Quantum limits in interferometric gravitational-wave antennas in the presence of even and odd coherent states

Nadeem A. Ansari, L. Di Fiore, M. A. Man'ko,* V.I. Man'ko,* S. Solimeno, and F. Zaccaria

Dipartimento di Scienze Fisiche, Università di Napoli "Federico II," Istituto Nazionale di Fisica Nucleare, Sezione di Napoli Mostra, d'Oltremare, Padia 20, 80125 Napoli, Italy

(Received 25 June 1993)

We discuss a model for interferometric gravitational-wave antennas without dissipative or active elements. It is predicted that the even and odd coherent states may play an alternative role to squeezed vacuum states in reducing the optimal power of the input laser.

PACS number(s): 42.62.−b, 42.25.Hz

I. INTRODUCTION

The problem of detecting gravitational waves has been a subject of interest for many years [1]. Specifically, the quantum sensitivity of Michelson interferometric gravitational-wave detection (GWD) has been described in detail by Caves [2]. An important ingredient improving the sensitivity of such detectors (GWD) is using the appropriate states of light beam through the two ports of the Michelson interferometer. Caves [2] showed, in fact, that if one uses coherent light [3] from the first port of the interferometer, then the optimal sensitivity is limited by the vacuum fluctuations which enter through the unused port of the interferometer. In such a setup the lower limit on the optimal power of the input laser comes out to be quite large and of no experimental interest. Caves [2] suggested to reduce considerably the above limit by squeezing the vacuum field entering through the unused port [4].

The main purpose of this paper is to answer the following question: Are there any other nonclassical states different from squeezed states which can replace squeezed vacuum in GWD for a better quantum sensitivity of the Michelson interferometer? We predict a possible positive answer to this question in the use of even or odd coherent states [5]. Even coherent states are closely related to the squeezed vacuum states because they too are superposition of even number states, but with different coefficients. Different nonclassical properties of even and odd coherent states (which are called also Schrödinger cat states) and theoretical predictions for their possible generation have been discussed in detail in Refs. [6–11].

Another problem which we want to analyze is the possibility of presenting the description of light beams in the interferometer in such a form as to have the factorized expression for the noise contribution containing separately geometrical parameters of the interferometer and the parameters characterizing the field state influence. It should be noted that the most general analysis of nonclassical states in interferometry was done by Yurke, McCall and Klauder [12]. Here we present a different approach the aim of which is to express the noise error as a product of two factors with tensorial-like structure, each of the factors being related to the geometry of interferomter and light states correspondingly. The problem of achieving sensitivities better than the standard quantum limit by correlating the radiation pressure and photon counting noises has been analyzed in recent papers [13–15].

II. INPUT AND OUTPUT BEAMS

In the Michelson interferometer (Fig. 1) we have two incoming fields through ports P_i, $i = 1,2$, described by the operators $(a_i, a_i^\dagger)$ acting on a Hilbert space $\mathcal{H}^a = \mathcal{H}_1^a \otimes \mathcal{H}_2^a$. To them correspond two fields at the mirrors M_i described by $(b_i, b_i^\dagger)$ acting on $\mathcal{H}^b = \mathcal{H}_1^b \otimes \mathcal{H}_2^b$ and two outgoing fields at P_i described by $(c_i, c_i^\dagger)$ on $\mathcal{H}^c = \mathcal{H}_1^c \otimes \mathcal{H}_2^c$. The basis in $\mathcal{H}_i^\rho$, $\rho = a, b, c$, will be denoted $\{| n, i, \rho\rangle, n \in \mathbb{Z}^+\}$. We simplify the Michelson interferometer as a device with two arms at the end of which two outer

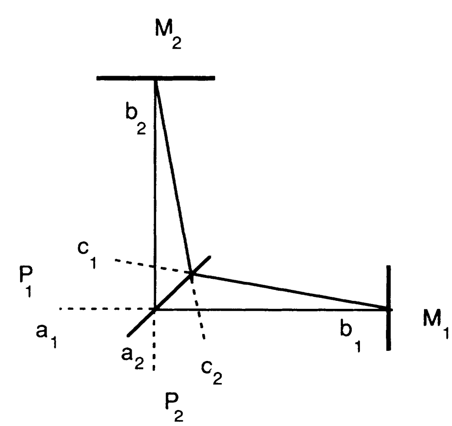

FIG. 1. Schemetic of the simple Michelson interferometer. a_1, a_2 and c_1, c_2 are, respectively, the input and output fields, while b_1, b_2 stand for the fields incident on mirrors M_1 and M_2.

