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Quantum Zeno effect in a double-well potential: A model of a physical measurement

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In this paper we study the quantum Zeno effect in real space due to a position measurement. The motion of a particle is decelerated or comes to a complete stop when its position is observed. Instead of using the von Neumann collapse hypothesis, we treat a real measurement process. The measurement consists of coupling a two-level atom in a double-well potential to a resonant laser beam. Subsequent resonance fluorescence can be used to determine the atom's position within the double well, provided the laser wavelength is short enough to ensure a resolvable scattering pattern of the fluorescence photons. Treating this process in the framework of dissipative quantum mechanics, we derive a master equation which describes the measurement process in all relevant details. Solving the master equation analytically as well as numerically we study the conditions for the decay of the nondiagonal elements. This leads directly to the inhibition of the center-of-mass motion, i.e., to a quantum Zeno effect. Because we treat the measurement process in detail we are able to investigate the conditions for a complete measurement. In particular, we study the role of the intensity and the wavelength of the probing laser field.

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I. INTRODUCTION

In quantum mechanics observation is a very subtle phenomenon: Even the most careful of all measurements will inevitably leave its trace on the observed system which is traditionally termed the "collapse of the wave function." In this paper we investigate the influence of the quantum-mechanical measurement process upon the dynamics of an observed system. The quantum Zeno effect describes the inhibition of a system's dynamic evolution when it is subjected to ideal measurements [1]. It is the apparently absurd consequence of the quantum Zeno effect that the supposed innocent observation will bring the dynamics to a halt.

In the framework of standard measurement theory this effect heavily relies on the von Neumann axiom of quantum mechanics [2]: a measurement projects the object onto one of its eigenstates. Since the interaction between object and apparatus are generally far to complex to be treated in detail, this postulate must remain only phenomenologically justified. This poses the question whether the quantum Zeno effect is only a consequence of the too idealized von Neumann approach. It is the purpose of this paper to investigate this problem in the case of a position measurement.

Cook proposed a scheme for studying the quantum Zeno effect in the V-shaped three-level system using energy measurements [3]. Experimental investigation by Itano *et al.* [4] as well as theoretical studies by Frerichs and Schenzle [5] and Block and Berman [6] confirmed the occurrence of a quantum Zeno effect in the V-shaped three-level system. In this paper we study position measurements and the corresponding Zeno effect. We will not use the collapse hypothesis, instead we will treat a particular real measurement process: Using dissipative quantum mechanics we investigate the nonselective measurement of an atom's position in a double well by a resonant laser beam and subsequent fluorescence. We will show that there is indeed a quantum Zeno effect which slows down the coherent tunneling of the center of mass between the left and right well. In the limit of a very fast response of the "apparatus" this motion stops altogether.

This result is especially interesting since Fearn and Lamb [7] denied the occurrence of a quantum Zeno effect due to the observation of a particle in a double-well potential. However, their approach to the problem of modeling a measurement is neither based on a first-principles treatment of the measurement as it is performed here, nor is it related to any real measurement interaction as it is actually used in the experiment. After this short introduction we summarize in Sec. II the dynamics of our object, i.e., the center-of-mass motion of a two-level atom in a double-well potential, and its interaction with the apparatus. Studying all relevant interactions we derive a general master equation which we simplify for our purposes in order to obtain a tractable equation. This is treated in Sec. III. Solving it analytically as well as numerically we find the expected inhibition of the system's center-of-mass motion. This is a quantum Zeno effect due to position measurements. Our approach enables us to treat the explicit dependence of the quantum Zeno effect on the experimental parameters. In Sec. IV we draw some general conjectures for measurement theory from the way measurement is handled here.

II. THE QUANTUM ZENO EFFECT IN COORDINATE SPACE

The idea of a quantum Zeno effect in coordinate space is to measure the position of a moving particle. If these measurements are performed in a dense sequence we expect the particle to be inhibited in its motion. To study this effect in greater detail using a real measurement process we would have to couple all possible position states to a measurement apparatus. However, in free space this would involve an infinite number of degrees of freedom. Here, in order to demonstrate the principle, we investigate the quantum Zeno effect in a double-well potential. However, the generalization to an extended multiwell structure is straightforward.

