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Ionization of muonic atoms by nonrelativistic electrons

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Analytical expressions for cross sections for ionization of muonic hydrogenlike atoms by nonrelativistic electrons are derived using the classical impulse approximation.

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I. INTRODUCTION

We study in this paper ionization of muonic hydrogenlike atoms by nonrelativistic electrons. The atoms consists of a muon μ^- moving in the Coulomb field of a nucleus of electric charge Ze (e is the elementary charge). The μ^- muon has mass $m_\mu = 206.76 m_e$ (m_e is the electron mass), and electric charge equal to the elementary charge. Muonic hydrogenlike atoms are produced when a muon μ^- is shown down in matter, it encounters a highly excited ordinary hydrogenlike atom, and replaces the atomic electron removing it from the atom. Subsequently, the bound muon cascades down to lower atomic levels.

Most of experimental studies of muonic atoms utilize high-energy μ^- beams coming from pion π^- decay generated by nuclear reactions. Some of the beam muons lose their energy by inelastic (ionization and excitation) collisions with particles in the medium. The electrons produced in the ionizing collisions have high energies, are still present in the medium when the muonic atoms are formed, and are able to ionize the muonic atoms. However, the efficiency of the electron-impact ionization of the muonic atoms in the experiments cannot be estimated because of lack of the ionization cross sections. [Scattering of electrons and other particles on muonic atoms has been studied (see Refs. [1–3] and references therein) but neither experimental nor theoretical cross sections for the electron-impact ionization of the atoms are available in literature.] Therefore, the main goal of this work is to derive the ionization cross sections for the collisions where the target atom is the simplest muonic atom, that is, an atom produced by replacing the electron in an ordinary hydrogenlike atom by a negative muon μ^-. We derive there cross sections using the classical impulse approximation [4] and compare them with the cross sections for the electron-impact ionization of ordinary hydrogenlike atoms.

Muonic hydrogenlike atoms such as muonic hydrogen ($\mu^- p$) and muonic helium (μ^- He)$^+$ and their isotopes are important systems for sensitive tests of QED and for study of the structure of nuclei with few nucleons [5]. Since our approach yields the cross sections for the ground state as well as for the excited atoms, it is convenient for studying the kinetics of the $n=2$ level of hydrogenlike muonic atoms (the $2s$ states are of special importance in spectroscopic studies of the atoms). In some experiments, the rate of depopulation of the $n=2$ level by electron-impact ionization may be comparable to, or greater than, the rate of quenching of the level by atomic collisions (for example [5], the cross section for the quenching in a He-(μ^- He)$^+$ collisions is less than 10^{-22} cm^2). By itself, the rate of the electron-impact ionization of the medium atoms by the muons in typical experiments is not high enough to produce electron density comparable with the density of the neutrals quenching the muonic atoms in the $2s$ states. (However, the electron density is much higher than expected on the basis of the moderate rate of the electron production by the muons because the electrons resulting from the ionization are confined to a very slender cylinder around the path of the muonic beam.) If the cross section for the electron-impact ionization of the $2s$ muonic atoms is much greater than the quenching cross section then the very large difference between the speeds of the quenching atoms and the ionizing electrons may be sufficient for the electron-impact ionization to be an important factor in the kinetics of the $2s$ state, especially when the density of the quenching atoms is not too high.

Even though we focus in this work primarily on the electron-impact ionization of hydrogenlike muonic atoms, many conclusions of the work can be used to investigate the ionization of other muonic atoms. (Some of such atoms are important in the muon catalized fusion—see Refs. [3] and [5].) The muonic ground-state orbital in a nonhydrogenlike atom is much closer to the atomic nucleus than the other (electronic) orbitals. Being mostly entirely inside the electronic orbitals, the ground-state orbital is weakly affected by the electronic orbitals. Therefore, the electron-impact ionization of the muonic ground-state orbital in nonhydrogenlike atoms can be described by the approach of the present work.

II. THE APPROACH

The approach of this work is based on the energy-transfer cross sections of the classical impulse approximation [4] which has been applied successfully to study a

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wide variety of atomic inelastic collisions [4,6–12]. (The approximation is especially suitable for the description of interaction of an electron with hydrogenlike atoms because of the very small number of subatomic species involved in the collision.) In the approximation, the target atom interacting with the incident electron is considered a group of localized particles (called hereafter "components" of the target atom) which are either electrons (in the case of ordinary atom) or muons μ^- (in the case of muonic atom) orbiting the nucleus in the field of a central force. Dynamic properties of the collision system (the incident electron plus the target atom) before the collision are defined [11,12] by a set C of n parameters $(C_1, C_2, \ldots, C_n)$ such as the particle masses, velocities, etc. During the collision, set C is transformed into a set C' ($C \rightarrow C'$), where the prime denotes the state of the collision system after the collision. The probability of transition $C \rightarrow C'$ can be determined if relationship between sets C and C' is known. This relationship can be given by a function F such that

$$C' = F(w, D, \Theta, C_4, \ldots, C_n) , \tag{1}$$

where we separated, for future convenience, three parameters [$C_1 \equiv w$ (the relative velocity of the incident electron and the target atom before the collision), $C_2 \equiv D$ (the impact parameter of the collision), and $C_3 \equiv \Theta$ (the azimuthal orientation of the "shot line" of the incident electron before the collision)] from the rest of the parameters of the set C. In the impulse approximation, the function $F(w, D, \Theta, C_4, \ldots, C_n)$ can be obtained, in principle, from the classical Hamiltonian for the scattering trajectory (with quantum-mechanical requirements imposed on the electronic energies of the target atom) if the potentials for the interactions between the incident electron and the components of the target atom are known. These potentials can be approximated by the central-force Coulomb potential, because the atomic components, having classical diameters of orders of magnitude smaller than the collision diameter, can be treated as point charges.

