Muon transfer from excited muonic hydrogen to helium

A. V. Kravtsov and A. I. Mikhailov
Petersburg Nuclear Physics Institute, Gatchina, 188350 St.Petersburg, Russia
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Direct muon transfer to helium from excited muonic hydrogen with principal quantum number n =2, 3, 4, and 5 is considered in a quasiclassical approximation. Reaction rates turn out to be $\sim 10^{11} - 10^{12}$ s^{-1} and should be taken into account when considering the cascade of excited muonic hydrogen in hydrogen-helium mixtures.

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During recent years muon-catalyzed fusion in the most effective deuterium-tritium mixture has been under intensive investigation [1]. Since hydrogen mixtures can contain impurities, it is very important to study muon transfer from muonic hydrogen to other nuclei, because this process results in a decrease of the number of cycles of catalysis per muon [2]. The transfer to helium is of special interest for muon-catalyzed fusion since both helium isotopes are produced continuously in the d-t mixture due to nuclear fusion reactions and tritium decay. The muon transfer from the ground state of muonic hydrogen to helium nuclei is strongly suppressed because the crossing point of the terms, corresponding to the initial and final states, turns out to be deep under the barrier for energies in question ($\leq$ 50 eV). For this reason the muon transfer from the ground state of muonic hydrogen proceeds mainly via the intermediate quasistationary molecular state [3–5].

One may expect that for excited muonic hydrogen this suppression is taken off [6], so the muon transfer

$$(\mathrm{H}\mu)^*_{\mathrm{n}} + \mathrm{He} \rightarrow (\mathrm{He}\,\mu)^*_{\mathrm{n'}} + \mathrm{H} \tag{1}$$

in the course of muonic hydrogen deexcitation plays an important role in the kinetics of cascade transitions. There exist experimental indications [4, 7] that a considerable fraction of muons is transferred from muonic hydrogen to helium nuclei during the cascade, although the effect could not be revealed clearly. An important observation was made in an experiment [8] where a pronounced pressure dependence of the fraction of muons found in Hμ and Heμ ground states has been measured. This confirms the importance of the reaction (1), since the capture ratio does not depend on pressure at a given helium concentration.

Theoretically the molecular and direct charge exchange of excited muonic hydrogen on helium was considered in [9] for $n = 2$ and 3. It was shown that even for $n = 2$ the rate of the direct muon transfer is much higher than that of the molecular transfer.

Here we investigate the process of the direct muon transfer from the excited muonic hydrogen for $n = 2$ and 3 (more correctly than in [9]) and for $n = 4$ and 5. As in [9] we use the quasiclassical approximation. The effective potential of the interaction of an excited muonic hydrogen atom having parabolic quantum numbers (n, n_1, n_2, m) with a nucleus of charge Z is deter-

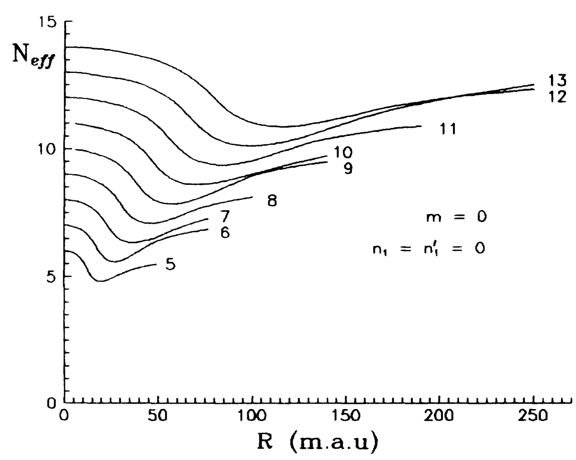

FIG. 1. Effective principal quantum number of the united atom $N_{\mathrm{eff}} = \{-9/[2u(R)]\}^{1/2}$ for σ terms with $n_1 = n_1' = 0$. The numbers at the curves denote l values. Terms with $l = 7, 10, 13$ correspond asymptotically to $(\mathrm{H}\mu)^*_n$ with $n = [l + m + 2(n_1 + 1)]/3$. Other terms correspond to $(\mathrm{He}\mu)^*_{n'}$, where n' satisfies the equation $n' + \mathcal{I}(n'/2) = l + m + 1 + 2n_1'$. $\mathcal{I}(A)$ is the largest integer that is not larger than A.

