Light-pressure force in N-atom systems

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An analytical description of long-range collisions between atoms in a laser cooling field is developed. We begin by considering an N-atom master equation. In the regime of low atomic densities (i.e., where the mean distance between two atoms is much larger than the laser wavelength) it is possible to treat the atom–atom interactions in perturbation theory. Furthermore we assume temperatures which allow a semiclassical treatment of the cooling process. The effect of the presence of other atoms can be separated analytically into two parts: an attenuation force due to the absorption of the laser beams in the atomic cloud similar to the results of Dalibard [Opt. Commun. **68**, 203 (1988)], which tends to compress the atomic cloud, and a two-atom force due to photon emission and absorption cycles between different atoms. This force proves to be repulsive for the configurations studied and prevents the cloud from collapsing. The result for the first-order perturbation expansion in collision strength generalizes the model proposed by Walker, Sesko, and Wieman [J. Opt. Soc. B **8**, 946 (1991)] by including additional terms, such as those associated with Raman couplings.

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I. INTRODUCTION

The trapping of neutral atoms has been extensively studied over the past few years. The deepest optical traps so far are the magneto-optical spontaneous force traps (MOT) which trap and cool at the same time [1–4]. This trap was first established by Raab *et al.* [5]. These laser cooling and trapping techniques enable temperatures below 1 mK and trap densities have already exceeded the range of 10^{11} atoms/cm^3 [6]. As long as the atomic gas is sufficiently dilute the neutral atoms in the trap behave as an ideal gas except when they undergo short-range collisions. At the high densities reached in some current experiments the atomic cloud becomes optically thick and collective effects give rise to a profound change in the characteristics of the atomic vapor.

So far the one-atom theory of laser cooling is well understood, both on the level of a semiclassical and a fully quantum mechanical treatment [7–10]. Very little theoretical work, however, has been done on the light-pressure force and diffusion in N-atom systems and the dynamics of dense atomic vapors in a trap. These many-atom couplings have mainly two effects. The first is a force that results from the local imbalance of the trap-laser amplitude, due to absorption in a random media. This force was discussed in the context of optical molasses in Refs. [11,12]. The second force has its origin in the process of reabsorption of fluorescence photons emitted by an atom by a second atom and will in general give rise to a repulsive force. This is basically the model used in Ref. [6] [called the Walker-Sesko-Wieman (WSW) model in the following] for the explanation of the global characteristics of their atomic cloud. The successful explanation of dark spot cooling [13] has recently proved the wide range of applicability of the model.

For an explanation of the interaction between atoms in the cloud it is necessary to understand ultracold collisions in the presence of the trapping light fields. Since the preparation of the atoms by the near-resonant laser beams and the collision itself can no longer be isolated, the dissipative aspect of spontaneous emission in these collisions has to be included in the description of spontaneous force traps. Burnett and co-workers [14–16] and Julienne and co-workers [17,18] have given a theoretical framework for the study of binary collisions in the ultracold regime based on a master equation description of the two-atom system. For the densities achieved so far, the global observables of the traps are nevertheless mainly characterized by the long-range regime of these cold collisions.

This paper reports on a detailed derivation of a generalized WSW-type model, including physical mechanisms not contained in the model of Ref. [6], that change the force between two atoms and including the change in temperature due to collective effects. We start our calculation with a derivation of the full N-atom master equation including center-of-mass (c.m.) motion and then derive an expression for the semiclassical forces between the atoms. Furthermore we derive a Fokker-Planck equation (FPE) for an interacting N-atom system, thus including many-atom corrections in the diffusion coefficient. We discuss the semiclassical force in the limit when the atoms are on the average far apart. This allows a perturbation theoretical treatment of the atom-atom interaction and mainly recovers the WSW model.

The model in its present form ignores localization effects of the atomic density on the scale of the laser

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wavelength as found, e.g., for optical molasses in a linearly cross-polarized counterpropagating laser configuration (lin⊥lin) [10]. We will discuss these effects in the context of a $\frac{1}{2}$-$\frac{3}{2}$ atomic system in a laser configuration in a forthcoming publication. For a magneto-optical trap such spatial order of the cold atoms has not yet been observed and we expect the conclusions from our model to be valid.

II. QUALITATIVE CONSIDERATIONS

The mechanism of damping and confinement of atoms in a MOT relies on the Doppler cooling of atoms with Zeeman degeneracy in a magnetic field with uniform field gradient [19]. In Fig. 1 we show a simple one-dimensional model of an angular momentum $F=0$ to $F'=1$ transition under the influence of a negative uniform magnetic field gradient in a σ^--σ^+ laser configuration. When the atom moves out of the origin (no magnetic field) the excited $F'=1$ states are split into their respective Zeeman energy levels, and for red laser detuning we induce a Doppler cooling light force by shifting the atom into resonance with one of the laser beams. This Zeeman-shift-induced force confines the atoms and establishes a harmonic trap potential near the origin, while at the same time cooling them. In addition to the Doppler cooling light force the experiments show a strong dependence on the spatial variation of the trapping beams, such as misalignment of the Gaussian intensity profiles. The gradient force that is induced by such configurations leads to all kinds of different cloud topologies [20].

Nevertheless the behavior of the atomic cloud strongly deviates from an ideal gas model due to long-range collective effects induced by optical thickness and photon exchange between atoms. In the following we give a qualitative discussion of these many-atom effects for the regime of low atomic saturation and a summary of the results we obtained for the high intensity limit including screening effects.

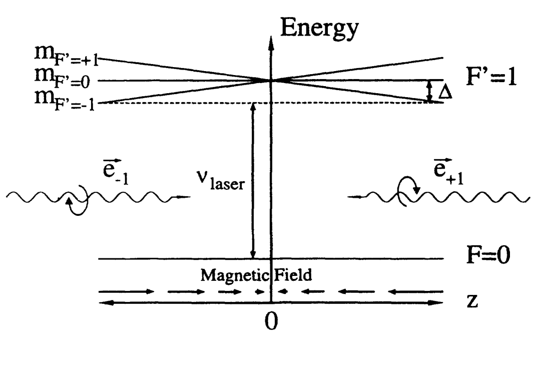

FIG. 1. One-dimensional model of a magneto-optical trap on an $F=0$ to $F'=1$ transition. The inhomogeneous magnetic field has the form $B(z)=-\beta z$ and the lasers are detuned to the red of the free-field atomic transition frequency.

A. Coherent and incoherent fluorescence from a N-atom system at low laser intensities

We consider Zeeman-degenerate two-level atoms at random positions $\vec{x}_\alpha$ ($\alpha=1,\ldots,N$) in a counterpropagating σ^--σ^+ plane wave laser field (frequency ω) along the z direction. The electric field operator in the Heisenberg picture at position $\vec{x}_\alpha$ of the atom α is given by

$$\hat{\vec{E}}^{(+)}(\vec{x}_\alpha,t)=\hat{\vec{E}}^{(+)}_{\rm vac}(\vec{x}_\alpha,t)+\vec{E}^{(+)}_{\rm cl}(\vec{x}_\alpha,t) -\sum_q \vec{e}_q\sum_{\beta\neq\alpha}\sum_{q'}\frac{\gamma}{2\mu}\phi^{(-)}_{qq'}(\vec{x}_\alpha-\vec{x}_\beta)\mathcal{A}^\dagger_{\beta,q'}(t), \tag{1}$$

where $\phi^{(-)}_{qq'}(\vec{x}_\alpha-\vec{x}_\beta)$ is the dipole scattering characteristics [defined in Sec. III, Eq. (39)], γ defines the spontaneous decay rate, μ is the reduced atomic dipole moment, and $\mathcal{A}_{\beta,q'}(t)$ is the atomic lowering operator of the atom at position $\vec{x}_\beta$ in a spherical basis $\vec{e}_\pm,\vec{e}_0$ [defined in Eq. (35)]. In Eq. (1) $\hat{\vec{E}}^{(+)}_{\rm vac}(\vec{x}_\alpha,t)$ is the vacuum field and $\vec{E}^{(+)}_{\rm cl}(\vec{x}_\alpha,t)$ denotes the incoming classical laser field.

The mean of the electric field operator, $\langle\hat{\vec{E}}^{(+)}(\vec{x}_\alpha,t)\rangle$, defines an effective electric field

$$\vec{E}^{(+)}_{\rm eff}(\vec{x}_\alpha,t)=\vec{E}^{(+)}_{\rm cl}(\vec{x}_\alpha,t) -\sum_q \vec{e}_q\sum_{\beta\neq\alpha}\sum_{q'}\frac{\gamma}{2\mu}\phi^{(-)}_{qq'}(\vec{x}_\alpha-\vec{x}_\beta) \times\langle\mathcal{A}^\dagger_{\beta,q'}(t)\rangle, \tag{2}$$

which is a random quantity with respect to the center-of-mass positions of all the atoms.

The resonance fluorescence spectrum, as seen by the atom α, is proportional to the Fourier transform of the electric field correlation function

$$\langle\hat{\vec{E}}^{(-)}(\vec{x}_\alpha,t)\hat{\vec{E}}^{(+)}(\vec{x}_\alpha,t')\rangle\ . \tag{3}$$

The spectrum of resonance fluorescence consists of a coherent δ-function contribution, proportional to $\langle\hat{\vec{E}}^{(-)}(\vec{x}_\alpha,t)\rangle\langle\hat{\vec{E}}^{(+)}(\vec{x}_\alpha,t')\rangle$, and an incoherent spectrum which is related to the quantum mechanical fluctuations of the electrical field, described by the operator

$$\delta\hat{E}^{(+)}(\vec{x}_\alpha,t)\equiv\hat{E}^{(+)}(\vec{x}_\alpha,t)-\langle\hat{E}^{(+)}(\vec{x}_\alpha,t)\rangle\ . \tag{4}$$

As long as the laser intensity is weak so that atomic saturation effects can be neglected, the coherent part is dominant and the incoherent part of the resonance fluorescence light emitted by the atoms can be neglected (which goes with the square of laser intensity). In this limit the light is scattered elastically from the atoms. Thus we can approximate the electrical field operator *in the low intensity regime* for coherent laser input fields by

$$\hat{E}^{(+)}(\vec{x}_\alpha,t)\simeq\hat{E}^{(+)}_{\rm vac}(\vec{x}_\alpha,t)+\vec{E}^{(+)}_{\rm eff}(\vec{x}_\alpha,t)\ . \tag{5}$$

The effective laser field is the sum of the incoming laser amplitude and the field scattered by the randomly distributed atoms in the atomic cloud. In the following we concentrate our discussion on the stationary limit of Eq. (2). For low atomic saturation the stationary mean atomic dipoles are proportional to the stationary local electric field and are given by

$$\langle \mathcal{A}_{\alpha,q} \rangle = \eta\mu E^{(-)}_{\text{eff},q}(\vec{x}_\alpha), \tag{6}$$

where $\eta\mu$ is the polarizability of the two-level atom. In writing Eq. (6) we find it convenient to drop the rapidly oscillating laser frequency (i.e. go to a rotating frame). The local electric field, averaged over the random positions of the atoms as indicated by $\langle\!\langle\,\rangle\!\rangle$, can be shown to obey the Twersky integral equation of classical electrodynamics [21]

$$\begin{aligned}\langle\!\langle E^{(-)}_{\text{eff},q}(\vec{x}_\alpha) \rangle\!\rangle = {} & E^{(-)}_{\text{cl},q}(\vec{x}_\alpha) \\ & + \int dx_\beta n(\vec{x}_\beta) \sum_{q'} u_{qq'}(\vec{x}_\alpha - \vec{x}_\beta) \\ & \times \langle\!\langle E^{(-)}_{\text{eff},q'}(\vec{x}_\beta) \rangle\!\rangle,\end{aligned} \tag{7}$$

with

$$u_{qq'}(\vec{x}_\alpha - \vec{x}_\beta) \equiv -\frac{\gamma}{2}\eta\Phi^{(+)}_{qq'}(\vec{x}_\alpha - \vec{x}_\beta) \tag{8}$$

and $n(\vec{x}_\beta)$ the position dependent atomic density in the cloud. Equation (7) expresses the local field as the sum of the incident laser field and the light fields scattered from the other atoms where the local field is calculated in a self-consistent way. This equation can be recognized as a Green function solution to the basic Maxwell-Bloch equation for the coherent field, where the polarization is expressed in terms of $E_{\text{eff},q}$. We have solved this integral equation for a $J = 0$ to $J = 1$ transition (see Sec. V) under the assumption that the effective field depends only on z (one-dimensional model). This provides us with an absorption coefficient for the laser and an index of refraction [compare Eq. (84)].

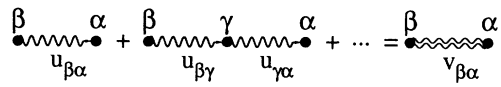

FIG. 2. Schematic representation of the screened scattering characteristic in terms of the multiple scattering series of the unscreened characteristic.

The tensor of the *total* local intensity at the position of the atom α is proportional to

$$\begin{aligned}I^{qq'}_{\text{tot}}(\vec{x}_\alpha) &= \langle\!\langle E^{(-)}_{\text{eff},q}(\vec{x}_\alpha) E^{(+)}_{\text{eff},q'}(\vec{x}_\alpha) \rangle\!\rangle \\ &= \langle\!\langle E^{(-)}_{\text{eff},q}(\vec{x}_\alpha) \rangle\!\rangle \, \langle\!\langle E^{(+)}_{\text{eff},q'}(\vec{x}_\alpha) \rangle\!\rangle + I^{qq'}_{\text{incoh}}(\vec{x}_\alpha),\end{aligned} \tag{9}$$

where we have averaged again over the random positions of the atoms as indicated by $\langle\!\langle\,\rangle\!\rangle$. This intensity consists of a *coherent* and *incoherent* contribution corresponding to the first and second term on the right-hand-side of Eq. (9). We note that in the present context the coherent and incoherent intensity is defined with respect to *position averages*. The intensity is a solution of the integral equation

$$\begin{aligned}(-1)^q \langle\!\langle E^{(-)}_{\text{eff},q}(\vec{x}_\alpha) E^{(+)}_{\text{eff},-q}(\vec{x}_\alpha) \rangle\!\rangle = {} & (-1)^q \langle\!\langle E^{(-)}_{\text{eff},q}(\vec{x}_\alpha) \rangle\!\rangle \, \langle\!\langle E^{(+)}_{\text{eff},-q}(\vec{x}_\alpha) \rangle\!\rangle + \sum_s \int d^3x_\beta v_{qs}(\vec{x}_\alpha - \vec{x}_\beta) v^*_{qs}(\vec{x}_\alpha - \vec{x}_\beta) n(\vec{x}_\beta)(-1)^s \\ & \times \langle\!\langle E^{(-)}_{\text{eff},s}(\vec{x}_\beta) E^{(+)}_{\text{eff},-s}(\vec{x}_\beta) \rangle\!\rangle .\end{aligned} \tag{10}$$

According to Eq. (10) the total intensity at the position of atom α is the sum of the coherent intensity at $\vec{x}_\alpha$ plus the contribution from the other atoms. The kernel of the integral equation (10) involves the function $v_{qs}(\vec{x}_\alpha - \vec{x}_\beta)$ which is defined by the integral equation [21] [this basically rewrites Eq. (7)]:

$$v_{qs}(\vec{x}_\alpha - \vec{x}_\beta) = u_{qs}(\vec{x}_\alpha - \vec{x}_\beta) + \sum_{s'} \int d^3x_\gamma u_{qs'}(\vec{x}_\alpha - \vec{x}_\gamma) v_{s's}(\vec{x}_\gamma - \vec{x}_\beta) n(\vec{x}_\gamma) . \tag{11}$$

The function v_{qs} is a *screened scattering characteristic* [21]. To see the physical significance of the integral equation for v_{qs} we iterate (11),

$$\begin{aligned}v_{qs}(\vec{x}_\alpha - \vec{x}_\beta) = {} & u_{qs}(\vec{x}_\alpha - \vec{x}_\beta) + \sum_{s'} \int d^3x_\gamma u_{qs'}(\vec{x}_\alpha - \vec{x}_\gamma) u_{s's}(\vec{x}_\gamma \vec{x}_\beta) n(\vec{x}_\gamma) \\ & + \sum_{r',s'} \int d^3x_\gamma d^3x_\delta u_{qs'}(\vec{x}_\alpha - \vec{x}_y) u_{s'r'}(\vec{x}_\gamma - \vec{x}_\delta) u_{r's}(\vec{x}_\delta - \vec{x}_\beta) n(\vec{x}_\delta) n(\vec{x}_\gamma) + \cdots .\end{aligned} \tag{12}$$

The first term, $u_{qs}(\vec{x}_\alpha - \vec{x}_\beta)$, represents the unscreened scattering characteristic of a scatterer β at position $\vec{x}_\alpha$. The second term is the correction to this scattering characteristic due to additional scattering from another atom at position $\vec{x}_\gamma$. Therefore $v_{qs}(\vec{x}_\alpha - \vec{x}_\beta)$ represents the screened scattering characteristic from atom β going through various other scatterers. In Fig. 2 we schematically show the expansion of the screened scattering characteristic into the multiple scattering series.

In summary, the two Twersky integral equations for the coherent light scattering (7) and the total intensity (10) constitute the basic equations for the light intensity in the atomic cloud [22].

B. Semiclassical forces in the low intensity limit

In the regime where the laser cooling can be treated semiclassically, the mean Doppler cooling force on the atom α at rest is given by

$$\vec{F}(\vec{x}_\alpha) = -\sum_q [\mathcal{A}_q \cdot \vec{\nabla} E^{(+)}_{\text{eff},q}(\vec{x}_\alpha)], \tag{13}$$

with $\sum_q \vec{e}_q \mathcal{A}_q$ the mean atomic dipole moment. If we insert the expression for the atomic dipole [Eq. (6)] we get for the mean force

$$\langle\!\langle \vec{F}(\vec{x}_\alpha)\rangle\!\rangle = \left\langle\!\!\left\langle \left(-\eta \sum_q (-1)^q \mu E^{(-)}_{\text{eff},q}(\vec{x}_\alpha) \frac{\partial}{\partial \vec{x}_\alpha} \mu E^{(+)}_{\text{eff},-q}(\vec{x}_\alpha) + \text{c.c.}\right)\right\rangle\!\!\right\rangle \tag{14}$$

where $\langle\!\langle\ \rangle\!\rangle$ again denotes the position average of the atoms in the vapor. The mean force is proportional to the negative gradient of the *total* intensity.