A particular cross section for a collision of the incident electron with the target atom is defined in the classical impulse approximation as a measure of the probability of a definite change in C during the collision, that is, the probability of a definite change in the state of the collision system. This change is characterized by change of one, or more, parameters of the set C. Subsequently, the cross section q_ξ with respect to a parameter ξ is defined as [12]

$$q_\xi(w, \xi, \ldots, C_n) = \int_0^{2\pi} \int_0^{\infty} \delta(\xi - F(w, D, \Theta, \xi, \ldots, C_n)) \times D \, dD \, d\Theta . \tag{2}$$

Adding all the cross sections that are nonzero in some practically useful range of ξ, one obtains the so-called differential cross section with respect to ξ,

$$\sigma_\xi(w, \ldots, C_n) = \int_{\xi_1}^{\xi_2} q_\xi(w, \xi, \ldots, C_n) d\xi . \tag{3}$$

Averaging the relationship (3) over some important parameters (denoted by the subscripts $j+1, \ldots, n$) of set C one obtains the average cross sections with respect to ξ,

$$Q_\xi(w, \ldots, C_j) = \int \cdots \int \sigma_\xi(w, \ldots, C_j, C_{j+1}, \ldots, C_n) f_{j+1}(C_{j+1}) \cdots f_n(C_n) dC_{j+1} \cdots dC_n , \tag{4}$$

where $f_{j+1}(C_{j+1}), \ldots, f_n(C_n)$ are distribution functions of parameters $C_{j+1}, \ldots, C_n$.

The complexity of the averaging (4) can be reduced significantly by assuming that collision of the incident electron with the target atom can be treated as a superposition of all the pairwise (binary) interactions between the components of the target atom and the incident electron. Averaging (4) over $\mathbf{v}_l$ (the velocity vector of the lth component of the target atom) one obtains

$$Q_\xi(w, \ldots, C_j) = \sum_{l=1}^{N_B} \int \int \int \sigma_{\xi,l}(w, \ldots, C_j, \mathbf{v}_l) f_B(\mathbf{v}_l) d\mathbf{v}_l , \tag{5}$$

where N_B is the number of the components in the target atom, and $f_B(\mathbf{v}_l)$ is the velocity distribution of the lth component of the atom.

If the spatial distribution $p_B(\theta, \vartheta)$ of the velocity vectors $\mathbf{v}_l$ in the target atom is assumed to be isotropic, then the distributions $p_B(\theta, \vartheta)$ and $f_B(\mathbf{v}_l)$ (in a spherical coordinate system, located at the center of the atom, with the angle θ measured from the vector of the relative velocity $\mathbf{w}$ before the collision) are, respectively,

$$p_B(\theta, \vartheta) d\theta \, d\vartheta = \sin\theta \, d\theta \, d\vartheta / 4\pi \tag{6}$$

and

$$f_B(\mathbf{v}_l) d\mathbf{v}_l = p_B(\theta, \vartheta) g_B(v_l) dv_l d\theta \, d\vartheta , \tag{7}$$

where $g_B(v_l)$ is the speed distribution of the lth component of the target atom.

Collision of the incident electron with a hydrogenlike atom can be reduced to one pairwise interaction between the incident electron and the atomic electron (if the target atom is an ordinary atom) or muon (if the target atom is a muonic atom). Subsequently, the expression (5) can be written as

$$Q_\xi(w, \ldots, C_j) = \int \int \int \sigma_{\xi,l}(w, \ldots, C_j, \mathbf{v}_l) f_B(\mathbf{v}_l) d\mathbf{v}_l , \tag{8}$$

where $\sigma_{\xi,l}$ is the cross section for the interaction between the incident electron and the only component of the target atom (this component is called hereafter the field particle).

In the present work, the scattering of an electron on a hydrogenlike atom is considered as an outcome of the interaction between the incident electron of mass $m_2 = m_e$

and velocity $\mathbf{v}_2$ with the atomic field particle to be removed from the atom as a result of ionization. The field particle (of mass m_1 and velocity $\mathbf{v}_1$) is either an electron or a muon μ^-. In the former case, $m_1=m_e$, while in the latter case $m_1=m_\mu$ where m_μ is the mass of the muon.

III. ENERGY TRANSFER IN THE IMPULSE APPROXIMATION

Efficiency of the energy transfer from the incident electron of mass m_2 and velocity $\mathbf{v}_2$ to the field particle of mass m_1 and velocity $\mathbf{v}_1$ is measured in the classical impulse approximation by the so-called energy-transfer cross section $q_{\Delta E}$ for the energy ΔE to be transferred during the collision from the incident electron to the field particle. In the case when the interaction potential between the incident particle and the field particle is a central-force potential, the cross section $\bar{q}_{\Delta E}$ can be written as [13,12]

$$\bar{q}_{\Delta E}(\Delta E)=\pi\int\int\frac{V}{v_2}p(\theta)\frac{F_s'(1/\cos^2\Psi_g,\theta)}{[W_{\Psi_g}(\Delta E,1/\cos^2\Psi_g,\theta)]^{1/2}}\times d(1/\cos^2\Psi_g)d\theta\ , \tag{9}$$

where Ψ_g is the scattering angle, V is the relative speed of the incident particle and the field particle, θ is the angle between the initial velocity vectors of the particles, and $p(\theta)$ is the distribution of the angle θ. The bar over $g_{\Delta E}$ in Eq. (9) indicates the fact that the cross section (9) is already averaged over the angle θ, whereas the cross section (2) is not averaged over the angle. Here, $F_s=D^2$ and

$$F_s'(1/\cos^2\Psi_g,\theta)=\frac{\partial F_s}{\partial(1/\cos^2\Psi_g,\theta)}\ , \tag{10}$$

$$W_{\Psi_g}=(2a\sin\Psi_g\cos\Psi_g)^2-(\Delta E+b\cos^2\Psi_g)^2\ , \tag{11}$$

$$a=\mu v_1 v_2\sin\theta\ , \tag{12}$$

$$b=\kappa\left[E_2-E_1+\frac{(m_1-m_2)}{2}v_1v_2\cos\theta\right]\ , \tag{13}$$

$$\mu=\frac{m_1m_2}{m_1+m_2}\ , \tag{14}$$

$$\kappa=\frac{4m_1m_2}{(m_1+m_2)^2}\ , \tag{15}$$

and

$$E_1=m_1v_1^2/2 \text{ and } E_2=m_2v_2^2/2\ . \tag{16}$$

In collisions of interest here the following inequality is often satisfied during electron-electron and electron-muon binary interactions:

$$(\Delta E+b\cos^2\Psi_g)^2 \gg (2a\sin\Psi_g\cos\Psi_g)^2\ . \tag{17}$$

To prove the validity of the relationship (17) we first rewrite it as

$$u \gg s\ , \tag{18}$$

where

$$u=\left|1+\frac{\Delta E}{b\cos^2\Psi_g}\right| \tag{19}$$

and

$$s=\frac{2a}{b}\tan\Psi_g\ . \tag{20}$$

[One should notice that the requirement (18) is stronger than the requirement (17).]