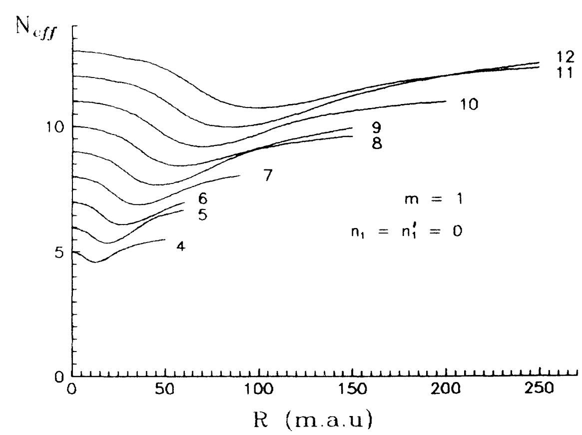

FIG. 2. N_{eff} for π terms with $n_1 = n_1' = 0$. Terms with $l = 6, 9, 12$ correspond to $(\mathrm{H}\mu)^*_n$, others to $(\mathrm{He}\mu)^*_{n'}$.

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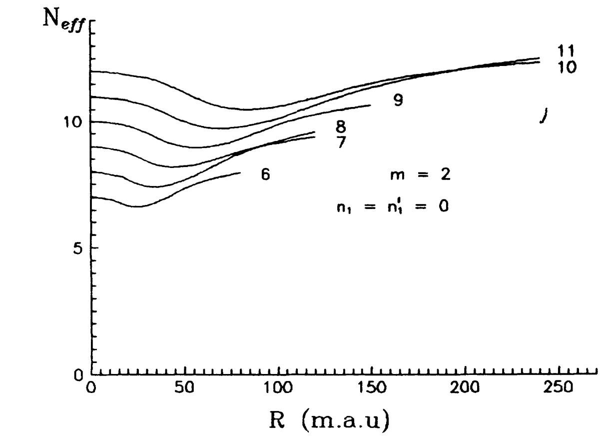

FIG. 3. $N_{\rm eff}$ for δ terms with $n_1 = n_1' = 0$. Terms with $l = 8, 11$ correspond to $(\mathrm{H}\mu)^*_n$, others to $(\mathrm{He}\mu)^*_{n'}$.

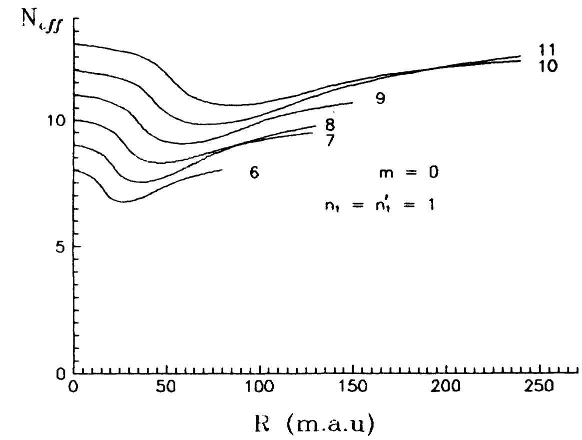

FIG. 5. $N_{\rm eff}$ for σ terms with $n_1 = n_1' = 1$. Terms with $l = 8, 11$ correspond to $(\mathrm{H}\mu)^*_n$, others to $(\mathrm{He}\mu)^*_{n'}$.

