Quantum noise reduction by radiation pressure

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We show that a linear Fabry-Pérot cavity with an oscillating end mirror can be used for quantum noise reduction. For a high-quality factor of the mechanical oscillator the output quantum fluctuations of the monochromatic light beam can be significantly squeezed at a frequency very close to that of the impinging light. The analysis is performed by taking into account the coupling of the system with the external world.

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I. INTRODUCTION

Even though, to date [1], high quadrature squeezing has been obtained with a transparent crystal having a $\chi^{(2)}$ nonlinear polarization, the search for high squeezing generated by materials with $\chi^{(3)}$ nonlinearities is motivated by the possibility of building up a quantum-noise-reduction device. In these materials, the squeezing is produced without frequency conversion, in contrast to the $\chi^{(2)}$ case, where it is usually generated in proximity to the first subharmonic frequency of the incident beam. Media with $\chi^{(3)}$ nonlinearities are the so-called Kerr media in nonlinear optics [2]. In Kerr media the effect is proportional to the intensity of the input beam and, because of small $\chi^{(3)}$ nonlinearities, an intense input beam is required. Of course, by using a resonant optical cavity one can enhance the effect. However, losses limit the squeezing obtainable both in the free-propagation configuration [3] and in the cavity configuration [4]. The advantages and drawbacks of $\chi^{(3)}$ media for the generation of squeezed light was recently assessed [5]. The squeezing of quantum fluctuations in a Kerr medium is a consequence of the optical-path dependence on the intensity of the light beam. An intensity-dependent optical path, however, is not related only to Kerr media. It was shown several years ago [6] that an empty cavity with a moving mirror in its steady state may mimic a Kerr medium when it is illuminated with coherent light. The effect is completely due to the radiation pressure force. Indeed, bistable behavior, analogous to that of a $\chi^{(3)}$ medium in a cavity, was experimentally demonstrated in the optical domain [7] as well as in the microwave domain [8] with such a system. More recently [9], the same configuration was used to select small signals, such as those due to the gravitational waves. Furthermore it has been shown [10] that such a cavity with a free oscillating mirror might be employed as a quantum nondemolition device for the photon number measurement. Finally, as a model for squeezing, it was first proposed by Stenholm [11] without taking into account the effect of thermal fluctuations on the oscillating mirror. A phenomenological model, including input field fluctuations and thermal noise, was recently considered in a semiclassical approximation by Hilico *et al.* [5], with the pessimistic conclusion that squeezing is possible only at extremely low temperatures, while as the temperature rises it disappears completely. In the present paper we will analyze the same system, starting from a Hamiltonian model which includes the coupling of the system to the external world. We will show that, in the so-called adiabatic limit and for high quantity factor Q_m of the mechanical resonator, i.e., the oscillating mirror, the obtainable squeezing, in a given frequency range, is not washed out by the thermal fluctuations and could be very large.

II. THE HAMILTONIAN OF THE MODEL

As is well known [2], a Kerr medium has a refraction index depending on the light intensity. When one considers the medium in a resonant cavity, since the optical path depends on the refraction index, the impinging radiation practically "sees" an equivalent empty cavity with variable length depending on its intensity. We then consider a linear Fabry-Pérot empty cavity with one fixed mirror with transmissivity T_r and one perfectly reflecting end mirror. The completely reflecting mirror can move, undergoing harmonic oscillations damped by the coupling to a thermal bath in equilibrium at temperature T. One could consider such a perfectly reflecting mirror coated over the surface of a quartz with a high mechanical quality factor Q_m. We assume that light impinging on the fixed mirror produces a small detuning with respect to one cavity mode. The cavity resonances are calculated in the absence of the impinging field. If L is the equilibrium cavity length, the resonant angular frequency of the cavity will be

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$$\omega_c = n\pi\frac{c}{L} , \tag{1}$$

where n is an arbitrary integer number and c the speed of light. Furthermore, we assume that at the frequency of the impinging field $\omega_0/2\pi$, the fixed mirror with transmissivity T_r does not introduce any excess noise except for the input field noise. We also assume that retardation effects, due to the oscillating mirror, in the intracavity field are negligible. We will use a field intensity such that the correction to the radiation pressure force, due to the Doppler frequency shift of the photons [12] on the moving mirror, is completely negligible. This means considering the damping coefficient of the oscillating mirror to be due only to the coupling with the thermal bath. We are thus able to write the system Hamiltonian [9,10] as

$$H_{\rm sys} = \hbar\omega_c(a^\dagger a + \tfrac{1}{2}) + \frac{p^2}{2m} + \tfrac{1}{2}m\omega_m^2 x^2 + H_{\rm int} , \tag{2}$$

where a and $a^\dagger$ are the boson operators of the resonant cavity mode at frequency $\omega_c/2\pi$; p and x are the momentum and the displacement, respectively, from the equilibrium position of the oscillating mirror with mass m and oscillation frequency $\omega_m/2\pi$. $H_{\rm int}$ accounts for the interaction between the cavity mode and the oscillating mirror. Since we have assumed no retardation effects, this is simply

$$H_{\rm int} = -\hbar g a^\dagger a x , \tag{3}$$

where the coupling constant is given by

$$g = \frac{\omega_c}{L} . \tag{4}$$

$H_{\rm int}$ represents the effect of the radiation pressure force $F_R = (\hbar\omega_c/L)a^\dagger a$ which causes the instantaneous displacement x of the mirror. $H_{\rm int}$ can be understood easily as follows. The cavity length will vary because of the radiation pressure force of the radiation inside the cavity. A variation of length will correspond to a variation of frequency through Eq. (1), i.e.,

$$\delta\omega_c = \frac{\partial\omega_c}{\partial L}\delta L = -\frac{\omega_c}{L}x , \tag{5}$$

thus

$$H_{\rm int} = -\hbar g(a^\dagger a + \tfrac{1}{2})x . \tag{6}$$

In Eq. (3) the very small vacuum contribution was neglected.

We have to take into account the damping of the oscillating mirror due to the coupling with the thermal bath. We model that bath with a large number of harmonic oscillators [13,14]. With this choice the frequency spectrum of the bath is dense and regular, and the coupling between system and bath can be considered linear, with the coupling constants κ_n being nonsingular functions of the frequency [14].

