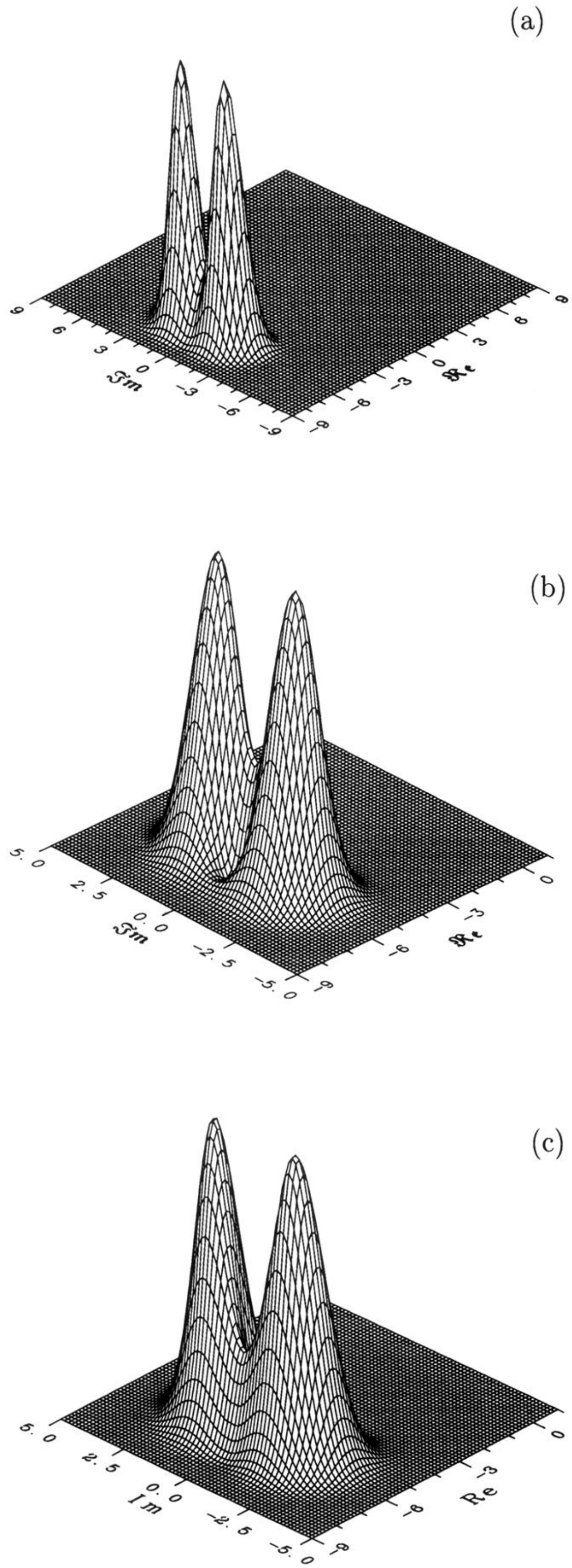

FIG. 3. (a) The Q distribution for the b mode in the long-time limit with the same parameters as in Fig. 2; and for the eigenvectors of the b mode in this limit, (b) Q_+ and (c) Q_-.