mined asymptotically by a linear Stark effect

$$u(R) \simeq \frac{3}{2}\frac{Z\,n\,(n_1 - n_2)}{R^2}, \tag{2}$$

where R is internuclear distance. The validity condition for the quasiclassical approximation reads here as

$$\frac{1}{2\pi}\frac{d\lambda}{dR} \simeq (3\,MZ\,n\,|n_1 - n_2|)^{-1/2} \ll 1, \tag{3}$$

where $M = M_{\rm He}M_{\rm H}/(M_{\rm He} + M_{\rm H})$ is the reduced mass of the nuclei and $M_{\rm H}$ and $M_{\rm He}$ are the masses of hydrogen and helium nuclei, respectively, with mesic atom units being used ($\hbar = m = e = 1$, $m^{-1} = m_\mu^{-1} + M_{\rm H}^{-1}$; m_μ is a muon mass). This condition is evidently fulfilled for $n_1 \neq n_2$. For this reason one may consider the motion of a mesic atom along a classical trajectory with the impact parameter ϱ.

We use here the asymptotic (for relative velocity of colliding particles $v \to 0$) theory of nonadiabatic transitions. The general theory can be found in the review article by Solovyov [10]. According to the theory, the transition probability is completely determined by analytic properties of terms, corresponding to the initial and final states of the system, in the complex R plane, being large mainly in the region of the quasicrossing of the terms in question, i.e., in the region close to the singularities (branch points) of the terms. It is exponentially small, the exponent being determined by a Massey parameter

$$\delta(\varrho) = \left|\mathrm{Im}\int_C p(R)\,dR\right| = \left|\mathrm{Im}\int_{\mathrm{Re}R_c}^{R_c}\frac{\Delta u(R)dR}{v_R}\right|, \tag{4}$$

where $p(R) = [2M(\varepsilon - u(R) - \varepsilon\varrho^2/R^2)]^{1/2}$ is the radial momentum; $\varepsilon = Mv^2/2$ is the relative energy of colliding particles; C is the integration contour, beginning and ending at the real axis and going around the complex crossing point of the terms R_c; and $v_R = [p_1(R) + p_2(R)]/(2M)$ is the mean radial velocity at the contour.

The transition region is passed twice: once on the way

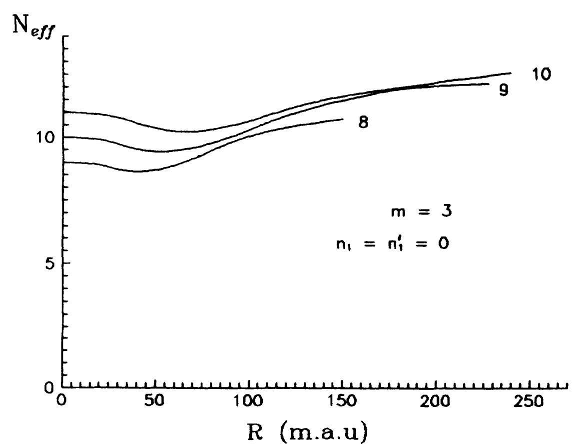

FIG. 4. $N_{\rm eff}$ for φ terms with $n_1 = n_1' = 0$. The term with $l = 10$ corresponds to $(\mathrm{H}\mu)^*_5$, others to $(\mathrm{He}\mu)^*_{n'}$ with $n' = 9$ and 8.

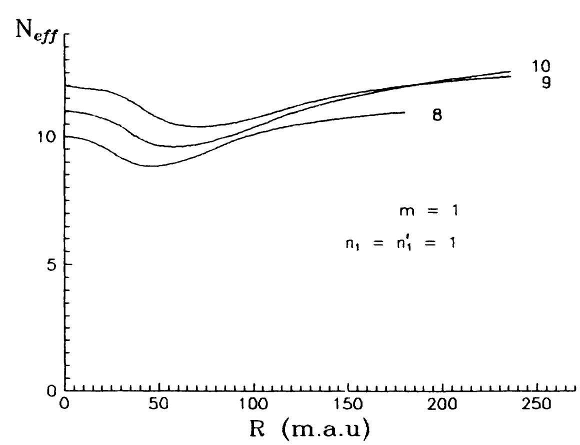

FIG. 6. $N_{\rm eff}$ for π terms with $n_1 = n_1' = 1$. The term with $l = 10$ corresponds to $(\mathrm{H}\mu)^*_5$, others to $(\mathrm{He}\mu)^*_{n'}$ with $n' = 9$ and 8.