Let us consider the "worst" situation when u in inequality (18) is very small ($u\approx 0$). This occurs when $\Delta E\approx -b\cos^2\Psi_g$. During ionization ΔE is close to U_k (U_k is the ionization energy of the target atom excited to the kth energy level) so that $u\approx 0$ when $-b\cos^2\Psi_g\approx U_k$. Consequently,

$$\tan\Psi_g=(\cos^{-2}\Psi_g-1)^{1/2}\approx\left[\frac{b}{U_k}-1\right]^{1/2} \tag{21}$$

and

$$s\approx 2a(bU_k)^{-1/2}\ . \tag{22}$$

Using relationships (12) and (13) one obtains from Eq. (21) the following:

(a) the electron-electron interaction ($m_1=m_2=m_e$),

$$s=\frac{2v_1\sin\theta}{v_k} \tag{23}$$

and

(b) the electron-muon μ^- interaction ($m_1\gg m_2=m_e$),

$$s=\frac{2v_1\sin\theta}{v_k}\left[1+\frac{m_1}{m_2}\left[\frac{E_1}{E_2}\right]^{1/2}\cos\theta\right]^{-1/2}\ , \tag{24}$$

where [14]

$$v_k=\langle v_1\rangle=\left[\frac{2U_k}{m_1}\right]^{1/2}\ . \tag{25}$$

It is obvious from Eqs. (23) and (25) that the requirement (18) is always fulfilled (for both electron-electron and electron-muon interactions) if the interaction of particles 1 and 2 occurs at small values of ratio v_1/v_k. This is always true in consistent classical (Kepler's) representation of elliptic orbits of the field particle (including the zero-angular-momentum orbit with eccentricity $\varepsilon\to 0$) which represents in Kepler's formalism the ground-state orbit of the field particle. But even if the Bohr (circular) orbit is assumed for an orbit of the field particle, the inequality (18) will still hold in the case when the field particle is a muon μ^-. This can be seen from the following considerations. When $\Psi_g\le\pi/2$ (the range of the angle Ψ_g), the condition (17) is the weakest when $b\approx 0$, that is, when $E_1\approx E_2$ and $\theta\approx\pi/2$. In such a case, the condition can be rewritten as

$$(\Delta E)^2\gg 4a^2\sin^2\Psi_g\cos^2\Psi_g\ . \tag{26}$$

If $(\Delta E)^2\gg 4a^2$ then the condition (26) is fulfilled for any value of the angle Ψ_g because $\sin^2\Psi_g\cos^2\Psi_g$ is never greater than one. Again, the condition $(\Delta E)^2 \gg 4a^2$ is the weakest when the term $4a^2$ has its maximum value, that is, when

$$4a^2 = 4(\mu v_1 v_2)^2 \simeq 16\frac{m_2}{m_1}E_1E_2 , \tag{27}$$

where the reduced mass of the electron-muon collision system is [see Eq. (14)] $\mu \simeq m_e$, $E_1 \approx U_k$, and $E_2 \approx E$ (E is the impact energy of the electron-atom collision). Thus, Eq. (27) can be rewritten in the case of the electron-muon system as

$$4a^2 \simeq 0.08 U_k E . \tag{28}$$

Since we assumed above that inequality (26) is the weakest when $E_2 \approx E_1 \approx U_k$, one can conclude that the requirement (26) [and, therefore, the requirement (17)] is fulfilled if

$$\left[\frac{\Delta E}{U_k}\right]^2 \gg 0.08 , \tag{29}$$

which is always true for the electron-muon interactions of interest in this work.

Neglecting the first term on the right-hand side of Eq. (11) means that practically only one value of the energy ΔE is possible ($\Delta E = -b\cos^2\Psi_g$) during collision [13,12] at a given scattering angle Ψ_g. In such a case,

$$W_{\Psi_g} = -(\Delta E + b\cos^2\Psi_g)^2 \rightarrow 0 , \tag{30}$$

and the concept of the δ function can be used [12],

$$\frac{1}{[-(\Delta E + b\cos^2\Psi_g)^2]^{1/2}} \simeq \frac{1}{\Delta E\cos^2\Psi_g}\delta\left[\frac{1}{\cos^2\Psi_g} + \frac{b}{\Delta E}\right] , \tag{31}$$

so that the energy-transfer cross section (9) reduces to

$$\bar{q}_{\Delta E} = \pi \int \frac{V}{v_2} p(\theta) \left[\frac{1}{\Delta E\cos^2\Psi_g} F_s'(1/\cos^2\Psi_g)\right]_{[1/\cos^2\Psi_g = -b/\Delta E]} d\theta . \tag{32}$$

Representation of motion of the localized field particle in the target atom is a difficult issue. Various distributions of kinetic energy of subatomic particles were proposed in the literature of the subject [8,14–17] and these distributions are discussed in Ref. [14]. It has been shown there that the quantum-classical distribution proposed in Ref. [14] is the closest to the distribution obtained from rigorous quantum-mechanical Hartree-Fock calculations, and that the distribution of speed of the field particle can be approximated, for purpose of studying the ionization of the target atom, by a δ function,

$$g_B(v_1) = \delta(v_1 - \langle v_1 \rangle) , \tag{33}$$

where $\langle v_1 \rangle$ is given by Eq. (25). Introducing a distribution other than the δ distribution for the field particle energies improves the agreement of the classical results with experimental data. This improvement does not, however, constitute any significant improvement over quantum-mechanical approximations [18,14].

Averaging the energy-transfer cross section (32) over the distribution (33) yields

$$\begin{aligned} Q'_{\Delta E}(E,\Delta E) &= \int \bar{q}_{\Delta E} g_B(v_1) dv_1 \\ &= \left[\pi \int \frac{V}{v_2} p(\theta) \frac{1}{\Delta E\cos^2\Psi_g} F_s'(1/\cos^2\Psi_g)\Big|_{[1/\cos^2\Psi_g = -b/\Delta E]} d\theta\right]_{(E_1, v_1)\rightarrow(U_k, \langle v_1\rangle)} . \end{aligned} \tag{34}$$

The cross section $Q_{ea}^{\text{ion}}(E)$ for a binary collision of the incident electron and the field particle when more than $\varepsilon_l = |\Delta E_l|$, but less than $\varepsilon_u = |\Delta E_u|$ energy is transferred to the field particle is

$$Q_{ea}(E) = \int_{\varepsilon_l}^{\varepsilon_u} Q'_\varepsilon(E,\varepsilon) d\varepsilon , \tag{35}$$

where $\varepsilon \equiv |\Delta E|$.