The bath Hamiltonian is then

$$H_b = \sum_n \left[\frac{P_n^2}{2M_n} + \kappa_n \frac{Q_n^2}{2} \right] , \tag{7}$$

while the coupling bath mirror within our assumptions will be

$$H_{bm} = -\sum_n \kappa_n Q_n x . \tag{8}$$

After a convenient canonical transformation of the bath variables, and considering a continuum spectrum for the bath frequency [14], we obtain the following thermal bath-mirror interaction Hamiltonian:

$$H_{bm} = \tfrac{1}{2}\int_0^\infty d\Omega\{[P(\Omega) - \kappa(\Omega)x]^2 + \Omega^2 Q^2(\Omega)\} . \tag{9}$$

We have also to take into account the cavity loss due to the coupling of the internal mode with all external modes of the radiation through the first mirror. This is usually obtained [15] in the rotating-wave approximation (RWA) as

$$H_{rc} = i\hbar\int_{-\infty}^{+\infty} d\omega\, s(\omega)[b^\dagger(\omega)a - b(\omega)a^\dagger] , \tag{10}$$

where $b(\omega)$ and $b^\dagger(\omega)$ are the Bose operators for the infinite modes of the external radiation field at frequency $\omega/2\pi$, and $s(\omega)$ is the coupling constant. The RWA is valid as long as the measurement time is large with respect to ω_c^{-1}, as it usually is in the optical regime. The RWA cannot be used for the coupling of the oscillating mirror to the thermal bath because the mechanical frequency $\omega_m/2\pi$ will be many orders of magnitude smaller than $\omega_c/2\pi$. This is the reason why we used the more realistic [14] interaction in Eq. (9). This choice of mechanical frequency also ensures that the number of photons generated by the Casimir effect [16] is completely negligible, and we actually are in the so-called adiabatic approximation, i.e., the cavity round trip time is much smaller than the mirror's period of oscillation. Finally, within the assumptions given above, the Hamiltonian of the system coupled to the "rest of Universe" [17] is

$$\begin{aligned} H = {} & \hbar\omega_c a^\dagger a + \frac{p^2}{2m} + \tfrac{1}{2}m\omega_m^2 x^2 - \hbar g a^\dagger a x \\ & + \tfrac{1}{2}\int_0^\infty d\Omega\{[P(\Omega) - \kappa(\Omega)x]^2 + \Omega^2 Q^2(\Omega)\} \\ & + i\hbar\int_{-\infty}^{+\infty} d\omega\, s(\omega)[b^\dagger(\omega)a - b(\omega)a^\dagger] \\ & + \hbar\int_{-\infty}^{+\infty} d\omega\, \omega b^\dagger(\omega)b(\omega) , \end{aligned} \tag{11}$$

and we have neglected the vacuum terms.

III. THE DYNAMICS OF THE SYSTEM

With the standard procedure, well described in Gardiner's book [14], of solving dynamical equations, we can eliminate the thermal bath variables and the variables of the radiation bath. In a frame rotating at the frequency $\omega_0/2\pi$ of the impinging field we can write the equations of motion for the cavity mode and the mechanical oscillator

$$\frac{da}{dt}=-i(\omega_c-\omega_0-gx)a-\frac{\gamma_c}{2}a+\sqrt{\gamma_c}a_{\rm in}(t)\ ,$$
$$\frac{da^\dagger}{dt}=i(\omega_c-\omega_0-gx)a^\dagger-\frac{\gamma_c}{2}a^\dagger+\sqrt{\gamma_c}a_{\rm in}^\dagger(t)\ , \tag{12}$$
$$\frac{dx}{dt}=\frac{p}{m}\ ,$$
$$\frac{dp}{dt}=-m\omega_m^2x+\hbar g a^\dagger a-\frac{\gamma_m}{2m}p-\sqrt{\gamma_m}\epsilon_{\rm in}(t)\ ,$$

These equations are obtained in the Markovian approximation [14], and have to be considered as coupled stochastic differential equations in the Stratonovich sense, and are valid in the low damping regime where $\gamma_m/m \ll \omega_m$. The constants $\gamma_c=2\pi s^2(\omega)$ and $\gamma_m/m=\pi\kappa^2(\Omega)/m$, respectively, represent the cavity linewidth and the damping constant of the mechanical oscillator due to the interaction with the external baths. They are constant in the so-called first Markovian limit [14]. Of course, γ_c accounts for the loss of photons in the cavity mode due to the interaction with the radiation field, i.e., $\gamma_c=cT_r/2L$, and $a_{\rm in}^\dagger(t)a_{\rm in}(t)$ is the flux of photons hitting the first cavity mirror in one second. In the rotating frame for an impinging monochromatic coherent field, we have

$$\begin{aligned}a_{\rm in}(t)&=\frac{-1}{\sqrt{2\pi}}\int_0^\infty d\omega\, e^{-i\omega t}b(\omega)\\&=\alpha_{\rm in}+\delta a_{\rm in}(t)\ ,\end{aligned} \tag{13}$$

where $\delta a_{\rm in}(t)$ represents the quantum fluctuation with respect the mean amplitude $\langle a_{\rm in}\rangle=\alpha_{\rm in}$ of the impinging mode. $\delta a_{\rm in}(t)$ can be considered "white quantum noise" [14], at least in a narrow bandwidth at ω_0 larger than γ_c, with expectations

$$\begin{aligned}&\langle\delta a_{\rm in}(t)\rangle=0\ ,\quad \langle\delta a_{\rm in}^\dagger(t)\delta a_{\rm in}(t')\rangle=\bar n\delta(t-t')\ ,\\&\langle\delta a_{\rm in}(t)\delta a_{\rm in}^\dagger(t')\rangle=(1+\bar n)\delta(t-t')\ ,\end{aligned} \tag{14}$$

where $\bar n$ represents the number of photons at the frequency of the impinging field, which is negligibly small at optical or infrared frequencies. The entire contribution of the thermal bath variables is included in $\sqrt{\gamma_m}\epsilon_{\rm in}(t)$, which can be considered a stochastic noise force depending only on the state of the thermal bath at time t_0, where t_0 is the initial time of the interaction. It can be shown [14] that this noise term at high temperature behaves as a classical noise, and has the following expectation values with respect to the equilibrium distribution of the thermal bath at time t_0;

$$\begin{aligned}&\langle\epsilon_{\rm in}(t)\rangle=0\ ,\\&\langle\epsilon_{\rm in}(t)\epsilon_{\rm in}(t')\rangle=K_BT\delta(t-t')\ .\end{aligned} \tag{15}$$

The high-temperature condition in the present case reads $\hbar\omega_m \ll K_BT$, where K_B is Boltzmann's constant, which is always satisfied in the microwave regime and for a temperature that is not extremely low. It means that we can disregard the quantum nature of the mirror's motion, and we can consider it a classical oscillator. Equations (12) are valid after an initial transient, and must then be considered for times $t>t_0$ [14].