We can evaluate Eq. (14) by use of Eqs. (9) and (11) and split the force $\langle\!\langle \vec{F}(\vec{x}_\alpha)\rangle\!\rangle$ into three different contributions,

$$\langle\!\langle \vec{F}(\vec{x}_\alpha)\rangle\!\rangle = \vec{F}_S(\vec{x}_\alpha) + \langle\!\langle \vec{F}_A(\vec{x}_\alpha)\rangle\!\rangle + \langle\!\langle \vec{F}_R(\vec{x}_\alpha)\rangle\!\rangle\ . \tag{15}$$

The first term, $\vec{F}_{\text{S}}$, is the familiar semiclassical laser cooling force in the limit of negligible trap laser attenuation (one atom theory). The second term, $\langle\!\langle \vec{F}_A\rangle\!\rangle$ is an attractive force due to the attenuation of the trapping laser beam *amplitude* in the cloud (proportional to the negative gradient of the *coherent* intensity). We will consider this force for atoms at rest,

$$\langle\!\langle \vec{F}_A(\vec{x}_\alpha)\rangle\!\rangle + \vec{F}_S(\vec{x}_\alpha) \equiv \left(-\eta \sum_q (-1)^q \langle\!\langle \mu E^{(-)}_{\text{eff},q}(\vec{x}_\alpha)\rangle\!\rangle \frac{\partial}{\partial \vec{x}_\alpha} \langle\!\langle \mu E^{(+)}_{\text{eff},-q}(\vec{x}_\alpha)\rangle\!\rangle + \text{c.c.}\right). \tag{16}$$

Since the absorption length of the coherent intensity is smaller than the corresponding absorption length of the total intensity the difference to the total force must give an additional *repulsive* force. There is of course a drift term corresponding to the force proportional to the lowest order in velocity. This velocity dependent drift term which determines the cooling time is considered in a future publication. The third term, $\langle\!\langle \vec{F}_{\text{R}}\rangle\!\rangle$, in Eq. (15) is a repulsive force,

$$\langle\!\langle \vec{F}_R(\vec{x}_\alpha)\rangle\!\rangle = -\eta \sum_{qq'} (-1)^{q'} \int d^3x_\beta v_{qq'}(\vec{x}_\alpha - \vec{x}_\beta) \frac{\partial}{\partial \vec{x}_\alpha} v^*_{qq'}(\vec{x}_\alpha - \vec{x}_\beta) \langle\!\langle \mu E^{(-)}_{\text{eff},q'}(\vec{x}_\beta) \mu E^{(+)}_{\text{eff},-q'}(\vec{x}_\beta)\rangle\!\rangle + \text{c.c.} \tag{17}$$

We will show that this is basically the (low intensity) repulsive force found in the WSW model [6].

Example: To be specific we consider now the case of an angular momentum $J = 0$ to $J = 1$ transition. Furthermore we limit ourselves to the weak absorption limit, defined below by Eq. (85). The result for the attenuation force is

$$\langle\!\langle \vec{F}_{\alpha,A}(\vec{x})\rangle\!\rangle = I_{\text{cl}} \epsilon_0 \langle \sigma_L \rangle^2 \left(\int_z^\infty dz' n(z') - \int_{-\infty}^z dz' n(z') \right) \vec{e}_z, \tag{18}$$

in agreement with Refs. [6,11]. The absorption cross section is defined by

$$\langle \sigma_L \rangle \equiv \frac{6\pi}{k_0^2} \left(\frac{\gamma^2/4}{\Delta^2 + \gamma^2/4} \right), \tag{19}$$

I_{cl} denotes the intensity of the incoming laser field, k_0 is the wave vector of the atomic transition, Δ is the detuning, and ϵ_0 is the permittivity of vacuum.

For the evaluation of the repulsive force we neglect for simplicity the multiple scattering processes and replace $v_{qs}(\vec{x}_\alpha - \vec{x}_\beta)$ by the unscreened scattering characteristic $u_{qs}(\vec{x}_\alpha - \vec{x}_\beta)$. Using this simplification the repulsive part is given by

$$\langle\!\langle \vec{F}_{\alpha,R}(\vec{x}_\alpha)\rangle\!\rangle \simeq \sum_q \int d^3x_\beta n(\vec{x}_\beta) \sum_s f_{q,s}(\theta,\phi) f^*_{q,s}(\theta,\phi) \frac{k_0(\vec{x}_\alpha - \vec{x}_\beta)}{k_0^2 |\vec{x}_\alpha - \vec{x}_\beta|^3} \left\{ \frac{\pi}{2} \int_{-\infty}^{\infty} d\nu S^{em}_{\text{el},s}(\nu) S^{ab}_q(\nu) \right\} \vec{e}_z, \tag{20}$$

where $f_{q,s}(\theta,\phi)$ defines the angular distribution part of the scattering characteristic $\phi^{(-)}_{qq'}(\vec{x}_\alpha - \vec{x}_\beta)$ [see Eq. (39)] and $S^{ab}_q(\nu)$ denotes the Mollow weak field absorption spectrum [23] for q polarized light defined by

$$S^{ab}_q(\nu) \equiv \frac{\gamma}{2\pi}\int_{-\infty}^{\infty} dt e^{i\nu t}\langle[\mathcal{A}_q, \mathcal{A}^\dagger_q(t)]\rangle \; . \tag{21}$$

The elastic part of the Mollow emission spectrum for an atom at position $\vec{x}_\alpha$ is given by

$$S^{em}_{\mathrm{el},s}(\nu) = (-1)^s \frac{\langle\langle \mu E^{(-)}_{\mathrm{eff},s}(\vec{x}_\alpha)\mu E^{(+)}_{\mathrm{eff},-s}(\vec{x}_\alpha)\rangle\rangle}{(\Delta^2+\gamma^2/4)}\delta(\nu). \tag{22}$$

Thus we see that the repulsive part is determined by the overlap integral of the emission and absorption spectrum of the two-level atoms and inversely proportional to the square of the distance between the atoms. We interpret this result as a force due to absorption of photons by atom α due to fluorescence photons emitted by atoms β (Fig. 3). Note that Eq.(20) can also be written in the WSW form [6]

$$\begin{aligned}\langle\langle \vec{F}_{\alpha,R}(\vec{x}_\alpha)\rangle\rangle = I_{\mathrm{cl}}\epsilon_0 \, &\langle\sigma_L\rangle\langle\sigma_F\rangle \\ &\times\sum_q \int d^3x_\beta n(\vec{x}_\beta)\sum_s f_{q,s}(\theta,\phi)f^*_{q,s}(\theta,\phi) \\ &\times\frac{(\vec{x}_\alpha - \vec{x}_\beta)}{4\pi k_0^2|\vec{x}_\alpha - \vec{x}_\beta|^3}\end{aligned} \tag{23}$$

with

$$\langle\sigma_F\rangle \equiv 2\pi^2 S^{ab}_q(\nu=0) \; . \tag{24}$$

C. High intensity effects, screening and the Vlasov equation

For higher intensities of the laser beams part of the fluorescence light is scattered inelastically from the atoms at frequencies that differ from the laser frequency. This leads to a number of new effects which are discussed in Sec. V in the limit of mean atomic distances much larger than the laser wavelength. We find that the attenuation force $\vec{F}_A$ decreases with saturation [see Eq. (88) in Sec. V]. In agreement with the WSW model the repulsive force $\vec{F}_R$ is again proportional to the emission and absorption spectrum as in Eq. (20) where

$$S^{em}_q(\nu) \equiv \frac{\gamma}{2\pi}\int_{-\infty}^{\infty} dt e^{-i\nu t}\langle\mathcal{A}^\dagger_q(t)\mathcal{A}_q\rangle \tag{25}$$

includes the inelastic components [see Eq. (89) in Sec. V]. Figure 4 shows a typical emission and absorption spectrum for a $J_g = 0$ to $J_e = 1$ transition in a counterpropagating σ^--σ^+ laser field (V system). In addition to the WSW model we get other contributions to the force. For the V system we find a Raman coupling between the σ^- and σ^+ transition; this has the tendency to reduce the repulsive force $\vec{F}_R$. In addition we find forward four-wave-mixing processes along the direction of laser propagation [24] which tend to decrease the repulsive force on an atom α. Nevertheless, the number of atoms that are involved in these processes is small compared to the slowly position dependent standard WSW and Raman coupling processes and we expect the modification of the two-atom force by these wave-mixing processes to be small. For the V system this inelastic force is repulsive, but we expect that for other atomic configurations the gain-absorption profile will show new interesting features [25,26].

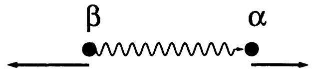

FIG. 3. Pair repulsion of atoms due to absorption of fluorescence photons.

The above discussion of the two-atom force has been limited to lowest-order perturbation theory (in a $1/kr$ expansion). In this approximation the repulsive force decreases with $1/r^2$. In Sec. VI we evaluate higher-order terms of the multiple scattering expansion. We find screening effects in the repulsive force on a scale given by the absorption length in the medium [see Eq. (97)]. In our calculations we neglect the changes in the density operator due to the incoherent background field. This approximation is shown to be valid for mean atomic distances much larger than the laser wavelength. Nevertheless, for very high densities the incoherent background field changes the atomic density operator and therefore the mean atomic dipole and spectra. A possible way to include these corrections more properly would be the use of radiative transfer equations to solve the one-atom problem [27]. Our result for the force is valid in the limit $k_l\langle\langle r_{\alpha\beta}\rangle\rangle \gg 1$ in the sense of an expansion in the inverse mean distance between the atoms.

In Sec. IV we derive a Fokker-Planck equation for the N-atom Wigner function (quasidistribution function for

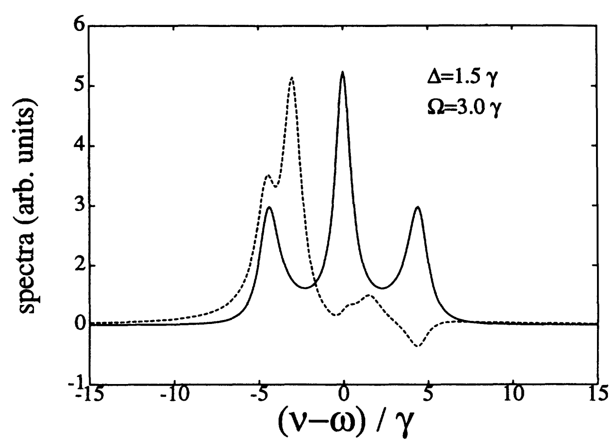

FIG. 4. Emission (solid line) and absorption (dashed line) spectrum for σ^+ polarized light as a function of the frequency ν for a $J_g = 0$ to $J_e = 1$ transition in a counterpropagating σ-σ^+ laser configuration. The parameters are $\Delta = 1.5\gamma$ and $\Omega \equiv \Omega_+ = \Omega_- = 3\gamma$. The negative regime in the absorption spectrum corresponds to gain.

positions and momenta). The drift term in this equation contains the one- and two-atom forces while the diffusion term (which is not explicitly calculated in the present paper) corresponds to fluctuations associated with these forces. In the simplest case one can derive an equation for the *one*-atom spatial distribution function $n(\vec{r})$. This can be extracted from the N-atom problem by a screened Vlasov approximation which only keeps pair correlations between the atoms (see Sec. VII). The equation derived in this way is equivalent to that used in Ref. [6] [Eq. (101) in Sec. VII],

$$\nabla_{\vec{r}} \cdot [\langle\!\langle \vec{F}(\vec{r}) \rangle\!\rangle n(\vec{r}) - k_b T_{\text{eff}} \nabla_{\vec{r}} n(\vec{r})] = 0 \,. \tag{26}$$

Here T_{eff} denotes the effective temperature of the atomic vapor determined by the atom-atom corrected diffusion constant and cooling time.

In summary, we have shown that we can derive a WSW-type model from an N-atom master equation under essentially three conditions. First, a semiclassical treatment must be valid. Second (mainly for convenience), the mean distances between the excited atoms have to be much larger than the trap laser wavelength. Finally, the collisional processes must be weak enough to ensure a small correlation between the atoms (i.e., all higher-order correlations are approximately given by a product of atom-pair correlations).

III. THE MODEL

A. Generalized optical bloch equations for a system of N two-level atoms

We consider mechanical light effects in a system of $\alpha = 1, \ldots, N$ distinguishable atoms strongly driven by laser light, and interacting via long-range dipole-dipole interactions. The internal degrees of freedom of each of these atoms are modeled by a two-level system with Zeeman ground states $|J_g, M_g\rangle$ and excited states $|J_e, M_e\rangle$ corresponding to an angular momentum $J_g \to J_e$ transition. We define a dipole transition operator from the excited to the ground state by $\hat{\mathbf{D}}^{\alpha}_{ge} \equiv \mathbf{P}_g \hat{\mu}_\alpha \mathbf{P}_e$ with $\mathbf{P}_g$ and $\mathbf{P}_e$ atomic projection operators, and $\hat{\mu}_\alpha$ the atomic dipole operator. The total Hamiltonian of the system is (we set $\hbar \equiv 1$ in the following)

$$H = H_{oA} + H_{oF} + H_{AF}. \tag{27}$$

Here

$$H_{oA} = \sum_\alpha \left(\frac{\hat{P}^2_\alpha}{2m} + \omega_{eg} \mathbf{P}_e \right) \tag{28}$$

is the free atomic Hamiltonian, where the first term corresponds to the kinetic energy of the atom, and the second term refers to the internal structure of the atom. The second term in (27) is the free Hamiltonian of the quantized radiation field,

$$H_{oF} = \sum\!\!\!\!\!\!\int_{\lambda} d^3q \, \omega_q b^\dagger_{\lambda q} b_{\lambda q}, \tag{29}$$

with $b_{\lambda q}$ the lowering operator of the mode with frequency ω_q, polarization λ, and wave vector q. The last term in Eq. (27),

$$H_{AF} = -\int d^3x \, \hat{\mathbf{D}}_{eg}(\vec{x}) [\vec{E}^{(+)}(\vec{x}) + \vec{E}^{(+)}_{\text{cl}}(\vec{x}, t)] + \text{H.c.}, \tag{30}$$

describes the interaction of the atoms with the light field at position $\hat{X}$ ($\hat{X}$ is the position operator), where

$$\hat{\mathbf{D}}_{eg}(\vec{x}) = \sum_\alpha \hat{\mathbf{D}}^{(\alpha)}_{eg} \delta(\vec{x} - \hat{\mathbf{X}}_\alpha) \equiv \sum_\alpha \hat{\mathbf{D}}^{(\alpha)}_{eg}(\vec{x}) \tag{31}$$

is the atomic dipole operator,

$$\vec{E}^{(+)}(\vec{x}) = -i \sum\!\!\!\!\!\!\int_{\lambda} d^3q \sqrt{\frac{\omega}{2\epsilon_0 (2\pi)^3}} \vec{\epsilon}_{\lambda q} b_{\lambda q} e^{-i\vec{q}\cdot\vec{x}} \tag{32}$$

the electric field operator, and $\vec{E}_{\text{cl}}(\vec{x}, t)$ the classical incident laser field. By $\vec{\epsilon}_{\lambda q}$ we denote the polarization vector for the mode characterized by λ and q.

Treating the radiation field as a reservoir, we derive a master equation for the N-atom reduced density operator $\rho(t)$ by tracing the total density operator $W(t)$ over the vacuum modes of the radiation field, $\rho(t) = \text{tr}_F W(t)$. We obtain

$$\begin{aligned} \frac{d\rho(t)}{dt} = &-i \sum_{\alpha \in A} [H^{(\alpha)}_{\text{eff}} \rho(t) - \rho(t) H^{\dagger(\alpha)}_{\text{eff}}] \\ &+\gamma \sum_{\alpha,\beta} \int d^3x d^3x' \hat{\mathbf{D}}^{(\alpha)}_{ge}(\vec{x}) \cdot \rho(t) \\ &\times \overleftrightarrow{F}(\vec{x} - \vec{x}') \cdot \hat{\mathbf{D}}^{(\beta)}_{eg}(\vec{x}'), \end{aligned}$$

with $H^{(\alpha)}_{\text{eff}}$ an effective non-Hermitian Hamiltonian

$$\begin{aligned} H^{(\alpha)}_{\text{eff}} \equiv \frac{\hat{P}^2_\alpha}{2m} + \left(-\Delta - i\frac{\gamma}{2}\right) \mathbf{P}^{(\alpha)}_e + \left(\int d^3x \hat{\mathbf{D}}^{(\alpha)}_{eg}(\vec{x}) \cdot \vec{E}^{(+)}_{\text{cl}}(\vec{x}, t) + \text{H.c.} \right) \\ -\frac{\gamma}{2} \sum_{\beta \neq \alpha} \int d^3x d^3x' \hat{\mathbf{D}}^{(\alpha)}_{eg}(\vec{x}) \cdot \overleftrightarrow{\phi}^{(-)}(\vec{x} - \vec{x}') \cdot \hat{\mathbf{D}}^{(\beta)}_{ge}(\vec{x}'). \end{aligned} \tag{33}$$

A derivation of this equation from the point of view of an Ito formalism of quantum stochastic differential equations can be found in Appendix A.

In Eq. (33) $\Delta = \omega - \omega_{eg}$ denotes the laser detuning, and γ is the spontaneous decay constant. The complex (tensor) interaction potentials $\overleftrightarrow{\phi}^{(-)} = \overleftrightarrow{G} + i\overleftrightarrow{F}$, as defined by

$$\overleftrightarrow{\phi}^{(-)}(\vec{x}-\vec{x}') \equiv \overleftrightarrow{G}(\vec{x}-\vec{x}') + i\overleftrightarrow{F}(\vec{x}-\vec{x}') = \lim_{\epsilon\to 0}\frac{3}{4\pi}\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\omega-\omega_{eg}-i\epsilon} \times \int d\Omega_{\vec{n}}(1-\vec{n}\otimes\vec{n})e^{i\vec{n}\cdot(\vec{x}-\vec{x}')k}, \quad (34)$$

describe the dipole-dipole and radiative interactions between the atoms located at $\vec{x}$ and $\vec{x}'$. These potentials are functions of the interatomic distance $|\vec{x}-\vec{x}'|$. For large interatomic separations $\overleftrightarrow{F}$, $\overleftrightarrow{G} \to \overleftrightarrow{0}$ and Eq. (33) reduces to the generalized optical Bloch equations for N noninteracting two-level atoms. Note that for $\vec{x} \to \vec{x}'$ (atomic self-interaction) the term $\gamma\overleftrightarrow{F}(0) = \gamma$ reduces to the radiative decay width of the atom while $\overleftrightarrow{G}(\vec{x}\to\vec{x}')$ contributes to the Lamb shift of the two-level atom (in the following we assume that this contribution is included in the transition frequency ω_{eg}). Explicit formulas for $\overleftrightarrow{F}$ and $\overleftrightarrow{G}$ can be found in Refs. [28–30] and are given below in Sec. II B.

Equation (33) is based on the Markov approximation which assumes that the time scale associated with retardation is short compared with the typical time scale of evolution of the system, i.e., $|\vec{x}-\vec{x}'|/c \ll \Delta t \approx 1/\gamma$ with $|\vec{x}-\vec{x}'| \approx L$ the dimension of the atomic sample, and c the speed of light. For a detailed discussion of the validity of the Markov approximation we refer to Refs. [31,32]. If the Markov approximation is not made, the effect of N-body interactions is to effectively screen the interaction potential [33]. However, this screening length is large compared to that associated with scattering of real photons (see later) provided that $\gamma|\vec{x}-\vec{x}'|/c \ll 1$ is again satisfied. Furthermore, in deriving Eq. (33) we have assumed a rotating-wave approximation (RWA). It can be shown that by extending all frequency integrations over the interval $-\infty < \omega < +\infty$, we obtain results which agree with calculations that include all the nonrotating terms [31,34].