The energy ΔE that can be transferred in a binary collision from the incident electron to the field particle depends on the properties (masses, velocities, etc.) of the colliding particles. As discussed in Ref. [13], the range of ΔE is

$$\Delta E^- \le \Delta E \le \Delta E^+ \tag{36}$$

with

$$\Delta E^{+,-} = -\frac{b}{2} \pm \left[\left[\frac{b}{2}\right]^2 + a^2\right]^{1/2} , \tag{37}$$

where a and b are given in Eqs. (12) and (13), respectively. The θ-averaged ΔE can be obtained from Eqs. (36) and (37) as a quantity close to ΔE calculated at $\theta = \pi/2$. Introducing a new parameter,

$$\lambda = \frac{m_1}{m_2} , \tag{38}$$

one obtains from Eq. (37),

$$\Delta E^+ = E_1 \text{ and } \Delta E^- = -E_2 \text{ (when } \lambda = 1) . \tag{39}$$

Similarly, assuming that $(\lambda-2)E_1E_2 \gg E_1^2$ and neglecting small quantities, one has

$$\Delta E^+ = \frac{2E_2}{\lambda}\left\{\left[1+(\lambda-2)\frac{E_1}{E_2}\right]^{1/2}+\frac{E_1}{E_2}-1\right\}, \quad \text{when } \lambda \gg 1, \tag{40}$$

$$\Delta E^- = -\frac{2E_2}{\lambda}\left\{1-\frac{E_1}{E_2}+\left[1+(\lambda-2)\frac{E_1}{E_2}\right]^{1/2}\right\}, \quad \text{when } \lambda \gg 1. \tag{41}$$

The energy ΔE transferred during atomic ionization must be greater than the energy threshold for ionization of the target atom excited to the kth level. For $\lambda=1$ (scattering of the incident electron on the field electron) the relationship (36) allows one to specify the "ionization" limits of integration in Eq. (35) as

$$\varepsilon_l = U_k \text{ and } \varepsilon_u = E_2 . \tag{42}$$

In the case when $\lambda \gg 1$ (scattering of the incident electron on the field muon μ^-) these limits are

$$\varepsilon_l = U_k \text{ and } \varepsilon_u = \frac{2E_2}{\lambda}\left\{1-\frac{U_k}{E_2}+\left[1+(\lambda-2)\frac{U_k}{E_2}\right]^{1/2}\right\}. \tag{43}$$

The threshold energy $(E_2)_{\text{th}}$ required for ionization of the target atom by electron can be found from the condition

$$\Delta E^- = -U_k . \tag{44}$$

Using Eqs. (39) and (41) one obtains from Eq. (44),

$$(E_2)_{\text{th}} = U_k , \quad \text{when } \lambda=1 \tag{45}$$

and

$$(E_2)_{\text{th}} \simeq \frac{\lambda}{8}U_k , \quad \text{when } \lambda \gg 1 \tag{46}$$

where we assumed in derivation of the last expression that

$$\lambda \gg 2 , \tag{47}$$

$$E_1 \simeq U_k , \tag{48}$$

$$\frac{\lambda+4}{\lambda+1} \simeq 1 , \tag{49}$$

$$\frac{4}{\lambda^2} \ll \frac{4}{\lambda} , \tag{50}$$

and

$$8E_2U_k \gg 4U_k^2 . \tag{51}$$

In the case of the electron-impact ionization of a muonic hydrogenlike atom, $\lambda=206.76$ and, according to Eq. (46), $(E_2)_{\text{th}} \simeq 25.8U_k$. Thus, the incident electron energy greater than $25.8U_k$ is required to ionize muonic hydrogenlike atom in the kth energy level.

IV. ELECTRON-FIELD PARTICLE INTERACTION POTENTIALS

The interaction potential of the incident electron and the field particle is the repulsive Coulomb potential,

$$U(r) = \alpha r^{-1} , \tag{52}$$

where r is the distance between the two charges, and $\alpha=e^2$.

Introducing a new variable,

$$g = 1/\cos^2\Psi_g , \tag{53}$$

the scattering angle Ψ_g for collision of two particles interacting through the potential (52) can be written as [19]

$$g = \cos^{-2}\left\{\int_{r_{\min}}^{\infty} Dr^{-2}\left[1-\left[\frac{D}{r}\right]^2\frac{U(r)}{E^*}\right]^{-1/2}dr\right\}, \tag{54}$$

where $E^*=\mu V^2/2$, and $r_{\min}$ is the distance of the closest approach. Introducing

$$y = D/r \text{ and } y_{\min} = D/r_{\min} = 2DE^*/\alpha \tag{55}$$

and using

$$\frac{\partial F(g)}{\partial g} = 2D\frac{\partial D}{\partial g} , \tag{56}$$

Eq. (54) can be rewritten as

$$g = \cos^{-2}\int_0^Y [1-y^2-2y_{\min}^{-1}y]^{-1/2}dy , \tag{57}$$

where Y is the least positive root of the expression in square brackets. Since

$$2D\frac{\partial D}{\partial g} = \left[\frac{\alpha}{2E^*}\right]^2\frac{\partial[y_{\min}^2(g)]}{\partial g} , \tag{58}$$

the derivative $\partial F/\partial g$ can be given as

$$\frac{\partial F(g)}{\partial g} = \left[\frac{\alpha}{2E^*}\right]^2 . \tag{59}$$

This derivative is used below to evaluate the integral (34).

V. ATOMIC LEVELS

A kth energy level $\varepsilon_k^{(a)}$ in hydrogenlike atom with principal quantum number n can be given as

$$\varepsilon_k^{(a)} = -\lambda\frac{Z^2\mathcal{R}}{n^2} , \tag{60}$$

where $\lambda=\mu_a/m_e \simeq m_1/m_e$ (μ_a is the reduced mass of the field particle and the atomic nucleus of charge Ze), and $\mathcal{R}$ is the Rydberg energy (13.6 eV).