IV. THE STEADY STATE

We are actually interested in the steady-state regime and in small fluctuations with respect to the steady state. Then, as is usual in the semiclassical approximation [18], we take the expectation with respect to the steady state of Eq. (12). Neglecting all the fluctuations, we set

$$\frac{d\langle a\rangle_s}{dt}=\frac{d\langle a^\dagger\rangle_s}{dt}=\frac{d\langle x\rangle_s}{dt}=\frac{d\langle p\rangle_s}{dt}=0\ . \tag{16}$$

Thus we obtain

$$\begin{aligned}&i(\omega_c-\omega_0-gx_s)\alpha_s+\frac{\gamma_c}{2}\alpha_s-\sqrt{\gamma_c}\alpha_{\rm in}=0\ ,\\&i(\omega_c-\omega_0-gx_s)\alpha_s^*-\frac{\gamma_c}{2}\alpha_s^*+\sqrt{\gamma_c}\alpha_{\rm in}^*=0\ ,\\&p_s=0\ ,\\&m\omega_m^2x_s-\hbar g\alpha_s^*\alpha_s=0\ ,\end{aligned} \tag{17}$$

where the subscript s is the steady-state value, i.e., $(\alpha_s=\langle a\rangle_s,\ \alpha_s^*=\langle a^\dagger\rangle_s,\ x_s=\langle x\rangle_s,\ p_s=\langle p\rangle_s)$. The steady-state solutions are

$$p_s=0\ ,\quad x_s=\frac{\hbar g}{m\omega_m^2}|\alpha_s|^2\ ,\quad \alpha_s=\frac{\sqrt{\gamma_c}\alpha_{\rm in}}{\frac{\gamma_c}{2}+i(\omega_c-\omega_0-gx_s)}\ . \tag{18}$$

Let us introduce the dimensionless quantity $y=x_s/L$. Thus, combining Eqs. (18), we obtain the equation of state

$$F(y)=E\ , \tag{19}$$

with

$$E=\frac{\gamma_cP_{\rm in}}{m\omega_c\omega_m^2\omega_0L^2} \tag{20}$$

and

$$F(y)=y^3-2\frac{\omega_c-\omega_0}{\omega_c}y^2+\left[\left[\frac{\omega_c-\omega_0}{\omega_c}\right]^2+\left[\frac{\gamma_c}{2\omega_c}\right]^2\right]y\ , \tag{21}$$

where we introduced the power of the input field:

$$P_{\rm in}=\hbar\omega_0|\alpha_{\rm in}|^2\ . \tag{22}$$

Equation (19) has three real solutions as long as $F'(y)=dF/dy=0$ is satisfied by two real values of y. This is possible for

$$|\omega_c-\omega_0|\rangle\frac{\sqrt{3}}{2}\gamma_c\ , \tag{23}$$

which represents the bistability condition. The steady-state values (18) are then consistent with the semiclassical approximation for values of y far from the turning points. In the vicinity of such points, one can no longer neglect fluctuations. Indeed, these might be large with respect to the steady-state solutions. In the following we will introduce a Ginzburg criterion [19] in order to be sure that fluctuations are negligibly small near the steady-state values chosen. Thus, away from the "critical points" the stable steady-state solutions are characterized by $F'(y)>0$, while $F'(y)<0$ gives the unstable solutions. However, one has to be careful in choosing the stable solutions even for $F'(y)>0$. This is not a sufficient condition because with higher input power $P_{\rm in}$ the solution may become unstable even for $F'(y)>0$. This is due to the fact that at input power higher than a threshold value $P_{\rm in}^{(\rm th)}$ the mirror's response becomes anomalous, and a stabilization of the mirror occurs [7] as a consequence of the radiation pressure force. This can be discussed quantitatively by means of the local Liapunov criterion [20]. In order to obtain equations that are not extremely cumbersome, and to simplify the presentation, in the following we will numerically study the stability of the steady-state solutions for a given set of parameters, and we will give the threshold power for the chosen set of parameters.

V. DYNAMICS OF SMALL FLUCTUATIONS

With the above in mind, we study the dynamics of small fluctuations near a stable steady state, and by writing

$$a=\alpha_s+\delta a\ , \quad x=x_s+\delta x\ , \tag{24}$$

we are able to write the evolution equations for the fluctuations up to first order:

$$\frac{d}{dt}\delta a=-i\left[\omega_c-\omega_0-\frac{\hbar g^2}{m\omega_m^2}|\alpha_s|^2\right]\delta a -\frac{\gamma_c}{2}\delta a+ig\alpha_s\delta x+\sqrt{\gamma_c}\delta a_{\rm in}\ , \tag{25a}$$

$$\frac{d^2}{d^2t}\delta x=-\omega_m^2\delta x+\frac{\hbar g}{m}\alpha_s^*\delta a+\frac{\hbar g}{m}\alpha_s\delta a^\dagger -\frac{\gamma_m}{2m}\frac{d}{dt}\delta x-\frac{\sqrt{\gamma_m}}{m}\epsilon_{\rm in}\ , \tag{25b}$$

and, obviously, the Hermitian conjugate of Eq. (25a).

From Eqs. (25) we see the effect of the coupling between the cavity mode and oscillating mirror. As the intracavity intensity field is $|\alpha_s|^2$, there will be an intensity-dependent phase shift. It is obvious that δx depends linearly on $\delta a^\dagger$, thus introducing a coupling between the fluctuation δa and its conjugate $\delta a^\dagger$. Thus, as a consequence of the dependence of δa on δx, the fluctuations of the internal field could be squeezed. However, δx depends on δa as well, so a further dynamical phase shift and damping is introduced by the coupling of the cavity mode to that mirror. Let us introduce the Fourier transform

$$f(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}e^{i\omega t}\tilde{f}(\omega)d\omega\ . \tag{26}$$

Then, in the frequency domain, Eqs. (25) become

$$\left[\frac{\gamma_c}{2}+i(\omega_c-\omega_0-\omega)-i\frac{\hbar g^2}{m\omega_m^2}|\alpha_s|^2\right]\delta\tilde{a}-ig\alpha_s\delta\tilde{x} =\sqrt{\gamma_c}\delta\tilde{a}_{\rm in}\ ,$$
$$-\frac{\hbar g}{m}\alpha_s\delta\tilde{a}e-\frac{\hbar g}{m}\alpha_s\delta\tilde{a}^\dagger+\left[(\omega_m^2-\omega^2)-i\frac{\gamma_m\omega}{2m}\right]\delta\tilde{x} =-\frac{\sqrt{\gamma_m}}{m}\tilde{\epsilon}_{\rm in}\ , \tag{27}$$

with $\delta\tilde{a}^\dagger(\omega)=[\delta\tilde{a}(-\omega)]^+$. We introduce the dimensionless dynamical response factor of the mirror:

$$\chi(\omega)=\frac{\omega_m^2}{(\omega_m^2-\omega^2)-i\dfrac{\gamma_m\omega}{2m}}=\chi_1(\omega)+i\chi_2(\omega)\ , \tag{28}$$

with $\chi^*(\omega)=\chi(-\omega)$. Eliminating $\delta\tilde{x}$ in Eq. (27), we can write

$$\frac{\gamma_c}{2}+i(\phi-\omega-\chi(\omega)|K|)\delta\tilde{a}-i\chi(\omega)K\delta\tilde{a}^\dagger =\sqrt{\gamma_c}\delta\tilde{a}_{\rm in}-\rho\chi(\omega)\frac{\sqrt{\gamma_m}}{m\omega_m^2}\tilde{\epsilon}_{\rm in} \tag{29}$$

and its Hermitian conjugate, where

$$K=\frac{\hbar g^2}{m\omega_m^2}\alpha_s^2\ , \quad \rho=ig\alpha_s\ , \tag{30a}$$

$$\phi=\omega_c-\omega_0-\frac{\hbar g^2}{m\omega_m^2}|\alpha_s|^2=\omega_c(1-y)-\omega_0\ , \tag{30b}$$

where we used Eqs. (18) and (14) together with the definition of $y=x_s/L$. We see that, besides an additional frequency-independent detuning, the interaction with the moving mirror introduces a frequency-dependent dissipative term in Eq. (29):

$$\gamma_a(\omega)=\chi_2(\omega)|K|\ , \tag{31}$$

which is maximum at $\omega=\omega_m$, while the frequency-dependent phase shift $\chi_1(\omega)|K|$ is zero at the same frequency. We want to avoid such additional dissipation, which might destroy the squeezing at any frequency, and in what follows we will require

$$\gamma_a(\omega)\ll\frac{\gamma_c}{2}\ . \tag{32}$$

Furthermore, we see from Eq. (29) that a thermal noise contribution is added to the usual radiation noise of the input field.

