Dissipative optical solitons

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It is found that dissipative types of stable soliton structures can exist in nonlinear optical media with broadband gain and group-velocity dispersion (GVD). These structures resemble ionization or combustion waves and are essentially self-accelerating pulses with a stationary-envelope form and a permanently shifting wave spectrum. Contrary to the conservative solitons, the dissipative ones exist for any sign of GVD. Being an attractor in the development of arbitrary initial distributions, the dissipative structures cause the fundamental Schrödinger solitons to disappear in the course of evolution in weakly nonconservative systems.

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Recent progress in developing ultrashort-laser techniques attracts the interest of studying interaction between powerful optical pulses and broadband active media. This problem is of particular importance for the dynamics of femtosecond laser generators [1], amplifiers of supershort pulses [2], and nonlinear active optical fibers [3]. A common theoretical aspect in these applications is the long-term evolution of wave packets in dispersive active media that exhibits simultaneously both conservative and dissipative nonlinearities. It is well known that the propagation of wave pulses in a lossless and dispersive dielectric medium with its self-action incorporated is described by the nonlinear Schrödinger (NLS) equation, which normally includes the conservative cubic nonlinearity of the refractive index. The interplay between nonlinearity and group-velocity dispersion (GVD) then may lead to soliton generation [4] or quasishock-wave formation [5]. On the other hand, in two-level optical media solitons of induced transparency can be formed due to the nonconservative character of the resonant light-matter interaction [6]. Specific nonconservative mechanisms of soliton generation, caused by a delicate balance between saturable absorption and amplification and/or transverse phase modulation and gain inhomogeneity, play an important role in the passively mode-locked short-pulse laser operation [1,7,8].

The purpose of the present paper is to show that the long-term evolution of a wave pulse propagating in a nonlinear two-level optical medium containing broadband gain elements results in the formation of new types of single soliton structures (named here dissipative optical soliton). These solitons are generated due to a balance between the GVD and the gain nonlinearity (dissipative nonlinearity) and are of particular interest of optical systems where the possibility exists to amplify and generate ultrashort-laser pulses so that the wave dispersion becomes the main limiting factor.

Our considerations are based on the Maxwell-Bloch equations describing the interaction of an intense electromagnetic radiation with a nonlinear dispersive medium containing two-level atoms [9,10],

$$-i\frac{\partial E}{\partial z}-\tfrac{1}{2}k_0''\frac{\partial^2 E}{\partial t^2}+\frac{\omega_0 n_2}{c}|E|^2E-i\frac{\alpha}{2}E+\frac{\omega_0^2}{2c^2\varepsilon_0 k_0}P_i=0\ , \quad (1)$$

$$\frac{\partial P_i}{\partial t}+\left[\frac{1}{\tau_2}+i(\omega_0-\omega_{12})\right]P_i=i\frac{d^2}{\hbar}NE\ , \quad (2)$$

$$\frac{\partial N}{\partial t}+\frac{1}{\tau_1}(N-N_0)=\frac{i}{2\hbar}(P_iE^*-P_i^*E)\ . \quad (3)$$

Here, Eq. (1) describes propagation of a wave pulse with the slowly varying complex amplitude E and the carrier frequency ω_0 in a nonlinear dispersive medium. The derivation of the Schrödinger-type equation, Eq. (1), from the full-wave equation is based on the standard procedure of slowly varying amplitudes and considering only dispersion effects to the second order. The notations are as follows: z is the distance of propagation in the medium (or the wave-pulse round trip in a laser cavity), $t=t'-z/v_{gr}$ is the time in a frame of reference moving at the group velocity $v_{gr}(\omega_0)$, $k_0=k(\omega_0)$ is the wave number at the carrier frequency, $k_0''\equiv(\partial^2k/\partial\omega^2)_{\omega_0}$ accounts for the group velocity dispersion, n_2 is the nonlinear Kerr coefficient, and α is the linear damping rate of the medium. The dynamics of the complex nonlinear polarization amplitude P_i associated with active ions is described by the two-level approximation with the help of Eqs. (2) and (3). Here, N_0 is the density of the inverse population created by pumping in the absence of the amplified wave pulse, N is the inverse population density in the presence of the wave pulse, d is the dipole moment of the resonant transition, and ω_{12} is the resonant frequency. The time τ_2 is the transverse relaxation time associated with the amplification bandwidth ($\tau_2=1/\pi c\Delta v_a$), and the longitudinal relaxation time τ_1 determines the relaxation of the excited level.