The fine structure of the levels of a hydrogenlike atom can be obtained from the solution of the Dirac equation for a fermion μ^- in Coulomb field of the nucleus of charge Ze. The resulting levels are

$$\varepsilon_k^{(a)} = -\lambda\frac{Z^2\mathcal{R}}{n^2}\left[1+\left[\frac{n}{j+1/2}-\frac{3}{4}\right]\left[\frac{Z\alpha}{n}\right]^2\right] , \tag{61}$$

where $\alpha=1/137.036$ is the fine-structure constant,

$j=l\pm 1/2$, and l is the orbital angular momentum quantum number of the field particle in the (n,l)th shell. (More accurate values of the energy levels of muonic atoms can be found in Ref. [20].) In our model of target atom we assume that $E_1=U_k=|\varepsilon_k^{(a)}|$, and that the energy of the kth atomic level is specified by the principal quantum number n $(k\equiv n)$.

In heavy (large value of Z), ordinary hydrogenlike atoms, the sizes of the electron orbits are much larger than the sizes of the nuclei of the atoms; as a result, the nuclei can be treated as point charges. It can be seen from Eq. (61) that the energy levels in the hydrogenlike muonic atom are proportional to a large factor $\lambda\simeq 207$. Thus, the corresponding sizes of the orbits of the field muon are greatly reduced (by the same factor), compared to the ordinary hydrogenlike atom where $\lambda=1$. Therefore, the sizes of the lowest orbits of heavy muonic hydrogenlike atoms can be compared with the sizes of the nuclei of the atoms. Since the nuclear radius is of the order of $1.2\times 10^{-13}\ A^{1/3}$ cm (A is the total number of nucleons in the atomic nucleus), the size of the orbit of the muon is smaller than the size of the nucleus when $Z\gtrsim 30$. Then, the departure of the nuclear electrostatic field from the field of a point charge is quite significant and must be taken into account.

As can be seen from Eq. (61), the ratio Z/n for the atom should be small enough to justify use of nonrelativistic mechanics. The relativistic effects associated with the field particle can be neglected if the particle orbital speed $v_k\equiv v_k^{(a)}$, is more than about five times smaller than the speed of light c because then the relativistic momentum of the field particle $\mathbf{p}_k=m_\mu \mathbf{v}_k/\sqrt{1-(v_k/c)^2}$ differs by less than few percent from the particle classical momentum $m_\mu \mathbf{v}_k$. If $v_k<c/5$ then

$$\frac{Z}{n}<\frac{1}{5\alpha}\simeq 27\ . \tag{62}$$

As discussed previously, the size of the lowest muonic orbit in a hydrogenlike muonic atom is greater than the size of the atomic nucleus when the nucleus charge number $Z<30$. Taking this and the relationship (62) into account, one can say that the approach of the present work is valid when the hydrogenlike muonic atom participating in the collision with the incident electron has $Z<30$ (however, see discussion in the last section).

VI. IONIZATION CROSS SECTIONS

The energy-dependent cross section Q_{kc}^{ion} for electron-impact ionization of a hydrogenlike atom (ordinary or muonic) excited to the kth energy level are calculated as the averaged (over the energy distribution of the field particle) cross section (35) for transfer of energy ΔE $(\varepsilon_l\le\Delta E\le\varepsilon_u)$ from the incident electron to the atomic field particle. Using the cross sections of Ref. [12], assuming $p(\theta)=(\sin\theta)/2$, and integrating over θ (from $\theta_{\min}$ to π) and over ε (from $\varepsilon_l=U_k$ to ε_u) one obtains

$$Q_{kc}^{\text{ion}}(E)=\frac{2\pi e^4}{U_k^2}\frac{\lambda^{1/2}(1+\lambda)^2}{(1+\lambda x)^{3/2}}\frac{(x-1)}{x^{1/2}}\left[1-\frac{U_k}{\varepsilon_u}\right]\ , \tag{63}$$

where $\pi e^4=6.56\times 10^{-14}\ \text{cm}^2\,\text{eV}^2$,

$$x=E/U_k\ , \tag{64}$$

and, as before, $E\approx E_2$ is the impact energy of the electron-atom collision. We used, when evaluating Eq. (63), the fact that $\theta_{\min}=0$ for the electron trajectory in the field of Coulomb force (see Ref. [12]). Also, we assumed that when integrating over θ the mean relative speed of the incident electron and the field particle is equal to

$$V=(v_2^2+\langle v_1\rangle^2)^{1/2}\ . \tag{65}$$

In the case of electron-impact ionization of an ordinary hydrogenlike atom ($\lambda=1$ and $U_k/\varepsilon_u=x^{-1}$), relationship (63) leads to the following cross section for ionization of the atom from the kth energy level:

$$Q_{kc}^{\text{ion}}(x)=\frac{2\pi e^4}{U_k^2}\frac{(x-1)^2}{x^{3/2}(1+x)^{3/2}}\ . \tag{66}$$

In the case of electron-impact ionization of a muonic hydrogenlike atom excited to the kth energy level [$\lambda\gg 1$ and ε_u is given by Eq. (43)] the ionization cross section obtained from relationship (63) is

$$Q_{kc}^{\text{ion}}(x)=\frac{2\pi e^4}{U_k^2}\frac{\lambda^{1/2}(1+\lambda)^2}{(1+\lambda x)^{3/2}}\frac{(x-1)}{x^{1/2}}\times\left\{1-\frac{\lambda}{2[x-1+(x^2+(\lambda-2)x)^{1/2}]}\right\}\ , \tag{67}$$

with the threshold value of x

$$x_{\text{th}}\simeq\frac{\lambda}{8}\ . \tag{68}$$

Since $\lambda\gg 1$ and $x>1$, one has

$$\lambda^{1/2}(1+\lambda)^2\simeq\lambda^{5/2} \tag{69}$$

and

$$(1+\lambda x)^{3/2}\simeq(\lambda x)^{3/2}\ . \tag{70}$$

Using relationships (69) and (70), the cross section (67) can be written in a somewhat simpler form as

$$Q_{kc}^{\text{ion}}(x)=\frac{2\pi e^4}{U_k^2}\frac{\lambda(x-1)}{x^2}\left\{1-\frac{\lambda}{2[x-1(x^2+\lambda x)^{1/2}]}\right\}\ . \tag{71}$$

At the threshold (where $x=x_{\text{th}}=\lambda/8$), the cross section (71) is

$$Q_{kc}^{\text{ion}}(x=\lambda/8)=\frac{16\pi e^4}{U_k^2}\frac{(\lambda-8)}{\lambda}\left[1-\frac{4\lambda}{4\lambda-8}\right]\ . \tag{72}$$

The deviation of the value of the second term in the square brackets in Eq. (72) from unity is a measure of inaccuracy (resulting from the simplifications made in this section) of the cross section (71). For $\lambda=206.76$ one has

$$\left|1-\frac{4\lambda}{4\lambda-8}\right|=0.009 \ . \tag{73}$$

Thus, the simplifications made in this section are well justified.