VI. THE OUTPUT QUADRATURE NOISE SPECTRUM

The input-output theory [21] gives the following relation among the incoming field, internal field, and output field as a consequence of boundary conditions at the fixed mirror surface. Within the assumption for the validity of such theory, we obtain

$$\delta\tilde{a}_{\rm out}+\delta\tilde{a}_{\rm in}=\sqrt{\gamma_c}\,\delta\tilde{a}\ , \tag{33}$$

And, finally, by using Eq. (29) and its conjugate, we can write

$$\delta\tilde{a}_{\rm out}(\omega)=\zeta(\omega)\delta\tilde{a}_{\rm in}(\omega)+\eta(\omega)\delta\tilde{a}^{\dagger}_{\rm in}(\omega)+\xi(\omega)\tilde{\epsilon}_{\rm in}(\omega) \tag{34}$$

and $\delta\tilde{a}^{\dagger}_{\rm out}(\omega)=[\delta\tilde{a}_{\rm out}(-\omega)]^{+}$, where

$$\zeta(\omega)=\frac{\omega^2+\left[\frac{\gamma_c}{2}-i\phi\right]\left[\frac{\gamma_c}{2}-2\chi_2(\omega)|K|+i(2\chi_1(\omega)|K|-\phi)\right]}{\Delta(\omega)}\ , \tag{35}$$

$$\eta(\omega)=\frac{i\gamma_c\chi(\omega)K}{\Delta(\omega)}=-\eta^{*}(-\omega)\ , \tag{36}$$

$$\xi(\omega)=\frac{-\rho\chi(\omega)\sqrt{\gamma_c\gamma_m}\left[\frac{\gamma_c}{2}-i(\phi+\omega)\right]}{m\omega_m^2\Delta(\omega)}\ , \tag{37}$$

and

$$\Delta(\omega)=\left[\frac{\gamma_c^2}{4}+\phi^2-\omega^2-2\phi\chi_1(\omega)|K|\right]-i(\gamma_c\omega+2\phi\chi_2(\omega)|K|)\ , \tag{38}$$

with $\Delta(-\omega)=\Delta^{*}(\omega)$. The Fourier transform of the output quadrature is

$$X_\theta^{\rm out}(\omega)=\tfrac{1}{2}[e^{-i\theta}a_{\rm out}(\omega)+e^{i\theta}a^{\dagger}_{\rm out}(\omega)]\ , \tag{39}$$

where θ is controlled by the experimenter in a homodyne detection scheme [22]. Then the spectral function of the output quadrature fluctuations is

$$\begin{aligned}\langle\delta X_\theta^{\rm out}(\omega)\delta X_\theta^{\rm out}(\omega')\rangle=\tfrac{1}{4}[&e^{-2i\theta}\langle\delta a_{\rm out}(\omega)\delta a_{\rm out}(\omega')\rangle+\langle\delta a_{\rm out}(\omega)\delta a^{\dagger}_{\rm out}(\omega')\rangle+\langle\delta a^{\dagger}_{\rm out}(\omega)\delta a_{\rm out}(\omega')\rangle\\&+e^{2i\theta}\langle\delta a^{\dagger}_{\rm out}(\omega)\delta a^{\dagger}_{\rm out}(\omega')\rangle]\ .\end{aligned} \tag{40}$$

By using Eqs. (33)–(38) together with $\delta\tilde{a}^{\dagger}(\omega)=[\delta\tilde{a}(-\omega)]^{+}$, $[\delta\tilde{a}^{\dagger}(-\omega)]^{+}=\delta\tilde{a}(\omega)$, $\tilde{\epsilon}_{\rm in}(\omega)=\tilde{\epsilon}^{*}_{\rm in}(-\omega)$, and the Fourier transforms of Eqs. (14) and (15), we obtain

$$\begin{aligned}\langle\delta X_\theta^{\rm out}(\omega)\delta X_\theta^{\rm out}(\omega')\rangle=\tfrac{1}{4}(&e^{-2i\theta}\{\zeta(\omega)\eta(-\omega)+\bar{n}[\zeta(\omega)+\zeta(-\omega)\eta(\omega)]+\xi(\omega)\xi(-\omega)K_BT\}+(1+\bar{n})[|\zeta(\omega)|^2+|\eta(-\omega)|^2]\\&+\bar{n}[|\zeta(-\omega)|^2+|\eta(\omega)|^2]+K_BT[|\xi(\omega)|^2+|\xi(-\omega)|^2]\\&+e^{2i\theta}\{\zeta^{*}(\omega)\eta^{*}(-\omega)+\bar{n}[\zeta^{*}(\omega)\eta^{*}(-\omega)+\zeta^{*}(-\omega)\eta^{*}(\omega)]+\xi^{*}(\omega)\xi^{*}(-\omega)K_BT\})\delta(\omega+\omega')\ .\end{aligned} \tag{41}$$

The quadrature noise spectrum normalized to the input shot noise [23] is then

$$\begin{aligned}S_\theta(\omega)&=\int d\omega'\frac{\langle\delta X_\theta^{\rm out}(\omega)\delta X_\theta^{\rm out}(\omega')\rangle}{\frac{1}{4}(1+2\bar{n})}\\&=\frac{1}{1+2\bar{n}}((1+\bar{n})|\zeta(\omega)|^2+\bar{n}|\zeta(-\omega)|^2+(1+2\bar{n})|\eta(\omega)|^2\\&\qquad+2\cos(2\theta)\{(1+\bar{n})[\zeta_1(\omega)\eta_1(-\omega)-\zeta_2(\omega)\eta_2(-\omega)]+\bar{n}[\zeta_1(-\omega)\eta_1(\omega)-\zeta_2(-\omega)\eta_2(\omega)]\}\\&\qquad+\sin(2\theta)\{(1+\bar{n})[\zeta_1(\omega)\eta_2(-\omega)+\zeta_2(\omega)\eta_1(-\omega)]+\bar{n}[\zeta_1(-\omega)\eta_2(\omega)+\zeta_2(-\omega)\eta_1(\omega)]\}\\&\qquad+K_BT\{2\cos(2\theta)[\xi_1(\omega)\xi_1(-\omega)-\xi_2(\omega)\xi_2(-\omega)]\\&\qquad+2\sin(2\theta)[\xi_1(\omega)\xi_2(-\omega)+\xi_1(-\omega)\xi_2(\omega)]+|\xi(\omega)|^2+|\xi(-\omega)|^2\})\ ,\end{aligned} \tag{42}$$