*On leave from Lebedev Physics Institute, Moscow, Russia.

mirrors M_i are attached to some string, thus behaving as two pendula, without considering Fabry-Pérot cavities and beam delaying optics into these arms.The positions of the mirrors are controlled by the joint actions of the restoring forces and the radiation pressure [16]. We will suppose that in all processes the dissipative and active effects are negligible so that conservation of energy in ensured. The Hamiltonian in $\mathcal{H}^\rho$ is taken to be

$$H^\rho = \hbar\omega(\rho_1^\dagger\rho_1 + \rho_2^\dagger\rho_2) \tag{1}$$

with ω the frequency and h the Planck's constant. Implicit here is the assumption of equal frequencies for modes 1 and 2. This can be achieved by introducing some degree of interaction among the two modes, which anyhow can be ignored in a first approximation, as in Ref. [2]. All $\mathcal{H}^\rho$ are unitarily equivalent and the operators H^ρ are connected to each other by 2×2 unitary matrices, the elements of which depend on the physical and geometrical parameters of the interferometer. For instance, we will write

$$b = V\ a, \quad b^\dagger = a^\dagger\ V^\dagger\ , \tag{2}$$

where

$$a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}\ ;\ b = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}\ ,$$
$$a^\dagger = (a_1^\dagger\ \ a_2^\dagger)\ ;\ b^\dagger = (b_1^\dagger\ \ b_2^\dagger)\ , \tag{3}$$

and $V \in \mathrm{U}(2)$ group. We conveniently write

$$V = \Phi K \tag{4}$$

with

$$\Phi = \begin{pmatrix} e^{i\phi_1} & 0 \\ 0 & e^{i\phi_2} \end{pmatrix}\ ,$$
$$K = \begin{pmatrix} \alpha_1 & \beta_2 \\ \beta_1 & \alpha_2 \end{pmatrix}\ . \tag{5}$$

In the above α_i and β_i are the complex transmittivity and reflectivity parameters of the beam splitter (BS) arbitrarily oriented for the ith input field mode, respectively, and ϕ_i is the phase distance between BS and the mirror M_i. Also

$$c = \mathrm{U}\ a\ ;\ \ c^\dagger = a^\dagger \mathrm{U}^\dagger \tag{6}$$

and

$$\mathrm{U} = -K^T\Phi^2 K = -V^T V\ . \tag{7}$$

The presence of a negative sign in the above equation is due to the phase change on reflections at the mirrors. Thus all the information about influence of optical elements of the interferometer on the light acting on the mirrors is contained in the unitary matrix V described in a generic case by four independent parameters. The influence of the interferometer elements (beam splitter, mirrors, etc.) on the outgoing field is described by the unitary matrix U which in the case under consideration is completely determined by the matrix V due to the relation (7).

III. NOISE IN FACTORIZED FORM

Following [2], we have two sources of errors which set the lower quantum limit Δz on the sensitivity z of GW antennas: (i) radiation pressure (RP) on M_i and (ii) photon counting (PC) noise due to the fluctuations in the number of photons in the input fields

$$\Delta z = \sqrt{(\Delta z_{\mathrm{RP}})^2 + (\Delta z_{\mathrm{PC}})^2}\ . \tag{8}$$

In this

$$(\Delta z_{\mathrm{RP}})^2 = \sigma_{\mathrm{RP}}^2(\hbar\omega\tau/mc)^2\ , \tag{9}$$

where

$$\sigma_{\mathrm{RP}}^2 = \langle(b^\dagger\sigma_3 b)^2\rangle - \langle b^\dagger\sigma_3 b\rangle^2 \tag{10}$$

and

$$(\Delta z_{\mathrm{PC}})^2 = \sigma_{\mathrm{PC}}^2\left(\frac{\partial\langle c^\dagger\sigma_3 c\rangle}{\partial(\phi_2-\phi_1)}\right)^{-2}\ , \tag{11}$$

where

$$\sigma_{\mathrm{PC}}^2 = \langle(c^\dagger\sigma_3 c)^2\rangle - \langle c^\dagger\sigma_3 c\rangle^2\ . \tag{12}$$