At very large impact energies (that is, very large value of x), the ionization cross section (63) for the electron-impact ionization of the ordinary hydrogenlike atom excited to the kth energy level becomes

$$Q_{kc}^{\text{ion}}(x\to\infty)=\frac{8\pi e^4}{U_k^2}x^{-1} \ , \tag{74}$$

and in the case of the electron-impact ionization of the muonic hydrogenlike atom,

$$Q_{kc}^{\text{ion}}(x\to\infty)=\frac{8\pi e^4}{U_k^2}\frac{\lambda}{4x+\lambda} \ . \tag{75}$$

The relationships (74) and (75) show the traditional weakness of the classical impulse approximation in description of collisions with large impact energies when the target atom is weakly excited. According to Eqs. (74) and (75), $Q_{kc}^{\text{ion}}(E\to\infty)\sim E^{-1}$, while the Bethe-Borne approximation give $Q_{kc}^{\text{ion}}(E\to\infty)\sim E^{-1}\ln E$. However, it was shown by Garcia [21] and by Omidvar [22] that at large impact energies E the cross section for electron-impact ionization of ordinary hydrogen atom should go smoothly from the Born approximation $\ln E/E$ behavior to the classical $1/E$ as the atomic principal quantum number n increases.

VII. RESULTS AND DISCUSSION

The validity of the classical impulse approximation is discussed in detail in literature (see, for example, Refs. [23,24,15,18,8,7]. Therefore, we discuss below only a few aspects of it which are directly related to the fact that the field particle in the case of ionization of muonic atoms is not an electron but a muon μ^-.

It was shown by Vriens [23] that the main weakness of the classical impulse approximation results from the fact that the requirement of conservation of linear momentum gives different values of the minimum momentum transferred in the binary collision (of the incident electron with the field particle) as compared with the minimum momentum transferred in a many-body collision (of the incident electron with several field particles). (The classical impulse approximation requires full conservation of momentum and energy between the incident electron and the field particle, while the quantum-mechanical approach requires only conservation of energy because the nucleus can take up momentum.) Thus, the ionizing collision of an electron with a hydrogenlike atom (ordinary or muonic) is the best candidate to study the electron-atom collisions using the formalism of the classical impulse approximation.

Another weakness of the classical impulse approximation is the assumption that the incident electron and the field electron are distinguishable during the electron collision with an ordinary atom. However, this weakness does not occur in the description of the electron-muonic atom collisions.

One of the goals of this section is to clarify the expectations of the classical impulse approximation when applying it to ionization of the muonic hydrogenlike atoms. A direct comparison of the corresponding cross sections with their experimental or quantum-mechanical values cannot be made because such data are not available. Therefore, we test the present approach indirectly by comparing our cross sections for the electron-impact ionization of some ordinary hydrogenlike atoms with the corresponding experimental cross sections which are available in the literature of the subject. We show the cross sections for the electron-impact ionization of ordinary H(1s) and H(2s) atoms in Figs. 1 and 2, respectively. In both cases the present cross sections [Eq. (66)] are in good agreement with the measured cross sections [25–29], and they are closer to the measured values than the other theoretical cross sections [30, 31, 32, 25]. We also compared the present cross section for the electron-impact ionization of singly ionized ordinary helium in the ground state (see Fig. 3) with the available measurements [33–38]. Our cross section for the ionization of the helium ion is also in good agreement with the experimental values.

Summarizing the above, one can say that the present approach yields analytical cross sections for electron-impact ionization of ordinary hydrogenlike atoms (with small values of Z) which are in good agreement with experimental data. Therefore, it seems that the approach should give reasonable values of the cross sections for the electron-impact ionization of muonic hydrogenlike atoms with small values of Z. At large values of Z (but not greater than 30—see Sec. V), an important effect not accounted for in the present approach must be taken into consideration. Due to the field of the target ion with a large value of Z, the trajectory and speed of the incident electron change significantly before they interact with the

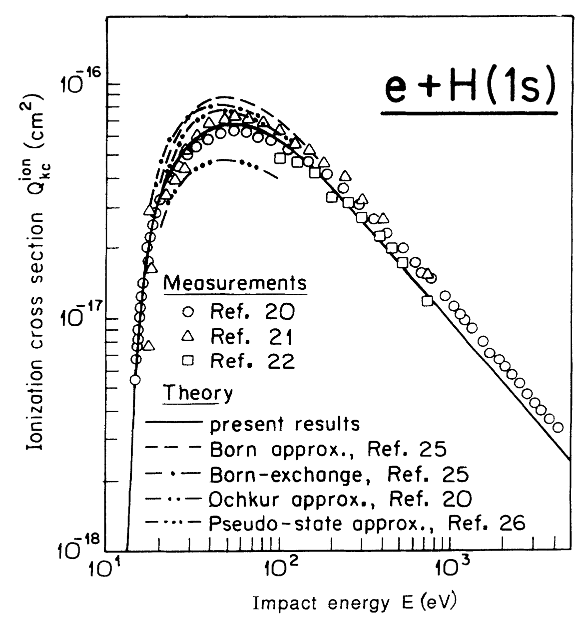

FIG. 1. The cross section for the electron-impact ionization of the H(1s) atom.

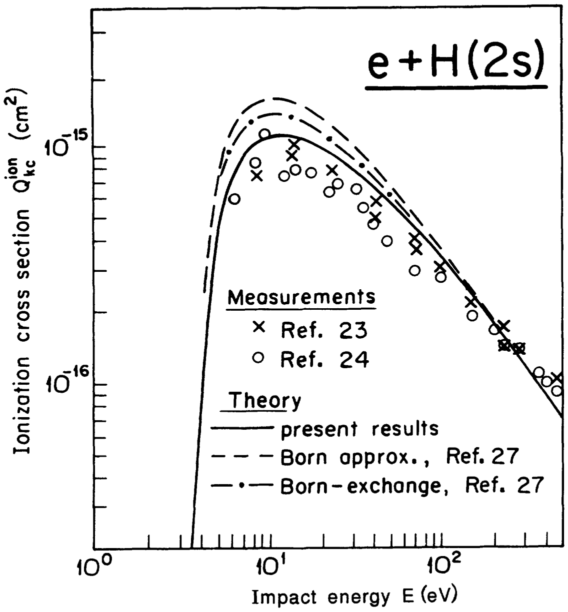

FIG. 2. The cross section for the electron-impact ionization of the H(2s) atom.