where the subscripts 1 and 2 represent the real and imaginary parts of the function, respectively. With some easy algebra, by using Eqs. (35)–(38), Eq. (42) can be written as

$$S_\theta(\omega)=1+\frac{2\gamma_c K}{(1+2\bar{n})|\Delta(\omega)|^2}\Bigg\{(1+2\bar{n})[\gamma_c K|\chi(\omega)|^2-2\omega\phi\chi_2(\omega)]-\chi_2(\omega)(\gamma_c^2/4+\omega^2+\phi^2)$$
$$+\cos(2\theta)\{\gamma_c(1+2\bar{n})[\phi\chi_1(\omega)-K|\chi(\omega)|^2]+\chi_2(\omega)(\gamma_c^2/4+\omega^2-\phi^2)\}$$
$$+\sin(2\theta)\{(1+2\bar{n})[\chi_1(\omega)(\gamma_c^2/4+\omega^2-\phi^2)+2\phi K|\chi(\omega)|^2]-\gamma_c\phi\chi_2(\omega)\}$$
$$+\frac{|\chi(\omega)|^2K_BT}{Q_m\hbar\omega_m}[\gamma_c^2/4+\omega^2+\phi^2-(\gamma_c^2/4+\omega^2-\phi^2)\cos(2\theta)+\gamma_c\phi\sin(2\theta)]\Bigg\} \tag{43}$$

where we introduced the mechanical quality factor $Q_m=m\omega_m/\gamma_m$, and chose the arbitrary phase of the impinging field such that α_s is real (this accounts for considering $|K|=K$). We have to choose the various values of the external parameters in order to satisfy the quadrature squeezing condition. However, it is better to define the optimum quadrature squeezing $S_{\rm opt}(\omega)$ by choosing $\theta(\omega)$ in such a way that $dS_\theta(\omega)/d\theta=0$, which gives

$$\theta(\omega)=\tfrac{1}{2}\tan^{-1}\left[\frac{B_2(\omega)}{B_1(\omega)}\right], \tag{44}$$

with

$$B_1(\omega)=\frac{2\gamma_c K}{|\Delta(\omega)|^2}\left[\gamma_c(1+2\bar{n})(\phi\chi_1(\omega)-K|\chi(\omega)|^2)+(\gamma_c^2/4+\omega^2-\phi^2)\left[\chi_2(\omega)-\frac{|\chi(\omega)|^2K_BT}{Q_m\hbar\omega_m}\right]\right], \tag{45a}$$

$$B_2(\omega)=\frac{2\gamma_c K}{|\Delta(\omega)|^2}\left[(1+2\bar{n})[\chi_1(\omega)(\gamma_c^2/4+\omega^2-\phi^2)+2\phi K|\chi(\omega)|^2]-\gamma_c\phi\left[\chi_2(\omega)-\frac{|\chi(\omega)|^2K_BT}{Q_m\hbar\omega_m}\right]\right]. \tag{45b}$$

Then, substituting back into Eq. (43), we obtain

$$S_{\rm opt}(\omega)=1+\frac{2\gamma_c K}{(1+2\bar{n})|\Delta(\omega)|^2}\Bigg[(1+2\bar{n})[\gamma_c K|\chi(\omega)|^2-2\omega\phi\chi_2(\omega)]+(\gamma_c^2/4+\omega^2+\phi^2)\left[\frac{|\chi(\omega)|^2K_BT}{Q_m\hbar\omega_m}-\chi_2(\omega)\right]$$
$$-\Bigg\{\Bigg[(1+2\bar{n})\gamma_c[K|\chi(\omega)|^2-\phi\chi_1(\omega)]+(\gamma_c^2/4+\omega^2-\phi^2)\left[\frac{|\chi(\omega)|^2K_BT}{Q_m\hbar\omega_m}-\chi_2(\omega)\right]\Bigg]^2$$
$$+\Bigg[(1+2\bar{n})[2\phi K|\chi(\omega)|^2+\chi_1(\omega)(\gamma_c^2/4+\omega^2-\phi^2)]$$
$$+\gamma_c\phi\left[\frac{|\chi(\omega)|^2K_BT}{Q_m\hbar\omega_m}-\chi_2(\omega)\right]\Bigg]^2\Bigg\}^{1/2}\Bigg]. \tag{46}$$

In order to obtain squeezing, the second term in the right-hand side must be negative and greater or equal to -1.

VII. INTENSITY SQUEEZING SPECTRUM

It is worthwhile also to consider the output intensity spectrum $S_I(\omega)$ [5], which is defined by

$$S_I(\omega)=\frac{1}{|\alpha_{\rm out}|^2}\int d\omega'\langle\delta I_{\rm out}(\omega)\delta I_{\rm out}(\omega')\rangle, \tag{47}$$

where

$$\delta I_{\rm out}(\omega)=\alpha_{\rm out}^*\delta a_{\rm out}(\omega)+\alpha_{\rm out}\delta a_{\rm out}^\dagger(\omega) \tag{48}$$

and

$$\alpha_{\rm out}=\sqrt{\gamma_c}\,\alpha_s-\alpha_{\rm in} \tag{49}$$

as a consequence of boundary conditions at the fixed mirror surface. By using Eqs. (33)–(38) and the Fourier transforms of Eqs. (14) and (15), as above, we obtain

$$S_I(\omega)=(1+\bar{n})|\zeta(\omega)|^2+\bar{n}|\zeta(-\omega)|^2+(1+2\bar{n})|\eta(\omega)|^2$$
$$+2\frac{\gamma_c^2/4-\phi^2}{\gamma_c^2/4+\phi^2}\{(1+\bar{n})[\zeta_1(\omega)\eta_1(-\omega)-\zeta_2(\omega)\eta_2(-\omega)]+\bar{n}[\zeta_1(-\omega)\eta_1(\omega)-\zeta_2(-\omega)\eta_2(\omega)]\}$$
$$-\frac{2\gamma_c\phi}{\gamma_c^2/4+\phi^2}\{(1+\bar{n})[\zeta_1(\omega)\eta_2(-\omega)+\zeta_2(\omega)\eta_1(-\omega)]+\bar{n}[\zeta_1(-\omega)\eta_2(\omega)+\zeta_2(-\omega)\eta_1(\omega)]\}$$
$$+\frac{K_B T}{\gamma_c^2/4+\phi^2}\left\{\left[\frac{\gamma_c}{2}[\xi_1(\omega)+\xi_1(-\omega)]-\phi[\xi_2(\omega)+\xi_2(-\omega)]\right]^2 + \left[\phi[\xi_1(\omega)-\xi_1(-\omega)]+\frac{\gamma_c}{2}[\xi_2(\omega)-\xi_2(-\omega)]\right]^2\right\}, \tag{50}$$

and finally we have

$$S_I(\omega)=1-\frac{4\omega K\phi\gamma_c}{(1+2\bar{n})(\gamma_c^2/4+\phi^2)|\Delta(\omega)|^2}\{\chi_2(\omega)[(1+2\bar{n})(\gamma_c^2/4+\phi^2)+\omega\phi]+(1/2+\bar{n})\omega\gamma_c\chi_1(\omega)\}$$
$$+\frac{4K\phi^2\omega^2\gamma_c}{(1+2\bar{n})(\gamma_c^2/4+\phi^2)|\Delta(\omega)|^2}\frac{|\chi(\omega)|^2 K_B T}{Q_m\hbar\omega_m}. \tag{51}$$