A. The double-well potential

The two lowest states of the symmetric double-well potential are the symmetric ground state $|s\rangle$ and the antisymmetric first excited state $|a\rangle$. We can superpose these two states in a symmetric (antisymmetric) combination. The resulting rate $|L\rangle$ ($|R\rangle$) is mainly concentrated in the left (right) well. Their dynamics exhibits "coherent tunneling," i.e., oscillations between the left and right well. If we choose the left state $|L\rangle$ as the initial state $|\psi,t=0\rangle$, then we find for the time evolution of $|\psi,t\rangle$:

$$|\psi,t\rangle = e^{-\phi(t)}\left\{|L\rangle\cos\left[\frac{T}{\hbar}t\right] + i|R\rangle\sin\left[\frac{T}{\hbar}t\right]\right\}. \quad (1)$$

Here, $\phi(t)=(E_a+E_s)/2\hbar$, is a physically irrelevant phase, whereas $T/\hbar=(E_a-E_s)/2\hbar$, the tunneling frequency, will be used as a central parameter in the following.

The basic idea of the quantum Zeno effect in the double well is to continuously monitor the coherent tunneling and study the measurement induced inhibition of the dynamics. We will use a realistic physical process to realize the monitoring. Suppose we place a two-level atom in the double-well potential. If we illuminate the potential with a resonant laser beam the two-level atom is electronically excited. Provided the spontaneous decay rate in the two-level atom is large compared to the tunneling frequency, many resonance fluorescence photons are emitted before the atom starts to move. If the fluorescence wavelength is shorter than the separation of the wells it is possible to conclude whether the two-level atom is in the left or right well by observing the origin of the fluorescence (cf. Fig. 1).

The physical process of illuminating the potential and subsequently observing the spontaneously emitted photon is more than a straightforward way to model a measurement of coherent tunneling. Rather, it is a real measurement, which we can describe in all its important features. In the following sections we will formulate the problem in more detail and we will derive and solve the corresponding master equation. This solution will answer the question whether there indeed is a quantum Zeno effect in the double-well potential.

B. Measurement interactions

In order to study the quantum Zeno effect in a double-well potential it is necessary to treat the combined system of the object interacting with the apparatus. In our case the object is the center of mass of the two-level atom moving in a one-dimensional double-well potential. We can separate the object dynamics in a longitudinal part along the direction of the potential minima and in a transverse part perpendicular to this direction. The longitudinal motion can be described by

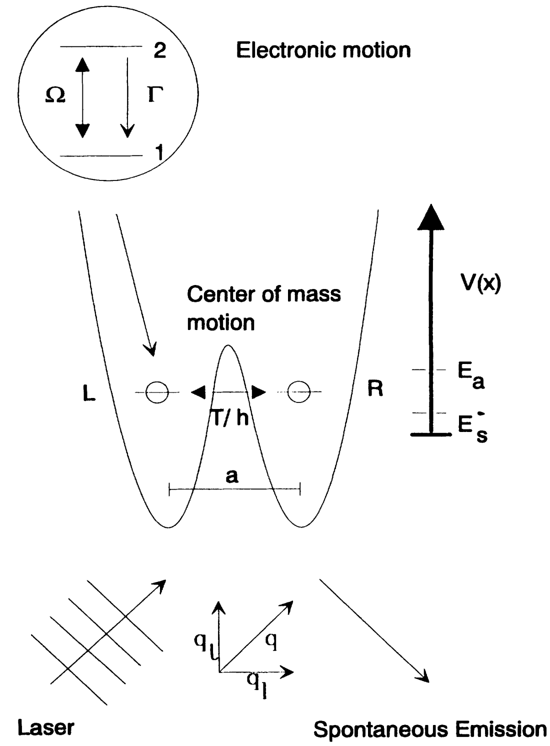

FIG. 1. The principle setup for the quantum Zeno effect in a double-well potential, using a two-level atom.

$$H_{\text{tunnel}}=E_L|L\rangle\langle L|+E_R|R\rangle\langle R|+T|L\rangle\langle R|+T|R\rangle\langle L|. \quad (2)$$

Since we regard the perpendicular motion as essentially free we will use a basis of two-dimensional plane-wave states $|\mathbf{k}_t\rangle$ for the transverse center-of-mass motion. The corresponding Hamiltonian H_{trans} is given by the kinetic energy of the transverse states.