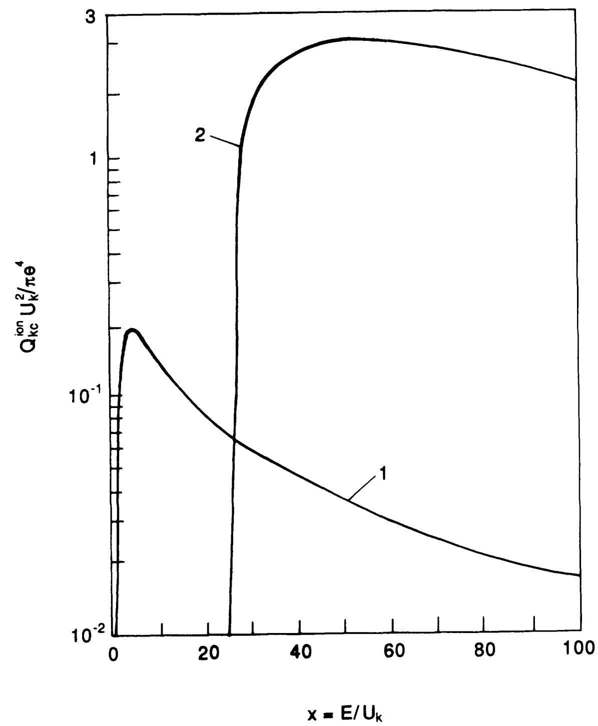

FIG. 4. The "scaled" cross sections, $Q_{kc}^{\rm ion}U_k^2/\pi e^4$, of the present work for the electron-impact ionization of ordinary [curve 1, Eq. (66)] and muonic [curve 2, Eq. (67)] hydrogenlike particles excited to the kth level; U_k is the ionization potential of the kth level and E is the impact energy of the electron-atom collision.

field particle. This effect can be taken into account, using the classical impulse approximation, in the way proposed by Thomas and Garcia in Ref. [39] or by Gryzinski and Kunc in Ref. [9].

The "scaled" cross section, $Q_{kc}^{\rm ion}U_k^2/\pi e^4$, for the electron-impact ionization of muonic hyrogenlike atoms [Eq. (67)] is shown in Fig. 4 together with the scaled cross section for the electron-impact ionization of ordinary hydrogenlike atoms [Eq. (66)]. The cross sections differ significantly both qualitatively and quantitatively.

The ratio of the maximum value of the cross section for the electron-impacted ionization of a muonic hydrogenlike atom excited to the level with the principal quantum number n to the maximum value of the corresponding cross section for ionization of an ordinary hydrogenlike atom excited to the level with the same value of n is

$$\frac{(Q_{nc}^{\rm ion})_{\max}^{\rm muonic}}{(Q_{nc}^{\rm ion})_{\max}^{\rm ordinary}} \approx 10\left[\frac{U_n}{U_n'}\right]^2 , \qquad (76)$$

where U_n and U_n' are ionization potentials for the ordinary and muonic atoms, respectively, excited to energy levels with the principal quantum number n. Thus, the maximum value of the cross section for the electron-impact ionization of the muonic atom is much smaller than the corresponding cross section for the ionization of the ordinary atom; if the atoms are in the ground states then the ratio (76) is $10/\lambda^2 \approx 2\times 10^{-4}$. However, as can be seen in Fig. 4, at impact energies E greater than the energy of the cross section maximum, the cross section for the ionization of the muonic hydrogenlike atoms decreases with energy much slower than the cross section for the ionization of the ordinary hydrogenlike atoms.

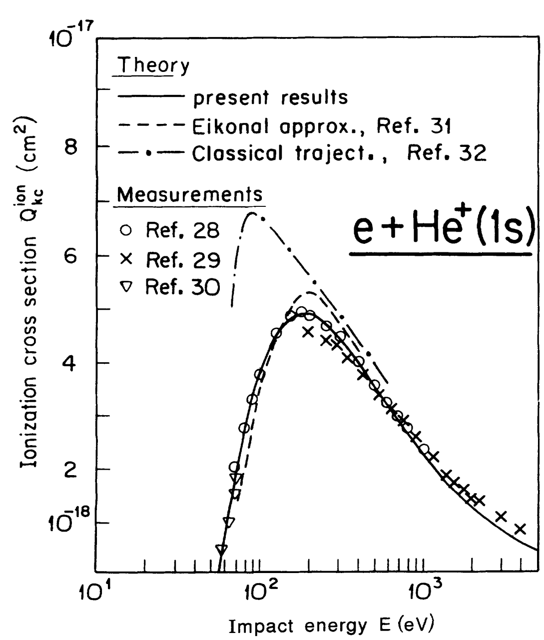

FIG. 3. The cross section for the electron-impact ionization of the He$^+$(1s) ion.

One should add that the ratio of the maximum value $|\Delta E|_{\max}$ of the energy which can be transferred in a binary Coulomb collision from the incident electron of energy E_2 to the field particle (an electron or a muon μ^-) of energy $E_1 = U_k$ is [see Eqs. (42) and (43)]

$$\frac{|\Delta E|_{\max}}{E_1}=\frac{E_2}{U_k}(\text{when } \lambda=1) \quad (77)$$

and

$$\frac{|\Delta E|_{\max}}{E_1}=\frac{2E_2}{\lambda U_k}\left\{1-\frac{U_k}{E_2}+\left[1+(\lambda-2)\frac{U_k}{E_2}\right]^{1/2}\right\}, \quad \text{when } \lambda \gg 1 . \quad (78)$$

As can be seen in Fig. 4, at energies E_2 corresponding to the maximum values ("peaks") of the ionization cross sections Q_{kc}^{ion} one has ($E_2/E_1 \approx E/U_k$ because $E_2 \approx E$ and $E_1=U_k$)

$$\left[\frac{E}{U_k}\right]_{\text{peak}} \approx 5 , \quad \text{when } \lambda=1$$

$$\left[\frac{E}{U_k}\right]_{\text{peak}} \approx 50 , \quad \text{when } \lambda=206.76 . \quad (79)$$