TABLE I. Branch points of Q series for the transition $(eZ_1) \to (eZ_2)$ and reduced Massey parameters $\tilde{\delta} = \delta/\sqrt{2M}$, calculated for $\varepsilon = 0.04$ eV and $\varrho = 0$. Asterisk denotes values calculated by Solovyov [13]. $(\mathrm{H}\mu)^*_n + \mathrm{He} \to (\mathrm{He}\mu)^*_{n'} + H$.

(Nlm)	(nn_1n_2)	(Nlm)	$(n'n'_1n'_2)$	$R_c(R_0)$	$\tilde{\delta}$	$\tilde{\delta}_{LZ}$
$(5g\sigma)$	(201)	$(4f\sigma)$	$(3'0'2')$	16.6;4.7	0.38	0.30
$(8k\sigma)$	(302)	$(7i\sigma)$	$(5'0'4')$	49.7;6.6	0.13	0.11
$(7i\pi)$	(301)	$(6h\pi)$	$(5'0'3')$	42.5;5.8	0.12	0.11
$(11n\sigma)$	(403)	$(10m\sigma)$	$(7'0'6')$	106.7		0.014
$(10m\pi)$	(402)	$(9l\pi)$	$(7'0'5')$	98.4		9.0×10^{-3}
$(9l\delta)$	(401)	$(8k\delta)$	$(7'0'4')$	89.0; 2.2*	3.9×10^{-3}	4.7×10^{-3}
$(10l\sigma)$	(412)	$(9k\sigma)$	$(7'1'5')$	90.5		8.2×10^{-3}
$(14r\sigma)$	(504)	$(13q\sigma)$	$(9'0'8')$	209		2.9×10^{-6}
$(13q\pi)$	(503)	$(12o\pi)$	$(9'0'7')$	201		1.6×10^{-6}
$(12o\delta)$	(502)	$(11n\delta)$	$(9'0'6')$	195		1.5×10^{-6}
$(11n\varphi)$	(501)	$(10m\varphi)$	$(9'0'5')$	191		1.8×10^{-6}
$(13o\sigma)$	(513)	$(12n\sigma)$	$(9'1'7')$	192		1.7×10^{-6}
$(12n\pi)$	(512)	$(11m\pi)$	$(9'1'6')$	185		1.8×10^{-6}

in and once on the way out, so the total transition probability is

$$w(\varrho) = 2\exp(-2\delta)\,[1-\exp(-2\delta)], \tag{5}$$

the cross section being equal to

$$\sigma = \pi\int_0^{\varrho^2_{\max}} w(\varrho)\,d\varrho^2, \tag{6}$$

where the maximum impact parameter is determined by the requirement that $p(R)$ be real at the trajectory, i.e., for $R \geq \mathrm{Re}R_c$. The reaction rate reduced to the liquid-hydrogen density $N_0 = 4.25\times10^{22}$ cm^{-3} is equal to

$$\lambda = N_0\,\sigma\,v, \tag{7}$$

where v is relative velocity of colliding particles.

According to Solovyov and co-workers [11, 12], who investigated analytical properties of the terms of the problem of two Coulomb centers, the main contribution to the transfer process (1) is connected with two types of the branch points of the terms, namely, the Q series and isolated branch points. The Q-type branch points arise when the term touches the top of the barrier in the angular equation of the two-center problem. These branch points connect the terms with the quantum numbers (in united-atom classification) (Nlm) and $(N+k, l+k, m)$ or (in classification of separated atoms) the terms with $\tilde{n}_1 = n_1$ and $\tilde{n}_2 \neq n_2$. The branch points closest to the real axis (which give the largest contribution to the cross section due to the small value of the Massey parameter) correspond to the crossing of the terms (Nlm) and $(N+1, l+1, m)$. The terms, connected by a Q-type branch point, may belong to either different asymptotic muon localization (eZ_1 and eZ_2) or to the same one, e.g. both terms may be of eZ_2 – type.

