Noise-induced switching and stochastic resonance in optically nonlinear CdS crystals

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We investigate the influence of noise present in the periodic input signal of a passive optically bistable element. The thermally induced absorption nonlinearity of CdS crystals is used to achieve optically bistable behavior. By measuring the signal-to-noise ratio (S/N) of the noisy modulated input signal and of the output signal the phenomenon of stochastic resonance is found: With increasing noise strength the S/N increases up to a maximum value corresponding to the above resonance and then decreases again. The S/N of a noisy input can be enhanced by up to approximately 6 dB with this system. Furthermore, the cases of sinusoidal and rectangular modulation of the bistable element are compared.

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I. INTRODUCTION

The properties of bistable optical systems have attracted attention in many areas, e.g., regarding their potential of optical data handling or the possible realization of neural networks. The influence of noise on the switching processes of such bistable systems also has been studied extensively in the literature [1–3]. An effect using noise-induced switching processes is the so-called stochastic resonance [4–9]. It describes the fact that noise present in the input signal of a modulated bistable system can *enhance* the signal-to-noise ratio (S/N) of the output signal, which certainly could bring about considerable improvements for the fields mentioned above. This effect has been observed also in, e.g., electronic systems [5,6] or active optical systems [7] showing bistable behavior. In this paper we investigate the case of a passive optically bistable system with modulated input intensity. The passive system has the advantage of requiring no external connections at all, and works just by incorporating it into the optical path. The system is driven inside the bistable region of the hysteresis loop. Only when noise is added is switching between the two possible states of transmission observed, leading to an output signal with the same basic frequency as that of the modulated input signal. In the following we first shall describe our experimental system. Then results obtained for the S/N of the output signals under different excitation conditions are given.

As nonlinear optical elements in our experiments we used CdS crystals with a thickness of about 6 μm showing thermally induced absorptive optical bistability (OB) under illumination with an Ar^+ laser ($\lambda = 514.5$ nm) as described in some detail in [3,10]. We use a two-beam setup for our experiments, both beams being incident on the same spot of the crystal and having an equal diameter of approximately 100 μm. The transmission state of the crystal is read by a constant beam with an input intensity I_{C1} that is too small to induce nonlinear behavior itself. The intensity of the second beam consists of a constant part I_{C2} to hold the system at the working point, and of a periodic signal part $I_S = Ag(2\pi f t)$. Moreover, the signal is perturbed by additional noise I_N. We discuss the cases of sinusoidal, i.e., $g(x)=\sin(x)$, and rectangular signals, i.e., $g(x)=\mathrm{sgn}(\sin(x))$. The modulation frequency f is in all cases much lower than the inverse relaxation time of the nonlinearity, which has a value of some ms. Nevertheless, the switching time of the bistable crystal may be longer than this relaxation time due to the effect of critical slowing down if the input intensity is near one of the transition points [11,12]. The values of I_{C2}, I_S, and I_N are set by a personal computer connected with a linear electro-optical modulator (EOM). The output of the bistable device, i.e., the transmitted part of I_{C1}, is digitized with a sampling rate of 39 ms.

The noise is represented by a sequence of random numbers causing a sequence of random light fluctuations. The characteristic time determining the highest-frequency component of the noise is the time τ_N, after which a new random number is generated and fed to the EOM. The Fourier spectrum of the noise is nearly constant for frequencies lower than τ_N^{-1}. The autocorrelation function $\langle I_N(t) I_N(0) \rangle$ of the noise decays to zero with a time constant given by τ_N. The random values have an approximately Gaussian distribution around 0, with a variance of $2D$, so that D determines the noise strength in comparison with the signal component.

I_{C1} in principle may be varied from nearly 0 (acting as a probe beam) up to a value near the back switching intensity $I_\uparrow$ to provide the best possible fan-out conditions for further use of the signal.

In every case the conditions $I_\uparrow < I_{C1}+I_{C2}-A$ and $I_{C1}+I_{C2}+A < I_\downarrow$ must hold, because otherwise the system switches even without any additional noise. In general, the dynamic switching intensities may be shifted compared to the quasistatic case, and thus $I_\uparrow$ and $I_\downarrow$ are functions of the modulation and noise frequency [13]. Here, the time average of the sum $\langle I_{C1}+I_{C2}+I_S+I_N \rangle$ was always situated in the center of the bistable loop.

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II. EXPERIMENTAL RESULTS

To perform quantitative measurements of the signal quality, we will compute the S/N of the output for different parameters by calculating the power spectrum of the time series and determining the intensity of the peak at the modulation frequency f and that of the background noise in the vicinity of this frequency. The difference between these two values, i.e., the S/N of the output signal at the modulation frequency, is plotted in Fig. 1 in a logarithmic scale as a function of the noise amplitude D. The period time of the modulation in this case was $T=f^{-1}=2.8$ s, and the noise time constant τ_N was approximately 90 ms. First, the modulation amplitude had a value of $A=490$ units, whereas the bistable loop had a width of 1000 units, i.e., without noise no switching processes were possible.