Using relationships (79) in Eqs. (77) and (78) one obtains, respectively,

$$\left[\frac{|\Delta E|_{\max}}{E_1}\right]_{\text{peak}} \approx 5, \quad \text{when } \lambda=1$$

$$\left[\frac{|\Delta E|_{\max}}{E_1}\right]_{\text{peak}} \approx 2, \quad \text{when } \lambda=206.76 . \quad (80)$$

Thus, the value of $(|\Delta E|_{\max}/E_1)_{\text{peak}}$ for electron-muon μ^- binary collision is smaller than the corresponding value for the electron-electron collision. However, in the case of the ionizing collisions, the collisions with $|\Delta E|/E_1<2$ are the most important ones [23,7]. Therefore, the accuracy (associated with the magnitude of the energy transferred during the binary collision) of our treatment of the electron-muon μ^- interaction should not differ from the accuracy of our treatment of the electron-electron interaction.

In some cases the cross sections of the present work can be used for approximate calculations of the cross sections for electron-impact ionization of muonic atoms other than hydrogenlike atoms. The cross section $\bar{Q}_{kc}^{\text{ion}}$ for the electron-impact ionization of such atoms can be given as

$$\bar{Q}_{kc}^{\text{ion}}=\sum_k \eta_k [Q_{kc}^{\text{ion}}]_k , \quad (81)$$

where $[Q_{kc}^{\text{ion}}]_k$ is the electron-impact ionization cross section [given in Eq. (67)] for removing one field particle (the muon μ^-) belonging to the atomic kth shell characterized by two (n and l) quantum numbers, and η_k is the number of the field particles in the shell. In general, accuracy of the approximation (81) depends on the collision system under consideration [4,8,9].

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[1] M. Yu. Amusia and V. A. Kharchenko, J. Phys. B **14**, L219 (1981).
[2] M. Yu. Amusia and A. V. Solovyov, J. Phys. B **18**, 3663 (1985).
[3] S. E. Jones, J. Rafelski, and H. J. Monkhorst, *Muon-Catalyzed Fusion* (AIP, New York, 1989).
[4] M. Gryzinski, Phys. Rev. A **138**, 336 (1965).
[5] V. W. Hughes and C. S. Wu, *Muon Physics* (Academic, New York, 1977).
[6] A. Burgess and I. C. Percival, Adv. At. Mol. Phys. **4**, 145 (1968).
[7] L. Vriens, in *Case Studies in Atomic Collision Physics,* edited by E. W. McDaniel and M. R. McDowell (North-Holland, Amsterdam, 1970), p. 335.
[8] D. R. Bates and A. E. Kingston, Adv. At. Mol. Phys. **6**, 269 (1970).
[9] M. Gryzinski and J. A. Kunc, J. Phys. B **19**, 2479 (1986).
[10] D. A. Erwin and J. A. Kunc, Phys. Rev. A **38**, 4135 (1988).
[11] J. A. Kunc, in *Progress in Astronautics and Aeronautics,* edited by E. P. Muntz, D. P. Weaver, and D. H. Campbell (AIAA, Washington, DC, 1989), Vol. 116.
[12] J. A. Kunc and W. H. Soon, J. Chem. Phys. **95**, 5738 (1991).
[13] M. Gryzinski, Phys. Rev. A **138**, 305 (1965).
[14] J. A. Kunc, J. Phys. B **21**, 3619 (1988).
[15] R. Abrines, I. C. Percival, and N. A. Valentine, Proc. Phys. Soc. **89**, 515 (1966).
[16] M. Gryzinski, Phys. Lett. **44A**, 131 (1973).
[17] J. A. Kunc, J. Phys. B **13**, 587 (1980).
[18] H. S. W. Massey, E. S. W. Burhop, and H. B. Gilbody, *Electronic and Ionic Impact Phenomena* (Oxford University Press, London, 1969).
[19] M. V. Berry and K. E. Mount, Rep. Prog. Phys. **35**, 315 (1972).
[20] E. Borie and G. A. Rinker, Rev. Mod. Phys. **54**, 67 (1982).
[21] J. D. Garcia, Phys. Rev. **159**, 39 (1967).
[22] K. Omidvar, Phys. Rev. A **26**, 140 (1965).
[23] L. Vriens, Phys. Lett. **9**, 295 (1964).
[24] R. Abrines and I. C. Percival, Proc. Phys. Soc. **88**, 861 (1966).
[25] M. B. Shah, D. S. Elliott, and H. G. Gilbody, J. Phys. B **20**, 3501 (1987).
[26] W. L. Fite and R. T. Brackmann, Phys. Rev. **112**, 1141 (1958).
[27] E. W. Rothe, L. L. Marino, R. H. Neynaber, and S. M. Trujillo, Phys. Rev. **125**, 582 (1962).
[28] A. J. Dixon, A. von Engel, and M. F. Harrison, Proc. R. Soc. London, Ser. A **343**, (1975).
[29] P. Defrance, W. Claeys, A. Cornet, and G. Poulaert, J. Phys. B **14**, 111 (1981).
[30] G. Peach, Proc. Phys. Soc. **87**, 381 (1966).
[31] D. F. Gallaher, J. Phys. B **7**, 362 (1974).
[32] S. S. Prasad, Proc. Phys. Soc. **87**, 393 (1966).
[33] K. T. Dolder, M. F. Harrison, and P. C. Thonemann, Proc. R. Soc. London Ser. A **264**, 367 (1961).

[34] B. Peart, D. S. Walton, and K. T. Dolder, J. Phys. B **2**, 1347 (1969).
[35] P. Defrance, F. Brouillard, W. Claeys, and G. van Wassenhove, J. Phys. B **14**, 103 (1981).
[36] A. Tsuji, H. Kotegawa, and H. Narumi, J. Phys. Soc. Jpn. **48**, 2062 (1980).
[37] D. Banks and L. G. Boesten, J. Phys. B **11**, 2209 (1978).
[38] E. D. Donets and V. P. Ovsyannikov, Zh. Eksp. Teor. Fiz. **80**, 916 (1981) [Sov. Phys. JETP **53**, 466 (1981)].
[39] B. K. Thomas and J. D. Garcia, Phys. Rev. **179**, 94 (1969).