PHYSICAL REVIEW A VOLUME 49, NUMBER 4 APRIL 1994

Contractive states of a free atom

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(Received 13 August 1993)

A position-measurement scheme is discussed which prepares an atom in a contractive state. Two operating regimes are analyzed. In the first, the quadratic form of the interaction potential is responsible for the focusing. In the second regime the potential varies approximately linearly with position, and the focusing results from the measurement itself. In this second regime the scheme provides a very close realization of the Einstein-Podolsky-Rosen *Gedankenexperiment*, an analogy that allows some insight into the production of measurement-induced contractive states.

PACS number(s): 03.65.Bz, 32.80.−t, 42.50.−p

I. INTRODUCTION

Quantum mechanics imposes a fundamental limitation on the precision of position measurements of a free mass since such measurements inevitably introduce momentum uncertainty into the system. This momentum uncertainty feeds into the position of the system, affecting the position uncertainty at a later time and limiting the resolution of a subsequent position measurement [1]. This has important practical consequences in gravitational wave detectors where the signal is comparable in magnitude to the quantum noise of the measurement. In an attempt to evade this so called standard quantum limit (SQL) Yuen [2] suggested the preparation of contractive states, a proposal which has initiated much debate (see [3,4] and references therein).

In this paper we discuss an experimentally realizable method to measure the position of an atom while simultaneously preparing it in a contractive state. The physical arrangement, which has been described previously [5,6], involves a standing light wave that is highly detuned from an atomic resonance. As the atom passes through the standing wave it induces a phase shift on the field that depends on the atom's transverse position. The atomic position can then be inferred by making a phase-sensitive measurement on the field.

An alternative method to measure the position of an atom has recently been demonstrated by Gardner *et al.* [7]. They obtain suboptical wavelength position resolution of atoms using Raman induced resonance imaging, in which a spatially varying light shift correlates an atomic resonance frequency with the atomic position. However, since the de Broglie wavelength of their atoms is much less than the position resolution they are in the limit of a classical measurement.

We first consider position measurements made in the region of a field antinode, where the potential is approximately quadratic. The production of contractive states in this region was mentioned briefly in Ref. [5].

Secondly we consider position measurements made in the region midway between the nodes and antinodes of the standing wave, where the potential is approximately linear. Whether or not the atom contracts in this case depends on the type of field measurement chosen. This choice may be made after the atom-field interaction, when the atom can no longer be physically manipulated. The system is shown to provide a very close realization of the *Gedankenexperiment* described by Einstein, Podolsky, and Rosen (EPR).

II. CONTRACTIVE STATES

The standard quantum limit, as it was originally formulated by Caves *et al.* [1], states that in two successive position measurements on a free mass m made a time τ apart, the result of the second measurement cannot be predicted with uncertainty smaller than $(\hbar\tau/m)^{1/2}$. A heuristic argument for this limit runs as follows. The uncertainty in the position of a mass at a time τ after a position measurement will arise from two sources: There will be some uncertainty due to the finite resolution $\Delta x(0)$ of

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the first measurement, and there will be a contribution $\Delta p(0)\tau/m$ from the momentum uncertainty introduced by the measurement. The resulting uncertainty obtained by combining these two contributions is

$$\Delta x(\tau) = \sqrt{\Delta x^2(0) + \Delta p^2(0)\left(\frac{\tau}{m}\right)^2}. \quad (1)$$

$\Delta x(\tau)$ can be minimized by varying the initial position and momentum uncertainties within the constraint imposed by the uncertainty principle. According to this argument, the position uncertainty at time τ must therefore be greater than a minimum given by

$$\langle \Delta x^2(\tau)\rangle_{\rm SQL} = \frac{\hbar\tau}{m}. \quad (2)$$

Yuen [2] pointed out a serious flaw in this heuristic argument for the standard quantum limit. A rigorous treatment of the evolution of a free mass in the Heisenberg picture shows that the position varies as

$$x(t) = x(0) + p(0)\frac{t}{m}, \quad (3)$$

where x and p are now quantum-mechanical operators. The position uncertainty at time t is therefore

$$\langle \Delta x^2(t)\rangle = \langle \Delta x^2(0)\rangle + \langle \Delta p^2(0)\rangle\frac{t^2}{m^2} + \langle \Delta x(0)\Delta p(0) + \Delta p(0)\Delta x(0)\rangle\frac{t}{m}. \quad (4)$$