The S/N indeed first increases with increasing noise amplitude, reaches a maximum, and then decreases again. So by increasing the noise in the input signal of this bistable element it is possible to increase the S/N of the output signal. A qualitatively very similar picture is observed for smaller modulation amplitudes, as can be seen in Fig. 1 for $A=445$, 400, and 300 units. The absolute values of the maximum S/N decrease with decreasing modulation signal from 28 dB for $A=490$ units, down to 25 dB for $A=300$ units. For small modulation amplitude one observes very low values of the S/N for small noise, whereas in the case of high modulation amplitude it is sufficient to add a small amount of noise to switch the element and thus achieve better S/N.

A point that should be taken into account is the S/N of the input signal itself. It obviously decreases with decreasing signal or increasing noise amplitude, respectively. To compare the S/N of the input signal with the S/N of the output signal, we plot in Fig. 2 the effective signal-to-noise ratio, i.e., $(S/N)_{\mathrm{eff}}=(S/N)_{\mathrm{out}}-(S/N)_{\mathrm{in}}$ as a function of D. Two facts are remarkable here: (1) The qualitative behavior of the curves for different modulation amplitudes is almost identical. (2) For noise amplitudes higher than approximately 400 units, the value of $(S/N)_{\mathrm{eff}}$ is positive, i.e., the S/N of the output signal is improved in comparison with that of the input signal, and we observe gain in the S/N. So this bistable device can act as a filter that enhances the S/N of the signal due to the presence of the noise itself. In the gain region the values vary, and no significant trend as a function of the modulation amplitude can be noticed. The maximum gain of the different curves ranges from approximately 2 to 5 dB. The values of $(S/N)_{\mathrm{eff}}$ decrease only very slowly for increasing noise amplitude after the maximum has been reached. This may be due to the fact that for high noise amplitude the system switches very frequently between the two states of transmission. Nevertheless, the probability of finding it in the high or low states still depends on the actual value of the slowly varying input intensity, producing a still prominent frequency component at the modulation frequency f in the spectrum.

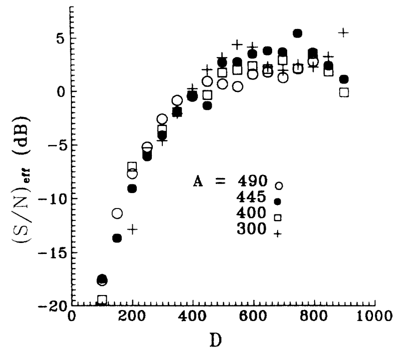

FIG. 2. The effective S/N as a function of the noise strength for the different modulation amplitudes of Fig. 1.

Similar experiments have been carried out for different values of τ_N using the same modulation frequency f, as before, and the highest modulation amplitude. The results are shown in Fig. 3, where $(S/N)_{\mathrm{eff}}$ is plotted as a function of D.

In the cases of $\tau_N=45$ and 90 ms, only low gain is observed, and the maximum of the effective S/N is around 1 dB. For higher values, namely $\tau_N=180$ ms, we observe a gain of around 4 dB, and, finally, in the case of $\tau_N=360$ ms the S/N of the output signal is up to around 6 dB

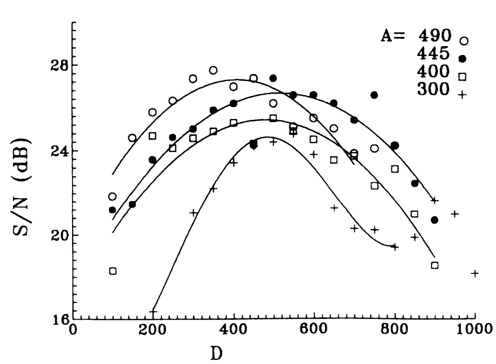

FIG. 1. The S/N of the output signal of the bistable device as a function of the noise strength for different modulation amplitudes A of the input signal. $A=490$, 445, 400, and 300 units (the width of the bistable loop is 1000 units). The lines are to guide the eye.

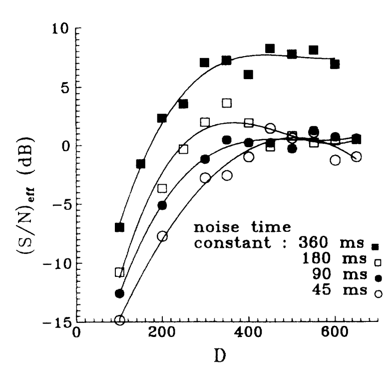

FIG. 3. The effective S/N as a function of the noise strength for different values of the noise time constant τ_N.

higher than that of the input signal. In summary, the S/N gain is larger for low-frequency noise. The reason may be that in the case of small values of τ_N the system frequently does not switch into the new state given by the input signal due to, e.g., critical slowing down leading to long switching times. Another effect that contributes is the following. The integral power of the noise, described by the value of D, is distributed over the frequency interval $[0,\tau_N^{-1}]$. This means that for constant D an increase of τ_N results in a higher background noise level in the input near the modulation frequency f (note that $f<\tau_N^{-1}$) because the constant noise power is distributed over a smaller frequency interval. The consequence is a decrease of $(S/N)_{\rm in}$ and thus a corresponding increase of $(S/N)_{\rm eff}$.