Here, we consider the propagation of wave pulses with a duration τ_p satisfying $\tau_2 \ll \tau_p \ll \tau_1$. Assuming also that $\Delta\omega=0$, where $\Delta\omega=\omega_0-\omega_{12}$ is the detuning of the carrier frequency from the resonance one, as well as that $(d^2\tau_2/h)\int_{-\infty}^{t}|E|^2dt' \ll 1$, the equation system (1)–(3) can be reduced to the following evolution equation for the pulse-field amplitude, cf. [11]:

$$-i\frac{\partial E}{\partial z}-\tfrac{1}{2}k_0''\frac{\partial^2 E}{\partial t^2}+\frac{\omega n_2}{c}|E|^2E+i\tfrac{1}{2}(g_0-\alpha)E -i\frac{g_0c^2k_0}{16\pi W_s\omega_0}E\int_{-\infty}^{t}|E|^2dt'=0 , \quad (4)$$

where $g_0=(d^2\tau_2N_0\omega_0^2)/(\hbar c^2\varepsilon_0k_0)$ is the coefficient of linear amplification and $W_s=(\hbar^2c^2k_0)/(8\pi d^2\omega_0\tau_2)$ is the saturation energy. Note that Eq. (4) is valid for $W \ll W_s$, where $W=(k_0c^2/8\pi\omega_0)\int_{-\infty}^{+\infty}|E|^2dt$ is the energy of the wave pulse.

Introducing now normalized variables according to

$$z'=\tfrac{1}{2}(g_0-\alpha)z ,$$
$$\tau=\frac{g_0c^3k_0}{16\pi n_2\omega_0^2W_s}t , \quad (5)$$
$$A=\left[\frac{2\omega_0n_2}{c(g_0-\alpha)}\right]^{1/2}E ,$$

where $g_0>\alpha$, we obtain Eq. (4) in the dimensionless form as

$$-i\frac{\partial A}{\partial z'}+\delta\frac{\partial^2 A}{\partial \tau^2}+|A|^2A+iA\left[1-\int_{-\infty}^{\tau}|A|^2d\tau'\right]=0 . \quad (6)$$

Here

$$\delta=-\frac{k_0''}{(g_0-\alpha)}\left[\frac{g_0c^3k_0}{16\pi n_2\omega_0^2W_s}\right]^2 \quad (7)$$

is the dimensionless parameter that characterizes dispersive properties of the medium; $\delta>0$ ($\delta<0$) corresponds to the case of negative (positive) GVD. The normalized pulse energy is given by $\overline{W}=\int_{-\infty}^{+\infty}|A|^2d\tau=[g_0/(g_0-\alpha)](W/W_s)$. Note that in the following we will write z instead of z' for simplicity.

For wave pulses with $A(\tau=\pm\infty)=0$, it follows from Eq. (6) that the pulse energy, $\overline{W}$, satisfies the Riccati equation

$$\frac{d\overline{W}}{dz}=2\overline{W}-\overline{W}^2 , \quad (8)$$

having the solution

$$\overline{W}(z)=\overline{W}(0)\frac{\exp(2z)}{1+\frac{1}{2}\overline{W}(0)[\exp(2z)-1]} . \quad (9)$$

This shows that the pulse energy reaches the asymptotic value $\overline{W}_\infty=2$ as $z\to\infty$ independently of the initial value $\overline{W}(0)$. However, the structural evolution of the wave pulse will still continue at $\overline{W}_\infty=2$. It will be demonstrated below that the pulse field, although having a constant value of the total energy, may undergo a drastic redistribution. The final result of the long-term wave dynamics will be the formation of a pulse with a stationary amplitude form. Thus, one is motivated to analyze Eq. (6) from the point of view of stationary-wave solutions.