We see that the thermal contribution might destroy the squeezing, and thus we have to be careful in choosing the range of frequency and the values of the external parameters. As will become clear in Sec. VIII, the squeezing is less sensitive to the thermal noise for $\omega \ll \omega_m$. Thus, for low ω, and setting $\bar{n}=0$ for simplicity, we obtain

$$S_I(\omega)\simeq 1-\frac{4K\phi\gamma_c^2\omega^2}{(\gamma_c^2/4+\phi^2)|\bar{\Delta}(\omega)|^2}\left[\frac{1}{2}-\frac{\phi n_{\rm th}}{\gamma_c Q_m}\right], \tag{52}$$

where $n_{\rm th}=K_B T/\hbar\omega_m$ represents the number of thermal excitations due to the interaction with the thermal bath, and $\bar{\Delta}(\omega)$ is the value of Eq. (38), where we considered $\chi_2(\omega \ll \omega_m)=0$ and $\chi_1(\omega \ll \omega_m)=1$. This function has a minimum at

$$\bar{\omega}=\pm(\tfrac{1}{4}+\varphi^2-2\varphi\bar{K})^{1/2}, \tag{53}$$

where all the frequencies are in units of γ_c (i.e., $\varphi=\phi/\gamma_c$; $\bar{A}=A/\gamma_c$). The minimum of Eq. (52) is then

$$S_I(\bar{\omega})=1-2\bar{K}\varphi(\tfrac{1}{4}+\varphi^2)^{-1}\left[1-2\varphi\frac{n_{\rm th}}{Q_m}\right]. \tag{54}$$

We see that, for

$$\varphi \ll \frac{Q_m}{n_{\rm th}}, \tag{55}$$

the intensity squeezing becomes independent of the thermal noise.

VIII. SQUEEZING CONDITIONS

We need some criteria for choosing the values of the various external parameters. In Eq. (32) we already met one condition that must be satisfied to be sure that the frequency-dependent dissipation does not spoil the squeezing. We also must ensure that the thermal noise contribution has no great influence in the squeezing spectrum. From Eq. (29) we can extract the "strength" of the added thermal noise, which can be written as

$$\gamma_{\rm th}=|\rho|^2|\chi(\omega)|^2\frac{\gamma_m}{m^2\omega_m^4}K_B T$$
$$=\frac{n_{\rm th}|K|}{Q_m}|\chi(\omega)|^2, \tag{56}$$

where we used Eqs. (28) and (30). We will then require that the strength of the thermal noise is small with respect to the damping constant of the internal mode, i.e.,

$$\gamma_{\rm th} \ll \gamma_c . \tag{57}$$

This condition means that the additional loss of photons due to the thermal dissipation is negligible.

Since the number $n_{\rm th}$ of thermal excitations at frequency ω_m could be large at room temperature in the MHz range, we have to choose a high mechanical quality factor Q_m. We also see that the thermal fluctuations become important for $\omega \simeq \omega_m$, while they are less important for low ω. Thus the squeezing, if there is any, will be strongly dependent on the thermal noise for $\omega \simeq \omega_m$. Finally, we have to choose the various external parameters in order to have a value of $|K|$, which represents the frequency-independent phase shift due to the radiation pressure, such that Eq. (32) holds. By the way, for high Q_m and low ω Eq. (32) can easily be satisfied. Furthermore, from Eq. (54) we know that for $\phi \ll \gamma_c Q_m/n_{\rm th}$ the thermal contribution does not have any influence on the minimum value of the output intensity squeezing at low frequency. From Eqs. (30a) and (30b), we have $\phi=\phi_0-K$ with $\phi_0=\omega_c-\omega_0$.

In the following, we shall consider the case of high mechanical quality factor

$$\frac{\gamma_m}{m} \ll \omega_m , \tag{58}$$

and we further require that the mechanical angular frequency be larger than the cavity linewidth. This is because, otherwise, the cavity photon does not "see" any coherently variable path but only fluctuations in the path length. We will then require

$$\gamma_c \ll \omega_m \ . \tag{59}$$

In order to obtain a relatively small γ_c, the transmissivity of the fixed mirror has to be very small. Rempe *et al.* [24] have shown that it is possible to obtain values as small as $T_r \simeq 10^{-6}$. We will then choose a mechanical frequency in the MHz range with a good quality factor which can be obtained easily with a quartz resonator.

If we go back to Eq. (54), with the above-specified choices for the various external parameters, and set the value $T=300$ K for the external temperature, which gives $n_{\text{th}} \simeq 10^7$, with $Q_m = 10^6$, Eq. (55) gives $\varphi \ll 0.1$. Then Eq. (54) can be further approximated, giving

$$S_I(\bar{\omega}) \simeq 1 - 8\bar{K}\varphi = 1 - 8\varphi(\varphi_0 - \varphi) \ , \tag{60}$$

thus the maximum squeezing will easily be realized for $\varphi_0 \gg \varphi$ at

$$\varphi = \frac{1}{8\varphi_0} \ . \tag{61}$$

Thus, once $\varphi_0 = (\omega_c - \omega_0)/\gamma_c$ has been fixed, the value of φ, which depends on the input power through Eqs. (30b) and (19), has to satisfy Eq. (55) and the stability condition.

IX. THE GINZBURG CRITERION

Once the range of frequencies and the quality factor of the mechanical oscillator and, consequently, the cavity linewidth have been chosen, one has to satisfy the stability condition. To this end, as we mentioned above, it is worthwhile to introduce a Ginzburg criterion [19]. The steady-state values in Eqs. (18) have to be far from the "critical points." This can be quantitatively assessed by considering the ratios

$$\frac{\langle \delta a^\dagger \delta a \rangle}{|\alpha_s|^2} \ll 1 \quad \text{and} \quad \frac{\langle \delta x \delta a \rangle}{|x_s \alpha_s|} \ll 1 \ , \tag{62}$$

where the equal-time expectations are considered with respect to the steady state and the distributions of the noises.