On the right-hand side of the master equation (33) we have contributions involving the non-Hermitian Hamiltonian $H_{\rm eff}^{(\alpha)}$ (the first two terms), while the last term describes the return of electrons to the ground state after a photon emission [but modified by two-body interactions via $\overleftrightarrow{F}(\vec{x}-\vec{x}')$]. The non-Hermitian Hamiltonian $H_{\rm eff}^{(\alpha)}$ in Eq. (33) consists of a kinetic energy term, the free two-level Hamiltonian including radiative damping, the atom-laser interaction term, and the dipole and radiative interactions between different atoms. This interpretation is supported by rewriting Eq. (33) in terms of n-photon contributions to the density matrix $\rho(t)$: introducing a projection operator $P^{(n)}$ on an n-photon subspace we define a reduced density operator $\rho^{(n)}(t) = {\rm tr}_F P^{(n)} W(t)$, so that $\mathbf{P}^{(n)}(t) = {\rm tr}_A \rho^{(n)}(t)$ give the probability that the system has emitted n photons at time t. For $\rho^{(n)}(t)$ we find the equation of motion

$$\frac{d}{dt}\rho^{(n)}(t) = -i\sum_{\alpha}[H_{\rm eff}^{(\alpha)}\rho^{(n)}(t) - \rho^{(n)}(t)H_{\rm eff}^{\dagger(\alpha)}] + \gamma\frac{3}{4\pi}\int d\Omega_{\vec{n}}\left(\sum_{\alpha}\int d^3x\, e^{-ik_0\vec{n}\cdot\vec{x}}\,\hat{\mathbf{D}}_{ge}^{(\alpha)}(\vec{x})\right)\cdot(1-\vec{n}\otimes\vec{n})\rho^{(n-1)}(t)\cdot\left(\sum_{\beta}\int d^3x'\, e^{ik_0\vec{n}\cdot\vec{x}'}\hat{\mathbf{D}}_{eg}^{(\beta)}(\vec{x}')\right).$$

The last term in this equation involves $\rho^{(n-1)}(t)$ and is thus a source term for the equation describing the time evolution of $\rho^{(n)}(t)$. It describes the collective quantum jump of the N-atom system which is associated with the spatially coherent emission of photons. So we see, that if the system is observed to have emitted a photon in direction $\vec{n}$, the quantum jump projects the system onto a quite complicated superposition state of all the atoms. For the problem of two two-level atoms this state is a coherent superposition of the superradiant and subradiant vector state [28,29] if we initially start with both atoms in the excited state. The relative phase factor strongly depends on the relative position of the two atoms.

B. Angular momentum decomposition

For specific applications we choose a spherical vector basis $\vec{e}_q$ with $q = 0, \pm 1$ and diagonalize with respect to $\vec{e}_o \cdot \vec{J}_{e,g} \equiv J_{e,g}^z$. The corresponding eigenvectors are denoted by $|gm_g\rangle$ and $|em_g\rangle$, respectively, and we introduce the atomic lowering operators

$$\mathcal{A}_q \equiv \sum_{m_e,m_g}\langle J_g m_g 1q|J_e m_e\rangle|gm_g\rangle\langle em_e|, \quad (35)$$

with Clebsch-Gordan coefficients $\langle J_e m_e 1q|J_g m_g\rangle$. Expanding the electric field amplitude in the basis $\vec{e}_q$,

$$\vec{\mathcal{E}}^{(+)}\left(\hat{\mathbf{X}}_\alpha\right) = \int d^3x \sum_{q=0,\pm} E_q(\vec{x}_\alpha)\delta^3(\vec{x}_\alpha - \hat{\mathbf{X}}_\alpha)\vec{e}_q^{\,*}$$

we can write the master equation as

$$\frac{d\rho(t)}{dt} = -i\sum_\alpha [H_{\text{eff}}^{(\alpha)}\rho(t) - \rho(t)H_{\text{eff}}^{(\alpha)\dagger}] + \frac{\gamma}{2\pi}\sum_\alpha\sum_q \int d\Omega_{\vec{n}} N_q(\vec{n}) e^{-ik_0\vec{n}\hat{\mathbf{X}}_\alpha} \mathcal{A}_q^{(\alpha)}\rho(t)\mathcal{A}_q^{(\alpha)\dagger} e^{+ik_0\vec{n}\hat{\mathbf{X}}_\alpha}$$
$$+\gamma \sum_{\substack{(\alpha,\beta)\\ \alpha\neq\beta}}\sum_{q,q'} \mathcal{A}_q^{(\alpha)}\left(\int d^3x d^3x' \delta^3(\vec{x}-\hat{\mathbf{X}}_\alpha)F_{qq'}(\vec{x}-\vec{x}')\rho(t)\delta^3(\vec{x}'-\hat{\mathbf{X}}_\beta)\right)\mathcal{A}_{q'}^{(\beta)\dagger},$$

with

$$\phi_{qq'}^{(-)}(\vec{x}-\vec{x}') \equiv \lim_{\epsilon\to 0}\frac{3}{4\pi}\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{d\omega}{\omega-\omega_{eg}-i\epsilon}\int d\Omega_{\vec{n}}\left(\delta_{qq'} - n_{-q}n_{-q'}^*\right) e^{i\vec{n}\cdot(\vec{x}-\vec{x}')k_0}(-1)^{q+q'}$$
$$F_{qq'}(\vec{x}-\vec{x}') \equiv \text{Im}[\phi_{qq'}^{(-)}(\vec{x}-\vec{x}')], \tag{36}$$

and the angular distributions

$$N_0(\vec{n}) = \frac{3}{4}[1-(\hat{n}\cdot\vec{e}_0)^2]$$
$$N_\pm(\vec{n}) = \frac{3}{8}[1+(\hat{n}\cdot\vec{e}_0)^2]. \tag{37}$$

The effective Hamiltonian of the system is given by

$$H_{\text{eff}}^{(\alpha)} = \left(-\Delta - i\frac{\gamma}{2}\right)\sum_{m_e}|j_e m_e\rangle\langle j_e m_e| + \frac{\hat{P}_\alpha^2}{2m} + \left(\mu\sum_q\int d^3x\delta^3(\vec{x}-\hat{\mathbf{X}}_\alpha)\mathcal{A}_q^{(\alpha)}(-1)^q\mathcal{E}_{-q}(\vec{x}) + \text{H.c.}\right)$$
$$-\frac{\gamma}{2}\sum_{\beta\neq\alpha}\sum_{q,q'}\int d^3x d^3x'\delta^3(\vec{x}-\hat{\mathbf{X}}_\alpha)\delta^3(\vec{x}'-\hat{\mathbf{X}}_\beta)\phi_{qq'}^{(-)}(\vec{x}-\vec{x}')\mathcal{A}_q^{(\alpha)\dagger}\mathcal{A}_{q'}^{(\beta)}.$$

The atomic dipole moment μ is defined by

$$\mu \equiv \langle e \parallel D \parallel g\rangle/\sqrt{2J_e+1}\,. \tag{38}$$

For a discussion of the function $\phi_{qq'}^{(-)}(\vec{x}-\vec{x}')$ we rewrite the $\delta_{qq'} - n_{-q}n_{-q'}^*$ term in terms of spherical harmonics,

$$N_{qq'}(\vec{n}) = \delta_{qq'} - n_{-q}n_{-q'}^* = \delta_{qq'}\left(\frac{4\pi}{3}\right)Y_1^{-q}(\hat{n})Y_1^{-q'*}(\hat{n}),$$

where $\hat{n} \equiv (\vartheta,\varphi)$ are the spherical coordinates of $\vec{n}$.

If one notes that the exponentials in the integral of $\phi_{qq'}^{(-)}$ can be expressed by

$$e^{i\vec{n}\cdot\vec{r}k} = 4\pi\sum_{\ell,m} i^\ell j_\ell(k|\vec{r}|)\,Y_\ell^m(\hat{n})Y_\ell^{m*}(\hat{n}'),$$

with spherical coordinates $\hat{n}' \equiv (\vartheta',\varphi')$ of $\vec{r}$ and j_ℓ denoting spherical Bessel functions [35] we can evaluate the functions $\phi_{qq'}^{(-)}$ explicitly (see also [32]).

$$\phi_{qq'}^{(-)}(\vec{x}-\vec{x}') = i\left\{\delta_{qq'}h_0^{(1)}(k_0|\vec{x}-\vec{x}'|) + (-1)^q Y_2^{q'-q}(\hat{n}')\sqrt{6\pi}\begin{pmatrix}1 & 1 & 2\\ -q & q' & q-q'\end{pmatrix}\times h_2^{(1)}(k_0|\vec{x}-\vec{x}'|)\right\}$$
$$\longrightarrow f_{qq'}(\hat{n}')\frac{e^{-ik_0|\vec{x}-\vec{x}'|}}{k_0|\,\vec{x}-\vec{x}'\,|}\quad\text{for}\quad k_0|\,\vec{x}-\vec{x}'\,|\to\infty. \tag{39}$$

$h_k^{(1)}$ denotes the Hankel function of the first kind of order k. For $k_0|\vec{x}-\vec{x}'| \gg 1$ we find $h_0^{(1)}(k_0|\vec{x}-\vec{x}'|) \simeq -h_2^{(1)}(k_0|\vec{x}-\vec{x}'|) = -i\frac{\exp(-ik_0|\vec{x}-\vec{x}'|)}{k_0|\vec{x}-\vec{x}'|}$ and the expression for $\phi_{qq'}^{(-)}(\vec{x}-\vec{x}')$ can be written as a product of an angular distribution $f_{qq'}(\hat{n}')$ and a spherical wave function. The angular distributions we need in Sec. V are given by

$$f_{1,1}(\hat{n}') = f_{-1,-1}(\hat{n}') = \frac{3}{2}\left(\frac{1}{2}(1+\cos^2\theta')\right),$$

$$f_{1,-1}(\hat{n}') = f^*_{-1,1}(\hat{n}') = \frac{3}{2}\left(\frac{1}{2}e^{-2i\phi'}\sin^2\theta'\right). \tag{40}$$

C. Equation of motion for the Wigner function

For a semiclassical solution of the problem we use the phase space methods commonly used in laser cooling of atoms instead of classical trajectory descriptions often used in collision physics. For further reference we review the most important relations. The Wigner operator equivalent to the density operator of a system with c.m. dynamics is defined according to Wigner by

$$W(\vec{x}_1,\ldots,\vec{x}_N,\vec{p}_1,\ldots,\vec{p}_N,t) \equiv \int \prod_\alpha \frac{d^3 r_\alpha}{(2\pi i)^3}\, e^{i\vec{p}_\alpha\cdot\vec{r}_\alpha}\left\langle \vec{x}_1 - \frac{1}{2}\vec{r}_1,\ldots|\rho(t)|\vec{x}_1+\frac{1}{2}\vec{r}_1,\ldots\right\rangle.$$

Note that W is still an operator for internal degrees of freedom. This formula has an inverse relation given by

$$\langle\vec{x}_1,\ldots|\rho(t)|\vec{x}_1',\ldots\rangle = \int \prod_\alpha d^3p_\alpha e^{i\vec{p}_\alpha\cdot(\vec{x}_\alpha-\vec{x}_\alpha')}W\left(\frac{1}{2}(\vec{x}_1+\vec{x}_1'),\ldots,\vec{p}_1,\ldots,t\right).$$

Our aim is to derive a differential equation for the Wigner operator. This equation will be the starting point of our semiclassical expansion and can be easily written down if one uses some rules of general validity, which have been derived for translating the master equation to the Wigner representation.

Density operator ρ_A(t)		Wigner function $W(\vec{x},\vec{p},t)$
$\frac{d}{dt}\,\rho_A - \frac{i}{\hbar}\left[\frac{\hat{p}^2}{2m},\rho_A\right]$	$\longrightarrow$	$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\cdot\nabla_{\vec{x}}\right)W(\vec{x},\vec{p},t)$
$e^{i\vec{k}\cdot\hat{X}}\,\rho_A$	$\longrightarrow$	$e^{i\vec{k}\cdot\vec{x}}\,W(\vec{x},\vec{p}-\frac{1}{2}\hbar\vec{k},t)$
$\rho_A e^{i\vec{k}\cdot\hat{X}}$	$\longrightarrow$	$e^{i\vec{k}\cdot\vec{x}}\,W(\vec{x},\vec{p}+\frac{1}{2}\hbar\vec{k},t)$
$F(\hat{X})\rho_A$	$\longrightarrow$	$\int d^3k\,\tilde{F}(\vec{k})\,e^{i\vec{k}\cdot\vec{x}}\,e^{-i\frac{1}{2}\vec{k}\cdot(-i\nabla_{\vec{p}})}\,W(\vec{x},\vec{p},t)$
$\rho_A F(\hat{X})$	$\longrightarrow$	$\int d^3k\,\tilde{F}(\vec{k})\,e^{i\vec{k}\cdot\vec{x}}\,e^{i\frac{1}{2}\vec{k}\cdot(-i\nabla_{\vec{p}})}\,W(\vec{x},\vec{p},t)$

Using the notation on the right-hand side of the above tabulation we can write the Wigner operator equation in a rather compact form

$$\begin{aligned}\frac{\partial W(t)}{\partial t} + \sum_\alpha \frac{\vec{P}_\alpha}{m}\cdot\vec{\nabla}_{x_\alpha}W(t) &= -i\sum_\alpha[h_{\text{eff}}^{(\alpha)}W(t) - W(t)h_{\text{eff}}^{\dagger(\alpha)}]\\ &+\gamma\sum_\alpha\sum_{qq'}\mathcal{A}_q^{(\alpha)}\int d\Omega_{\vec{n}}N_q(\vec{n})e^{-ik_0\vec{n}\cdot[(-i)\nabla_{\vec{p}_\alpha}]}W(t)\mathcal{A}_q^{(\alpha)\dagger}\\ &+\gamma\sum_{\substack{(\alpha,\beta)\\ \alpha\neq\beta}}\sum_{qq'}\mathcal{A}_q^{(\alpha)}\int d\Omega_{\vec{n}}N_{qq'}(\vec{n})e^{-i\frac{1}{2}k_0\vec{n}\cdot[(-i)\nabla_{\vec{p}_\alpha}+(-i)\nabla_{\vec{x}_\beta}]}W(t)\mathcal{A}_{q'}^{(\beta)\dagger},\end{aligned} \tag{41}$$

with

$$h_{\rm eff}^{(\alpha)} \equiv \left(-\Delta - i\frac{\gamma}{2}\right) P_e^{(\alpha)} + \mu \sum_q \mathcal{A}_q^{(\alpha)} \left[\sum\!\!\!\!\!\!\int_{\lambda} d^3k \mathcal{E}(\vec{k},\lambda)\vec{\epsilon}_{\vec{k},\lambda} e^{i\vec{k}\cdot\vec{x}_\alpha} e^{i\frac{1}{2}\vec{k}\cdot(-i\hbar\nabla_{\vec{p}_\alpha})} + \text{H.c.}\right]$$
$$-\frac{\gamma}{2} \sum_{\substack{(\alpha,\beta)\\ \beta\neq\alpha}} \sum_{qq'} \mathcal{A}_q^{(\alpha)} \left(\lim_{\epsilon\to 0} \frac{3}{4\pi}\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{d\omega}{\omega - \omega_{eg} - i\epsilon} \int d\Omega_{\vec{n}} N_{qq'}(\vec{n}) e^{ik_0\vec{n}\cdot(\vec{x}_\alpha - \vec{x}_\beta)} e^{i\frac{1}{2}k_0\vec{n}\cdot(-i\hbar\nabla_{\vec{p}_\alpha})}\right.$$
$$\left. \times e^{-i\frac{1}{2}k_0\vec{n}\cdot(-i\hbar\nabla_{\vec{p}_\beta})}\right) \mathcal{A}_{q'}^{(\beta)\dagger} . \tag{42}$$

By $\mathcal{E}(\vec{k},\lambda)\vec{\epsilon}_{\vec{k},\lambda}$ we denote the Fourier components of the classical laser field. Note that the displacement operators in Eq. (42) are a signature for the processes of momentum exchange between laser and atoms and atoms with atoms due to the recoil effect of spontaneous emission. So we see that, in fact, the momentum kicks in the recycling term [last two lines in Eq. (42)] have both the same direction and show the change in the c.m. momentum. In the effective Hamiltonian we find the opposite direction. This shows the change in the relative momentum.

IV. DISCUSSION OF THE SEMICLASSICAL FORCE

In the preceding section we have derived the master equation for an N-atom system in a Wigner function representation for the c.m. coordinates. Below we will consider the semiclassical limit of this equation, and derive a Fokker-Planck equation for the N-atom distribution function

$$f(x,p,t) = \text{tr}_{\rm int} W(x,p,t) \tag{43}$$

by adiabatic elimination of the atomic internal degrees of freedom. $\text{tr}_{\rm int}(\,)$ denotes the trace with respect to the internal degrees of freedom. This will provide us with expressions for the mechanical light forces between the atoms. The semiclassical approximation is based on the assumption that the width of the atomic momentum distribution Δp is broad compared with the momentum transfers $\hbar k$ associated with photon emissions and absorptions, $\Delta p \gg \hbar k$. Adiabatic elimination of the internal degrees of freedom assumes that the atomic distribution changes slowly on the scale given by the internal atomic dynamics. Although our derivation of the Fokker-Planck equation for $f(x,p,t)$ is fairly general, our discussion will focus on the atomic light forces (drift terms in the FPE), and we leave a more detailed study of the diffusion terms to a future publication.

A. Semiclassical expansion

In a semiclassical analysis the Wigner function is expanded as

$$W(\ldots,\vec{p}_\alpha + \hbar\vec{k}_\alpha,\ldots) = \left[1 + \sum_\alpha \sum_i \hbar k_\alpha^i \cdot \frac{\partial}{\partial p_\alpha^i} + \frac{1}{2}\sum_{\alpha,\beta}\sum_{ij} \hbar^2 k_\alpha^i k_\beta^j \frac{\partial^2}{\partial p_\alpha^i \partial p_\beta^j} + \ldots\right] \times W(\ldots,\vec{p}_\alpha,\ldots) \tag{44}$$

keeping terms up to second order in $\hbar k$. In this approximation the equation of motion for $W(x,p,t)$ takes on the form

$$\left(\frac{\partial}{\partial t} + \sum_\alpha \frac{\vec{p}_\alpha}{m}\cdot\frac{\partial}{\partial \vec{x}_\alpha}\right) W(x,p,t) = (\mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2)\, W(x,p,t), \tag{45}$$

where $\mathcal{L}_0$, $\mathcal{L}_1$, and $\mathcal{L}_2$ are, respectively, the zeroth-, first-, and second-order terms in the $\hbar k$ expansion.