The apparatus consists of the two electronic levels $\{|1\rangle,|2\rangle\}$ and all the modes of the electromagnetic field $\{|\mathbf{q},\sigma\rangle\}$. Here, $\mathbf{q}$ is the photon's momentum and σ its polarization. The apparatus is described by the free electronic Hamiltonian H_{elec} and the free field Hamiltonian H_{field}.

The measurement interactions are the interaction between the two-level atom and the laser field as well as the interaction with the vacuum field, which causes the atom to decay spontaneously. We will treat the coupling between the atom and the light field in the electric-dipole approximation. We express the electric field in the form

$$\mathbf{E}(\mathbf{r})=\sum_{q,\sigma}E_q^0\epsilon_{q\sigma}(a_{q\sigma}e^{i\mathbf{q}\cdot\mathbf{R}}+a_{q\sigma}^\dagger e^{-i\mathbf{q}\cdot\mathbf{R}}) . \quad (3)$$

Here, $\epsilon_{q\sigma}$ is the polarization vector and E_q^0 is the electric field per photon. Note that we couple the photon's momentum to the center-of-mass motion of the atom by the exponential $e^{i\mathbf{q}\cdot\mathbf{R}}$, $\mathbf{R}$ being the center-of-mass coordinate. Because we differentiate between a longitudinal and a transverse center-of-mass motion, it is useful to separate the photon's momentum in a transverse and a longitudinal part,

$$\mathbf{q}=\mathbf{q}_t+\mathbf{q}_l \ . \tag{4}$$

Thus, we can write for the interaction between the light field and the atom,

$$H_{\text{light-atom}}=-\sum_{i,j,q,\sigma} E_q^0|i\rangle\boldsymbol{\mu}_{ij}\cdot\boldsymbol{\epsilon}_{q\sigma}\langle j|\left[a_{q\sigma}\int d^2\mathbf{k}_t|\mathbf{k}_t\rangle\langle\mathbf{k}_t-\mathbf{q}_t|\sum_{M,M'}|M\rangle S_{MM'}(q_1)\langle M'| + a_{q\sigma}^\dagger\int d^2\mathbf{k}_t|\mathbf{k}_t\rangle\langle\mathbf{k}_t+\mathbf{q}_t|\sum_{M,M'}|M\rangle S_{MM'}(-q_1)\langle M'|\right] \ . \tag{5}$$

Here, $\boldsymbol{\mu}_{ij}$ is the matrix element of the dipole operator between the electronic states. The Hamiltonian $H_{\text{light-atom}}$ includes the transfer of the photon's momentum due to absorption and emission. In the following we will especially pay attention at the operator $S(q_l)$ and its matrix elements which describe the longitudinal transfer of momentum,

$$S(q_l)=e^{iq_lx} \ , \tag{6}$$

$$S_{MM'}(q_l)=\langle M|S(q_l)|M'\rangle=\int_{-\infty}^{+\infty}dx\,e^{iq_lx}\psi_M^*(x)\psi_{M'}(x) \ . \tag{7}$$

We will refer to $S_{MM'}(q_l)$ as the matrix element of the boost operator.

The interaction between the external laser field and atom is treated semiclassically as

$$H_{\text{laser-atom}}=-\hbar\Omega\int d^2\mathbf{k}_t\sum_{M,M'}(e^{-i\omega_Q t}|2\rangle\langle 1|\,|\mathbf{k}_t\rangle\langle\mathbf{k}_t-\mathbf{Q}_t|\,|M\rangle S_{MM'}(Q_l)\langle M'|+\text{H.c.}) \ , \tag{8}$$

Here, Ω is the Rabi frequency induced by the laser field, E is the classical field amplitude, $\mathbf{Q}$ is the wave vector, and $\boldsymbol{\Sigma}$ is the polarization of the laser mode

$$\Omega=\frac{1}{\hbar}E\boldsymbol{\mu}_{ij}\cdot\boldsymbol{\epsilon}_{Q\Sigma} \ . \tag{9}$$