Let us first consider a stationary-wave pulse in the inertial reference frame, i.e., $A(z,\tau)=|A(\tau)|\exp[i\kappa_0z+i\int_{-\infty}^{\tau}\Omega(\tau')d\tau']$. The energy flux equation following from Eq. (6) is then

$$\frac{\partial|A|^2}{\partial z}-2\delta\frac{\partial}{\partial\tau}(\Omega|A|^2)=2|A|^2\left[1-\int_{-\infty}^{\tau}|A|^2d\tau'\right] . \quad (10)$$

The right-hand side of Eq. (10) shows that at the leading front of the pulse, where $U(z,\tau)=\int_{-\infty}^{\tau}|A(z,\tau')|^2d\tau'<1$, the energy flows from the medium to the pulse. On the other hand, at the trailing front of the pulse, where $U>1$, dissipation of pulse energy takes place. Consequently, in order to maintain a fixed pulse structure $|A(\tau)|^2$, an energy flux through the pulse is required. This can be achieved by means of the frequency modulation, $\Omega(\tau)$, since according to Eq. (10) the rate at which a certain part of the pulse is shifting along the τ axis is $V_\tau=-2\delta\Omega$. Similar dissipative stationary-wave pulses with a symmetrical distribution of $|A(\tau)|$ and $\Omega(\tau)=\Omega_0=\text{const}$ have been first considered in Ref. [11], where the exact solution of Eq. (6) has been obtained in the form ($\delta>0$),

$$A(z,\tau)=\frac{1}{\sqrt{2\delta}}\,\text{sech}\left[\frac{\tau}{2\delta}\right]\exp\left[i\left[\delta-\frac{1}{4\delta}\right]z-i\tau\right] . \quad (11)$$

However, a simple qualitative consideration will demonstrate that the soliton structures discussed above and in Ref. [11] are unstable. Let us consider the case $\delta>0$ and assume a perturbation with $\Omega>0$, which arises at the leading front of the pulse. This perturbation having a group velocity greater than that of the pulse center will outrun the soliton structure in a dispersive medium with gain and will shift to the region with $U\to 0$ where conditions preferable for amplification exist. Consequently, the fast perturbation will grow in the course of propagation whereas the bulk distribution will experience a pump-depletion effect. Clearly, a wave pulse that is formed in a gain medium has a frequency spectrum being permanently shifted towards the range of greater group velocities. In real space, this corresponds to the formation of a self-accelerating dissipative soliton structure. Therefore, such a soliton structure has to be investigated in the noninertial reference frame moving with the acceleration $a(z)=-2\delta\, d\overline{\Omega}(z)/dz$, where $\overline{\Omega}(z)$ is the central frequency of the pulse. In terms of Eq. (6), $a(z)$ corresponds to an acceleration in real space as well. The transformation to the accelerating reference frame introduces an artificial inertial force corresponding to an effective potential $\sim a(z)\tau$, so that the wave pulse seems to be moving in a linearly inhomogeneous time profile. In order to transform properly Eq. (6) to the accelerating reference frame, we use the following substitution:

$$A(z,\tau)=B\left[z,\tau+2\delta\int_0^z\overline{\Omega}(z')dz'\right]\times\exp\left[i\overline{\Omega}(z)\tau+i\frac{\delta}{3}\int_0^z\overline{\Omega}^2(z')dz'\right], \quad (12)$$

which yields

$$-i\frac{\partial B}{\partial z}+\delta\frac{\partial^2 B}{\partial\overline{\tau}^2}+B|B|^2+\frac{d\overline{\Omega}(z)}{dz}\overline{\tau}'B+iB\left[1-\int_{-\infty}^{\overline{\tau}}|B|^2d\overline{\tau}'\right]=0 , \quad (13)$$

where $\overline{\tau}=\tau+2\delta\int_0^z\overline{\Omega}(z')dz$. The inhomogeneous term $(d\overline{\Omega}/dz)\tau'B$ appearing on the left-hand side of Eq. (13) changes essentially the character of the above stability discussion. In this case, a perturbation tending to overtake the field distribution will be reflected from the wave evanescence zone at $\tau'<0$ (i.e., in the maximum gain region). Consequently, the effective inhomogeneity of the medium will stabilize the dissipative soliton structure.