1. Zeroth order

The zeroth-order term $\mathcal{L}_0$ is the *optical Bloch operator* for the N-atom system (see Lehmberg's discussion of the N-atom master equation [28]). We find it convenient to write this operator as the sum of two contributions,

$$\mathcal{L}_0 = \mathcal{L}_0^{(1)} + \mathcal{L}_0^{(2)}, \tag{46}$$

a *single-atom* term corresponding to a *semiclassical mean field theory*, and a two-particle term which describes the deviations from this mean field picture. The single-atom term $\mathcal{L}_0^{(1)}$ is

$$\mathcal{L}_0^{(1)} W \equiv i\left\{ \sum_\alpha \left[\left(\Delta + i\frac{\gamma}{2}\right)\mathbf{P}_e^{(\alpha)} - \sum_q [\mathcal{A}_\alpha^{q^\dagger} \mu E_{\rm eff}^{(+)^q}(\vec{x}_\alpha)^* + \text{H.c.}]\right], W\right\} + \gamma \sum_{\alpha,q} \mathcal{A}_\alpha^q W \mathcal{A}_\alpha^{q^\dagger} , \tag{47}$$

with $\vec{E}_{\rm eff}^{(+)}(x,t)$ an *effective field* determined by a *mean atomic polarization* $\langle \mathcal{A}_\alpha^q \rangle_\sigma$ [which will be defined below in Eq. (49) as the self-consistent solution of the Maxwell-Bloch equations for the propagation of the incident laser field in the atomic medium]. The term $\mathcal{L}_0^{(2)}$ is a *two-atom* interaction

$$\mathcal{L}_0^{(2)}W \equiv -i\frac{\gamma}{2}\sum_{\alpha\neq\beta}\sum_{qq'}\phi_{qq'}^{(-)}(\vec{x}_\alpha-\vec{x}_\beta)(\tilde{\mathcal{A}}_\alpha^q W\tilde{\mathcal{A}}_\beta^{q'\dagger}-\tilde{\mathcal{A}}_\alpha^{q\dagger}\tilde{\mathcal{A}}_\beta^{q'}W)-i\frac{\gamma}{2}\sum_{\alpha\neq\beta}\sum_{qq'}\phi_{qq'}^{(+)}(\vec{x}_\alpha-\vec{x}_\beta)(W\tilde{\mathcal{A}}_\alpha^{q\dagger}\tilde{\mathcal{A}}_\beta^{q'}-\tilde{\mathcal{A}}_\alpha^q W\tilde{\mathcal{A}}_\beta^{q'\dagger}).$$

Here

$$\tilde{\mathcal{A}}_\alpha^q \equiv \mathcal{A}_\alpha^q - \langle\mathcal{A}_\alpha^q\rangle_\sigma \tag{48}$$

is defined as the atomic lowering operator with its mean value subtracted, $\Phi_{qq'}^{(+)}$ denotes the complex conjugate of $\Phi_{qq'}^{(-)}$, and

$$\mu\vec{E}_{\text{eff}}^{(+)}(\vec{x}_\alpha)=\mu\vec{E}_{\text{cl}}^{(+)}(\vec{x}_\alpha)-\sum_q \vec{e}_q\sum_{\beta\neq\alpha}\sum_{q'}\frac{\gamma}{2}\phi_{qq'}^{(-)}(\vec{x}_\alpha-\vec{x}_\beta)\langle\mathcal{A}_\beta^{q'\dagger}\rangle_\sigma \tag{49}$$

is the mean field which is the sum of the incident field and the field radiated by the atomic dipoles. Note that by $\langle\ \rangle_\sigma$ we denote the expectation with respect to an *independent particle density operator* σ defined as the steady state solution $\mathcal{L}_0^{(1)}\sigma=0$ [see Eq. (69) below].

2. First order

The first-order term in the $\hbar k$ expansion is

$$\begin{aligned}\mathcal{L}_1 W \equiv & -\frac{1}{2}\sum_\alpha\frac{\partial}{\partial\vec{p}_\alpha}\{\hat{\vec{F}}_\alpha^{(1)}(\vec{x}_\alpha),W\}-\frac{\gamma}{4}\sum_{\alpha,\beta}\left(\frac{\partial}{\partial\vec{x}_\alpha}G_{qq'}(\vec{x}_\alpha-\vec{x}_\beta)\right)\cdot\frac{\partial}{\partial\vec{p}_\alpha}\{\mathcal{A}_\alpha^q\tilde{\mathcal{A}}_\beta^{q'\dagger}+\tilde{\mathcal{A}}_\beta^{q'}\mathcal{A}_\alpha^{q\dagger},W\}\\ & -i\frac{\gamma}{4}\sum_{\alpha,\beta}\left(\frac{\partial}{\partial\vec{x}_\alpha}F_{qq'}(\vec{x}_\alpha-\vec{x}_\beta)\right)\cdot\frac{\partial}{\partial\vec{p}_\alpha}[\mathcal{A}_\alpha^q\tilde{\mathcal{A}}_\beta^{q'\dagger}+\tilde{\mathcal{A}}_\beta^{q'}\mathcal{A}_\alpha^{q\dagger},W]\\ & +i\frac{\gamma}{4}\sum_{\alpha,\beta}\left(\frac{\partial}{\partial\vec{x}_\alpha}F_{qq'}(\vec{x}_\alpha-\vec{x}_\beta)\right)\cdot\frac{\partial}{\partial\vec{p}_\alpha}(2\mathcal{A}_\alpha^q W\tilde{\mathcal{A}}_\beta^{q'\dagger}-2\tilde{\mathcal{A}}_\beta^{q'}W\mathcal{A}_\alpha^{q\dagger})\end{aligned} \tag{50}$$

where

$$\hat{\vec{F}}_\alpha^{(1)}(\vec{x}_\alpha)\equiv\left(\sum_q\mathcal{A}_\alpha^{q\dagger}\cdot\frac{\partial}{\partial\vec{x}_\alpha}\mu E_{\text{eff}}^{(+)^{q*}}(\vec{x}_\alpha)+\text{H.c.}\right) \tag{51}$$

will be identified below in the Fokker-Planck equation for $f(x,p,t)$ as the *single-atom force* due to the *mean field* $\vec{E}_{\text{eff}}^{(+)}$, and the last three lines will give rise below to *two-atom forces.*

3. Second order

Finally, the second-order terms are

$$\begin{aligned}\mathcal{L}_2 W \equiv\ & i\frac{1}{4}\sum_\alpha\sum_{ij}\frac{\partial^2}{\partial p_\alpha^i\partial p_\alpha^j}\left[\frac{\partial^2}{\partial x_\alpha^i\partial x_\alpha^j}\left(\sum_q\mathcal{A}_\alpha^{q\dagger}\mu E_{\text{cl}}^{(+)^{q*}}(\vec{x}_\alpha)+\text{H.c.}\right),W\right]\\ & +\left[-\sum_{\alpha\neq\beta}\sum_{ij}\frac{1}{8}\left(\frac{\partial}{\partial p_\alpha^i}-\frac{\partial}{\partial p_\beta^i}\right)\left(\frac{\partial}{\partial p_\alpha^j}-\frac{\partial}{\partial p_\beta^j}\right)\frac{\partial^2}{\partial x_\alpha^i\partial x_\beta^j}\left(\sum_{qq'}\frac{\gamma}{2}\phi_{qq'}^{(-)}(\vec{x}_\alpha-\vec{x}_\beta)\mathcal{A}_\alpha^q\mathcal{A}_\beta^{q'\dagger}\right)W+\text{H.c.}\right]\\ & +\sum_\alpha\sum_{ij}\frac{\partial^2}{\partial p_\alpha^i\partial p_\alpha^j}\left(\frac{\gamma k_l^2}{2}\sum_q\kappa_{ij}^q\mathcal{A}_\alpha^q W\mathcal{A}_\alpha^{q\dagger}\right)\\ & +i\sum_{\alpha\neq\beta}\sum_{ij}\frac{1}{8}\left(\frac{\partial}{\partial p_\alpha^i}+\frac{\partial}{\partial p_\beta^i}\right)\left(\frac{\partial}{\partial p_\alpha^j}+\frac{\partial}{\partial p_\beta^j}\right)\frac{\partial^2}{\partial x_\alpha^i\partial x_\beta^j}\left(\sum_{qq'}\gamma F_{qq'}(\vec{x}_\alpha-\vec{x}_\beta)\mathcal{A}_\alpha^q W\mathcal{A}_\beta^{q'\dagger}\right),\end{aligned} \tag{52}$$

$$\kappa_{ij}^q \equiv \int d\Omega_{\vec{n}}n^i n^j N_q(\vec{n}). \tag{53}$$

κ^q_{ij} is the average value of $n^i n^j$ on line q, where $\hat{n}$ is the angular direction of the momentum of the fluorescence photon.

4. Discussion

Before we discuss the main features of the adiabatic elimination procedure, we comment on these terms again.

(1) $\mathcal{L}_0^{(1)}$ is identical to the Bloch problem of N "independent" atoms. The atom α located at x_α is affected by the effective stochastic field $E_{\rm eff}$, which is the sum of the incident laser field and the mean radiated field of all the other atoms β.

(2) $\mathcal{L}_0^{(2)}$ are the two-atom contributions that give a correction to the mean field picture. $\mathcal{L}_0 = \mathcal{L}_0^{(1)} + \mathcal{L}_0^{(2)}$ corresponds to the Lehmberg Bloch operator [28,29] in the zero-velocity regime ($\vec{v} = \vec{0}$) and describes a system of N interacting atoms without any mechanical light effects.

(3) $\mathcal{L}_1$ is first order in $\hbar k$ and associated with the atomic light force operator. The first term, $F_\alpha^{(1)}(\vec{x}_\alpha)$, describes the usual one atom force operator in an effective (mean) laser field.

(4) The rest of $\mathcal{L}_1$ is associated with incoherent radiative interactions between atoms.

B. Adiabatic elimination of internal degrees of freedom

If the internal dynamics and the time scale of collisions is fast compared with the cooling time one can adiabatically eliminate the internal degrees of freedom and derive an equation of motion for the Wigner function $f(x,p,t) \equiv {\rm tr}_{\rm int}\{W(x,p,t)\}$, which completely determines the c.m. motion of the atoms. The adiabatic elimination is done by the projection operator formalism worked out in detail in Ref. [36] although we are careful to include atomic motion in our projection operator. The choice of our reference state is fixed by the $\hbar\vec{k} = \vec{0}$ terms and is therefore defined by the solution of the N-atom Bloch equation without any mechanical light effects, since one expects the deviation from this steady state to be rather small. Explicitly the reference state for adiabatic elimination is given by

$$\mathcal{L}_{\rm Bloch}\rho_{\rm ref}(x,p) \equiv \mathcal{L}_0\rho_{\rm ref}(x,p) - \sum_\alpha \frac{\vec{p}_\alpha}{m}\cdot\frac{\partial}{\partial\vec{x}_\alpha}\rho_{\rm ref}(x,p) = 0\,. \tag{54}$$

The projection operator is defined by

$$[\mathcal{P}X](x,p) \equiv \rho_{\rm ref}(x,p){\rm tr}_{\rm int}\{X(x,p)\}\,, \tag{55}$$

$$f(x,p,t) \equiv {\rm tr}_{\rm int}\{W(x,p,t)\}\,, \tag{56}$$

$$[\mathcal{P}W](x,p,t) = \rho_{\rm ref}(x,p)f(x,p,t). \tag{57}$$

Obviously $\mathcal{P}$ has the characteristics of a projection operator

$$\mathcal{P}^2X = \mathcal{P}X. \tag{58}$$

Using these definitions, we can write Eq. (45) as

$$\left(\frac{\partial}{\partial t} + \sum_\alpha \frac{\vec{p}_\alpha}{m}\cdot\frac{\partial}{\partial\vec{x}_\alpha}\right)W(x,p,t) = (\mathcal{L}_{\rm Bloch} + \mathcal{L}_1' + \mathcal{L}_2')\,W(x,p,t) \tag{59}$$

with

$$\begin{aligned}\mathcal{L}_{\rm Bloch}W &\equiv \mathcal{L}_0W - \sum_\alpha\left[\frac{\vec{p}_\alpha}{m}\cdot\frac{\partial}{\partial\vec{x}_\alpha}\rho_{\rm ref}(x,p)\right]{\rm tr}_{\rm int}\{W\}\,,\\ \mathcal{L}_1'W &\equiv (\langle\mathcal{L}_1\rangle_{\rho_{\rm ref}} + \langle\mathcal{L}_2\rangle_{\rho_{\rm ref}})\,W\\ &\quad -\rho_{\rm ref}(x,p)\sum_\alpha\frac{\vec{p}_\alpha}{m}\frac{\partial}{\partial\vec{x}_\alpha}{\rm tr}_{\rm int}\{W\}\\ &\quad -\sum_\alpha\frac{\vec{p}_\alpha}{m}\frac{\partial}{\partial\vec{x}_\alpha}(1-\mathcal{P})W,\\ \mathcal{L}_2'W &\equiv (\mathcal{L}_1 - \langle\mathcal{L}_1\rangle_{\rho_{\rm ref}} + \mathcal{L}_2 - \langle\mathcal{L}_2\rangle_{\rho_{\rm ref}})\,W\,.\end{aligned} \tag{60}$$

Obviously we have $\mathcal{P}\mathcal{L}_{\rm Bloch} = \mathcal{L}_{\rm Bloch}\mathcal{P} \equiv 0$ and $\mathcal{P}\mathcal{L}_2\mathcal{P} \equiv 0$. Note also that we have included the c.m. motion through the $\frac{\vec{p}_\alpha}{m}$ terms. Up to second-order perturbation theory in $\hbar k$ we get for the equation of motion for the Wigner function $f(x,p,t)$ (see [37]) (assuming that the time scale of cooling is much longer than any other time scale so that the Markovian approximation can be made)

$$\frac{\partial}{\partial t}f(x,p,t) + \sum_\alpha\frac{\vec{p}_\alpha}{m}\frac{\partial}{\partial\vec{x}_\alpha}f(x,p,t) = [\langle\mathcal{L}_1\rangle_{\rho_{\rm ref}} + \langle\mathcal{L}_2\rangle_{\rho_{\rm ref}}]f(x,p,t) + \int_0^\infty d\tau\,{\rm tr}_{\rm int}\{[\mathcal{L}_1 - \langle\mathcal{L}_1\rangle_{\rho_{\rm ref}}]\,e^{(\mathcal{L}_0 - \sum_\alpha\frac{\vec{p}_\alpha}{m}\frac{\partial}{\partial\vec{x}_\alpha})\tau} \times[\mathcal{L}_1 - \langle\mathcal{L}_1\rangle_{\rho_{\rm ref}}]\rho_{\rm ref}(x,p)\}f(x,p,t), \tag{61}$$

with $\langle\mathcal{L}_1\rangle_{\rho_{\rm ref}}$ given by

$$\begin{aligned}\langle\mathcal{L}_1\rangle_{\rho_{\rm ref}} = &-\sum_\alpha\frac{\partial}{\partial\vec{p}_\alpha}{\rm tr}_{\rm int}[\hat{\vec{F}}_\alpha^{(1)}(\vec{x}_\alpha)\rho_{\rm ref}] - \frac{\gamma}{4}\sum_{\alpha,\beta}\left(\frac{\partial}{\partial\vec{x}_\alpha}\Phi_{qq'}^{(+)}(\vec{x}_\alpha - \vec{x}_\beta)\right)\cdot\frac{\partial}{\partial\vec{p}_\alpha}{\rm tr}_{\rm int}\left(\mathcal{A}_\alpha^q\tilde{\mathcal{A}}_\beta^{q'\dagger}\rho_{\rm ref}\right)\\ &-\frac{\gamma}{4}\sum_{\alpha,\beta}\left(\frac{\partial}{\partial\vec{x}_\alpha}\Phi_{qq'}^{(-)}(\vec{x}_\alpha - \vec{x}_\beta)\right)\cdot\frac{\partial}{\partial\vec{p}_\alpha}{\rm tr}_{\rm int}\left(\tilde{\mathcal{A}}_\beta^{q'}\mathcal{A}_\alpha^{q\dagger}\rho_{\rm ref}\right)\end{aligned} \tag{62}$$

and $\langle \mathcal{L}_2\rangle_{\rho_{\rm ref}}$ given by

$$\langle \mathcal{L}_2\rangle_{\rho_{\rm ref}} = \sum_{\alpha}\sum_{ij}\frac{\partial^2}{\partial p^i_\alpha \partial p^j_\alpha}\left(\frac{\gamma k_l^2}{2}\sum_q \kappa^q_{ij}\langle \mathcal{A}^{q^\dagger}_\alpha \mathcal{A}^q_\alpha\rangle_{\rho_{\rm ref}}\right) + \sum_{\alpha\neq\beta}\sum_{ij}\frac{1}{4}\frac{\partial^2}{\partial p^i_\alpha \partial p^j_\beta}\left(\frac{\partial^2}{\partial x^i_\alpha \partial x^j_\beta}\left[\sum_{qq'}\gamma F_{qq'}(\vec{x}_\alpha - \vec{x}_\beta)\langle \mathcal{A}^q_\alpha \mathcal{A}^{q'^\dagger}_\beta + \mathcal{A}^{q'}_\beta \mathcal{A}^{q^\dagger}_\alpha\rangle_{\rho_{\rm ref}}\right]\right) .$$

Equation (61) contains several types of terms: a velocity dependent drift part and diffusion terms. Note that the diffusion term [last term in Eq. (61)] actually contains *two* time scales, a short one which is determined by the spontaneous decay time and a longer one that can be identified with a collision time that has its origin in the kinetic term in the exponential, which leads to the evolution of $\mathcal{L}_1$ under a velocity $\frac{\vec{p}_\alpha}{m}$ for time τ. Physically this means that the force between two atoms shows a long term correlation since an atom can run through several correlated photon absorption and emission processes before they separate from each other. For small velocities one can expand the drift term up to first order in the velocity and obtain a static force and a linear damping term.