Let us now introduce the column vectors

$$\mathbf{v} = \begin{bmatrix} \delta a \\ \delta a^\dagger \\ \delta x \\ \delta p \end{bmatrix} , \quad \mathbf{w} = \begin{bmatrix} \sqrt{\gamma_c}\,\delta a_{\text{in}} \\ \sqrt{\gamma_c}\,\delta a_{\text{in}}^\dagger \\ 0 \\ -\sqrt{\gamma_m}\,\epsilon_{\text{in}} \end{bmatrix} , \tag{63}$$

and the matrix

$$\underline{A} = \begin{bmatrix} -\frac{\gamma_c}{2} - i\phi & 0 & ig\alpha_s & 0 \\ 0 & -\frac{\gamma_c}{2} + i\phi & -ig\alpha_s^* & 0 \\ 0 & 0 & 0 & \frac{1}{m} \\ \hbar g\alpha_s^* & \hbar g\alpha_s & -m\omega_m^2 & -\frac{\gamma_m}{2m} \end{bmatrix} . \tag{64}$$

Thus, Eq. (12) can be linearized with respect to the steady state by writing

$$\frac{d\mathbf{v}}{dt} = \underline{A}\mathbf{v} + \mathbf{w} \ . \tag{65}$$

In order to solve Eq. (65), we introduce the unitary matrix $\underline{U}$ such that $\underline{U}\underline{U}^{-1} = \underline{1}$. The diagonal matrix is then

$$\underline{\Lambda} = \underline{U}^{-1}\underline{A}\underline{U} \ , \tag{66}$$

and the eigenvectors of the matrix $\underline{A}$ form the columns of the unitary matrix. For the stability condition the real part of all the eigenvalues of the matrix $\underline{A}$ have to be negative [21]. Finally,

$$\mathbf{v}(t) = \underline{U}e^{\Delta t}\underline{U}^{-1}\mathbf{v}(0) + \int_0^t \underline{U}e^{\Delta(t-t')}\underline{U}^{-1}\mathbf{w}(t')dt' \ . \tag{67}$$

Denoting the transpose matrix with $\underline{B}^T$, for the steady state ($t \to \infty$) we obtain

$$\langle \mathbf{v}(t)\mathbf{v}(t)^T \rangle = \underline{U}\underline{\Gamma}\underline{U}^T \ , \tag{68}$$

with

$$\underline{\Gamma} = \lim_{t\to\infty} \int_0^t dt' e^{\Delta t'} \underline{G} e^{\Delta t'} \tag{69}$$

and

$$\underline{G} = \underline{U}^{-1}\underline{M}(\underline{U}^{-1})^T \ . \tag{70}$$

The noise correlation matrix $\underline{M}$ is given by

$$\underline{M} = \langle \mathbf{w}(t)\mathbf{w}(t)^T \rangle$$

$$= \begin{bmatrix} 0 & (1+\bar{n})\gamma_c & 0 & 0 \\ \gamma_c\bar{n} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_m K_B T \end{bmatrix} \tag{71}$$

obtained by using Eqs. (14) and (15). Then the elements of the $\underline{\Gamma}$ matrix are easily obtained as

$$\Gamma_{ij} = -\frac{G_{ij}}{\lambda_i + \lambda_j} \ , \tag{72}$$

with λ_i representing the eigenvalues of $\underline{A}$. The secular equation for the matrix $\underline{A}$ is the fourth-order equation

$$\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0 \ , \tag{73}$$

with

$$a_0=\omega_m^2\frac{\gamma_c^2}{4}+\omega^2\phi^2-2\hbar g^2\frac{|\alpha_s|^2}{m}\phi\ ,$$
$$a_1=\frac{\gamma_c^2\gamma_m}{8m}+\frac{\gamma_m}{2m}\phi^2+\gamma_c\omega_m^2\ , \qquad (74)$$
$$a_2=\frac{\gamma_c^2}{4}+\phi^2+\frac{\gamma_c\gamma_m}{2m}+\omega_m^2\ ,\quad a_3=\gamma_c+\frac{\gamma_m}{2m}\ .$$

Even though the roots could be obtained analytically [25], the expressions are so involved that one can hardly extract useful information. On the other hand, the choice of various external parameters has to be made in such a way that the Ginzburg criterion in Eq. (62) is satisfied. This can be achieved easily with computer aid, by imposing that the set of external parameters we choose satisfies not only the criteria in Eqs. (32), (57), and (59), but also the Ginzburg criterion given in Eq. (62). The relevant expectation values in Eq. (62) are obtained easily by using Eqs. (68) and (72), with λ_i obtained by solving the eigenvalues equation (73) for matrix $\underline{A}$.

X. RESULTS

Once we have chosen the frequency of the impinging mode on the first mirror, of course we must choose ω_c by fixing the cavity length L. The only relevant external parameter which remains to be fixed is the input power $P_{\rm in}$ defined in Eq. (22). Indeed, all the steady-state values are implicitly dependent on $P_{\rm in}$ through Eqs. (18) and (30). For the stability condition, as we mentioned above, the real part of all the eigenvalues λ_i must be negative, and the steady-state value have to be consistent with Eq. (55).

We finally fix the various external parameters as

$$\omega_m=10^6\ {\rm s}^{-1}\ ,\quad m=10^{-9}\ {\rm kg}\ ,\quad Q_m=10^6\ ,$$
$$T_r=1.6\times10^{-4}\ ,\quad \gamma_c=10^5\ {\rm s}^{-1}\ . \qquad (75)$$

Of course this choice is arbitrary, because other choices satisfying the above-mentioned criteria can be made giving the same qualitative results. In Fig. 1(a) and 1(b) we plot the real part of the eigenvalues when we choose $\lambda_0=0.6328\times10^{-6}$ m ($\omega_0=2\pi c/\lambda_0$) for a He-Ne input laser, and consequently $\lambda_c=0.632\,799\times10^{-6}$ m. We see that a threshold value exists for $P_{\rm in}$ below which all the eigenvalues have negative real parts and the solutions are stable. There is also a lower limit in the range of the possible input power, and this is connected with the already discussed Ginzburg criterion. For lower input power, indeed, the steady-state solutions are too near to the turning points, and the linearization procedure fails. However, this value is practically indistinguishable from the value below which Eq. (19) has only one real solution, and no bistability occurs. In Fig. 2(a) we show the equation of state (19) as a function of $P_{\rm in}$, with the other external parameters given in Eq. (75). The stabilization phenomenon of the oscillating mirror [7] appears clearly in the figure. In order to have a further, almost insignificant, enhancement of the mirror's oscillation amplitude, a very high input power is needed. In Fig. 2(b) we show the magnification of the local minimum of Eq.