C. A general master equation

From all the ingredients derived in the previous section we could construct a reversible von Neumann equation for our problem. However, we are not interested in the infinite number of degrees of freedom that are involved in the evolution of the electromagnetic field because we investigate nonselective measurements. Therefore, we will remove these variables from our problem by an adiabatic elimination procedure which will lead to a master equation for the reduced density operator of the atomic system only. Of course, the reduced density operator refers to an ensemble average over the reservoir. However, quantum mechanics is essentially a statistical theory. For instance, a wave function only refers to an ensemble of identically prepared systems. All predictions which can be inferred from this wave function can only be tested experimentally using such an ensemble. However, in our example the possible meter readings are continuously distributed. Therefore, if we find a particular meter reading, i.e., a particular state of the reservoir realized as consequence of the measurement interaction, this is a unique event. There is no ensemble which corresponds to this wave function. Hence, any predictions which are experimentally verifiable must be inferred from a stochastic ensemble of not identically prepared systems, as given by the reduced density operator. This, in turn, is obtained by the solution of the master equation. Since physics as a science only makes statements about phenomena which are, in principle, observable, the description of any physical meaningful effect regarding the small system can be deduced from the reduced density operator.

We think of our system as divisible in a small system (the atom) and a large system (the field modes). In the following we refer to the atom by the subscript A and to the heat bath of the field modes by the subscript R (for the reservoir). The division yields

$$H_A=H_{\text{elec}}+H_{\text{tunnel}}+H_{\text{trans}}+H_{\text{laser-atom}} \ , \tag{10}$$

$$H_R=H_{\text{field}} \ , \tag{11}$$

$$V_I=H_{\text{light-atom}} \ , \tag{12}$$

Here, V_I is the interaction which mediates the influence of the bath to the atom. We define the bath operators as R^p, the atomic operators as A^p, and p as a multiple index. Comparison with Eq. (5) yields

$$R^p=E_q^0a_{q\sigma}a_{q\sigma}, \quad A^p=|i\rangle\langle j|\,|\mathbf{k}_t\rangle\langle\mathbf{k}_t-\mathbf{q}_t|\,|M\rangle S_{MM'}(q_l)\langle M'| \quad \text{if } p=\{q,\sigma,\mathbf{k}_t,i,j,M,M',-\} \tag{13}$$

and

$$R^p=E_q^0a_{q\sigma}^\dagger, \quad A^p=|i\rangle\langle j|\,|\mathbf{k}_t\rangle\langle\mathbf{k}_t+\mathbf{q}_t|\,|M\rangle S_{MM'}(q_l)\langle M'| \quad \text{if } p=\{q,\sigma,\mathbf{k}_t,i,j,M,M',+\} \ . \tag{14}$$

The "−" sign ("+" sign) corresponds to the absorption (emission) of a momentum $\mathbf{k}_t$ in Eq. (5).

Using these abbreviations the general master equation in an interaction picture for H_A ($\tilde{\rho}_A = \mathrm{Tr}_R \tilde{\rho}$) reads as

$$\frac{\partial}{\partial t}\tilde{\rho}_A(t) = -\frac{1}{\hbar^2}\int_0^\infty dt' \Bigg[\sum_{p,p'} \mathrm{Tr}_R[\tilde{R}^{p}(t)\tilde{R}^{p'}(t-t')\rho_R(0)]\{\tilde{A}^{p}(t)\tilde{A}^{p'}(t-t')\tilde{\rho}_A(t) - \tilde{A}^{p'}(t-t')\tilde{\rho}_A(t)\tilde{A}^{p}(t)\} + \sum_{p,p'} \mathrm{Tr}_R[\rho_R(0)\tilde{R}^{p}(t-t')\tilde{R}^{p'}(t)]\{\tilde{\rho}_A(t)\tilde{A}^{p}(t-t')\tilde{A}^{p'}(t) - \tilde{A}^{p'}(t)\tilde{\rho}_A(t)\tilde{A}^{p}(t-t')\} \Bigg] . \quad (15)$$

In this equation the bath operators only occur in separate factors which can be evaluated to give the correlation function of the bath. As pointed out in Appendix A, the correlation time is approximately a/c (a: well separation). The finite correlation time is due to the inclusion of recoil effects in the terms in curly brackets in Eq. (15). Nevertheless, the correlations between the bath and the atom in the double well will vanish rapidly on the relevant atomic time scale.

In principle, we can now execute the time integrals to calculate relaxation rates and Lamb-shift terms. However, since all transverse states as well as all longitudinal states are coupled by the transfer of momentum, the resulting system of differential equations would contain an infinite number of equations. Therefore it is convenient to restrict the space in which the master equations acts. In the next section we will motivate such a restriction.