Here we are mainly interested in the static drift terms ($\vec{v} = \vec{0}$). So for the present we formally drop the kinetic part in $\mathcal{L}_{\rm Bloch}$. This defines a Liouville-type equation for the N-atom Wigner function

$$\frac{\partial}{\partial t}f(x,p,t) + \sum_\alpha \frac{\vec{p}_\alpha}{m}\frac{\partial}{\partial \vec{x}_\alpha}f(x,p,t) = \langle \mathcal{L}_1\rangle_{\rho_{\rm ref}} f(x,p,t) \tag{63}$$

with

$$\langle \mathcal{L}_1\rangle_{\rho_{\rm ref}} = -\sum_\alpha [\vec{F}^{(1)}_\alpha(\vec{x}_\alpha) + \vec{F}^{(2)}_\alpha(\vec{x}_\alpha) + \vec{F}^{(3)}_\alpha(\vec{x}_\alpha)]\cdot\frac{\partial}{\partial \vec{p}_\alpha} \tag{64}$$

and

$$\begin{aligned}
\vec{F}^{(1)}_\alpha(\vec{x}_\alpha) &\equiv \Big(\sum_q (-1)^q \langle \mathcal{A}^\dagger_{\alpha,q}\rangle_\sigma \frac{\partial}{\partial \vec{x}_\alpha} \times \mu E^{(-)}_{{\rm eff},-q}(\vec{x}_\alpha) + {\rm c.c.}\Big),\\
\vec{F}^{(2)}_\alpha(\vec{x}_\alpha) &\equiv \Big(\sum_\beta\sum_{qq'}\frac{\gamma}{2}\frac{\partial}{\partial \vec{x}_\alpha}\phi^{(-)}_{qq'}(\vec{x}_\alpha - \vec{x}_\beta) \times \langle \tilde{\mathcal{A}}_{\beta,q'}\tilde{\mathcal{A}}^\dagger_{\alpha,q}\rangle_{\rho_{\rm ref}} + {\rm c.c.}\Big),\\
\vec{F}^{(3)}_\alpha(\vec{x}_\alpha) &\equiv \Big(\sum_\beta\sum_{qq'}\frac{\gamma}{2}\frac{\partial}{\partial \vec{x}_\alpha}\phi^{(-)}_{qq'}(\vec{x}_\alpha - \vec{x}_\beta)\langle \mathcal{A}_{\alpha,q'}\rangle_\sigma \times\langle \tilde{\mathcal{A}}^\dagger_{\beta,q}\rangle_{\rho_{\rm ref}} + {\rm c.c.}\Big)\\
&\quad + \Big(\sum_q (-1)^q \langle \tilde{\mathcal{A}}^{q^\dagger}_\alpha\rangle_{\rho_{\rm ref}}\frac{\partial}{\partial \vec{x}_\alpha} \times \mu E^{(-)}_{{\rm eff},-q}(\vec{x}_\alpha) + {\rm c.c.}\Big).
\end{aligned} \tag{65}$$

Here $\vec{F}^{(1)}_\alpha$ is the single-atom force in the mean field (49). $\vec{F}^{(2)}_\alpha$ is a two-atom force which is proportional to the gradient of the mean potential energy between the atomic dipoles in the reference state of Eq. (54) (note that the interaction of the mean dipoles was already taken into account into the single-atom force). Finally, $\vec{F}^{(3)}_\alpha$ is a small correction to the one-atom force which arises from the changes in the atomic coherences due to the incoherent back ground field (for a more detailed discussion see Sec. V A). $\vec{F}^{(2)}_\alpha$ can then be written in the form

$$\vec{F}^{(2)}_\alpha(\vec{x}_\alpha) = -\left(\sum_\beta\sum_q (-1)^q \times\left\langle \tilde{\mathcal{A}}^{q^\dagger}_\alpha \frac{\partial}{\partial \vec{x}_\alpha}\mu\delta E^{(-)}_{-q}(\vec{x}_\alpha)\right\rangle_{\rho_{\rm ref}} + {\rm c.c.}\right) \tag{66}$$

with $\delta E^{(-)}_q(\vec{x}_\alpha)$ defined by the quantum fluctuations of the electric field,

$$\delta E^{(-)}_q(\vec{x}_\alpha) \equiv -\sum_\beta\sum_{q'}\frac{\gamma}{2}\phi^{(-)}_{qq'}(\vec{x}_\alpha - \vec{x}_\beta)\tilde{\mathcal{A}}^{q'}_\beta . \tag{67}$$

Note that due to the random distribution of the atoms, the effective mean field and the force are still random quantities in the position variables $\vec{x}_\beta$ of the atoms.

C. Semiclassical N-atom force in perturbation theory

In the preceding section we derived expressions for the one- and two-atom forces [see Eq. (65)]. These forces were defined with respect to the reference state $\rho_{\rm ref}$ [Eq. (54) in the limit of zero velocity]. This reference state is the solution of the N-atom Lehmberg-Bloch problem. In this section we construct this reference state in perturbation theory starting from N independent particles. Since the asymptotic behavior of $\mathcal{L}^{(2)}_0$ is dominated by the $1/r$ characteristic of $\Phi^{(\pm)}(\vec{x}_\alpha - \vec{x}_\beta)$, we seek a solution by treating these interaction terms in perturbation theory, which is certainly permissible if the mean distance between two atoms is much larger than the laser wavelength. We will see that the lowest-order term contains the WSW model [6]. We illustrate the formal results derived in the following subsection by the example of a V system in a σ^--σ^+ laser configuration.

A perturbation expansion for $\rho_{\rm ref}$ is derived by iterating the integral equation

$$\rho_{\rm ref} = \sigma + \int_0^\infty dt \quad e^{\sum_\alpha \mathcal{L}^{(1)}_{0,\alpha} t} \sum_{\alpha,\beta} \mathcal{L}^{(2)}_{0,\alpha\beta}\rho_{\rm ref}$$

$$= \sigma + \int_0^\infty dt \quad e^{\sum_\alpha \mathcal{L}^{(1)}_{0,\alpha} t} \sum_{\alpha,\beta} \mathcal{L}^{(2)}_{0,\alpha\beta}\sigma + \cdots, \qquad (68)$$

with σ the solution of the independent particle equation

$$\mathcal{L}^{(1)}_0 \sigma = \sum_\alpha \mathcal{L}^{(1)}_{0,\alpha}\sigma = 0. \qquad (69)$$

[Note that $\mathcal{L}^{(1)}_0$ contains the effective field $\vec{E}_{\rm eff}$, (49),(47).] In *first* order we obtain for the one-atom force due to the mean field

$$\vec{F}^{(1)}_\alpha \equiv \left(\sum_q \langle \mathcal{A}^\dagger_{\alpha,q}\rangle_\sigma \frac{\partial}{\partial \vec{x}_\alpha} \mu E^{(+)^{q^*}}_{\rm eff}(\vec{x}_\alpha) + {\rm c.c.} \right). \qquad (70)$$

The two-atom force is

$$\vec{F}^{(2)}_\alpha = \frac{\gamma}{2} \sum_{\beta\neq\alpha} \sum_{q,q'} \frac{\partial}{\partial \vec{x}_{\alpha\beta}} \phi^{(+)\alpha\beta}_{qq'} {\rm tr}_{\rm int}\{\tilde{\mathcal{A}}^\dagger_{\alpha,q}\tilde{\mathcal{A}}_{\beta,q'}\rho_{\rm ref}\}$$

$$+\frac{\gamma}{2} \sum_{\beta\neq\alpha} \sum_{q,q'} \frac{\partial}{\partial \vec{x}_{\alpha\beta}} \phi^{(-)\alpha\beta}_{qq'} {\rm tr}_{\rm int}\{\tilde{\mathcal{A}}^\dagger_{\beta,q}\tilde{\mathcal{A}}_{\alpha,q'}\rho_{\rm ref}\},$$

with $\phi^{(\pm)\alpha\beta}_{qq'} \equiv G^{\alpha\beta}_{qq'} \pm iF^{\alpha\beta}_{qq'}$ and

$$\begin{aligned} {\rm tr}_{\rm int}\{\tilde{\mathcal{A}}^\dagger_{\alpha,q}\tilde{\mathcal{A}}_{\beta,q'}\rho_{\rm ref}\} = \langle \mathcal{A}^\dagger_{\alpha,q}\rangle\langle \mathcal{A}_{\beta,q'}\rangle - i\frac{\gamma}{2}\sum_{s,s'}\phi^{(+)\alpha\beta}_{ss'} \Bigg\{ \int_0^\infty dt \{ & \langle \tilde{\mathcal{A}}^\dagger_{\alpha,q}(t)\tilde{\mathcal{A}}_{\alpha,s}\rangle_\sigma \langle [\tilde{\mathcal{A}}^\dagger_{\beta,s'}, \tilde{\mathcal{A}}_{\beta,q'}(t)]_\sigma\rangle \\ & + \langle \tilde{\mathcal{A}}_{\beta,q'}(t)\tilde{\mathcal{A}}_{\beta,s}\rangle_\sigma \langle [\tilde{\mathcal{A}}^\dagger_{\alpha,q}, \tilde{\mathcal{A}}^\dagger_{\alpha,s'}(t)]\rangle_\sigma \} \Bigg\} \\ + i\frac{\gamma}{2}\sum_{s,s'}\phi^{(-)\alpha\beta}_{ss'} \Bigg\{ \int_0^\infty dt \{ & \langle \tilde{\mathcal{A}}^\dagger_{\beta,s}\tilde{\mathcal{A}}_{\beta,q'}(t)\rangle_\sigma \langle [\tilde{\mathcal{A}}^\dagger_{\alpha,q}(t), \tilde{\mathcal{A}}_{\alpha,s'}]\rangle_\sigma \\ & + \langle \tilde{\mathcal{A}}^\dagger_{\alpha,s}\tilde{\mathcal{A}}^\dagger_{\alpha,q}(t)\rangle_\sigma \langle [\tilde{\mathcal{A}}_{\beta,q'}(t), \tilde{\mathcal{A}}_{\beta,s'}]\rangle_\sigma \} \Bigg\} . \end{aligned} \qquad (71)$$

In deriving these results we have converted the perturbation expression involving the time-evolution operator of the density matrix to atomic correlation functions via the quantum regression theorem. In first order, $\vec{F}^{(3)}_\alpha \equiv \vec{0}$.

The central result of this section is the expressions (70) and (71) for the asymptotic (perturbative) one- and the two-atom force, respectively. In particular, the two-atom force (71) is proportional to the convolution of (single) atom correlation functions describing the emission and absorption of fluorescence photons: the Fourier transform of

$$\langle \tilde{\mathcal{A}}^\dagger_{\alpha,q}(t)\tilde{\mathcal{A}}_{\alpha,s}\rangle_\sigma \qquad (72)$$

is proportional to the *incoherent part of the resonance fluorescence spectrum* [38], while the Fourier transform of

$$\langle [\tilde{\mathcal{A}}^\dagger_{\beta,s'}, \tilde{\mathcal{A}}_{\beta,q'}(t)]\rangle_\sigma \qquad (73)$$

is related to the *weak field absorption spectrum* of the atom [23]. We emphasize that in perturbation theory these correlation functions are the familiar Mollow correlation functions defined for a single (independent) atom in the state σ.

For low laser intensities (below saturation) the one-atom force (70) is proportional to the intensity of the local laser field. On the other hand, the two-atom force (71) is proportional to the *incoherent* part of the spectrum of resonance fluorescence and thus will scale like the intensity squared, i.e., will under these low intensity conditions be small in comparison with respect to $\vec{F}^{(1)}$.

V. DISCUSSION: *N*-ATOM FORCES IN A V SYSTEM

In this section we will give a detailed discussion of the one- and two-atom forces for a V system ($J_g = 0$ to $J_e = 1$ transition) in a σ^--σ^+ plane wave laser configuration (see Fig. 5). We will calculate average forces under the assumption that the atoms are randomly and uniformly distributed in space on the scale given by the

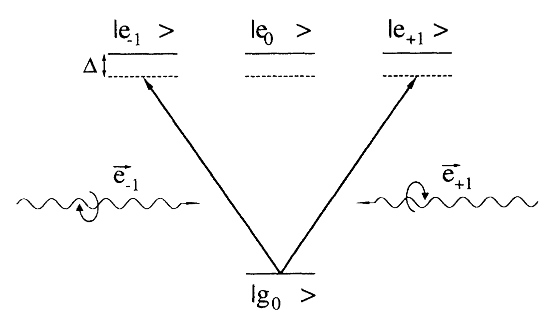

FIG. 5. Atomic-level scheme for a $J_g = 0$ to $J_e = 1$ transition in a σ^--σ^+ configuration.

wavelength of the light (see the discussion of the Vlasov equation in Sec. VII). First we will discuss the low intensity regime (in this context see [11]), which is characterized by the condition that the Rabi frequency $\Omega_\pm$ for each wave is smaller than the natural linewidth γ. In a second step we discuss the regime of strong saturation under the assumption that the dimension of the atomic cloud is not larger than the absorption length. We emphasize that for convenience our discussion considers the force for zero velocity ($\vec{v} = \vec{0}$).

A. The effective one-atom force

For low laser intensities light scattering is essentially elastic and $\vec{F}_\alpha^{(2)}(\vec{x}_\alpha)$ can be neglected. The atomic coherences in the stationary state are given by

$$\langle \mathcal{A}_{\alpha,q}\rangle_\sigma \simeq \mu E^{(-)}_{\text{eff},q}(\vec{x}_\alpha)\frac{\Delta - i\gamma/2}{\Delta^2+\gamma^2/4} \tag{74}$$

and the force $\vec{F}_\alpha^{(1)}$, averaged over random atomic positions $\langle\langle\,\rangle\rangle$, is

$$\langle\langle \vec{F}_\alpha^{(1)}\rangle\rangle \equiv \left(\sum_q (-1)^q \frac{\Delta - i\gamma/2}{\Delta^2+\gamma^2/4} \times\left\langle\left\langle \mu E^{(-)}_{\text{eff},q}(\vec{x}_\alpha)\frac{\partial}{\partial \vec{x}_\alpha}\mu E^{(+)}_{\text{eff},-q}(\vec{x}_\alpha)\right\rangle\right\rangle + \text{c.c.}\right) . \tag{75}$$

We write this mean force as the sum of three contributions [6], the familiar spontaneous light force (as calculated in the theory of laser cooling and trapping for single atoms [39]), an attenuation force [11], and a repulsive part,

$$\langle\langle \vec{F}^{(1)}\rangle\rangle = \langle\langle \vec{F}_S^{(1)}\rangle\rangle + \langle\langle \vec{F}_A^{(1)}\rangle\rangle + \langle\langle \vec{F}_R^{(1)}\rangle\rangle \,. \tag{76}$$

The spontaneous force due to the incident laser field $E^{(-)}_{\text{cl}}$ is

$$\vec{F}^{(1)}_{\alpha,S} \equiv \left(\sum_q (-1)^q \frac{\Delta - i\gamma/2}{\Delta^2+\gamma^2/4}\mu E^{(-)}_{\text{cl},q}(\vec{x}_\alpha)\frac{\partial}{\partial \vec{x}_\alpha} \times \mu E^{(+)}_{\text{cl},-q}(\vec{x}_\alpha) + \text{c.c.}\right) . \tag{77}$$

We define the attenuation force by

$$\langle\langle \vec{F}^{(1)}_{\alpha,A}(\vec{x})\rangle\rangle + \vec{F}^{(1)}_{\alpha,S}(\vec{x}) \equiv \sum_q (-1)^q \left\{ \langle\langle \mu E^{(-)}_{\text{eff},q}(\vec{x})\rangle\rangle \nabla_{\vec{x}} \langle\langle \mu E^{(+)}_{\text{eff},-q}(\vec{x})\rangle\rangle \frac{\Delta - i\gamma/2}{\Delta^2+\gamma^2/4} + \text{c.c.}\right\}, \tag{78}$$

which differs from (75) by factorizing the average over the atomic positions (coherent part). Finally the repulsive force is defined as the remaining incoherent contribution according to Eq. (76). We will identify the attenuation force $\langle\langle \vec{F}_A^{(1)}\rangle\rangle$ as a *compressive* force due to the absorption (attenuation) of the incident laser amplitude in the atomic cloud which results in a different Rabi frequency from the left and right propagating laser beams. On the other hand, $\langle\langle \vec{F}_R^{(1)}\rangle\rangle$ will be shown to give a *repulsive* force due to photon exchange. Defining the "scattering characteristic" [21]

$$u_{qq'}(\vec{x}_\alpha - \vec{x}_\beta) \equiv \Phi^{(+)}_{qq'}(\vec{x}_\alpha - \vec{x}_\beta)\frac{\frac{\gamma}{2}(-\Delta + i\frac{\gamma}{2})}{\Delta^2 + \frac{\gamma^2}{4}} \tag{79}$$

we can write the effective field in the form

$$E^{(-)}_{\text{eff},q}(\vec{x}_\alpha) = E^{(-)}_{\text{cl},q}(\vec{x}_\alpha) + \sum_{\beta\neq\alpha}\sum_{q'} u_{qq'}(\vec{x}_\alpha - \vec{x}_\beta)E^{(-)}_{\text{eff},q'}(\vec{x}_\beta) \tag{80}$$

by using Eq. (74). We can evaluate this expression following classical multiple scattering theory [21],

$$\langle\langle \vec{F}_\alpha^{(1)}\rangle\rangle \equiv \sum_q (-1)^q \left(\langle\langle \mu E^{(-)}_{\text{eff},q}(\vec{x}_\alpha)\rangle\rangle \frac{\partial}{\partial \vec{x}_\alpha}\langle\langle \mu E^{(+)}_{\text{eff},-q}(\vec{x}_\alpha)\rangle\rangle + \sum_s (-1)^{s-q}\int v_{qs}(\vec{x}_\alpha - \vec{x}_\beta)\frac{\partial}{\partial \vec{x}_\alpha}v^*_{qs}(\vec{x}_\alpha - \vec{x}_\beta)\langle\langle \mu E^{(-)}_{\text{eff},s}(\vec{x}_\beta)\mu E^{(+)}_{\text{eff},-s}(\vec{x}_\beta)\rangle\rangle \times n(\vec{x}_\beta)d^3x_\beta + \text{c.c.}\right)\frac{\Delta - i\gamma/2}{\Delta^2+\gamma^2/4} + \text{c.c.}, \tag{81}$$

with $n(\vec{x})$ the spatial atomic density. Here $v_{qq'}(\vec{x}_\alpha - \vec{x}_\beta)$ is defined by the solution of the integral equation [see Eq. (14–29) in Ref. [21]]. (As indicated in Sec. II this is just a Green function solution of the Maxwell equation for the field with the polarization written in terms of the local field variables)

$$v_{qq'}(\vec{x}_\alpha - \vec{x}_\beta) = u_{qq'}(\vec{x}_\alpha - \vec{x}_\beta) + \sum_s \int u_{qs}(\vec{x}_\alpha - \vec{x}_\gamma) v^*_{sq'}(\vec{x}_\gamma - \vec{x}_\beta) \times n(\vec{x}_\gamma) d^3x_\gamma. \quad (82)$$

According to classical multiple scattering theory we interpret $v_{qq'}(\vec{x}_\alpha - \vec{x}_\beta)$ as the screened scattering characteristic from $\vec{x}_\alpha$ to $\vec{x}_\beta$ going through all other scatterers. An iterative solution of Eq. (82) gives the whole series of multiple scattering.