Besides the Q series, at large $R > \mathrm{Re}R_Q$ isolated branch points R_I may exist, which arise when energy levels in two pits of the effective potential of the angular equation coincide with each other. For this reason these branch points always connect the terms with different muon localization (eZ_1 and eZ_2), which have quantum numbers $n'_1 = n_1$ and $n'_2 \neq n_2$.

There exists also the S series of the branch points, related to the reorganization of the potential of the radial equation of the two-center problem at small R and the corresponding reorganization of muon wave function

TABLE II. Branch points of Q series for the transition $(eZ_2) \to (eZ_2)'$ and reduced Massey parameters calculated for $\varepsilon = 0.04$ eV and $\varrho = 0$.

(Nlm)	(nn_1n_2)	$(Nlm)'$	$(n'n'_1n'_2)'$	$R_c(R_0)$	$\tilde{\delta}$	$\tilde{\delta}_{LZ}$
$(7i\sigma)$	$(5'0'4')$	$(6h\sigma)$	$(4'0'3')$	30.6;9.4	0.57	0.67
$(6h\pi)$	$(5'0'3')$	$(5g\pi)$	$(4'0'2')$	24.2;8.6	0.58	0.87
$(10m\sigma)$	$(7'0'6')$	$(9l\sigma)$	$(6'0'5')$	69.9;13.6	0.34	0.28
$(9l\pi)$	$(7'0'5')$	$(8k\pi)$	$(6'0'4')$	60.8;12.7	0.33	0.30
$(8k\delta)$	$(7'0'4')$	$(7i\delta)$	$(6'0'3')$	51.1;11.7	0.35	0.31
$(9k\sigma)$	$(7'1'5)$	$(8i\sigma)$	$(6'1'4')$	52.4;11.8	0.36	0.41
$(13q\sigma)$	$(9'0'8')$	$(12o\sigma)$	$(8'0'7')$	129		0.15
$(12o\pi)$	$(9'0'7')$	$(11n\pi)$	$(8'0'6')$	117		0.14
$(11n\delta)$	$(9'0'6')$	$(10m\delta)$	$(8'0'5')$	105		0.13
$(10m\varphi)$	$(9'0'5')$	$(9l\varphi)$	$(8'0'4')$	94		0.13
$(12n\sigma)$	$(9'1'7')$	$(11m\sigma)$	$(8'1'6')$	106		0.14
$(11m\pi)$	$(9'1'6')$	$(10l\pi)$	$(8'1'5')$	94		0.16

from the wave function of the united atom to molecular wave function. These branch points connect the terms (Nlm) and $(N+1,l,m)$ and are located three to four times closer to the origin than R_Q and R_I. For this reason, though the S-branch points may relate to the muon transfer, they are not important because they are deep under the Coulomb barrier for the energies in question ($\varepsilon \leq 50$ eV).

When calculating the cross section of the process (1) for $n=2$ and 3 [9] we used the branch points of the Q series, calculated with the help of the Solovyov's program [11]. For higher states ($n \geq 4$) the process of calculating of R_Q (and even the calculating of the terms for real large R, corresponding to the transition point) becomes laborious and time consuming [13], which makes the calculation impossible with the computers that we have. Nevertheless the quasicrossings of the corresponding terms are clearly seen at real R (Figs. 1–6). For this reason we used the quasiclassical approximation for the terms of the two–center problem [14] to calculate the necessary terms at real R.[1] This allowed us to use the Landau-Zener formula [10] when calculating the Massey parameter:

TABLE III. Rates (10^{11} s^{-1}) of muon transfer to the bare helium nucleus reduced to liquid-hydrogen density.