In the above, τ is the observation time and m the mass of each end mirror. Here we consider fixed BS as in Ref. [2]. After a little algebra we can write

$$\sigma_{\mathrm{RP}}^2 = (\mathrm{U}^\dagger\sigma_3\mathrm{U})_{ik}(\mathrm{U}^\dagger\sigma_3\mathrm{U})_{mn}\ , T_{ikmn}\ ,$$
$$\sigma_{\mathrm{PC}}^2 = (V^\dagger\sigma_3 V)_{ik}(V^\dagger\sigma_3 V)_{mn}T_{ikmn} \tag{13}$$

with the summation over repeated indices taken from 1 to 2 and

$$T_{ikmn} = \langle a_i^\dagger a_k a_m^\dagger a_n\rangle - \langle ai^\dagger a_k\rangle\langle a_m^\dagger a_n\rangle\ . \tag{14}$$

This allows an easy comparison between situations arising from the use of different types of input fields. Combining Eqs. (8)–(13) yields

$$(\Delta z)^2 = X_{ikmn}\ T_{ikmn}\ (ikmn = 1,2)\ , \tag{15}$$

where X_{ikmn} contain the geometrical and physical properties of the antenna while the second factors T_{ikmn} depend only on the incoming fields.

For a simple Michelson interferometer, Caves suggested using squeezed vacuum light in order to minimize the input laser power [2]. Equation (15) permits us to investigate very general states of the input field like even and odd coherent states, correlated states, states with higher order squeezing, etc. In particular, we will illustrate in this communication the dependence of the optimal Δz on the characteristic parameters of the even or odd coherent states from the second port of the interferometer.

First, we will evaluate the matrix X_{ikmn} as far as the geometrical and physical parameters of the Michelson interferometer are concerned. If we consider a 50–50 % BS, then the elements of the matrix K are

$$\alpha_1 = \alpha_2 = \frac{e^{i\delta}}{\sqrt{2}}\,,$$
$$\beta_1 = \beta_2 = \frac{e^{i\mu}}{\sqrt{2}}\,, \tag{16}$$

where δ is the phase because of the BS which can be set to zero for an ideally thin BS while μ is the phase introduced by the BS between the reflected and transmitted waves and for simplicity we take $\mu = \pi/2$. Then

$$V^\dagger \sigma_3 V = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \tag{17}$$

and

$$\mathrm{U}^\dagger \sigma_3 \mathrm{U} = \begin{pmatrix} -\cos\phi & -\sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \tag{18}$$

in $\phi = 2(\phi_2 - \phi_1)$. Also

$$\langle c^\dagger \sigma_3 c\rangle = \langle a^\dagger \sigma_3 a\rangle \cos\phi\ . \tag{19}$$

If the interferometer has to operate in a dark fringe then the arm's lengths can be adjusted to have $\phi = \frac{(2n+1)\pi}{2}$ and dark fringes correspond to the situation where no field contributions are present in the difference of the output photon numbers. In such cases we have

$$\mathrm{U}^\dagger \sigma_3 \mathrm{U} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\ . \tag{20}$$

Then X_{ikmn} become

$$X_{1212} = X_{2121} = -A^2 + B^2\,,$$
$$X_{1221} = X_{2112} = A^2 + B^2\,, \tag{21}$$

where

$$A = \left(\frac{\hbar\omega\tau}{mc}\right)\,,$$
$$B = \left(\frac{\partial I}{\partial Z}\right)^{-1} \tag{22}$$

and

$$I = \langle c^\dagger \sigma_3 c\rangle\,,$$
$$Z = \phi\frac{c}{2\omega}\ . \tag{23}$$

The variable Z corresponds to the difference between the displacements of the two outer mirrors caused by the radiation pressure with respect to their mean positions in the absence of any field.

IV. FIELD STATE PARAMETERS

We will now evaluate the factors T_{ikmn} in (i) Caves' setup and (ii) a new one which replaces the squeezed light with even or odd coherent light. In order to evaluate the contribution of the fields which are applied to the two ports of the interferometer for GW detection, we will assume a coherent light for the field relative to port 1 of the interferometer while for the second port we will consider the two situations (i) by squeezing the vacuum fluctuations (the situation considered by the Caves [2]) and (ii) by applying even or odd coherent states. We will show the important role played by these states in order to get a better detection sensitivity and to reduce the optimal input laser power.