This full treatment reveals a third contribution to the position uncertainty that depends on the correlation between the position and momentum. This correlation was implicitly assumed to be nonnegative in the heuristic treatment of the SQL. Indeed it is identically zero if the measurement leaves the mass in a minimum uncertainty state (which, for the purposes of this paper, we have taken to mean a state satisfying $\Delta x^2\Delta p^2 = \hbar^2/4$).

Yuen described a class of states which can breach the SQL. These states, which he called "contractive states," have a negative position-momentum correlation that causes them to contract with time. This contraction does not occur indefinitely, but stops when a certain minimum position variance is reached, after which time the state spreads out in the usual manner.

Yuen defined one set of such states, which he termed "twisted coherent states," in analogy with the squeezed states of the electromagnetic field. These twisted coherent states may alternatively be thought of as minimum uncertainty states of a free mass that have been propagated backwards in time.

Suppose that at time $t = 0$ a free mass is in a minimum uncertainty state with position uncertainty σ. Then at an arbitrary time t the correlation between the position and momentum of the mass is given by

$$\langle \Delta x\Delta p + \Delta p\Delta x\rangle = \frac{\hbar^2 t}{2m\sigma^2}, \quad (5)$$

which for $t < 0$ is negative, as required for a contractive state. In fact the states of the mass for $t < 0$ form the set of twisted coherent states. Their position uncertainty decreases while t is negative, reaching a minimum at $t = 0$. This minimum position uncertainty is analogous to a beam waist in optics, and we denote it by w. For positive t the distribution spreads out again.

We may define the "contraction" C of the state in terms of the fraction by which the position variance decreases,

$$C \equiv -\mathrm{sgn}\left(\langle \Delta x\Delta p + \Delta p\Delta x\rangle\right)\sqrt{\frac{\langle\Delta x^2\rangle - w^2}{w^2}}. \quad (6)$$

For a Gaussian distribution w is given by the minimum uncertainty relation, and the contraction is therefore

$$C = -\mathrm{sgn}\left(\langle \Delta x\Delta p + \Delta p\Delta x\rangle\right)\sqrt{\frac{4\langle\Delta p^2\rangle\langle\Delta x^2\rangle}{\hbar^2} - 1}. \quad (7)$$

Hence for a twisted coherent state

$$C = \frac{-\hbar t}{2m\sigma^2}. \quad (8)$$

An expression for the twisted coherent states can be derived as follows. Given the minimum uncertainty state of the free mass at time $t = 0$,

$$|\phi(0)\rangle = \frac{1}{\sqrt{\sigma}\sqrt[4]{2\pi}}\int dx\, \exp\left[-\frac{(x-x_0)^2}{4\sigma^2} + ik_0\left(x - \frac{x_0}{2}\right)\right]|x\rangle, \quad (9)$$

the state of the system at time t is

$$|\phi(t)\rangle = e^{\frac{-iHt}{\hbar}}|\phi(0)\rangle = \frac{\sqrt{\sigma}}{\sqrt[4]{2\pi}\sqrt{\sigma^2 + i\frac{\hbar t}{2m}}}\int dx\, \exp\left\{-\frac{[x - x_0(t)]^2}{4\left(\sigma^2 + i\frac{\hbar t}{2m}\right)} + ik_0\left[x - \frac{x_0(t)}{2}\right]\right\}|x\rangle, \quad (10)$$

where

$$x_0(t) = x_0 + \frac{\hbar t}{m}k_0. \quad (11)$$

The states $|\phi(t < 0)\rangle$ are the set of twisted coherent states.