Effective field and attenuation force. The mean effective laser field $\langle\langle E^{(+)}_{\text{eff},q}(z)\rangle\rangle$ [with the polarization index $q = -1$ ($q = +1$) denoting the right (left) propagating $\sigma^\pm$ laser wave] obeys the Maxwell equation (see Sec. 14–5 of Ref. [21])

$$\frac{\partial \langle\langle E^{(+)}_{\text{eff},q}(z)\rangle\rangle}{\partial z} \simeq [\text{sgn}(q)][ik_l + \alpha(z)]\langle\langle E^{(+)}_{\text{eff},q}(z)\rangle\rangle, \quad (83)$$

where

$$\alpha(z) = \alpha_0 n(z) \equiv \frac{3\pi\gamma}{2k_0^2}\frac{\gamma/2 - i\Delta}{\Delta^2 + \gamma^2/4} n(z) \,. \quad (84)$$

The real part of $\alpha(z)$ is the absorption coefficient and k_l is the laser wave vector. In writing Eq. (83) we have restricted ourselves to a one-dimensional model with z dependence.

In the weak absorption regime, i.e., when the dimension of the atomic cloud is less than the absorption length, we can expand the solution up to first order,

$$\langle\langle E^{(+)}_{\text{eff},+}(z)\rangle\rangle = e^{-\int_z^\infty dz'\alpha(z')} E_{\text{cl},+}(z) \simeq \left(1 - \int_z^\infty dz'\alpha(z')\right) E_{\text{cl},+}(z),$$

$$\langle\langle E^{(+)}_{\text{eff},-}(z)\rangle\rangle = e^{-\int_{-\infty}^z dz'\alpha(z')} E_{\text{cl},-}(z) \simeq \left(1 - \int_{-\infty}^z dz'\alpha(z')\right) E_{\text{cl},-}(z), \quad (85)$$

and we find the following result for the attenuation force:

$$\langle\langle \vec{F}^{(1)}_{\alpha,A}(\vec{x})\rangle\rangle \simeq \frac{1}{2}k_l\gamma\frac{|\Omega_{\text{cl}}|^2/2}{\Delta^2 + \gamma^2/4} 2\text{Re}[\alpha_0] \times \left(\int_z^\infty dz' n(z') - \int_{-\infty}^z dz' n(z')\right)\vec{e}_z, \quad (86)$$

which gives the force due to the imbalance of the local Rabi frequencies. This expression is identical to the one found in Eq. (24) of Ref. [11] and Eq. (4) of Ref. [6].

Repulsive force. In the weak absorption limit we can approximately write $v_{qq'}(\vec{x}_\alpha - \vec{x}_\beta) \simeq u_{qq'}(\vec{x}_\alpha - \vec{x}_\beta)$ and obtain for the mean field repulsive force

$$\langle\langle \vec{F}^{(1)}_{\alpha,R}(\vec{x}_\alpha)\rangle\rangle \simeq \sum_q k_0 \int d^3x_\beta n(\vec{x}_\beta) \sum_s \frac{\gamma^2}{4} f_{q,s}(\theta,\phi) f^*_{q,s}(\theta,\phi) \frac{(\vec{x} - \vec{x}_\beta)}{k_0^2|\vec{x} - \vec{x}_\beta|^3} \times \left\{(-1)^s \frac{\langle\langle \mu E^{(-)}_{\text{eff},s}(\vec{x}_\beta)\mu E^{(+)}_{\text{eff},-s}(\vec{x}_\beta)\rangle\rangle}{(\Delta^2 + \gamma^2/4)} \int_{-\infty}^\infty dt \left\langle \left[\mathcal{A}_{\alpha,q}(t), \mathcal{A}^\dagger_{\alpha,q}\right]\right\rangle_\sigma \right\}, \quad (87)$$

where $f_{q,s}(\theta,\phi)$ denotes the angular distribution of $\Phi^{(-)}(\vec{x} - \vec{x}')$ given in Eq. (40). The expression in angular brackets is the product of the elastic δ peak of the Mollow resonance fluorescence spectrum [38] (which is proportional to the total intensity) and the Mollow weak field absorption spectrum [23] evaluated at the laser frequency.

Solution of the integral equation (82) for $v_{qq'}(\vec{x}_\alpha - \vec{x}_\beta)$ has been discussed in Sec. 14-6 of Ref. [21]. This amounts to summing all terms due to multiple scattering of the elastic portion of light emitted from an atom α which is scattered from all the other atoms while traveling to atom β. Equation (87) is the zeroth-order term, neglecting all these multiple scattering processes. Inclusion of these processes gives rise to an absorption length for the total intensity propagating through the cloud (see also Sec. VI). In particular, we point out that according to Ref. [21] the absorption length of the coherent intensity of the light field is shorter than for the total intensity.

Finally, for laser intensities close to saturation the atomic dipole is given by

$$\langle \mathcal{A}_{\alpha,q}\rangle_\sigma = \mu E^{(-)}_{\text{eff},q}(\vec{x})\frac{\Delta - i\gamma/2}{\Delta^2 + \gamma^2/4 + |\,\Omega_{\text{eff}}(\vec{x}_\alpha)\,|^2}, \quad (88)$$

where the effective Rabi frequency is defined by $|\,\Omega_{\text{eff}}(\vec{x}_\alpha)\,|^2 \equiv \frac{1}{2}[|\,\Omega_{\text{eff},+}(\vec{x}_\alpha)\,|^2 + |\,\Omega_{\text{eff},-}(\vec{x}_\alpha)\,|^2]$. Thus the absorption coefficient defined in Eq. (84) (and therefore the force $\vec{F}^{(1)}$) decreases with higher laser intensity.

B. The two-atom force

For high intensities the two-atom force $\langle\langle \vec{F}^{(2)}\rangle\rangle$ must be considered. This term is related to the incoherent part of the Mollow spectrum of resonance fluorescence. We find

$$\langle\langle \vec{F}^{(2)}_{\alpha}(\vec{x}_\alpha)\rangle\rangle \simeq \sum_{qq'}\int d^3x_\beta n(\vec{x}_\beta)\sum_{s,s'} f_{q,q'}(\theta,\phi) f^*_{s,s'}(\theta,\phi)\frac{k_0(\vec{x}_\alpha-\vec{x}_\beta)}{k_0^2\,|\vec{x}_\alpha-\vec{x}_\beta|^3}$$
$$\times\frac{\gamma^2}{4}\Bigg\{\Big\langle\Big\langle\int_0^\infty dt\langle\tilde{\mathcal{A}}^\dagger_{\beta,s'}\tilde{\mathcal{A}}_{\beta,q'}(t)\rangle_\sigma\langle[\tilde{\mathcal{A}}_{\alpha,s},\tilde{\mathcal{A}}^\dagger_{\alpha,q}(t)]\rangle_\sigma\Big\rangle\Big\rangle+\Big\langle\Big\langle\int_0^\infty dt\langle\tilde{\mathcal{A}}^\dagger_{\alpha,s}\tilde{\mathcal{A}}^\dagger_{\alpha,q}(t)\rangle_\sigma\langle[\tilde{\mathcal{A}}_{\beta,s'},\tilde{\mathcal{A}}_{\beta,q'}(t)]\rangle_\sigma\Big\rangle\Big\rangle$$
$$-\Big\langle\Big\langle\int_0^\infty dt\langle\tilde{\mathcal{A}}_{\beta,q'}(t)\tilde{\mathcal{A}}_{\beta,s'}\rangle_\sigma\langle[\tilde{\mathcal{A}}^\dagger_{\alpha,s}(t),\tilde{\mathcal{A}}^\dagger_{\alpha,q}]\rangle_\sigma e^{-2ik_0|\vec{x}_\alpha-\vec{x}_\beta|}\Big\rangle\Big\rangle$$
$$+\Big\langle\Big\langle\int_0^\infty dt\langle\tilde{\mathcal{A}}^\dagger_{\alpha,q}(t)\tilde{\mathcal{A}}_{\alpha,s}\rangle_\sigma\langle[\tilde{\mathcal{A}}_{\beta,q'}(t),\tilde{\mathcal{A}}^\dagger_{\beta,s'}]\rangle_\sigma e^{-2ik_0|\vec{x}_\alpha-\vec{x}_\beta|}\Big\rangle\Big\rangle\Bigg\}+\text{c.c.} \qquad (89)$$

Consider the first term in parentheses in Eq. (89). In view of the expressions for the incoherent emission spectrum and the atomic weak field absorption spectrum,

$$\tilde{S}^{em}_{q,s}(\nu)=\frac{\gamma}{2\pi}\int_{-\infty}^{\infty} dt e^{i\nu t}\langle\tilde{\mathcal{A}}^\dagger_q(t)\tilde{\mathcal{A}}_s\rangle$$
$$\equiv\frac{\gamma}{2\pi}\int_{-\infty}^{\infty} dt e^{i\nu t}[\langle\mathcal{A}^\dagger_q(t)\mathcal{A}_s\rangle-\langle\mathcal{A}^\dagger_q(t)\rangle\langle\mathcal{A}_s\rangle],$$
$$\tilde{S}^{ab}_{q,s}(\nu)=\frac{\gamma}{2\pi}\int_{-\infty}^{\infty} dt e^{i\nu t}\langle[\tilde{\mathcal{A}}_q,\tilde{\mathcal{A}}^\dagger_s(t)]\rangle$$
$$\equiv\frac{\gamma}{2\pi}\int_{-\infty}^{\infty} dt e^{i\nu t}\langle[\mathcal{A}_q,\mathcal{A}^\dagger_s(t)]\rangle, \qquad (90)$$

we rewrite the first term as

$$\gamma^2\Big\langle\Big\langle\int_0^\infty dt\langle\tilde{\mathcal{A}}^\dagger_{\beta,q'}\tilde{\mathcal{A}}_{\beta,q'}(t)\rangle_\sigma\langle[\tilde{\mathcal{A}}_{\alpha,q},\tilde{\mathcal{A}}^\dagger_{\alpha,q}(t)]\rangle_\sigma\Big\rangle\Big\rangle$$
$$+\text{c.c.}=2\pi\Big\langle\Big\langle\Big(\int_{-\infty}^{\infty} d\nu\,\tilde{S}^{em}_{q',q'}(\nu)\tilde{S}^{ab}_{q,q}(\nu)\Big)\Big\rangle\Big\rangle, \qquad (91)$$

which is the mean of the overlap of incoherent emission and absorption spectrum in agreement with the WSW model [6]. Physically, this term corresponds to the transfer of momentum by the emission of photons by an atom at $\vec{x}_\beta$ and subsequent absorption by the atom located at $\vec{x}_\alpha$. The weak field absorption spectrum has positive (absorption) and negative parts (gain). Typically the absorption part will dominate so that this portion of the two-atom force will be repulsive (see Fig. 4); it would be of considerable interest to find atomic and laser configurations where the two-atom force is attractive. For low laser intensities this term scales proportional to the square of the laser intensity. The second term in Eq. (89) shows the Raman coupling between the σ^- and σ^+ resonance fluorescence line of a V system. For low intensities one can interpret these two-photon correlations in terms of the Feynman graph shown in Fig. 6(a). In Fig. 6(b) we plotted the relative size of this part in comparison with the total position independent force (including the mean field part) as a function of the Rabi frequency for various detunings. For small laser intensities and high detunings we find a rather large (i.e., $\simeq -20\%$) deviation from the standard WSW model of Ref. [6]. Note that this two-photon correlation part has opposite sign compared to the standard WSW term and the total repulsive force is actually smaller.

The third and fourth terms in Eq. (89) are strongly oscillating functions of the position of the atoms $\vec{x}_\beta$, except for a small cone along the propagation directions of the two laser beams. These forward effects can be interpreted as four-wave-mixing processes. The factor $\langle\tilde{\mathcal{A}}_{\beta,q}(t)\tilde{\mathcal{A}}_{\beta,q}\rangle_\sigma$ is related to the two-photon correlation of the individual σ^- and σ^+ resonance fluorescence line found in the photon number fluctuation of a forward direction balanced homodyne detection experiment using a local oscillator with σ^- (respectively, σ^+) polarization (see also Ref. [40,41]). As such it is a quantity which can be related to the "squeezing" of the field. The commuta-

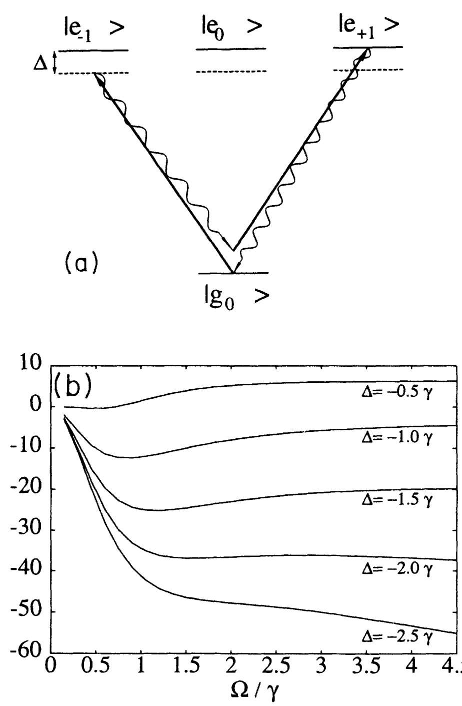

FIG. 6. In (a) we indicate schematically the lowest-order Feynman graph that shows the two-photon correlation between the σ^- and σ^+ resonance fluorescence line. In (b) the ratio R of the two-photon correlation part and the total force (including the one-atom force) is plotted as a function of the Rabi frequency $\Omega\equiv\Omega_+=\Omega_-$. We have neglected all forward terms.

tor $\langle[\tilde{\mathcal{A}}^\dagger_{\alpha,q}, \tilde{\mathcal{A}}^\dagger_{\alpha,q}(t)]\rangle_\sigma$ is related to the absorption coefficient appearing in four-wave-mixing processes due to correlations in the absorption (or emission) of two photons of frequency ω_l and subsequent emission (absorption) of one photon of frequency $\omega_l+\nu$ and another of $\omega_l-\nu$, when the laser and the resonance fluorescence photons emitted into the two Mollow sidebands are in phase (which happens only in the direction of the laser propagation [40]). The absorption spectrum of a two-level system in a probe field with frequencies $\omega_l \pm \nu$ and a pump field of frequency ω_l is discussed in some detail in Ref. [24]. A discussion of the correlated sideband emission in the laser propagation direction can be found in Ref. [42].

The last term is a four-wave-mixing backward term related to correlations between the σ^- and σ^+ lines. Note that in the linear absorption limit all the spectra can be calculated by replacing the local mean fields by the unattenuated classical incoming fields.

To summarize this section, we have seen that the force on an atom at rest in a σ^--σ^+ laser field decouples into an attenuation part, which tends to compress the atomic cloud and a force due to photon emission and absorption processes, that is, of repulsive nature for the discussed configuration. The absorptive regime in the Mollow weak field absorption spectrum for a V system dominates the gain region for the parameters used in the experiments. We have also shown that the Raman coupling terms are important and have to be included in the calculation of the force.

VI. HIGHER-ORDER PERTURBATION THEORY

So far we have restricted our discussion of the interaction between the atoms in the high intensity limit to the lowest-order perturbation theory in the interatomic interaction. A consideration of higher-order terms is necessary because the field of one atom at position $\vec{x}_\alpha$ falls off like $1/r$ so that a calculation of the diffusion coefficient would actually diverge. We have already taken into account the medium to some extent by including the mean field emitted by the atoms. The main effect of the medium is to cause absorption (screening) of the fields. However, if there is a considerable number of excited atoms in the medium (i.e., if it is close to saturation) then the strong incoherent background field can actually change the populations (density matrix elements). In general the self-consistent solution taking into account incoherent fields would be very difficult. One such approximate method would be to use radiative transfer for the incoherent part and Maxwell-Bloch for the coherent (see Ref. [27]).

As a first approximation we will assume that the medium is rather dilute ($k_0\langle\langle| \vec{x}_\alpha - \vec{x}_\beta |\rangle\rangle \gg 1$) and the laser intensity is not too high, so that modifications of the mean atomic dipoles and spectra due to the incoherent background field are small. Then the modification of interactions is primarily due to scattering from atoms, and (as we see below in Sec. VI) leads to screening of the dipole-dipole interaction by effectively an absorption length.

For higher-order perturbation theory $\vec{F}^{(3)}(\vec{x})$ is no longer zero due to "quantum corrections" to the classical mean field picture. As mentioned above, for very high saturation and atomic densities the incoherent background field actually changes the density matrix elements. $\vec{F}^{(3)}(\vec{x})$ is the strong field correction to the mean field force $\vec{F}^{(1)}(\vec{x})$ due to the change of the atomic coherences. We restrict the discussion of this term to the second-order perturbation theory and furthermore we only consider the case of a nondegenerate two-level system in a running wave configuration for notational simplification.

A. Corrections to the one-atom force

In the second-order perturbation $\vec{F}^{(3)}_\alpha$ (65) is determined by the second-order correction to the atomic coherence $\langle \tilde{a}^\dagger_\alpha\rangle_{\rho_{\rm ref}}$ given by

$$\begin{aligned}\langle \tilde{a}^\dagger_\alpha\rangle_{\rho_{\rm ref}} \simeq & -\frac{\gamma^2}{4}\lim_{t\to\infty}\int_0^t dt'\int_0^{t'} ds\sum_\gamma \Phi^{(+)}_{\alpha\gamma}\Phi^{(-)}_{\alpha\gamma}\{\langle \tilde{a}^\dagger_\gamma(t')\tilde{a}_\gamma(s)\rangle_\sigma\langle[\tilde{a}^\dagger_\alpha(s),[\tilde{a}_\alpha(t'),\tilde{a}^\dagger_\alpha(t)]]\rangle_\sigma \\ & +\langle \tilde{a}^\dagger_\gamma(s)\tilde{a}_\gamma(t')\rangle_\sigma\langle[\tilde{a}_\alpha(s),[\tilde{a}^\dagger_\alpha(t'),\tilde{a}^\dagger_\alpha(t)]]\rangle_\sigma\} \\ & -\frac{\gamma^2}{4}\lim_{t\to\infty}\int_0^t dt'\int_0^{t'} ds\sum_\gamma \Phi^{(+)}_{\alpha\gamma}\Phi^{(+)}_{\alpha\gamma}\langle \tilde{a}^\dagger_\gamma(s)\tilde{a}^\dagger_\gamma(t')\rangle_\sigma\langle[\tilde{a}_\alpha(s),[\tilde{a}_\alpha(t'),\tilde{a}^\dagger_\alpha(t)]]\rangle_\sigma \\ & -\frac{\gamma^2}{4}\lim_{t\to\infty}\int_0^t dt'\int_0^{t'} ds\sum_\gamma \Phi^{(-)}_{\alpha\gamma}\Phi^{(-)}_{\alpha\gamma}\langle \tilde{a}_\gamma(t')\tilde{a}_\gamma(s)\rangle_\sigma\langle[\tilde{a}^\dagger_\alpha(s),[\tilde{a}^\dagger_\alpha(t'),\tilde{a}^\dagger_\alpha(t)]]\rangle_\sigma,\end{aligned} \tag{92}$$

where we have dropped all the rapidly oscillating terms that average to zero.