When coherent laser light from port one and squeezed vacuum from the other port of the interferometer are applied, the two fields are anticorrelated. The states of $\mathcal{H}^a$ can be written as

$$\mid\psi\rangle = \mathcal{D}_1(\alpha) \mid 0,1,a\rangle e^{\frac{\xi a_2^{\dagger 2} - \xi^* a_2^2}{2}} \mid 0,2,a\rangle\ , \tag{24}$$

where $\mathcal{D}_i(\alpha) = e^{(\alpha a_i^\dagger - \alpha^* a_i)}$ i=1,2, $\alpha \in \mathbb{C}$, and $\xi = re^{i\theta_1}$. It is easy to see that in such states $\langle a_1 a_2\rangle$,$\langle a_1^\dagger a_2\rangle$, etc., are equal to zero.

If we take α to be real for simplicity then we have

$$\begin{aligned}
T_{1111} &= \alpha^2\,,\\
T_{1122} &= 0\,,\\
T_{1212} &= -\alpha^2 \sinh r \cosh r\, e^{i\theta_1}\,,\\
T_{1221} &= \alpha^2 \sinh^2 r + \alpha^2\,,\\
T_{2112} &= \alpha^2 \sinh^2 r + \sinh^2 r\,,\\
T_{2121} &= -\alpha^2 \sinh r \cosh r\, e^{-i\theta_1}\,,\\
T_{2211} &= 0\,,\\
T_{2222} &= 2\sinh^2 r\ .
\end{aligned} \tag{25}$$

Thus we have calculated noise factor depending on the input field state for the case of coherent light at port one and the squeezed vaccuum light at the other port. This tensorial-like factor corresponds to the case considered in Ref. [2].

V. FIELD FACTOR FOR EVEN AND ODD COHERENT STATES

When even or odd coherent states replace squeezed vacuum in port two the states of $\mathcal{H}^a$ to be taken into account are, with $\beta \in \mathbb{C}$,

$$\begin{aligned}\mid\psi\rangle &= \mid \alpha, \beta_\pm\rangle\\ &= \mathcal{D}_1(\alpha) \mid 0,1,a\rangle N_\pm[\mathcal{D}_2(\beta) \pm \mathcal{D}_2(-\beta)] \mid 0,2,a\rangle\ ,\end{aligned} \tag{26}$$

where $+,-$ signs correspond to even and odd coherent states, respectively, and their normalization constants are

$$N_+ = \frac{1}{2e^{-\frac{|\beta|^2}{2}}\sqrt{\cosh\mid\beta\mid^2}}\,,$$
$$N_- = \frac{1}{2e^{-\frac{|\beta|^2}{2}}\sqrt{\sinh\mid\beta\mid^2}}\ . \tag{27}$$

For the even light, coefficients T_{ikmn} take the following values:

$$\begin{aligned}
T_{1111} &= \alpha^2\,,\\
T_{1122} &= 0\\
T_{1212} &= \alpha^2 \mid\beta\mid^2 e^{2i\theta_2}\,,\\
T_{1221} &= \alpha^2 \mid\beta\mid^2 \tanh\mid\beta\mid^2 + \alpha^2\,,\\
T_{2112} &= \alpha^2 \mid\beta\mid^2 \tanh\mid\beta\mid^2 + \mid\beta\mid^2 \tanh\mid\beta\mid^2\,,\\
T_{2121} &= \alpha^2 \mid\beta\mid^2 e^{-2i\theta_2}\,,\\
T_{2211} &= 0\,,\\
T_{2222} &= \mid\beta\mid^4 - \mid\beta\mid^4 \tanh^2\ \mid\beta\mid^2 + \mid\beta\mid^2 \tanh\mid\beta\mid^2\,,
\end{aligned} \tag{28}$$