III. PRODUCING CONTRACTIVE STATES WITH A QUADRATIC POTENTIAL

The position of an atom may be measured by passing it through a standing light wave that is highly detuned from the atomic resonance [5,6]. The atom alters the phase of the field by an amount that depends on its position in the cavity mode. Position information can then be recovered by measuring a phase-sensitive quantity of the field such as the quadrature phase.

The atom is assumed to enter the field in the ground state. Because of its large detuning from the atomic resonance, the field is very unlikely to induce an atomic transition from the ground to the excited state, and hence the probability of spontaneous emission is negligible. The Hamiltonian in this regime can be derived using the method outlined in Ref. [6], and may be written (in a frame rotating at the optical frequency) as

$$H = \hbar\frac{|g|^2}{\Delta}\left[\sigma_z(2a^\dagger a+1)+\tfrac{1}{2}\right]\cos^2(kx+\xi)+\hbar\Delta\sigma_z . \tag{12}$$

Here a and $a^\dagger$ are the annihilation and creation operators of the cavity field, and $k \equiv 2\pi/\lambda$ denotes the field wave number. σ_z is the inversion of the atom, which is equal to $-1/2$ since the atom is in the ground state. $|g|$ is the coupling constant (equal to the one-photon Rabi frequency) and Δ is the detuning of the atomic transition frequency from the cavity frequency. Since the detuning is chosen to be positive, the antinodes of the field act as potential wells. Before the interaction the field is prepared in a coherent state $|\alpha\rangle$ with α real, and the atom has a position distribution denoted by $\kappa(x)$. Taking the interaction time to be sufficiently brief that the atom's transverse motion in the cavity may be neglected (the Raman Nath condition), we find that the state of the system after the interaction is given by

$$\begin{aligned}|\Psi\rangle &= \int dx\,\kappa(x)e^{-\frac{iHt}{\hbar}}|\alpha\rangle\otimes|x\rangle\\ &= e^{\frac{i\Delta t}{2}}\int dx\,\kappa(x)|\alpha e^{i\frac{|g|^2t}{\Delta}\cos^2(kx+\xi)}\rangle\otimes|x\rangle .\end{aligned} \tag{13}$$

The interaction establishes a correlation between the position of the atom, and the phase of the field. Information about the atomic position may then be obtained by measuring the quadrature phase $X_\theta = ae^{-i\theta}+a^\dagger e^{i\theta}$ using balanced homodyne detection [8]. Denoting the result of the field quadrature measurement by χ_θ, we can calculate the state of the atom after the field measurement by projecting the system onto the quadrature phase eigenstate $|\chi_\theta\rangle$.

$$\begin{aligned}|\psi\rangle_{\rm atom} &= N\langle\chi_\theta|\Psi\rangle\\ &= N\int dx\,\kappa(x)\left\langle\chi_\theta\left|\alpha e^{i\frac{|g|^2t}{\Delta}\cos^2(kx+\xi)}\right.\right\rangle|x\rangle\\ &= N\int dx\,\kappa(x)\frac{1}{\sqrt[4]{2\pi}}\exp\left\{-\left[\left(\alpha_1-\frac{\chi_\theta}{2}\right)^2\right.\right.\\ &\qquad\left.\left.+i\alpha_2(\alpha_1-\chi_\theta)\right]\right\}|x\rangle ,\end{aligned} \tag{14}$$

where

$$\alpha_1+i\alpha_2 \equiv \alpha\exp\left\{i\left[\frac{|g|^2t}{\Delta}\cos^2(kx+\xi)-\theta\right]\right\} \tag{15}$$

and N is a normalization factor.

Provided the atomic beam is not too rapidly diverging as it enters the cavity, a field measurement that localizes the atom in the region of a field antinode will simultaneously project it into a contractive state [5]. If a higher intensity field is chosen the atom will be better localized. The potential sampled by the atom will then be more nearly quadratic, and the contractive state into which the atom is projected will more closely approximate Yuen's ideal twisted coherent state.