Looking for the solution of Eq. (13) in the form of a self-frequency shifting stationary-wave pulse, we introduce

$$B(z,\tau')=\frac{D(\xi)}{|\delta|^{1/4}}\exp\left[i\,\mathrm{sgn}(\delta)\int_{-\infty}^{\xi}\Omega(\xi')d\xi'\right], \quad (14)$$

where $\xi=\overline{\tau}/\sqrt{|\delta|}$, and assume a linear dependence of the central frequency $\overline{\Omega}$ on z, i.e.,

$$\overline{\Omega}(z)=\frac{a}{2\delta}z . \quad (15)$$

[Note that there exist only two types of localized stationary-wave solutions of Eq. (13): with $d\overline{\Omega}/dz=0$ or with $d\overline{\Omega}/dz=\text{const.}$] Substituting (14) and (15) into (13) and separating the real and imaginary parts, we obtain

$$\frac{d^2D}{d\xi^2}-\Omega^2D+\mu D^3+\xi\tilde{a}D=0 , \quad (16)$$

$$\frac{dU}{d\xi}=D^2 , \quad (17)$$

$$U-\frac{U^2}{2}=-\Omega D^2 , \quad (18)$$

where $\mu=\sqrt{|\delta|}/\delta$ is the parameter characterizing the nonlinear and dispersive properties of the medium, and $\tilde{a}=a/(2\sqrt{|\delta|})$ is the normalized acceleration of the soliton. Note that $\tilde{a}=\tilde{a}(\mu)$ is the eigenvalue for the problem of finding the localized field distribution $D(\xi)$ having the total energy $\int_{-\infty}^{+\infty}D^2d\xi=U(+\infty)=2$. Equation (18) describes the above discussed fact of energy equilibrium that is established over the soliton structure: a balance between pumping and dissipation due to energy flux (frequency modulation) through the soliton. The asymptotic behavior of the solution at the leading front ($\xi\to-\infty$) is determined by the Airy function for the field amplitude $D\sim\exp(-\frac{4}{3}\tilde{a}^{1/2}|\xi|^{3/2})/|\xi|^{1/4}$ and the power law for the frequency tending to its maximum value ($\Omega=0$) is $\Omega=-1/(2\tilde{a}^{1/2}|\xi|^{1/2})$. At the trailing edge ($\xi\to+\infty$) the envelope is localized in accordance with $D\approx(\xi\tilde{a})^{-1/4}\exp(-\sqrt{\xi/\tilde{a}})$ and the frequency is varying as $\Omega=-\sqrt{\tilde{a}\xi}$.

It is convenient to start the analysis of Eqs. (16)–(18) by considering the case $\mu=0$ ($\delta\to\infty$) where the soliton properties are defined by the dissipative nonlinearity of the medium. Let us assume that due to a strong frequency modulation providing energy balance over the structure, the first term in Eq. (16) can be omitted everywhere except at the leading front where $\Omega\to0$. Then Eqs. (16)–(18) can be integrated exactly to yield

$$U=1+\tanh(\sqrt{\xi/\tilde{a}}+\beta) , \quad (19a)$$

$$D=[\sqrt{2}(\tilde{a}\xi)^{1/4}\cosh(\sqrt{\xi/\tilde{a}}+\beta)]^{-1} , \quad (19b)$$

$$\Omega=-\sqrt{\tilde{a}}\,\xi . \quad (19c)$$

Taking into account the term $d^2D/d\xi^2$, the solution (19) breaks at $\xi^3\tilde{a}\le0.3$ and it has to be matched in the region $\xi<0$ to the Airy asymptotics of the linearized system. This matching will define the constants $\tilde{a}$ and β which are arbitrary in the expressions (19a)–(19c). The numerical solution of the eigenvalue yields the field distribution shown in Fig. 1 which coincides with Eq. (19) almost in the whole energy-containing region [compare $U(\xi)$ from analytical and numerical solutions in Fig. 1(c)]. The mode parameters are $\tilde{a}(0)=0.478$, $\beta(0)=2.96$. Hence, in broadband media with wave dispersion, self-accelerating localized structures (autosolitons) exist. These structures are characterized by a stationary profile and a spectrum that is continuously shifting towards the regions of greater group velocities. The existence and properties of the autosolitons do not depend on the sign of the GVD. It is seen from Eqs. (14) and (15) that the change $\delta\to-\delta$ will only result in a change of the frequency sign (perturbations with smaller Ω will be ahead).