n	ε (eV)	$p\mu{}^3$He	$p\mu{}^4$He	$d\mu{}^3$He	$d\mu{}^4$He	$t\mu{}^3$He	$t\mu{}^4$He
2	0.01	8.8	7.7	3.1	2.4	1.8	1.2
	0.04	4.4	3.9	1.6	1.2	0.90	0.62
	0.1	2.8	2.5	1.00	0.77	0.57	0.40
	0.5	1.28	1.12	0.45	0.35	0.26	0.18
	1	0.92	0.80	0.32	0.25	0.19	0.13
	5	0.44	0.39	0.16	0.12	0.090	0.062
	10	0.34	0.30	0.12	0.093	0.069	0.048
	20	0.28	0.24	0.100	0.077	0.058	0.040
	30	0.26	0.23	0.094	0.073	0.055	0.039
	50	0.26	0.23	0.095	0.074	0.056	0.040
3	0.01	135	128	86	77	67	58
	0.04	68	65	43	39	34	29
	0.1	43	41	27	25	22	19
	0.5	19	18	12.4	11.1	9.7	8.4
	1	13.9	13.2	8.8	8.0	7.0	6.0
	5	6.8	6.4	4.3	3.9	3.4	3.0
	10	5.2	5.0	3.4	3.1	2.7	2.4
	20	4.3	4.1	2.8	2.6	2.3	2.0
	30	3.9	3.8	2.6	2.4	2.2	1.9
	50	3.7	3.6	2.6	2.4	2.1	1.9
4	0.01	189	182	141	134	125	119
	0.04	97	94	72	69	64	61
	0.1	63	61	47	45	42	40
	0.5	31	30	23	22	20	19
	1	23	22	17	16	15	14
	5	11.3	10.9	8.3	7.9	7.4	7.0
	10	8.5	8.2	6.2	5.9	5.5	5.2
	20	6.9	6.6	4.8	4.6	4.2	4.0
	30	6.5	6.2	4.4	4.1	3.8	3.5
	50	6.6	6.2	4.2	3.9	3.5	3.3
5	0.01	419	395	255	227	197	168
	0.04	214	202	130	116	100	85
	0.1	139	131	84	75	65	55
	0.5	69	65	42	37	32	27
	1	53	50	32	28	24	20
	5	31	29	19	17	14	12
	10	28	26	17	15	13	11
	20	28	27	18	16	14	12
	30	30	29	19	17	15	13
	50	34	33	22	21	18	16

[1] Unfortunately, the quasiclassical approximation obtained in [14] does not allow one to calculate complex branch points [13].

$$\delta_{\rm LZ}(\varrho) = \frac{\pi\,(\Delta u_{\rm min})^2}{4\,v_c(\varrho)\Delta F}, \tag{8}$$

where $\Delta u_{\rm min} = u_1(R_0) - u_2(R_0)$ is the minimal distance between the quasicrossing adiabatic terms, $\Delta F = |F_1(R_0) - F_2(R_0)|$ is the difference between derivatives of crossing diabatic terms at the crossing point R_0, and $v_c = [p_1(R_0) + p_2(R_0)]/2M$ is the mean radial velocity at R_0. When the complex branch points R_c were known, we used the following approximation for difference between terms [12] describing the square-root behavior of the terms in the vicinity of the branch point:

$$\Delta u(R) \simeq \frac{\Delta u({\rm Re}R_c)}{{\rm Im}R_c}\sqrt{(R-R_c)(R-R_c^*)}, \tag{9}$$

which allowed one to calculate the Massey parameter as

$$\delta(\varrho) \simeq \frac{\pi\Delta u({\rm Re}R_c)}{4v_c(\varrho)}\,{\rm Im}R_c. \tag{10}$$

Only attractive terms were considered, which gave the largest contribution to the cross section due to the "focusing" of the particles. The cross section for a given n was obtained as a sum of the cross sections (6) over all the attractive terms with the given n, having the statistic weight $(2-\delta_{0m})/n^2$.