Figure 1(a) shows the Wigner distribution [9] of the atom after a measurement of the field quadrature has localized it at an antinode. The Wigner distribution clearly shows the negative correlation between the position and momentum of the atom. Figure 1(b) shows the variance of the atomic distribution as it propagates freely after leaving the cavity. The distribution contracts until it reaches a "waist," beyond which it spreads out again. This is compared with the ideal focusing achieved by a twisted coherent state with the same momentum variance

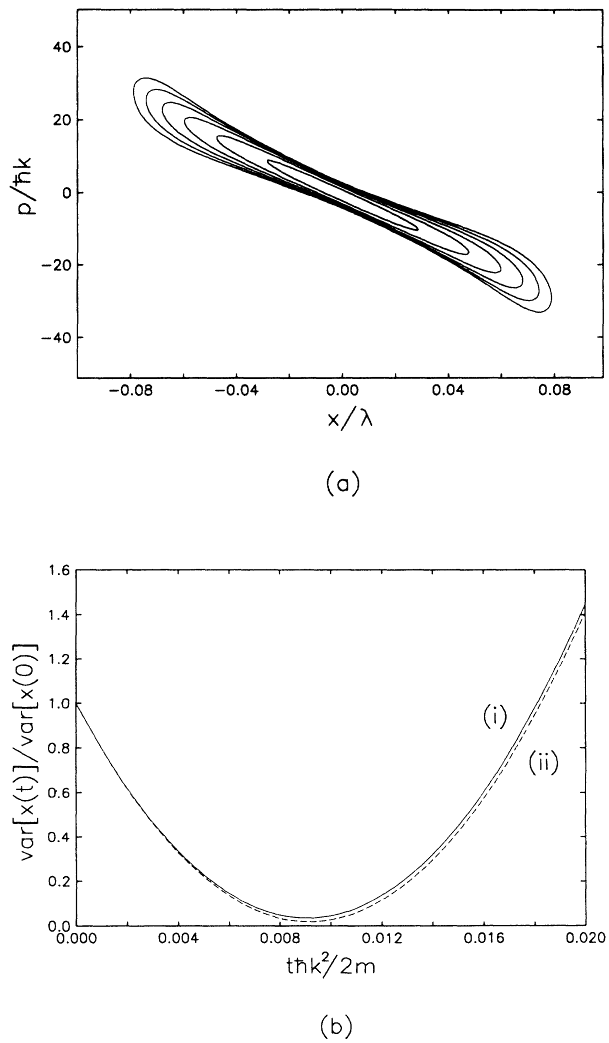

FIG. 1. (a) The atomic Wigner distribution after the field quadrature measurement $X_0 = -2\alpha$, for $\alpha = \sqrt{8}$ and $|g|^2t/\Delta = \pi$. The initial atomic state was chosen to be a minimum uncertainty state centered at an antinode with position uncertainty $\sigma = \lambda/4\pi$. The contour levels are spaced logarithmically. (b) The evolution of the atomic position variance (i) is compared with the focusing of a twisted coherent state (ii).

and initial position variance. The effective focal length of the atomic lens provided by the field antinode is

$$f=\frac{\Delta}{2|g|^2t|\alpha|^2k^2}\frac{p}{\hbar}, \tag{16}$$

where p is the longitudinal momentum of the atom.

The focusing of the atom described above is largely independent of the field measurement, and essentially results from the quadratic nature of the effective potential at the field antinode. The focusing of atomic beams by standing light waves has been observed by Sleator *et al.* [10]. The standing wave used for their experiment was created by reflecting a traveling wave off a mirror at grazing incidence. The periodicity of the resulting intensity grating was $\approx 45\,\mu$m, much larger than the wavelenth of the light used. A lens aperture ($25\,\mu$m) was centered in one antinode of the standing wave, and irradiated through a $2\,\mu$m wide object structure. The atomic beam was focused down to a spot size of $4\,\mu$m.

IV. MEASUREMENT-INDUCED CONTRACTIVE STATES

In this section we show that position measurements made using a linear potential can also project a particle into a contractive state. We analyze the same scheme as was discussed in the preceding section, but now consider measurements that localize the atom in the region midway between a node and an antinode of the standing wave, where the potential varies linearly with position. In this case focusing may be exhibited by all the conditional atomic distributions (conditioned on the result of the field measurement), but not by the total distribution (obtained when no field measurement is made). We therefore describe the atomic focusing as being measurement induced. The degree of focusing depends on which field quadrature is measured, that is it depends on the phase chosen at the homodyne detector.