Every single atom α is probed by the incoherent field emitted by other atoms γ and the atomic coherence of atom α is changed in *second*-order response. Note that we have kept again the position dependence in the optical Bloch equations and correlation functions like $\langle \tilde{a}_\gamma(t')\tilde{a}_\gamma(s)\rangle_\sigma$ oscillate with $e^{-i2k_l r_\gamma}$.

In order to estimate the change of the mean atomic dipole by the incoherent background field we evaluated Eq. 92 in the dressed state basis using a secular approximation [43] in which we assume $\sqrt{\Omega^2+\Delta^2} \gg \gamma$. The result is

$$\langle \tilde{a}_\alpha^\dagger \rangle_{\rho_{\rm ref}} = -\langle a_\alpha^\dagger \rangle \frac{\Omega_{\rm cl}^4}{2(\Omega_{\rm cl}^2 + 2\delta^2)(3\Omega_{\rm cl}^2 + 2\delta^2)} \times \sum_\gamma f^2(\theta) \frac{1}{k_0^2 |\, \vec{x}_\alpha - \vec{x}_\gamma \,|^2} . \quad (93)$$

In the high field limit this correction is independent of the field intensity and can be neglected if the mean distance between the excited atoms is much larger than the laser wavelength. One could extend this model to the regime where this condition is not satisfied by calculating the coherences and populations in a self-consistent way with Maxwell-Bloch (coherent part) and radiative transfer equations that treat the incoherent background field as a broadband chaotic field [27].

B. Two-atom force–second-order expansion

The second-order correction to the two-atom force is given by the expression

$$\langle\langle \delta^{(2)} \vec{F}^{(2)}(\vec{x}_\alpha) \rangle\rangle \equiv -ik_0 \frac{\gamma}{2} \left\langle\left\langle \sum_\beta \frac{\vec{x}_\alpha - \vec{x}_\beta}{|\, \vec{x}_\alpha - \vec{x}_\beta \,|} \Phi_{\alpha\beta}^{(-)} {\rm tr}_{\rm int}\{\tilde{a}_\beta^\dagger \tilde{a}_\alpha \delta^{(2)} \rho_{\rm ref}\} + {\rm c.c.} \right\rangle\right\rangle$$

$${\rm tr}_{\rm int}\{\tilde{a}_\beta^\dagger \tilde{a}_\alpha \delta^{(2)} \rho_{\rm ref}\} = \sum_{\gamma\delta\epsilon\omega} \lim_{t\to\infty} \int_0^t dt' \int_0^{t'} ds \; {\rm tr}_{\rm int}\{\tilde{a}_\beta^\dagger \tilde{a}_\alpha e^{\mathcal{L}_0^{(1)}(t-t')} \mathcal{L}_{0,\gamma\delta}^{(2)} e^{\mathcal{L}_0^{(1)}(t'-s)} \mathcal{L}_{0,\epsilon\omega}^{(2)} \sigma\} . \quad (94)$$

By using the quantum regression theorem the remaining terms can again be added up to a sum of products of correlation functions of the individual atoms. We will drop all kinds of backscattering terms (which take into account the change of the atomic spectra by the incoherent background field). For notational reasons we drop any terms with forward scattering behavior and give the result for the second order part of the force that does not strongly fluctuate with position

$$\begin{aligned}\langle\langle \delta^{(2)} \vec{F}^{(2)}(\vec{x}_\alpha) \rangle\rangle = ik_0 \lim_{t\to\infty} \frac{\gamma^3}{8} \int_0^t dt' \int_0^{t'} ds \Bigg\langle\Bigg\langle \sum_{\beta,\gamma\neq\alpha} \frac{\vec{x}_\alpha - \vec{x}_\beta}{|\, \vec{x}_\alpha - \vec{x}_\beta \,|} \Bigg(&\Phi_{\alpha\beta}^{(-)} \Phi_{\alpha\gamma}^{(+)} \Phi_{\gamma\beta}^{(+)} \{\langle \tilde{a}_\beta^\dagger(t) \tilde{a}_\beta(s) \rangle \langle [\tilde{a}_\alpha^\dagger(t'), \tilde{a}_\alpha(t)] \rangle \\ &\times \langle [\tilde{a}_\gamma^\dagger(s), \tilde{a}_\gamma(t)] \rangle\} - \Phi_{\alpha\gamma}^{(-)} \Phi_{\alpha\beta}^{(+)} \Phi_{\beta\gamma}^{(-)} \{\langle \tilde{a}_\beta^\dagger(t') \tilde{a}_\beta(s) \rangle \langle [\tilde{a}_\gamma^\dagger(t), \tilde{a}_\gamma(t')] \rangle \langle [\tilde{a}_\alpha^\dagger(s), \tilde{a}_\alpha(t)] \rangle \\ &+ \langle \tilde{a}_\beta^\dagger(s) \tilde{a}_\beta(t') \rangle \langle [\tilde{a}_\gamma^\dagger(t), \tilde{a}_\gamma(s)] \rangle \langle [\tilde{a}_\alpha^\dagger(t'), \tilde{a}_\alpha(t)] \rangle\} \Bigg) \Bigg\rangle\Bigg\rangle + {\rm c.c.}\end{aligned} \quad (95)$$

By $\Phi_{\gamma\beta}^{(+)}$ we denote the complex conjugate of $\Phi_{\gamma\beta}^{(-)}$.

Second-order generalization of the WSW-type model. The summation over atom γ in Eq. (95) can be performed by using the method of stationary phase [21] if we replace the two-atom spatial distribution by the product of one-atom distributions (see Sec. VII for a discussion of the screened Vlasov approximation). In the limit of $k_0|\vec{r}_\alpha - \vec{r}_\beta| \gg 1$ the main contribution comes from the atoms on the line connecting the atoms α and β. We get

$$\langle\langle \delta^{(2)} \vec{F}^{(2)}(\vec{x}_\alpha) \rangle\rangle = \int d^3x_\beta n(\vec{x}_\beta) f^2(\theta) \frac{k_0(\vec{x}_\alpha - \vec{x}_\beta)}{k_0^2 |\vec{x}_\alpha - \vec{x}_\beta|^3} \frac{\pi}{2} \left\{ \int_{-\infty}^{\infty} d\nu \tilde{S}^{em}(\nu) \tilde{S}^{ab}(\nu) \frac{(-1)^1}{1!} n(\vec{x}_\alpha) \tilde{S}^{ab}(\nu) f(\theta) \frac{r_{\alpha\beta}}{k_0^2} \right\} \quad (96)$$

if we assume that the density is slowly varying on the distance $r_{\alpha\beta}$ and neglect the dependence of the spectra on the local effective field parameters. For convenience the subscripts have been dropped in the angular distribution $f(\theta)$ of the spontaneous emission. Obviously this term is the second-order extension of the WSW model (see Fig. 7). If the absorption profile dominates the gain behavior of the atoms in between α and β, the repulsive force on α is weakened. We will argue later in this section that the presence of additional atoms gives rise to a screening term, of which Eq. (96) is the appropriate first-order expansion (i.e. $e^x = 1 + x + \ldots$).

C. Two-atom force–third-order expansions

We evaluated the third-order expression in the perturbation series for nondegenerate two-level atoms. Although we did not evaluate all of the time integrals the r dependence and the number of terms agree with the second order of an effective screened two-atom interaction.

Screened WSW-type model. The calculations up to third order motivate a generalization of the WSW model by including screening effects. In the limit of $k_l r_{\alpha\beta} \gg 1$ one can hypothesize an analytical expression (an exponential damping as is expected from the physics of the

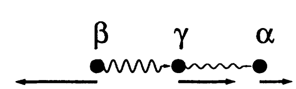

FIG. 7. Screening of repulsive force between a pair of atoms α, β due to absorption of resonance fluorescence photons by a third atom γ.

situation) for the whole series of terms that generalize the WSW terms. For simplicity we also assume that the atomic density does not change with position on the scale of some screening length and furthermore, dependencies of the spectra on the local effective field parameters are neglected. We thus expect the following model for a screened two-atom interaction that is consistent with our perturbation expansion

$$\langle\langle \vec{F}_\alpha^{(2)}(\vec{x}_\alpha)\rangle\rangle \simeq \int d^3x_\beta n(\vec{x}_\beta) f^2(\theta) \frac{k_0(\vec{x}_\alpha - \vec{x}_\beta)}{k_0^2 |\vec{x}_\alpha - \vec{x}_\beta|^3} \times \frac{\pi}{2} \int_{-\infty}^{\infty} d\nu S^{em}(\nu) S^{ab}(\nu) \times e^{-n(\vec{x}_\alpha) S^{ab}(\nu) f(\theta) \frac{r_{\alpha\beta}}{k_0^2}} . \quad (97)$$

Again $f(\theta)$ denotes the angular distribution of the spontaneous emission, θ is the angle between $\vec{x}_\alpha - \vec{x}_\beta$ and the direction of laser propagation, and $S^{ab}(\nu)$ is the absorption spectrum defined in Eq. (90). Note that for gain frequency ranges in the absorption spectrum the exponential factor amplifies the gain characteristics. Nevertheless, for high laser intensities the incoherent background field changes the fluorescence properties of the atoms and one has to correct this model (see [27]). In addition one would have to include the local dependency of the spectra on the effective field and replace the exponential in Eq. (97) by a Dyson series of some "mean" absorption coefficient and then perform the random average $\langle\langle\,\rangle\rangle$ of the whole expression.

VII. THE SCREENED VLASOV APPROXIMATION

So far we have concentrated our discussion on the N-atom Liouville-type equation for the Wigner function f and discussed the mean of the static forces in terms of a screened two-atom interaction. Equation (61) gives a N-atom Fokker-Planck-type equation. We will discuss the correction to the cooling time (via the term proportional to the velocity in the force–rather than $\vec{v} = \vec{0}$ as considered here) and diffusion term due to interactions with the other atoms in a later publication. For the calculation of the atomic density we need an equation for the effective *one*-atom spatial distribution function. In the screened Vlasov approximation for the associated BBKGY hierarchy of the N-atom Fokker-Planck equation we can derive an equation for the *one*-atom Wigner function $f_1(x,p,t)$. The screened Vlasov procedure assumes that the correlations between different atoms are small and can be approximated by two-atom correlations. This procedure corresponds to treating the systems as N-independent screened quasiparticles rather than N-independent atoms. The screening is of course due to the absorption as in Eq. (97). Formally this can be done by replacing the N-atom Wigner function in the N-atom FPE including screening terms by the product state

$$f(x,p,t) \simeq \otimes_\alpha f_1(\vec{x}_\alpha, \vec{p}_\alpha, t) \quad (98)$$

and integrating over $N-1$ pairs of quasiparticle coordinates. The result is similar to Ref. [11], including screening effects.

$$\frac{\partial f_1}{\partial t} + \vec{v}\cdot\nabla_{\vec{r}} f_1 = -\nabla_{\vec{v}} \cdot \left[\frac{1}{m}\left(\langle\langle \vec{F}(\vec{r})\rangle\rangle - \frac{\vec{v}}{\tau_{\text{eff}}}\right) f_1\right] + D_{\text{eff}} \nabla_{\vec{v}}^2 f_1 . \quad (99)$$

By $\langle\langle\,\rangle\rangle$ we denote the average with respect to the c.m. coordinates and τ_{eff} (respectively, D_{eff}) represents the corrected cooling time (respectively diffusion constant) due to multiatom interactions. The effect of the rest of the cloud on a single atom at position $\vec{r}$ is smeared out and modeled by an average force $\langle\langle \vec{F}(\vec{r})\rangle\rangle$. Adiabatic elimination (which is valid) of the kinetic degree of freedom gives a Smoluchowski equation for the spatial atomic density $n(\vec{r},t)$ [44,11]

$$m\frac{\partial n(\vec{r},s)}{\partial s} = -\nabla_{\vec{r}} \cdot \left[\langle\langle \vec{F}(\vec{r})\rangle\rangle n(\vec{r},s) - k_b T_{\text{eff}} \nabla_{\vec{r}} n(\vec{r},s)\right], \quad (100)$$

where T_{eff} has the dimension of a temperature determined by the atom-atom corrected mean diffusion D_{eff} and cooling time τ_{eff}. The time s is measured in units of τ_{eff}. The stationary spatial distribution is the solution of

$$\nabla_{\vec{r}} \cdot \left[\langle\langle \vec{F}(\vec{r})\rangle\rangle n(\vec{r}) - k_b T_{\text{eff}} \nabla_{\vec{r}} n(\vec{r})\right] = 0 . \quad (101)$$

Such an equation is used in Ref. [6] for the calculation of the spatial density of the atoms in the cloud without correcting the diffusion coefficient for multiatom effects.

To summarize, the WSW model of Ref. [6] makes use of two major approximations. The screened Vlasov approximation, which assumes that all N-atom correlations can be approximated by products of two-atom correlations and additionally the approximation that the stationary state of the N-atom Bloch problem can be construct using perturbation theory with respect to the atom-atom interaction.

VIII. CONCLUSIONS

In this work we have developed an analytical description of the semiclassical light force for atoms in an atomic cloud. In the Vlasov limit for screened interactions we reduced the N-atom equation to an equation for the one-particle Wigner function (actually a screened quasiparticle) and by adiabatic elimination of the velocity degree of freedom we end up with a Smoluchowski equation for the one-atom spatial distribution function [44,11]. The expression for the force occurring in this equation [Eq. (101)] is evaluated in perturbation theory. Our first-order perturbation expansion in terms of the long-range interatomic potential shows that the effect of the presence of other atoms can be separated analytically into two parts similar to the model of Walker, Sesko and Wieman although the absorption force derived in this paper generalizes their results to include Raman couplings among the Zeeman sublevels. The local imbalance of the laser fields due to absorption effects causes an attractive potential with its minimum in the center of the atomic cloud. The resulting attenuation force found is similar to the result of Dalibard [11] and tends to compress the cloud.

The origin of the other force is photon emission and absorption. Each atom is affected by the absorption of the resonance fluorescence photons emitted by the other atoms. For the V system discussed in Sec. VI the gain profile of the absorption spectrum is too small to compensate the absorptive part and leads to a repulsive force by simple momentum conservation. We expect that for other atomic configurations the gain-absorption profile will give rise to new interesting features for the incoherent two-atom force (see [25,26]). We have also given a discussion of terms found in higher-order perturbation expansions and argued that this gives rise to an exponential screening which corresponds to the absorption of the radiation in traveling from one atom to the other.

We point out that our procedure can easily be generalized to more realistic internal structures for the interacting atoms. The model is limited to the semiclassical regime. For the extremely low temperatures found in optical molasses the semiclassical approach breaks down and the c.m. degrees of freedom have to be treated quantum mechanically.

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APPENDIX A: DERIVATION OF THE N-ATOM MASTER EQUATION

For the derivation of the Ito equation [45,46] we neglect the laser term for the moment, since it is obvious how to build it in afterwards (see [47]). The interaction Hamiltonian can be written in the form

$$H_{\rm AF}^{(\alpha)} = i\eta \int\int d^3k_\alpha d^3k'_\alpha |k_\alpha\rangle\langle k'_\alpha| \left\{ \sum\!\!\!\!\!\!\int_\lambda d^3q g(\omega)\sqrt{\frac{\omega}{\omega_0}}[\hat{\mathbf{D}}_{ge}^{(\alpha)}\cdot\vec{\epsilon}^{\,*}_{\lambda q}\delta(k-k'+q)b^\dagger_{\lambda q} - \hat{\mathbf{D}}_{eg}^{(\alpha)}\cdot\vec{\epsilon}_{\lambda q}\delta(k-k'+q)b_{\lambda q}] \right\}, \tag{A1}$$

where we have defined $\eta \equiv \sqrt{\frac{\omega_0}{2\epsilon_0(2\pi)^3}}$ and $\hat{\mathbf{D}}_{eg}^{(\alpha)} \equiv \mathbf{P}_e\hat{\mu}\mathbf{P}_g$. In order to obtain a standard form we define the following vector of distributions:

$$\vec{g}_\alpha^{(+)}(\lambda,q,k,k') \equiv g(\omega)\sqrt{\frac{\omega}{\omega_0}}\vec{\epsilon}^{\,*}{}_{\lambda q}\delta(k-k'+q),$$

$$\vec{g}_\alpha^{*(-)}(\lambda,q,k,k') \equiv g(\omega)\sqrt{\frac{\omega}{\omega_0}}\vec{\epsilon}_{\lambda q}\delta(k-k'-q)\ .$$

Using this notation, we can write the interaction Hamiltonian in the standard form found in Ref. [48].

$$H_{\rm AF}^{(\alpha)} = i\eta\sum_{\alpha\in A}\int\int d^3k_\alpha d^3k'_\alpha|k_\alpha\rangle\langle k'_\alpha| \times[\hat{\mathbf{D}}_{ge}^{(\alpha)}b^\dagger(\vec{g}_\alpha^{(+)}) - \hat{\mathbf{D}}_{eg}^{(\alpha)}b(\vec{g}_\alpha^{(-)})]$$

with

$$b^\dagger(\vec{g}_\alpha^{(+)}) \equiv \sum\!\!\!\!\!\!\int_\lambda d^3q\, \vec{g}_\alpha^{(+)}(\lambda,q,k,k')b^\dagger_{\lambda q}.$$

For the Ito equation we need the time-evolution operators of the different terms for the uncoupled situation.

$$\text{(i) } S_t^{o,\rm int}\hat{\mathbf{D}}_{eg}^{(\alpha)} = e^{-i\omega_0 t}\hat{\mathbf{D}}_{eg}^{(\alpha)},$$

$$\text{(ii) } S_t^{o,\rm bath} = \int_0^\infty d\omega e^{-i\omega t}dE(\omega), \tag{A2}$$

$$\text{(iii) } S_t^{o,\rm c.m.}\delta(k-k'+q) = \delta(k-k'+q)e^{-i\epsilon_q t}e^{i\frac{k\cdot q}{m}t}\ ,$$

$$\epsilon_q \equiv \frac{q^2}{2m}.$$

If we collect these individual terms, we get the unperturbed time evolution of the distributions $\vec{g}_\alpha^{(+)}$ and $\vec{g}_\alpha^{(-)}$:

$$S_t^o\vec{g}_\alpha^{(+)}(\lambda,q,k,k') = \vec{g}_\alpha^{(+)}(\lambda,q,k,k')e^{i(\omega-\epsilon_q+\frac{kq}{m})t},$$
$$S_t^o\vec{g}_\alpha^{(-)}(\lambda,q,k,k') = \vec{g}_\alpha^{(-)}(\lambda,q,k,k')e^{i(\omega-\epsilon_q+\frac{kq}{m})t}. \tag{A3}$$