When calculating the cross section for the charge exchange on a helium atom, we took into account the effect of screening of the charge of helium nucleus by atomic

TABLE IV. Rates (10^{11} s^{-1}) of muon transfer to the helium atom (including electron screening) reduced to liquid-hydrogen density.

n	ε (eV)	$p\mu^3$He	$p\mu^4$He	$d\mu^3$He	$d\mu^4$He	$t\mu^3$He	$t\mu^4$He
2	0.01	1.22	1.08	0.49	0.39	0.30	0.21
	0.04	1.47	1.29	0.58	0.45	0.35	0.25
	0.1	1.51	1.33	0.59	0.46	0.35	0.25
	0.5	1.17	1.03	0.43	0.33	0.25	0.17
	1	0.90	0.79	0.32	0.25	0.18	0.13
	5	0.44	0.39	0.16	0.12	0.090	0.062
	10	0.34	0.30	0.12	0.093	0.069	0.048
	20	0.28	0.24	0.100	0.077	0.058	0.040
	30	0.26	0.23	0.094	0.073	0.055	0.039
	50	0.26	0.23	0.095	0.074	0.056	0.040
3	0.01	9.7	9.3	6.7	6.1	5.5	4.9
	0.04	12.3	11.7	8.5	7.8	7.0	6.1
	0.1	13.9	13.2	9.5	8.7	7.8	6.8
	0.5	14.6	13.8	9.6	8.7	7.8	6.8
	1	12.2	11.6	7.9	7.2	6.3	5.5
	5	6.7	6.3	4.3	3.9	3.4	3.0
	10	5.1	4.9	3.4	3.0	2.7	2.3
	20	4.2	4.0	2.8	2.6	2.3	2.0
	30	3.9	3.7	2.6	2.4	2.2	1.9
	50	3.6	3.5	2.5	2.4	2.1	1.9
4	0.01	3.2	2.8	1.31	1.04	5.4	5.1
	0.04	3.9	3.5	1.6	1.27	6.2	5.8
	0.1	4.3	3.8	1.8	1.39	6.9	6.5
	0.5	4.4	3.9	1.7	1.35	8.0	7.5
	1	4.0	3.5	1.5	1.20	8.0	7.6
	5	2.7	2.4	1.01	0.78	6.5	6.2
	10	2.4	2.1	0.91	0.72	5.0	4.8
	20	2.5	2.2	0.98	0.78	4.0	3.7
	30	2.8	2.5	1.12	0.90	3.6	3.4
	50	3.5	3.2	1.49	1.21	3.4	3.1
5	0.01	17	16	11	10	8.9	7.8
	0.04	21	20	14	12	11	9.5
	0.1	23	22	15	14	12	11
	0.5	27	26	18	16	14	12
	1	28	27	18	16	14	12
	5	27	26	17	15	13	11
	10	26	24	16	14	12	11
	20	27	25	17	15	13	12
	30	29	28	19	17	15	13
	50	33	32	22	20	18	16

electrons. The effective potential corresponding to the initial state was written as

$$u(R) = u_0(R) + \frac{3}{2}\, n\, (n_1 - n_2)\, [\mathcal{E}(R) - Z/R^2] \qquad (11)$$

where $u_0(R)$ is the term without screening and $\mathcal{E}(R)$ is the electric field of the helium atom [15]. The screening was ignored when calculating the branch points and Massey parameters, which may be justified by the fact that both R_c and δ depend mainly on a difference between the terms.

Table I contains the branch points R_c (or R_0) responsible for reaction (1) for n =2, 3, 4, and 5 as well as the corresponding Massey parameters (calculated for $\varrho = 0$ and $\varepsilon = 0.04$ eV), both accurate, i.e., calculated via formula (10) when the complex branch points R_c are known, and approximate $\delta_{\rm LZ}$ calculated in the frame of the Landau-Zener model (8). As seen from Table I the values $\delta_{\rm LZ}$ are close to the accurate parameters δ. It is also seen that for $n \geq 4$ the Massey parameters [and hence the transition probabilities (5)] become very small. For this reason, to calculate the charge exchange cross section for $n \geq 4$ one should consider also the two-step process $(eZ_1) \to (eZ_2) \to (eZ_2)'$ [e.g., $(11n\sigma) \to (10m\sigma) \to (9l\sigma)$ in Fig.1]. In this case the first (right-hand) crossing is passed by the system diabatically, that is, with the probability $w_1 \simeq 100\%$ (at least for $n > 4$). The total probability of the transition to either of terms (eZ_2) or $(eZ_2)'$ is then

$$w = 2\, w_1\, [1 - w_1 + w_1 w_2\, (1 - w_2)], \qquad (12)$$
$$w_i = \exp(-2\delta_i).$$