It is shown that the entangled atom-field state produced by the interaction is analogous to the entangled two-particle state at the heart of the EPR *Gedankenexperiment.* The atom plays the role of one of the particles in the EPR experiment and the field plays the role of the other. This analogy provides some insight into the reason that atomic focusing results from certain quadrature phase measurements.

A. Producing contractive states with a linear potential

We suppose that the atom passes through the standing light wave in the region midway between a node and an antinode. If the initial atomic distribution $\kappa(x)$ is sufficiently narrow we need consider only the linear component of the potential. We therefore approximate the field mode by the linear function

$$\cos^2\left(kx-\tfrac{\pi}{4}\right)\approx kx+\frac{1}{2}. \tag{17}$$

In our calculation of the atomic state after the interaction and field measurement we will assume that $|g|^2t/\Delta=\pi$. Substituting approximation (17) into Eq. (15) then gives

$$\begin{aligned}\alpha_1&=-\alpha\sin(\pi kx-\theta),\\ \alpha_2&=\alpha\cos(\pi kx-\theta).\end{aligned} \tag{18}$$

The state into which the atom is projected by the field measurement is calculated by inserting these expressions for α_1 and α_2 into Eq. (14). In keeping with our assumption that the initial atomic distribution $\kappa(x)$ is very narrow, we neglect terms of higher than second order in x in the exponent. For large α we find that the atomic state after the field measurement is given by

$$\begin{aligned}|\psi\rangle_{\rm atom}=N\int dx\,\kappa(x)\\ \times\exp\left\{-\frac{\left[\alpha(\pi kx-\tan\theta)+\frac{\chi_\theta}{2\cos\theta}\right]^2}{1+i\tan\theta}\right\}\\ \times\exp\left\{i\alpha\left[\alpha(\pi kx-\tan\theta)+\frac{\chi_\theta}{\cos\theta}\right]\right\}|x\rangle.\end{aligned} \tag{19}$$

If the initial distribution $\kappa(x)$ is Gaussian then this expression is of the same form as (10), and it is clear that the state into which the atom is projected by the field measurement is precisely a twisted coherent state. Calculating the resolution of the position measurement from Eq. (19) gives

$$\delta x=\frac{1}{|2\alpha\pi k\cos\theta|}. \tag{20}$$

The best resolution is obtained for an amplitude quadrature measurement (corresponding to $\theta=0$). The resolution worsens as θ is increased, until in the limit of a phase quadrature measurement ($\theta=\pi/2$) no position information is obtained from the field measurement. Hence in the parameter regime considered in this section we cannot rightly describe a perfect phase quadrature measurement as a position measurement of the atom.

We will take the initial atomic wave function $\kappa(x)$ to be that of a minimum uncertainty state, with momentum uncertainty σ_p,

$$\kappa(x)=\sqrt[4]{\frac{2\sigma_p^2}{\pi\hbar^2}}\exp\left[-\left(\frac{x\sigma_p}{\hbar}\right)^2\right]. \tag{21}$$

If θ is chosen so that $\tan\theta$ is negative, then $|\psi\rangle_{\rm atom}$ in Eq. (19) is a contractive state. Since it has a Gaussian position distribution, the contraction and effective focal length can easily be calculated. The focal length (in terms of the longitudinal momentum p) is given by

$$f=\frac{-\tan\theta}{2(\alpha\pi k)^2}\frac{p}{\hbar}, \tag{22}$$

where it must be remembered that we have set $|g|^2t/\Delta=$

π. From Eq. (7) the contraction is found to be

$$C = \frac{-\alpha^2 \tan\theta}{\left(\frac{\sigma_p}{\pi\hbar k}\right)^2 \sec^2\theta + \alpha^2}. \tag{23}$$

Note that the contraction depends on the phase θ of the measured field quadrature, although it is independent of the particular result χ_θ of the field measurement. This phase θ is set at the homodyne detector, and may be chosen after the interaction.