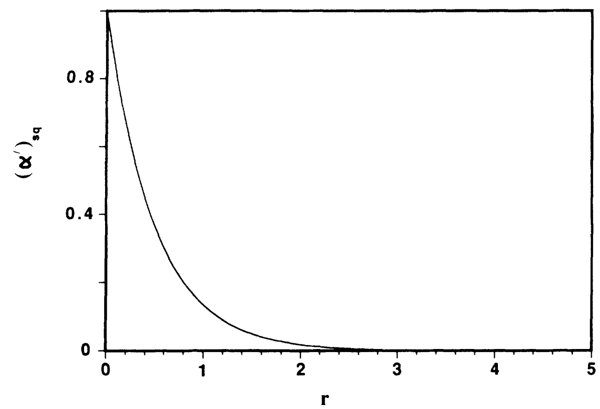

FIG. 2. Relative value $(\alpha')_{\rm sq}$ of the optimal laser intensity in the presence and of the squeezed vacuum mode a_2 versus the squeezing parameter r.

in which θ_2 is the phase of β. For odd coherent states we get the same expressions as above, except that $\tanh \mid \beta \mid^2$ should be replaced by $\coth \mid \beta \mid^2$.

VI. NOISE FOR EVEN AND ODD COHERENT STATES

The general expression for $(\Delta z)^2$, irrespective of the nature of the incoming fields, can now be written as

$$(\Delta z^2) = A^2(T_{1221} + T_{2112} - T_{1212} - T_{2121}) \\ +B^2(T_{1221} + T_{2112} + T_{1212} + T_{2121}) \ . \qquad (29)$$

This quantity depends on the incoming field through P_1 and we denote by $(\alpha^2_{\rm opt})^{(o)} = mc^2/(2\hbar\omega\tau)$ the intensity of this field which minimizes $(\Delta z)^2$ when at P_2 the ordinary vacuum is present. Then it can be seen that the value which minimizes $(\Delta z)^2$, which we call $(\alpha^2_{\rm opt})^{(\rm sq)}$, in presence of squeezed vacuum at P_2, under the condition $\alpha^2 >> \sinh^2 r$ and $\theta_1 = 0$, is

$$(\alpha^2_{\rm opt})^{(\rm sq)} = e^{-2r} \ (\alpha^2_{\rm opt})^{(o)} \ . \qquad (30)$$

This is the Caves result, which allows us to reduce the intensity of the input laser beam to values experimentally significant.

The analogous analysis for the cases of even or odd coherent states replacing squeezed vacuum, under the condition that $\alpha^2 >> \mid \beta \mid^2 \tanh \mid \beta \mid^2$, gives

$$(\alpha^2_{\rm opt})^{(\rm ev)} \\ = \sqrt{\frac{2 \mid \beta \mid^2 \tanh \mid \beta \mid^2 + 2 \mid \beta \mid^2 \cos 2\theta + 1}{2 \mid \beta \mid^2 \tanh \mid \beta \mid^2 - 2 \mid \beta \mid^2 \cos 2\theta + 1}} (\alpha^2_{\rm opt})^{(o)} \qquad (31)$$

and

$$(\alpha^2_{\rm opt})^{(\rm odd)} \\ = \sqrt{\frac{2 \mid \beta \mid^2 \coth \mid \beta \mid^2 + 2 \mid \beta \mid^2 \cos 2\theta + 1}{2 \mid \beta \mid^2 \coth \mid \beta \mid^2 - 2 \mid \beta \mid^2 \cos 2\theta + 1}} (\alpha^2_{\rm opt})^{(o)} \ . \qquad (32)$$

Thus using even coherent light, under the limit $1 \ll \mid \beta \mid^2 \ll \alpha^2$ and $\theta_2 = \pi/2$, yields

$$(\alpha^2_{\rm opt})^{(\rm ev)} = \frac{(\alpha^2_{\rm opt})^{(o)}}{2 \mid \beta \mid} \ . \qquad (33)$$

This result, which is also true for odd coherent states, allows an alternative way to decrease the optimal input power, i.e., an alternative way to increase the sensitivity of the interferometer. We have, therefore, given a positive answer to the question originally posed. The question whether this new way might be experimentally achievable or not is left open, depending on the actual physical generation of the even and odd coherent states.