The optical solitons found above do not have any analogy in the physics of conservative nonlinear waves. They resemble ionization or combustion autowaves [12] in which the dissipative properties determine both the wave structure and the only possible velocity of the wave propagation. For an ionization wave under the action of electromagnetic radiation with intensity I, the following simplest model can be used: $\partial N/\partial t=D_e\partial^2N/\partial x^2+(kI-\alpha)N$, where D_e and α are the coefficients of ambipolar diffusion and local particle losses, respectively. Taking into account electromagnetic dissipation in the plasma according to $\partial I/\partial x\sim NI_0$, one obtains a balance equation being analogous to Eq. (6) with a nonconservative nonlinearity. The difference will be in the nature of the diffusion coefficient which is imaginary for wave-packet evolution in dispersive media. As a result, instead of a single ionization-wave velocity $\approx\sqrt{D_e(kI_0-\alpha)}$ there is a single eigenvalue for acceleration of the optical soliton.

Let us now consider the case $\mu\neq0$ when the symmetry between the solitary solutions in media with positive and negative GVD is absent. Using the same idea to facilitate Eq. (16) for a slowly varying envelope, one obtains for the modulation frequency

$$\Omega=-\sqrt{\xi\tilde{a}+\mu D^2}\ . \tag{20}$$

If $\mu>0$, the term μD^2 in Eq. (20) increases the energy transport along the ξ coordinate in the region of large amplitudes. This results in a shift of the amplitude maximum towards larger ξ values. Consequently the eigenvalue of acceleration has to decrease, since taking only into account the shift of the maximum position ξ_m, the amplitude in the region $\xi<\xi_m$ decreases and therefore weakens the mechanism of the energy redistribution. Numerical analysis has shown that the contribution of the term μD^2 in Eq. (20) is small in comparison with the inhomogeneous term even for large values of μ. Therefore, the solution (19) appears to be valid for any $\mu>0$ provided one substitutes the corresponding values of $\tilde{a}(\mu)$ and $\beta(\mu)$ (see Fig. 2). The amplitude and frequency distributions of dissipative modes at $\mu=10$ are shown in Fig. 3. It can be seen that at $\mu\gg1$ the energy-containing

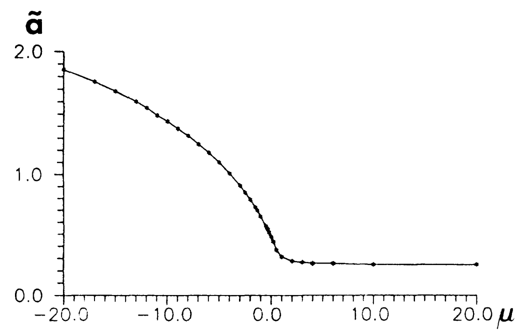

FIG. 2. Dependence of the eigenvalue of acceleration $\tilde{a}$ on the coefficient of conservative nonlinearity μ for the dissipative soliton.