Figures 2(a) and 2(b) show the evolution of the atomic position distribution for two different results of the field quadrature measurement. Note that the mean momentum differs between the two cases. If the atomic distribution is not conditioned on the result of the field measurement then no focusing is observed, as expected from a linear potential.

B. EPR analogy

The effect of the atom-field interaction at a linear part of the field mode is to correlate the position of the atom with the amplitude quadrature of the field. Similarly we will show that the momentum of the atom is correlated with the phase quadrature of the field. These correlations are reminiscent of the EPR *Gedankenexperiment* [11], in which two particles have correlated positions and correlated momenta. The analogy with the EPR state provides some insight into how contractive states arise in the atom-field interaction.

The correlation established between the amplitude quadrature of the field and the position of the atom is best revealed by expanding the entangled atom-field state $|\Psi\rangle$ in terms of the respective eigenstates of these observables. Using Eq. (19) and setting $\theta = 0$ we obtain

$$\begin{aligned} |\Psi\rangle &= \int d\chi_0 \, |\chi_0\rangle\langle\chi_0|\Psi\rangle \\ &= N \int dx \int d\chi_0 \, \exp\left[-\left(\frac{x\sigma_p}{\hbar}\right)^2 - \left(\alpha\pi k x + \frac{\chi_0}{2}\right)^2 + i\alpha\left(\alpha\pi k x + \chi_0\right)\right] |\chi_0\rangle \otimes |x\rangle, \end{aligned} \tag{24}$$

from which we can deduce that the best estimate of the position of the atom given the result χ_0 for the amplitude quadrature measurement is

$$x = \frac{-\frac{\chi_0}{2}\alpha\pi k}{\left(\frac{\sigma_p}{\hbar}\right)^2 + (\alpha\pi k)^2}. \tag{25}$$

Alternatively the system state may be expressed in terms of the eigenstates of momentum and phase quadrature, to reveal the correlation between these observables,

$$\begin{aligned} |\Psi\rangle &= \int dp \int d\chi_{\frac{\pi}{2}} \, |\chi_{\frac{\pi}{2}}\rangle \otimes |p\rangle\langle p| \otimes \langle\chi_{\frac{\pi}{2}}|\Psi\rangle \\ &= N \int dp \int d\chi_{\frac{\pi}{2}} \, \exp\left\{-\frac{1}{4\sigma_p^2}\left[p - \alpha\pi\hbar k\left(\chi_{\frac{\pi}{2}} - \alpha\right)\right]^2 - \left(\frac{\chi_{\frac{\pi}{2}}}{2} - \alpha\right)^2\right\} |\chi_{\frac{\pi}{2}}\rangle \otimes |p\rangle. \end{aligned} \tag{26}$$

The best estimate of the atomic momentum given the result $\chi_{\frac{\pi}{2}}$ for the phase quadrature measurement is

$$p = \alpha\pi\hbar k(\chi_{\frac{\pi}{2}} - \alpha). \tag{27}$$

We may make the following analogy between the atom-field system and the two-particle EPR system. The field plays the role of the first particle, on which the measurements are made, and the atom plays the role of the second particle, whose position and momentum are inferred. The EPR state with total momentum p_0 can be expressed in both the position and momentum representations as

$$|\text{EPR}\rangle = \int dx \, e^{\frac{ip_0 x}{\hbar}} |x\rangle_1 \otimes |x + x_0\rangle_2 \tag{28}$$

$$= \int dp \, e^{-\frac{ipx_0}{\hbar}} |p_0 - p\rangle_1 \otimes |p\rangle_2. \tag{29}$$

The position representation of the state (28) explicitly demonstrates the perfect correlation between the positions of the two particles. If the position of particle 1 is measured to be x_1, then it can be inferred that particle 2 is at position $x_1 + x_0$. Similarly, the momentum representation of the state (29) reveals the perfect momentum correlation of the two particles: if the momentum of particle 1 is measured to be p_1, then it can be inferred that particle 2 has momentum $p_0 - p_1$.