We wish now to consider more general situations: namely for case (i) ξ is arbitrary and for case (ii) β is arbitrary and in both situations α is real. $(\Delta z)^2$ is a function of such variables and we can look for its minimization with respect to α^2. This results for case (i) in $\theta_1 = 0$ and $(\alpha^2_{\rm opt})^{(\rm sq)}$ function of r and for case (ii) in

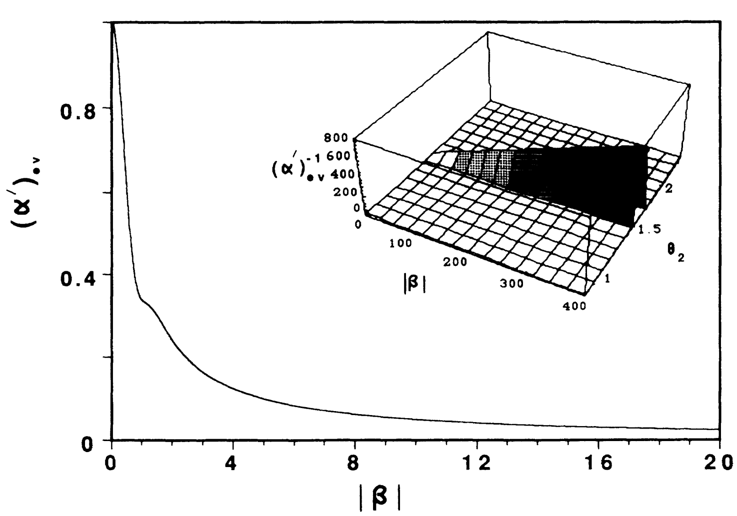

FIG. 3. (a) Three dimensional plot of the relative value $(\alpha')^{-1}_{\rm ev}$ of the optimal laser intensity in the presence of even coherent states versus $\mid \beta \mid$ and θ_2. (b) $(\alpha')_{\rm ev}$ versus $\mid \beta \mid$ for $\theta_2 = \pi/2$.

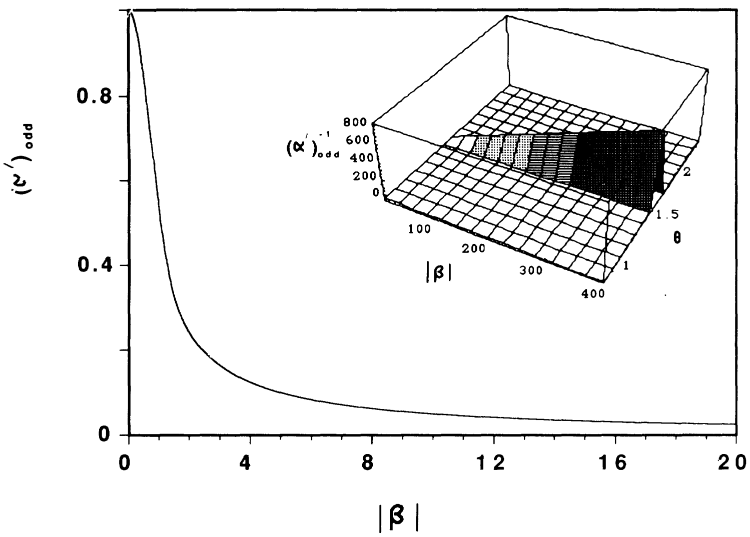

FIG. 4. (a) Three dimensional plot of the relative value $(\alpha')^{-1}_{\rm odd}$ of the optimal laser intensity in the presence and in the absence of odd coherent states versus $|\beta|$ and θ_2. (b) $(\alpha')_{\rm odd}$ versus $|\beta|$ for $\theta_2 = \pi/2$.

$\theta_2 = \pi/2$ and $(\alpha^2_{\rm opt})^{(\rm ev)}$, $(\alpha^2_{\rm opt})^{(\rm odd)}$ functions of $|\beta|$. In each case, $\alpha^2_{\rm opt}$ as function of the respective independent variable is given through a solution of the following equation in ξ:

$$\Gamma_1\xi^3 - \Gamma_2\xi - 2\Gamma_3(1+\Gamma_2) = 0 \tag{34}$$

in which

$$\xi = \frac{\alpha^2_{\rm opt}}{(\alpha^2_{\rm opt})^{(o)}} - \Gamma_3 \tag{35}$$

and

$$\Gamma_1 = e^{2r}\,,\quad \Gamma_2 = e^{-2r}\,,\quad \Gamma_3 = \frac{\sinh^2 r}{(\alpha^2_{\rm opt})^{(o)}} \tag{36}$$

for the squeezed vacuum, and

$$\begin{aligned}\Gamma_1 &= 2\,|\beta|^2 \tanh|\beta|^2 - 2\,|\beta|^2\cos 2\theta_2 + 1\,,\\ \Gamma_2 &= 2\,|\beta|^2 \tanh|\beta|^2 + 2\,|\beta|^2\cos 2\theta_2 + 1\,,\\ \Gamma_3 &= \frac{|\beta|^2\tanh|\beta|^2}{(\alpha^2_{\rm opt})^{(o)}}\end{aligned} \tag{37}$$

for the even coherent states. For the quantities relative to odd coherent states the same formulas apply with coth $|\beta|^2$ in place of tanh $|\beta|^2$.