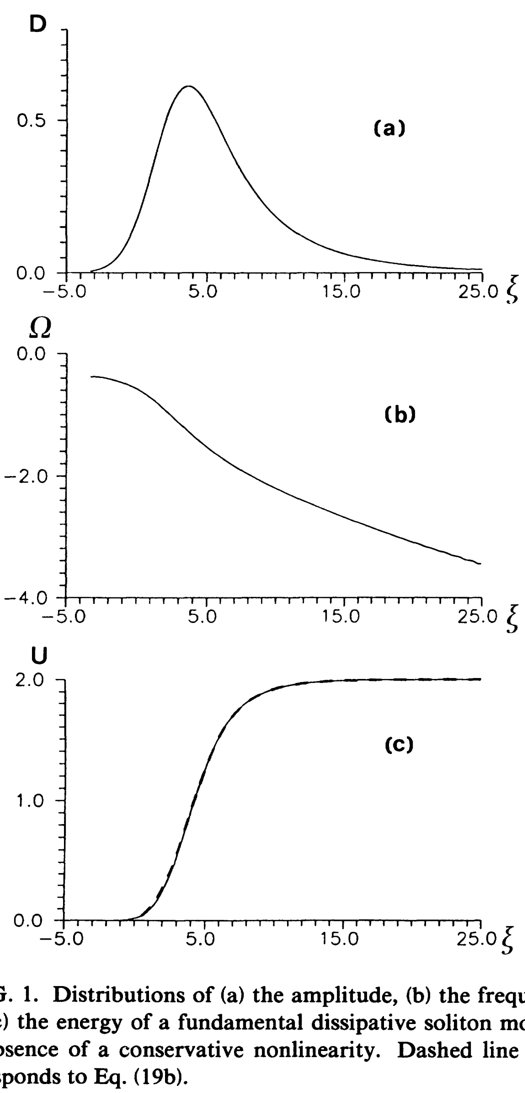

FIG. 1. Distributions of (a) the amplitude, (b) the frequency, and (c) the energy of a fundamental dissipative soliton mode in the absence of a conservative nonlinearity. Dashed line in (c) corresponds to Eq. (19b).

bulk shifts towards larger values of ξ so that it ceases to be affected by the time profile inhomogeneity. Because of this the soliton envelope becomes quasisymmetric and takes the canonical form $D=D_0/\cosh[(\xi-\xi_0)/\Delta\xi]$ with the amplitude $D_0\cong1/[12(\tilde{a}\xi_0)^{1/4}]$ and the width $\Delta\xi\cong2\sqrt{\tilde{a}\xi_0}$, where $\xi_0\cong4\beta^2\tilde{a}$.

Considering the positive sign of GVD corresponding to $\mu<0$, it can be expected that the behavior of the dissipative soliton is opposite to that of the case $\mu>0$ with regard to the form modification and the acceleration due to increasing conservative nonlinearity. Indeed, the structure is steepening at the leading front thus becoming more asymmetric and it experiences greater acceleration, see Figs. 2 and 3. However, it is important to note that even in this case the main part of the pulse is correctly described by Eq. (19). In the limit case $|\mu|\gg1$ we have shock wave with a sharp leading front (characterized by the dispersion scale ~1) and a smooth tail. The tail con-

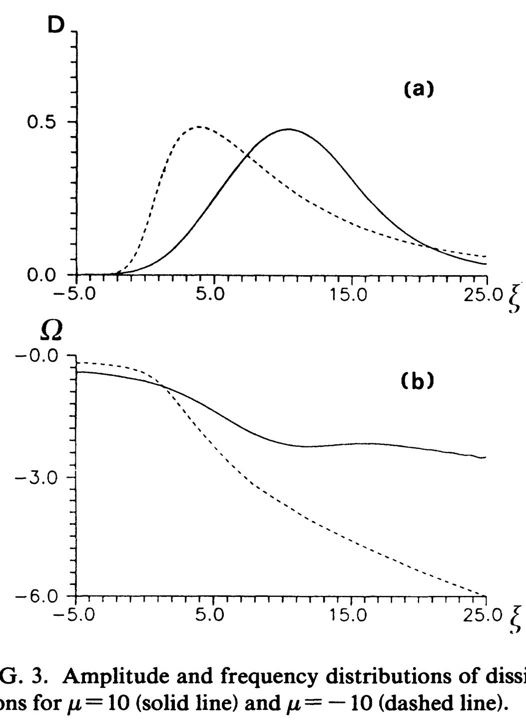

FIG. 3. Amplitude and frequency distributions of dissipative solitons for $\mu=10$ (solid line) and $\mu=-10$ (dashed line).

tains almost all of the pulse energy and its asymptotic is $D \approx \exp(-\sqrt{\xi/\tilde{a}})$. Obviously, a distinctive analogy exists between this limit solution and the structure form of diverging shock wave appearing in connection with optical pulse evolution in conservative nonlinear media with positive GVD [5].