If instead a measurement is made on particle 1 of a linear combination $\widehat{x_1} + \widehat{p_1}t/m$ of position and momentum, then particle 2 will collapse to an eigenstate of $\widehat{x_2} - \widehat{p_2}t/m$. For negative t, this resulting eigenstate will have the negative correlation between position and momentum that is characteristic of contractive states. The analogy in the atom-field system is the measurement of observables $X_0 \cos\theta + X_{\frac{\pi}{2}} \sin\theta = X_\theta$ with $\tan\theta$ negative. It is precisely these measurements which we have shown project the atom into a contractive state.

The atom-field system described in this paper provides an experimentally accessible realization of the EPR system, the essential difference being that the correlations established in the EPR system are perfect, whereas those of the atom-field system are not. This difference is unavoidable since a perfect position correlation requires infinite energy to establish, and is therefore impossible to realize exactly in an experimental situation. However, in order for the system to present a paradox, the correlations must be sufficiently good that the information inferred about the atom by means of field measurements has a higher precision than can be contained within a single-particle quantum description of the atom.

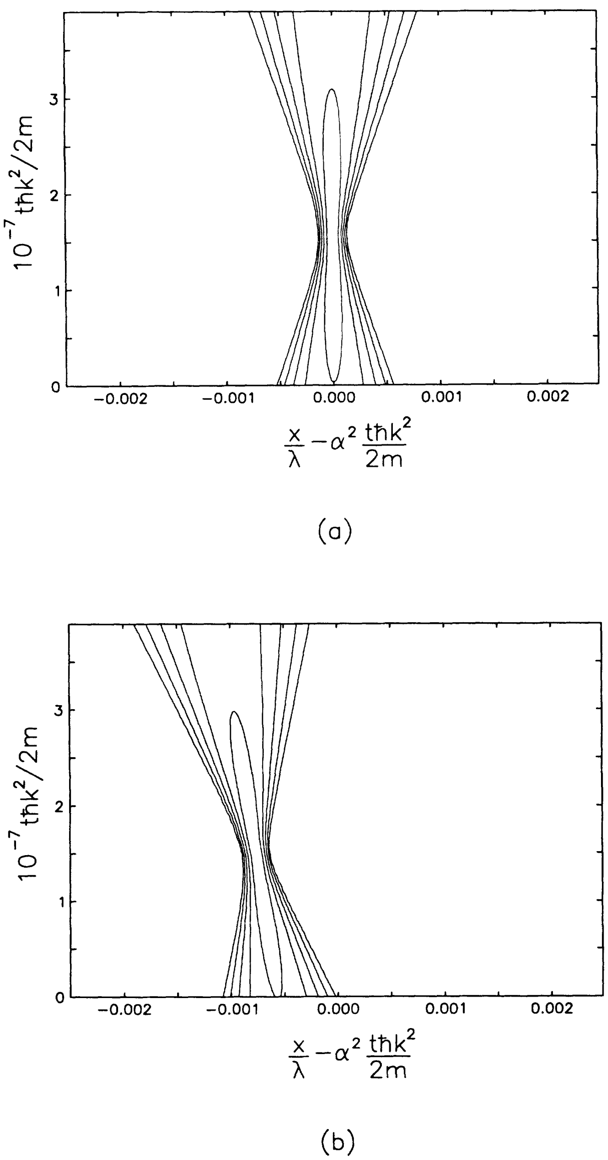

FIG. 2. The evolution of the atomic position distribution after the field quadrature measurements (a) $X_\theta = 2\alpha \sin\theta$ (the most probable result) and (b) $X_\theta = (2\alpha - 5)\sin\theta$ (a less probable result), for $\theta = -0.45\pi$, $\alpha = 10^3$, and $|g|^2 t/\Delta = \pi$. The initial atomic state was chosen to be a minimum uncertainty state centered midway between a node and an antinode, with momentum uncertainty $\sigma_p = \alpha\pi\hbar k/10$. The time dependence in the units used for the position axis is introduced to suppress the large deflection of $p = \alpha^2 \pi\hbar k$ produced by the optical potential. The contour levels are spaced logarithmically.