Equation (34) has three roots, two of them complex and the physical one has the following form:

$$\begin{aligned}\alpha' &\equiv \frac{\alpha^2_{\rm opt}}{(\alpha^2_{\rm opt})^{o}} = \frac{3^{\frac{3}{2}}\sqrt{\Gamma_1}\Gamma_4\Gamma_3 + 9\Gamma_2 + \Gamma_4^2}{3^{\frac{3}{2}}\sqrt{\Gamma_1}\Gamma_4}\,,\\ \Gamma_4 &= \Big[9\Gamma_3\sqrt{\Gamma_1}(1+\Gamma_2)\\ &\quad + \sqrt{81\Gamma_1\Gamma_3^2(1+\Gamma_2)^2 - \Gamma_2^3}\Big]^{\frac{1}{3}}.\end{aligned} \tag{38}$$

Figure (2), illustrates case (i) and α' is plotted versus the squeezing parameter r. For $r = 0$ we have the situation in which the only vacuum fluctuations enter from P_2 and in this case $\alpha'=1$. For large values of r, we have Caves' result [2], i.e., the optimal value of the input coherent field through P_1 can be decreased to a large amount.

With the same spirit, in Fig. 3(a), we have plotted $(\alpha')^{-1}_{\rm ev}$ versus $|\beta|$ and θ_2 and in Fig. 3(b) this quantity versus $|\beta|$ at $\theta_2 = \pi/2$, which is the value for all the local minima, when even coherent light enters from port 2. The analogous graphical analysis for odd coherent states is shown in Figs. 4(a) and 4(b). Such figures illustrate how the optimal values of the input coherent field in P_1 can be reduced considerably and allows us to predict an application of even or odd coherent fields.

In conclusion it has been shown that such states might offer a new technique to reduce the optimal power of the input coherent laser and for a better sensitivity of the interferometer. In frame of suggested formalism it is possible to analyze the sensitivity of interferometric gravitational wave antenna for other electromagnetic field states. We will do it in future presentations.

ACKNOWLEDGMENTS

M.A.M. and V.I.M. wish to thank the Dipartimento di Scienze Fisiche, of the Università di Napoli and INFN for the kind hospitality. The research of N.A.A. was supported by the International Center for Theoretical Physics Programme for Research and Training in Italian Laboratories, Trieste, Italy.

[1] See, for example, *The Detection of Gravitational Waves*, edited by D.G. Blair (Cambridge University Press, Cambridge, England, 1991), and the references therein.

[2] C.M. Caves, Phys. Rev. A **23**, (1981) 1693; Phys. Rev. Lett. **45**, 75 (1980).

[3] R.J. Glauber, Phys. Rev. Lett. **10**, 84 (1963).

[4] D. Stoler, Phys. Rev. D **4** 2309 (1971); J.N. Hollenhorst, Phys. Rev. D **19**, 1669 (1979); E.Y. Lu, Lett. Nuovo Cimento **3**, 585 (1972); D.P. Bertrand, K. Moy, and E.A. Mishkin, Phys. Rev. D **4**, 1909 (1971); H.P. Yuen, Phys.