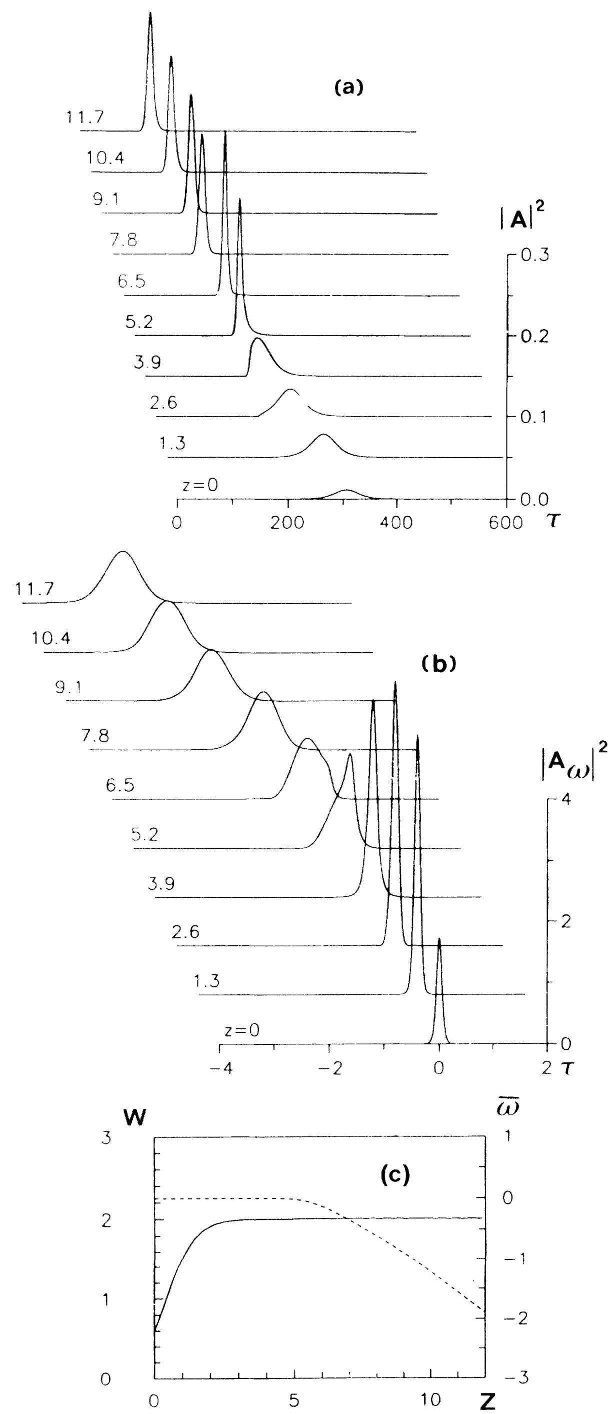

FIG. 4. Evolution of (a) the pulse amplitude, (b) the frequency spectrum, and (c) the central frequency of the spectrum $\bar{\omega}=\int_{-\infty}^{+\infty}\omega|A_\omega|^2 d\omega/\int_{-\infty}^{+\infty}|A_\omega|^2 d\omega$ (dashed line), and the pulse energy according to Eq. (6) with $\delta=-2$ ($\mu=-1/\sqrt{2}$) (solid line). The initial pulse form is $A(z=0)=(1/\sqrt{60})/\cosh(\tau/15)$.

The dissipative structures found above play a fundamental role in broadband gain media with wave dispersion by being attractors for the evolution of arbitrary field distributions. This fact has been demonstrated by numerical simulations of optical pulse dynamics for many values of the parameter μ. As an example, Fig. 4 shows the evolution of an initially bell-shaped and frequency-nonmodulated optical pulse with the initial energy $\overline{W}_0<2$ in the medium with $\mu=1/\sqrt{2}$ (comparable contributions of conservative and dissipative nonlinearities). As has been discussed earlier, the establishing of constant energy is followed by a complicated dynamics of the pulse resembling the collapse process in conservative media [13]. The analysis of possible regimes of the transient dynamics in Eq. (6) will be published elsewhere. Here, we only emphasize that, in the course of dynamical