According to a generalization of [48], the corresponding Ito equation for the interaction picture time-evolution operator U_t in the weak-coupling limit is given by

$$dU_t = \left\{ \sum_\alpha \int\int d^3k_\alpha d^3k'_\alpha|k_\alpha\rangle\langle k'_\alpha|[\hat{\mathbf{D}}_{ge}^{(\alpha)}\cdot dA_t^\dagger(\vec{g}_\alpha^{(+)}) - \hat{\mathbf{D}}_{eg}^{(\alpha)}\cdot dA_t(\vec{g}_\alpha^{(-)})] - \sum_\beta\int\int d^3k_\beta d^3k'_\beta|k_\beta\rangle\langle k'_\beta|\cdot[\hat{\mathbf{D}}_{eg}^{(\alpha)}\cdot(\vec{g}_\alpha^{(-)}|\vec{g}_\beta^{(+)})^-\cdot\hat{\mathbf{D}}_{ge}^{(\beta)}]dt \right\}U_t \tag{A4}$$

with the following tensor coefficients and Ito algebra:

$$(\vec{g}_\alpha^{(-)}|\vec{g}_\beta^{(+)})^- \equiv \int_{-\infty}^0 e^{-i\omega_o t}\langle\vec{g}_\alpha^{(-)}|S_t^o\vec{g}_\beta^{(+)}\rangle dt,$$
$$dA_t(g_\alpha^{i(-)})dA_t^\dagger(g_\beta^{j(+)}) = 2\,\mathrm{Re}(\vec{g}_\alpha^{(-)}|\vec{g}_\beta^{(+)})_{ij}^- dt, \tag{A5}$$
$$dA_t^\dagger(g_\alpha^{i(+)})dA_t(g_\beta^{j(-)}) = 0\,.$$

As mentioned before, this equation is valid in the weak-coupling limit which implies validity on a time scale determined by the coupling constant between bath and system which is much larger than the correlation time of the bath variables. Note that for simplicity we restrict to a zero-temperature bath. The effect of finite temperatures can be built in easily (see [48]). The Ito equation with an additional plane wave laser field is given by (see [47]).

$$dU_t = \Bigg\{\sum_{\alpha\in A}\int\int d^3k_\alpha d^3k'_\alpha|k_\alpha\rangle\langle k'_\alpha|\{\hat{\mathbf{D}}_{ge}^{(\alpha)}[dA_t^\dagger(\vec{g}_\alpha^{(+)}) - g\vec{\epsilon}_l{}^*\tilde{\mathcal{E}}^*(k_\alpha - k'_\alpha + k_l)dt]$$
$$-\hat{\mathbf{D}}_{eg}^{(\alpha)}[dA_t(\vec{g}_\alpha^{(-)}) - g\vec{\epsilon}_l\tilde{\mathcal{E}}^*(k_\alpha - k'_\alpha + k_l)dt]\} - i\sum_\alpha \hbar\Delta P_e^{(\alpha)}dt$$
$$-\sum_{(\alpha,\beta)\epsilon A\times A}\int\int d^3k_\alpha d^3k'_\alpha d^3k_\beta d^3k'_\beta|k_\alpha k_\beta\rangle\langle k'_\alpha k'_\beta|[\hat{\mathbf{D}}_{eg}^{(\alpha)}(\vec{g}_\alpha^{(-)}|\vec{g}_\beta^{(+)})^-\hat{\mathbf{D}}_{ge}^{(\beta)}]dt - \frac{\gamma}{2}\sum_\alpha\sum_{s\alpha}|s_\alpha\rangle\langle s_\alpha|dt\Bigg\}U_t. \tag{A6}$$

γ is the radiative decay rate of a single atom. For a more explicit form of the Ito equation we evaluate the tensor of distributions $(\vec{g}_\alpha^{(-)}|\vec{g}_\beta^{(+)})^-$, by using the spectral representation of S_t^o,

$$(\vec{g}_\alpha^{(-)}|\vec{g}_\beta^{(+)})^- = \int_0^\infty d\omega\frac{\omega^3}{\omega_o}|g(\omega)|^2\delta(k_\alpha + k_\beta - k'_\alpha - k'_\beta)$$
$$\times \sum\!\!\!\!\!\!\int_{\lambda,s^2} d\Omega_{\vec{n}}(1 - \vec{n}\otimes\vec{n})\delta(k_\beta - k'_\beta + q)\left[\pi\delta(\omega - \bar{\omega}_o) - iP\frac{1}{\omega - \bar{\omega}_o}\right], \tag{A7}$$

where we have used the Sokholsky-Penely formula and the definition $\bar{\omega}_o \equiv \omega_o - \epsilon_{q_o} - \frac{1}{2}\frac{\hbar}{mc}k_\beta\cdot q_o$ with $q_o \equiv \hat{n}\cdot\omega_o$. In an approximation, consistent with the "nonrelativistic" description of the atom, we used $\bar{\omega}_o \simeq \omega_0$.

If we write out all the terms explicitly and change from the momentum to the position representation we can write the Ito equation in a more familiar form:

$$dU_t = \Bigg\{\sum_{\alpha\in A}\Bigg(\hat{\mathbf{D}}_{ge}^{(\alpha)}\cdot\int d^3x\int d\Omega_{\vec{n}}\delta(\vec{x} - \hat{\mathbf{X}}_\alpha)e^{-i\vec{n}\cdot\vec{x}k_0}d\vec{B}_{\vec{n}}^\dagger(t)$$
$$-\hat{\mathbf{D}}_{eg}^{(\alpha)}\cdot\int d^3x\int d\Omega_{\vec{n}}\delta(\vec{x} - \hat{\mathbf{X}}_\alpha)e^{i\vec{n}\cdot\vec{x}k_0}d\vec{B}_{\vec{n}}(t)\Bigg)$$
$$+g\sum_{\alpha\in A}\left(\hat{\mathbf{D}}_{ge}^{(\alpha)}\cdot\int d^3x\delta^3(\vec{x} - \hat{\mathbf{X}}_\alpha)\vec{\mathcal{E}}(\vec{x}) - \hat{\mathbf{D}}_{eg}^{(\alpha)}\cdot\int d^3x\delta^3(\vec{x} - \hat{\mathbf{X}}_\alpha)\vec{\mathcal{E}}^*(\vec{x})\right)dt$$
$$-i\sum_{\alpha\in A}\left(-\hbar\Delta - i\frac{\gamma}{2}\right)\mathbf{P}_e^{(\alpha)}dt - i\sum_{\alpha\in A}\frac{\hat{P}_\alpha^2}{2m}dt$$
$$-\frac{\gamma}{2}\sum_{\alpha\neq\beta}\hat{\mathbf{D}}_{eg}^{(\alpha)}\cdot\int d^3x d^3x'\cdot\overleftrightarrow{\phi}^{(-)}(\vec{x} - \vec{x}')\delta^3(\vec{x} - \hat{\mathbf{X}}_\alpha)\delta^3(\vec{x}' - \hat{\mathbf{X}}_\beta)\cdot\hat{\mathbf{D}}_{ge}^{(\beta)}dt\Bigg\}U_t, \tag{A8}$$

with

$$\overleftrightarrow{\phi}^{(-)}(\vec{x} - \vec{x}') \equiv \overleftrightarrow{G}(\vec{x} - \vec{x}') + i\overleftrightarrow{F}(\vec{x} - \vec{x}')$$
$$\equiv \lim_{\epsilon\to 0}\frac{3}{4\pi}\frac{1}{\pi}\int_0^\infty\frac{d\omega}{\omega - \omega_0 - i\epsilon}\int_{s^2}d\Omega_{\vec{n}}(1 - \vec{n}\otimes\vec{n})e^{i\vec{n}\cdot(\vec{x}-\vec{x}')\omega}. \tag{A9}$$

In deriving Eq. (A8) we used a RWA. All the non-rotating terms can be included by simply extending the frequency integral in Eq. (A9) to $-\infty$ [31,32]. The Ito algebra of the noise differentials is defined by

$$(dB_{\vec{n}})_i\,(dB_{\vec{n}'})_j^\dagger \equiv 2\frac{3\gamma}{8\pi}\delta(\vec{n} - \vec{n}')\,(\delta_{ij} - n_i n_j)\,dt.$$

By use of standard Ito algebra one can derive a master equation for the N-atom system. Use of the operator version of the classical Ito formula allows us to evaluate the differential of the density operator $\bar{\rho}(t) \equiv U_t \bar{\rho}(0) U_t^\dagger$

$$d\bar{\rho}(t) = \left(dU_t U_t^\dagger\right) \bar{\rho}(t) + \bar{\rho}(t) \left(U_t dU_t^\dagger\right) + \left(dU_t U_t^\dagger\right) \bar{\rho}(t) \left(U_t dU_t^\dagger\right).$$

The N-atom system operator, defined by $\rho(t) \equiv \mathrm{tr}_{\mathbf{bath}} \{\bar{\rho}(t)\}$ solves the following master equation:

$$\frac{d\rho(t)}{dt} = -i \sum_{\alpha \in A} (H_{\mathrm{eff}}^{(\alpha)} \rho(t) - \rho(t) H_{\mathrm{eff}}^{(\alpha)\dagger}) + \gamma \sum_{(\alpha,\beta) \in A \times A} \hat{\mathbf{D}}_{ge}^{(\alpha)} \cdot \left(\int d^3x d^3x' \delta^3(\vec{x} - \hat{\mathbf{X}}_\alpha) \overleftrightarrow{F} (\vec{x} - \vec{x}') \rho(t) \delta^3(\vec{x}' - \hat{\mathbf{X}}_\beta) \right) \cdot \hat{\mathbf{D}}_{eg}^{(\beta)} \tag{A10}$$

where we have defined an effective Hamiltonian

$$H_{\mathrm{eff}}^{(\alpha)} \equiv \left(-\hbar\Delta - i\frac{\gamma}{2}\right) \mathbf{P}_e + \frac{\hat{P}_\alpha^2}{2m} + g\left(\int d^3x \delta^3(\vec{x} - \hat{\mathbf{X}}_\alpha) \hat{\mathbf{D}}_{ge}^{(\alpha)} \cdot \vec{\mathcal{E}}(\vec{x}) + \text{H.c.} \right) - \frac{\gamma}{2} \sum_{\beta \neq \alpha} \hat{\mathbf{D}}_{eg}^{(\alpha)} \cdot \left[\int d^3x d^3x' \delta^3(\vec{x} - \hat{\mathbf{X}}_\alpha) \overleftrightarrow{\phi}^{(-)} (\vec{x} - \vec{x}') \delta^3(\vec{x}' - \hat{\mathbf{X}}_\beta) \right] \cdot \hat{\mathbf{D}}_{ge}^{(\beta)}. \tag{A11}$$

We introduce a convenient notation

$$\hat{\mathbf{D}}_{ge}^{(\alpha)}(\vec{x}) \equiv \delta^3(\vec{x} - \hat{\mathbf{X}}_\alpha) \hat{\mathbf{D}}_{ge}^{(\alpha)}. \tag{A12}$$

The output field in direction $\vec{n}$ defined by

$$\vec{B}_{\vec{n}}^{\mathrm{out}}(t) \equiv U_t^\dagger \vec{B}_{\vec{n}}(t) U_t \tag{A13}$$

is given by

$$dB_{\vec{n},q}^{\mathrm{out}}(t) = dB_{\vec{n},q}(t) + \sum_{\beta,q'} \frac{3\gamma}{4\pi} N_{qq'}(\vec{n}) e^{-i\vec{n}\cdot\hat{\mathbf{X}}_\beta(t) k_0} \mathcal{A}_{\beta,q'}(t) dt \tag{A14}$$

so that the total output field

$$dA_{t,q}^{\mathrm{out}} = \int d\Omega_{\vec{n}} dB_{\vec{n},q}^{\mathrm{out}}(t) \tag{A15}$$

is given by

$$dA_{t,q}^{\mathrm{out}} = dA_{t,q} + \sum_{\beta,q'} \frac{3\gamma}{4\pi} \int d\Omega_{\vec{n}} N_{qq'}(\vec{n}) e^{-i\vec{n}\cdot\hat{\mathbf{X}}_\beta(t) k_0} \mathcal{A}_{\beta,q'}(t) dt. \tag{A16}$$

APPENDIX B: DIVISION INTO N-PHOTON STATES

Let us define a Schrödinger wave function by

$$|\vartheta, t\rangle \equiv U_t |\vartheta, 0\rangle,$$

where $|\vartheta, 0\rangle$ is some initial vector for the composite system and then divide that wave function into the linear combination

$$|\vartheta, t\rangle = \sum_{n=0}^{\infty} |\vartheta, n, t\rangle = \sum_{n=0}^{\infty} P^{(n)} |\vartheta, t\rangle,$$

where $P^{(n)}$ is the projection operator on the n-photon subspace. The set of $|\vartheta, n, t\rangle$ obeys the Ito equations

$$d|\vartheta,n,t\rangle = \sum_{\alpha\in A}\left(\int d^3x\hat{\mathbf{D}}_{ge}^{(\alpha)}(\vec{x})\cdot\int d\Omega_{\vec{n}}\, e^{-i\vec{n}\cdot\vec{x}k_0}d\vec{B}_{\vec{n}}^{\dagger}(t)|\vartheta,n-1,t\rangle\right.$$
$$\left.-\int d^3x\hat{\mathbf{D}}_{eg}^{(\alpha)}(\vec{x})\cdot\int d\Omega_{\vec{n}}\, e^{i\vec{n}\cdot\vec{x}k_0}d\vec{B}_{\vec{n}}(t)|\vartheta,n+1,t\rangle\right)$$
$$+g\sum_{\alpha\in A}\left\{\int d^3x(\hat{\mathbf{D}}_{ge}^{(\alpha)}(\vec{x})\cdot\vec{\mathcal{E}}(\vec{x})-\hat{\mathbf{D}}_{eg}^{(\alpha)}(\vec{x})\cdot\vec{\mathcal{E}}^*(\vec{x}))|\vartheta,n,t\rangle dt\right\}$$
$$-\frac{\gamma}{2}\sum_{\substack{(\alpha,\beta)\\ \alpha\neq\beta}}\int\int d^3xd^3x'\hat{\mathbf{D}}_{eg}^{(\alpha)}(\vec{x})\cdot\overleftrightarrow{\phi}^{(-)}(\vec{x}-\vec{x}')\cdot\hat{\mathbf{D}}_{ge}^{(\beta)}(\vec{x}')|\vartheta,n,t\rangle dt$$
$$-i\sum_{\alpha\in A}\left(\left[-\Delta-i\frac{\gamma}{2}\right]\mathbf{P}_e^{\alpha}+\frac{\hat{P}_{(\alpha)}^2}{2m}\right)|\vartheta,n,t\rangle dt\quad,\quad|\vartheta,-1,t\rangle\equiv 0 \tag{B1}$$

by construction.

Now define

$$\rho^n(t)=\mathrm{Tr}_{\mathrm{bath}}\{|\vartheta,n,t\rangle\langle\vartheta,n,t|\} \tag{B2}$$

and derive equations of motion for this "n-photon part" of the atomic density matrix. Obviously $\rho(t)=\sum_{n=0}^{\infty}\rho^n(t)$, $\mathrm{tr}_{\mathrm{A}}\{\rho^n(t)\}\leq 1$. By the use of the Ito relation

$$d(|\vartheta,n,t\rangle\langle\vartheta,n,t|)=d|\vartheta,n,t\rangle\cdot\langle\vartheta,n,t|+|\vartheta,n,t\rangle d\langle\vartheta,n,t|+d|\vartheta,n,t\rangle\cdot d\langle\vartheta,n,t| \tag{B3}$$

we get an equation for $\rho^n(t)$ of the form

$$\frac{d\rho^n(t)}{dt}=-iH_{\mathrm{eff}}\rho^n(t)+i\rho^n(t)H_{\mathrm{eff}}^{\dagger}+\gamma\sum_{(\alpha,\beta)}\int d^3xd^3x'\hat{\mathbf{D}}_{ge}^{(\alpha)}(\vec{x})\cdot\overleftrightarrow{F}(\vec{x}-\vec{x}')\rho^{(n-1)}(t)\cdot\hat{\mathbf{D}}_{eg}^{(\beta)}(\vec{x}'),$$
$$H_{\mathrm{eff}}\equiv\sum_{\alpha\in A}\left\{\left(-\Delta-i\frac{\gamma}{2}\right)\mathbf{P}_e^{(\alpha)}+\frac{\hat{P}_{\alpha}^2}{2m}+g\int d^3x[\hat{\mathbf{D}}_{eg}^{(\alpha)}(\vec{x})\cdot\vec{\mathcal{E}}(\vec{x})-\hat{\mathbf{D}}_{ge}^{(\alpha)}(\vec{x})\cdot\vec{\mathcal{E}}^*(\vec{x})]\right.$$
$$\left.-\frac{\gamma}{2}\sum_{\beta\neq\alpha}\int\int d^3xd^3x'\hat{\mathbf{D}}_{eg}^{(\alpha)}(\vec{x})\cdot\overleftrightarrow{\phi}^{(-)}(\vec{x}-\vec{x}')\cdot\hat{\mathbf{D}}_{ge}^{(\beta)}(\vec{x}')\right\}. \tag{B4}$$

Define $P^n(t)\equiv\mathrm{tr}_A[\rho^n(t)]$, then $P^n(t)$ is the probability that the bath is in an n-photon state (B2). To see the emergence of a rate process from Eq. (B4) we make the ansatz

$$\rho^n(t)=P^n(t)r_0, \tag{B5}$$

where r_0 is some equilibrium atomic density matrix with unit trace. Using this ansatz we obtain the rate equation for $P^n(t)$,

$$\frac{d}{dt}P^{(n)}(t)=-RP^{(n)}(t)+RP^{(n-1)}(t)$$

characteristic for a Poisson counting process. The rate R depends on the output number process $\Lambda(t)$ defined in Ref. [49]. It is given by the expression

$$R=\frac{3\gamma}{4\pi}\sum_{(\alpha,\beta)}\sum_{q,q'}\int d\Omega_{\vec{n}}(\delta_{qq'}-n_{-q}n_{-q'}^*)(-1)^{q+q'}$$
$$\times\mathrm{tr}_{\mathrm{int}}[e^{i\vec{n}(\hat{\mathbf{X}}_\alpha-\hat{\mathbf{X}}_\beta)k_0}\mathcal{A}_q^{(\beta)\,\dagger}\mathcal{A}_{q'}^{(\alpha)}r_0]. \tag{B6}$$

So R is equivalent to the time derivative of the expectation of the output number process $R=\frac{d}{dt}\langle\Lambda(t)\rangle$ associated with photon counting in the stationary limit (see Ref. [49]). We can therefore interpret it as the sum of rates of photodetection in detectors located all around the atomic cloud. The detection rate of any of these single photodetectors strongly depends on its relative position with respect to the atoms *and* on the relative positions of the atoms themselves, not only by the trivial angular distribution of the spontaneous emission of single two-level atoms. This shows the *spatial* coherent characteristics of the model.

If the atoms in the cloud are treated as independent particles in the sense that one can neglect all collective effects the rate R is given by

$$R=\frac{\gamma}{2\pi}\sum_{\alpha}\sum_{q}\int d\Omega_{\vec{n}}N_q(\hat{n})\mathrm{tr}_{\mathrm{int}}[\mathcal{A}_q^{(\alpha)\,\dagger}\mathcal{A}_q^{(\alpha)}r_0]\,. \tag{B7}$$

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