We will formulate this condition mathematically using the terminology introduced by Reid in Ref. [12]. In this paper Reid proposes another realization of the EPR *Gedankenexperiment*, which has since been demonstrated experimentally by Ou *et al.* [13]. Reid pointed out that the correlated state of the output beams of a nondegenerate parametric amplifier is analogous to the entangled state of the EPR *Gedankenexperiment*, both conjugate quadrature phases of the signal and idler being highly correlated.

Reid introduces "inference errors" to describe the uncertainty with which information about the signal beam can be determined by measurements on the idler beam. If the product of the inference errors for the two conjugate variables is less than the Heisenberg uncertainty product, then it is deemed that a paradox has been demonstrated.

In our scheme, as in the original EPR *Gedankenexperiment*, the conjugate variables are position and momentum, and we can calculate their inference errors as follows. If the momentum of the initial state is localized to within a variance of

$$V_p = \sigma_p^2, \tag{30}$$

then the variance in the inferred momentum after the interaction will be the same,

$$\Delta^2_{\text{inf}} p = \sigma_p^2. \tag{31}$$

If the atom is initially in a known minimum uncertainty state, then its initial position variance is

$$V_x = \left(\frac{\hbar}{2\sigma_p}\right)^2 \tag{32}$$

and the variance in the inferred position after the interaction will be

$$\Delta^2_{\text{inf}} x = \frac{1}{(2\pi k\alpha)^2 + \left(\frac{2\sigma_p}{\hbar}\right)^2}. \tag{33}$$

The product of these variances is

$$\Delta^2_{\text{inf}} x \Delta^2_{\text{inf}} p = \left(\frac{\hbar}{2}\right)^2 \frac{1}{1 + \left(\frac{\pi\hbar k\alpha}{\sigma_p}\right)^2}. \tag{34}$$

Thus to demonstrate a paradox we must have

$$\frac{\pi\hbar k\alpha}{\sigma_p} > 0, \tag{35}$$

a condition which is always satisfied.

In most experimental situations, however, the initial atomic state is unlikely to be a known minimum uncertainty state. The initial momentum variance V_p is easily controllable by collimation, but the position distribution is not likely to be well known. Our best estimate in this case for the variance in the inferred position will be

$$\Delta^2_{\text{inf}} x = \frac{1}{(2\pi k\alpha)^2}, \tag{36}$$

and the uncertainty product will be

$$\Delta^2_{\rm inf}x\Delta^2_{\rm inf}p = \left(\frac{\hbar}{2}\right)^2 \left(\frac{\sigma_p}{\pi\hbar k\alpha}\right)^2. \tag{37}$$

In this case a paradox is demonstrated if

$$\frac{\pi\hbar k\alpha}{\sigma_p} > 1. \tag{38}$$

Thus we see that for a sufficiently intense field, the Einstein-Podolsky-Rosen paradox can be demonstrated without preparing the atom initially in a minimum uncertainty state.

V. CONCLUSION

We have suggested an experimentally realizable position measurement scheme that projects an atom into a contractive state. The technique consists of measuring the phase shift induced on a standing light wave by its position-dependent interaction with the atom. Two regions of the standing wave were considered. The first was the region near an antinode, where the quadratic form of the potential was responsible for focusing the atom. The second was the region midway between a node and an antinode, where the potential was approximately linear and the focusing was found to be measurement-induced. In this region of linear potential, the scheme was found to provide a close realization of the Einstein-Podolsky-Rosen *Gedankenexperiment*, and the production of measurement-induced contractive states could be understood in terms of this analogy.

ACKNOWLEDGMENTS

This research has been supported by the University of Auckland Research Committee, the New Zealand Lottery Grants Board, the United States Office of Naval Research, and the New Zealand Foundation of Research, Science, and Technology. T.S. received support from the Alexander von Humboldt Foundation. He also acknowledges useful discussions with Paul R. Berman.

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