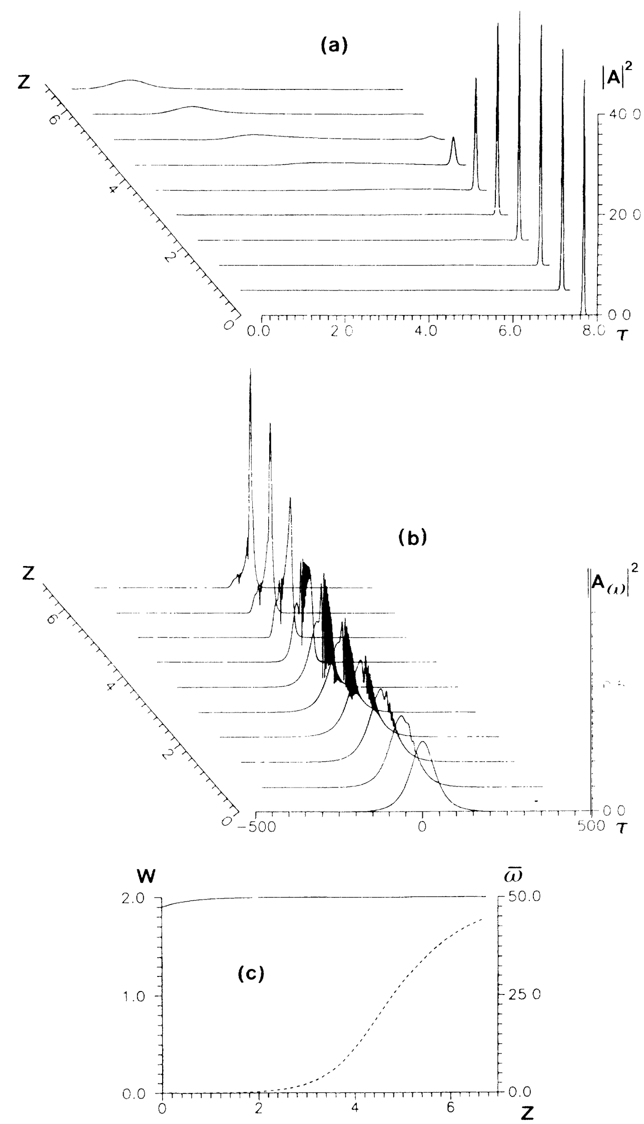

FIG. 5. Evolution of (a) the pulse amplitude, (b) the frequency spectrum, and (c) the central frequency of the spectrum (dashed line); and the pulse energy according to Eq. (6) with $\delta=10^{-2}$ (solid line) ($\mu=10$). The initial form is chosen to be the Schrödinger soliton according to Eq. (11) with the energy $\overline{W}=1.95$ and $|A(z=0,\tau)|=\sqrt{48.75}/\cosh(50\tau)$.

evolution, a dissipative soliton structure emerges, which demonstrates a stable accelerated propagation over distances available in the computer experiment.

We emphasize that the usual Schrödinger soliton defined by Eq. (11) exists in a medium with $\mu > 0$. Switching off the dissipation in our problem can be achieved by taking $\mu \rightarrow \infty$. Therefore, an interesting question arises about the relationship between the dissipative solitons (autosolitons) and the Schrödinger solitons as fundamental nonlinear structures in weakly nonconservative systems. In order to investigate this problem the following numerical experiment has been performed. For a medium with a rather large value of μ, an initial distribution in the form of a Schrödinger soliton, Eq. (11) has been considered with an energy close to the limit value $\overline{W}=2$. The corresponding evolution is shown in Fig. 5 for $\mu=10$ ($\delta=10^{-2}$) and $\overline{W}_0=1.95$. It has been observed that after the rapid establishing of the steady-state energy value, a long-pulse-propagation regime appeared with a quasiconstant amplitude but with a growing central frequency. This state has been followed by a passage to another temporal structure that is characterized by a smaller amplitude and corresponds exactly to the self-accelerating mode. Consequently, we conclude that the dissipative solitons represent fundamental nonlinear structures in weakly nonconservative systems.

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