Until now we have considered the case of sinusoidal modulation. For applications in digital optical information processing systems the natural kind of input signal would be a rectangular modulation. Thus we carried out similar investigations for this kind of input signal. To compare the results the modulation frequency again was chosen to be $f=(2.8\text{ s})^{-1}$, and the maximum modulation amplitude was the same for the two types of modulation, having a value so that the rectangular pulses without additional noise did not switch the system.

The results for $(S/N)_{\rm eff}$ as a function of D are shown in Fig. 4. One can see that the effective S/N in the case of rectangular modulation in the gain region is always lower than that in the case of sinusoidal modulation. The maximum is only slightly above zero, whereas for the sinusoidal case a gain of 3 dB can be observed.

The reason for this is twofold. First, the rectangular input signal has a better S/N than the sinusoidal one, because of the higher switching energy it contains compared with a sinusoidal signal with the same maximum amplitude. This yields a smaller $(S/N)_{\rm eff}$ in the case of the rectangular input signal. Second, in the case of rectangular modulation the switching probability is constant over the whole duration of the plateau in the input signal, thus leading to a break of the locking of the switching intervals to multiples of $f^{-1}/2$ causing a "jitter" in the output signal: the system switches reliably but the time intervals between the switching events are varying, thus leading to a higher noise background in the spectrum. In the case of sinusoidal modulation the switching probability has a maximum when I_S is at an extremum which reduces this effect.

III. NUMERICAL SIMULATION

To model our experimental results we use a mathematical description for the OB introduced in [10]. It is based on an ordinary differential equation for the temperature T in the laser spot:

$$\dot{T}=\frac{I_0(t)A(T)}{CL}-\frac{T-T_0}{\tau}\ , \tag{1}$$

where I_0 is the total input intensity, $A(T)$ is the absorption increasing faster than linearly with temperature [10], T_0 is the ambient temperature, τ is the temperature relaxation time, and C and L are the heat capacity and thickness of the sample. The detected signal $I(t)$ can be calculated by

$$I(t)=[1-A(T(t))]I_{C1}\ . \tag{2}$$

As an example we solved this equation under conditions corresponding to the experimental parameters of $f=2.8$ s, $\tau_N=90$ ms, and a full width modulation within the bistable region, i.e., corresponding to the experimental data that are given as the full points in Fig. 3. The results of the simulation can be seen in Fig. 5. The qualitative agreement is very good. The major difference is that for small noise strength ($D<300$ units) we observe higher values of $(S/N)_{\rm eff}$ in the experiment than in the simulation. With increasing D the effective S/N tends to a constant value around zero in both cases. Using this simple one-dimensional model one cannot expect to describe all details of the experiment in a quantitative manner. For example, in the simulation the increase of $(S/N)_{\rm eff}$ with increasing τ_N is less prominent than was found in the experiment. It is known [12,14] that for the dynamics of such a bistable element transverse effects are important. Thus a more detailed numerical treatment would involve the solution of a diffusion equation for the temperature, which is beyond the scope of this paper.

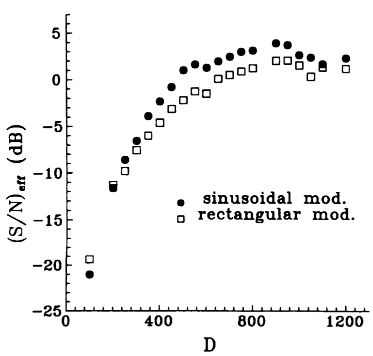

FIG. 4. The effective S/N as a function of the noise strength in the case of sinusoidal and rectangular modulations. The parameters τ_N and A are the same for both types of modulation.

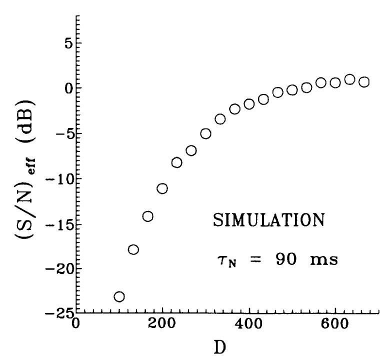

FIG. 5. Simulation: The effective S/N as a function of the noise strength for the experimental parameters corresponding to the full points in Fig. 3.

IV. CONCLUSIONS

We have shown experimentally the possibility of improving the S/N of a light signal near its modulation frequency by the use of a bistable device. With increasing noise amplitude that is added to the modulation signal we observed an increase of the S/N of the output signal that reached a maximum and decreased again. The best S/N of the output signal in the case of sinusoidal modulation has been found to be up to approximately 6 dB better than the S/N of the corresponding input signal. So, by choosing appropriate parameters of the hysteresis loop, it is possible to increase the S/N by increasing the noise. A further enhancement may be possible by using an input-output characteristic with a small bistable region but high switching contrast. In this case the signal amplification would be even higher, whereas the noise would not be amplified as long as one works in stochastic resonance. In the case of a rectangular modulation signal we also observed (for small noise) increasing S/N with increasing noise, but less gain of the S/N.

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