Here one should note one more circumstance concerning an electron shell of the helium atom. If the charge exchange takes place on the helium atom and both electrons remain bound after it, one must spend 51.8 eV of energy to reorganize the electron shell of the system [actually this value is a little bit smaller because the energy level of the (Heμe) "atom" is deeper than that of hydrogen atom]. The energy yield at the transition from $n = 4$ to $n' = 7$ amounts to 0.009 566 m.a.u., which corresponds to 48.4 eV for muonic protium, 51 eV for muonic deuterium, and 51.9 eV for muonic tritium. This means that this transition is energetically forbidden for muonic protium and deuterium and the transfer proceeds here only at the left-hand (second) quasicrossing. In this case the transition probability is equal to

$$w = 2\, w_1\, w_2\, (1 - w_2). \qquad (13)$$

Of course the same consideration holds also for $n > 4$, but for $n = 5$ Eqs. (12) and (13) give the same result because $w_1 \approx 1$.

TABLE V. Conversion factors k for calculating the cross sections. The cross section $\sigma(10^{-16}\ \mathrm{cm}^2) = k\lambda(10^{11}\ \mathrm{s}^{-1})/\sqrt{\varepsilon\ (eV)}$.

$p\mu^3$He	$p\mu^4$He	$d\mu^3$He	$d\mu^4$He	$t\mu^3$He	$t\mu^4$He
0.0147	0.0152	0.0186	0.0196	0.0208	0.0222

The branch points R_c (or R_0), corresponding to the second transition $(eZ_2) \to (eZ_2)'$, and the corresponding Massey parameters are given in Table II. One can see that for these quasicrossings $\delta_{\rm LZ}$ are also close to the accurate values δ (with the exception for the lowest transition $n' = 5 \to n' = 4$, which has the negligible probability). Table III contains the reduced reaction rates for the charge exchange of excited muonic hydrogen on the helium nucleus for $n = 2$, 3, 4, and 5. The rates for the charge exchange on the helium atom are given in Table IV.

According to Eq. (7), the cross sections for the reaction $\sigma = k\lambda/\sqrt{\varepsilon}$, where k are given in Table V.

As seen from Table III, the rates of muon transfer to the nucleus at low energy behave like $\sim 1/\sqrt{\varepsilon}$, and the corresponding cross sections $\sigma \sim 1/\varepsilon$. It is natural because for the potential $u(R) \sim -\alpha/R^2$ the square of the maximum impact parameter $\varrho^2_{\max} = \alpha/\varepsilon + R_0^2$, which gives for the cross section [for $w(\varrho) \simeq 0.5$]

$$\sigma \simeq \frac{\pi}{2}\left(\frac{\alpha}{\varepsilon} + R_0^2\right). \qquad (14)$$

When the screening is switched on, $\varrho_{\max}$ decreases sharply at low energy and the energy dependence of λ becomes much more flat.

The rates decrease at the transition from light isotopes to heavy ones, which is caused by the Massey parameter dependence $\delta \sim \sqrt{2M}$ and the behavior of the transition probability (5) for sufficiently large δ. Low values of rates (and the cross sections) for the transfer to the helium atom from the state $n = 4$ of the protium and deuterium are caused by the energetic veto for the transition at the right-hand (first) crossing. The obtained values of the muon transfer rates are in agreement with the experimental estimates (rather rough) [16], with the exception for the rate for $n = 3$, for which the experimental value $(2 \pm 7) \times 10^{10}\ \mathrm{s}^{-1}$ is much lower than the theoretical one.

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