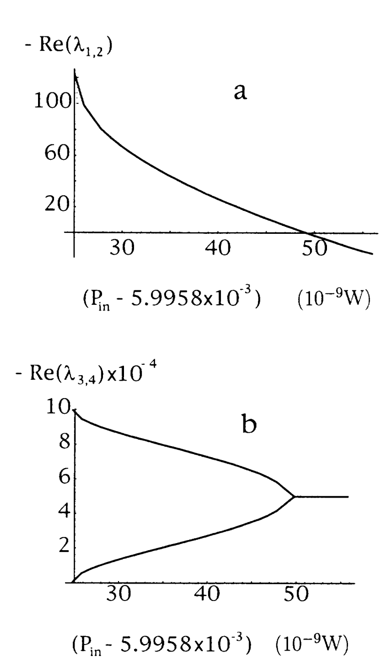

FIG. 1. (a) and (b) The real part of the eigenvalues of the matrix $\underline{A}$ is plotted vs the input power $P_{\rm in}$.

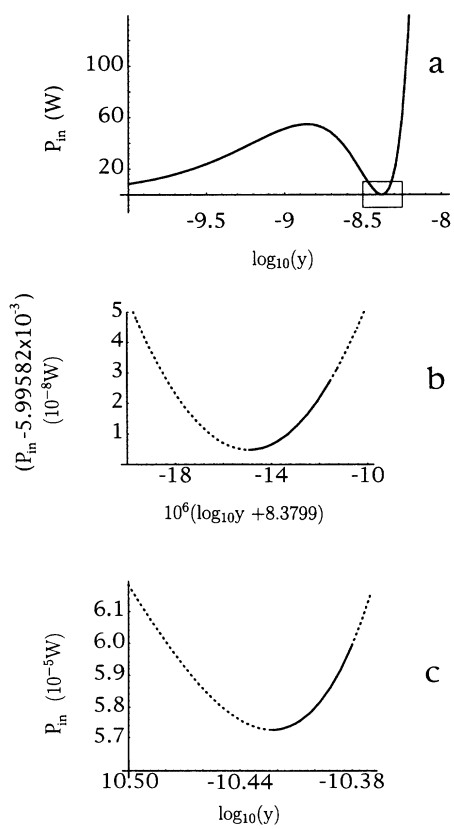

FIG. 2. (a) The semilog plot of the equation of state is shown. (b) The magnification of the local minimum. For both figures the temperature is $T=300$ K. (c) The magnification of the local minimum for $T=4$ K. In all the figures the dashed line represents the unstable solutions.

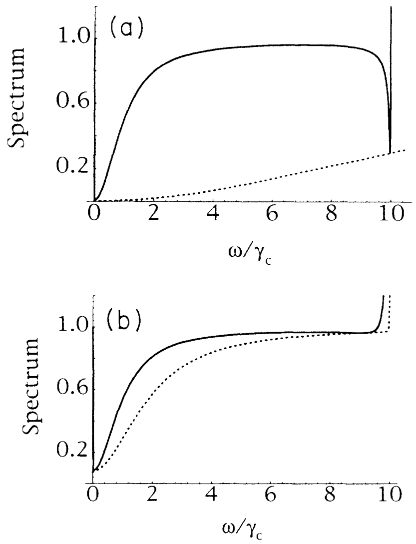

FIG. 3. (a) The two spectra $S_I(\omega)$ (full line) and $S_{\rm opt}(\omega)$ (dashed) vs ω/γ_c for $T=4$ K is shown. (b) The same as (a) for $T=300$ K.

(19). The dashed lines represent the unstable solutions. All the above curves are obtained with the thermal bath in equilibrium at $T=300$ K. We see that the range of possible input powers, when all the above conditions are satisfied, is very narrow, and one must use an input laser that is well stabilized in power. At lower temperature, as shown in Fig. 2(c) for $T=4$ K, that range is wider, and the power stabilization is less important. The subsequent figures are devoted to the squeezing spectra. At low temperature $T=4$ K, as shown in Fig. 3(a), the spectrum of the intensity squeezing in Eq. (51) has two minima at $\omega \simeq \omega_m$ and for low ω. As the temperature rises, the higher frequency minimum disappears. This is shown in Fig. 3(b), which is obtained for $T=300$ K. On the other hand, the low-frequency minimum is almost insensitive to the temperature. In both the above figures, the dashed line shows the optimum quadrature squeezing given in Eq. (46), while the full line represents the intensity squeezing in Eq. (51). We have also considered only the higher value of the steady-state bistable solutions which satisfies Eq. (55). In Fig. 4 we show the approximation for the intensity squeezing as given by Eq. (52) (the dashed line) for the case $T=300$ K. We obtain an almost perfect agreement within the range of validity of the approximation. Figure 5 is devoted to the spectrum of in-

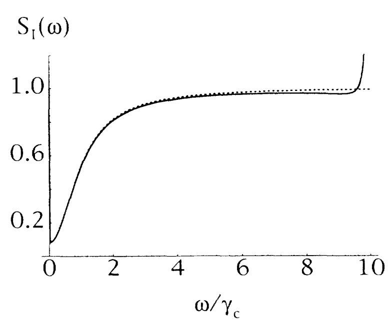

FIG. 4. A comparison between the approximate value (dashed line) and the complete behavior of $S_I(\omega)$ is shown.

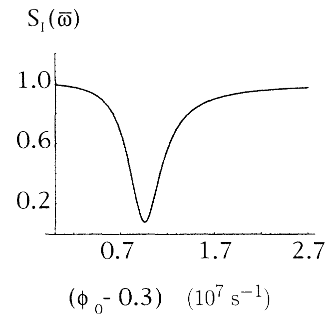

FIG. 5. (a) The intensity squeezing is shown for the particular value of $\omega=\varpi$ vs the detuning $\phi_0=\omega_c-\omega_0$.

tensity squeezing, at the particular frequency ϖ at which we obtain the maximum reduction of the intensity fluctuations, vs ϕ_0, to show that the squeezing is obtainable only when the bistability condition in Eq. (23) is satisfied; however, because we choose $T=300$ K, the squeezing is spoiled well before we reach the value of ϕ_0 given in Eq. (23), since the maximum squeezing is realized only when Eq. (61) holds.

XI. CONCLUDING REMARKS

With the chosen value of the input power $P_{\rm in}=5.995\,825\times 10^{-3}$ W the value of the phase shift is $\phi=100$ s^{-1} (i.e., $\varphi=10^{-3}$), and we have shown that the output light presents an almost perfect intensity and quadrature squeezing at a frequency very close to the input one. The effect of the thermal noise can be made completely negligible; however, the drawback is that one has to have a perfect control of the input power. We are led to the conclusion that with such a device quantum noise reduction can be realized.

After this work was completed, we became aware that Fabre and co-workers [26], by using the same model as in Ref. [5], revised their previously pessimistic conclusion, observing that with a high quality mechanical resonator the squeezing can be obtained at low frequency, which is in agreement with our result. In our analysis, however, we give a full description of the model, and show the relevant parameters to be controlled in order to obtain relevant output squeezing even at room temperature.

The practical realization of such a device should be an interesting challenge for an experimenter. Of course, in real experiments additional losses could be introduced by an imperfect longitudinal oscillation of the coated quartz surface, and additional heating due to an imperfectly reflecting oscillating mirror; however, we think that the up-to-date technology is mature enough to obtain an optomechanical control of the quantum fluctuations.

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