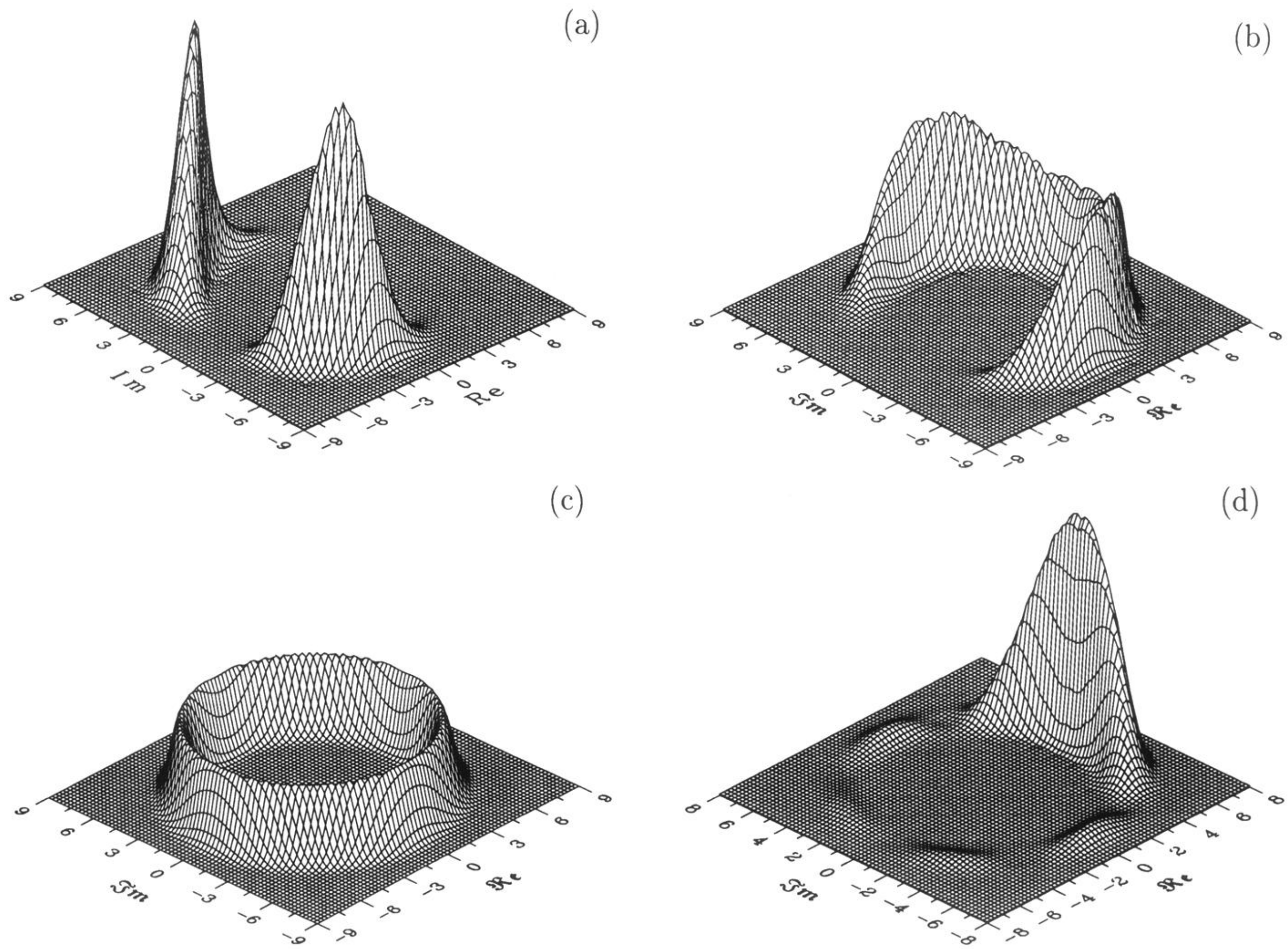

FIG. 5. The Q distribution of the b mode for the same situation as in Fig. 4, with (a) $\Gamma=3$, and (b) $\Gamma=1$. These are to be compared with the Q distribution f or (c) a number state $|40\rangle$, and (d) the state $\sum_{n=35}^{40}|n\rangle$.

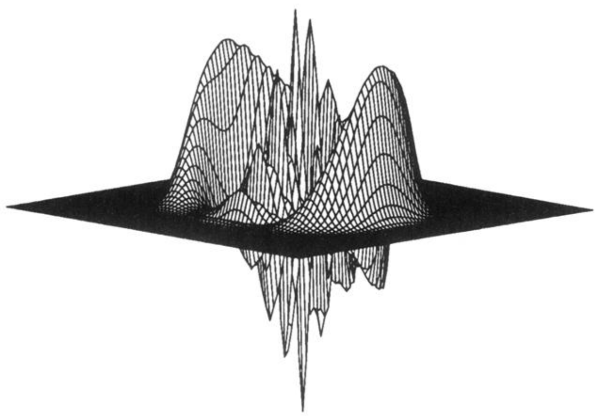

FIG. 6. The Wigner distribution of the b mode for the parameters $N=10$, $n=5$, and $\Gamma=1.5$.