Rev. A **13**, 2226 (1976); D.F. Walls, Nature **306**, 141 (1983).
[5] V.V. Dodonov, I.A. Malkin, and V.I. Man'ko, Physica **72**, 597 (1974).
[6] J. Gea-Banacloche, Phys. Rev. A **44**, 5913 (1991).
[7] C.C. Gerry and E.E. Hach III, Phys. Lett. A **74**, 185 (1993).
[8] V. Buzek, A. Vidiella-Barranco, and P.L. Knight, Phys. Rev. A **45**, 6570 (1992).
[9] J. Jansky and A.V. Vinogardov, Phys. Rev. Lett. **64**, 2771 (1990).
[10] J. Perina, *Quantum Statistics of Linear and Non-Linear Optical Phenomena* (Reidel, Dordrecht, 1984), p. 78.
[11] C.C. Gerry, Opt. Commun. **91**, 247 (1992).
[12] B. Yurke, S.L. McCall and J.R. Klauder, Phys. Rev. A **33**, 4033 (1986).
[13] M.T. Jackel and S. Reynaud, Europhys. Lett. **13**, 301 (1990).
[14] A. Luis and L.L. Sanchez-Soto, Opt. Commun. **89**, 140 (1992).
[15] A. Luis and L.L. Sanchez-Soto, Phys. Rev. A **45**, 8228 (1992).
[16] S. Solimeno, F. Barone, C. de Lisio, L. Di Fiore, L. Milano, and G. Russo, Phys. Rev. A **43**, 6227 (1991).

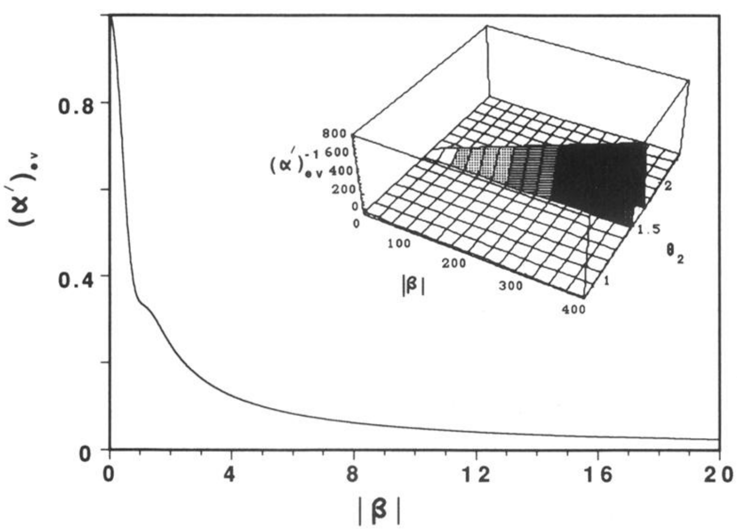

FIG. 3. (a) Three dimensional plot of the relative value $(\alpha')_{\mathrm{ev}}^{-1}$ of the optimal laser intensity in the presence of even coherent states versus $|\beta|$ and θ_2. (b) $(\alpha')_{\mathrm{ev}}$ versus $|\beta|$ for $\theta_2 = \pi/2$.

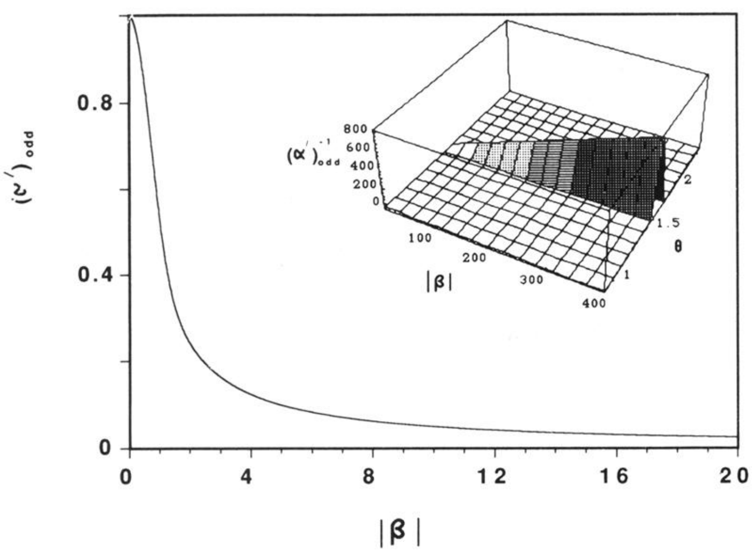

FIG. 4. (a) Three dimensional plot of the relative value $(\alpha')^{-1}_{\text{odd}}$ of the optimal laser intensity in the presence and in the absence of odd coherent states versus $|\ \beta\ |$ and θ_2. (b) $(\alpha')_{\text{odd}}$ versus $|\ \beta\ |$ for $\theta_2 = \pi/2$.