D. The restriction of the master equation

In order to come to a tractable master equation we have to remove most of the participating states from Eq. (15). In the first place we are interested in the measurement's influence upon the dynamics of the lower longitudinal states. Therefore we average over all transverse states. Furthermore, under certain conditions we can remove the higher longitudinal states.

1. Elimination of the transverse degrees of freedom

We average the master equation by tracing over the transverse space. Performing the trace we will find in the irreversible part of the master equation only diagonal elements of the transverse states, while we find in the reversible part diagonal as well as nondiagonal elements of the transverse states. We replace all matrix elements by a unique average value. As pointed out in Appendix B, in the case of the nondiagonal elements this involves the essentially semiclassical condition $Q_t \ll k_t$, i.e., the transverse transfer of the laser photon momentum has to be small compared to typical values of the atomic momentum. This relation can be fulfilled for wavelengths smaller than 0.1 μm even for small atomic numbers (cf. Appendix B).

2. Restriction of the longitudinal space

In our general master equation (15) all longitudinal states are coupled by matrix elements from Eq. (7). Physically, this means that the recoil due to absorption or emission of a photon excites all other center-of-mass states in the double-well potential. To understand the relevance of the transfer of momentum for the shape of the kicked wave function, imagine a wave function in momentum space. Kicking this wave function by a momentum q amounts to shifting the wave function by q, thereby preserving its shape in momentum space. It is clear that in the case of very deep wells in our potential the corresponding states $|L\rangle$ and $|R\rangle$ will be very well localized in one of the wells. This implies a very broad momentum spectrum, which again means that the kicked wave function will hardly change. Thus, if we limit our considerations to relatively deep wells it should be possible to neglect the coupling to higher eigenstates of the potential in the master equation.

In Appendix C we derive conditions for the validity of two-state and four-state approximations. In a two-state approximation it must generally be valid that

$$1 \simeq |S_{LL}(q_l)|^2 + |S_{LR}(q_l)|^2 . \quad (16)$$

Figure 2 illustrates that Eq. (16) is approximately valid for deep potentials and $q < 2q_0$ ($q_0 \equiv 2\pi/a$). As shown in Appendix C this requires the parameter $b^2 = 2(\sigma q_l/2)^2$ to be small compared to unity (σ is the width of the wave function). Analogously, a four-state approximation requires $b^4 \ll 1$.

We have done calculations in a two-state approximation as well as a four-state approximation. For potentials which are not too shallow the essential results are the same [8]. However, from these studies we find another argument which encourages us to perform a "few-state" approximation. Comparison of the rates within a four-state system reveals that the rates connecting the upper pair of states with the lower pair are diminished by the factor b^2 compared to the rates within the upper, or respectively, lower pair of states. Thus, if we choose the duration of the laser pulse to lie between these two time

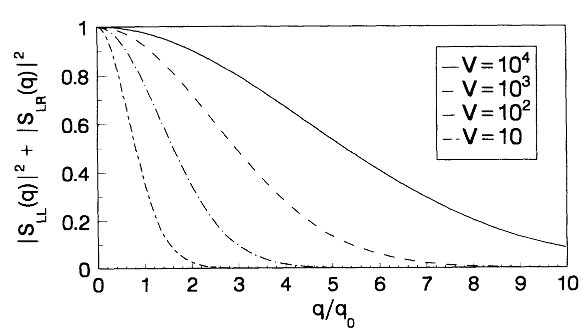

FIG. 2. The norm of the kicked wave function in a two-state approximation for four different potentials [$V = V_0(-2x^2 + x^4)$, $V_0 = 10$, 10^2, 10^3, and 10^4] versus the momentum transfer q in units $q \equiv 2\pi/a$.

scales we can suppress population transfer between the two pairs of states. Therefore, if we start in the lower pair of states we may neglect the excitation of higher potential states.

A few-state approximation again amounts to a semiclassical treatment. Nevertheless, although we neglect the transfer of momentum in most respects we will keep track of the changes in the phase of the wave function due to recoil. These changes appear in the reversible part of the master equation as well as in the irreversible part and will finally lead to the emergence of the quantum Zeno effect.

E. The master equation in the *LR* system

In this section we derive the master equation in the two-state approximation. Because only the left state, $|L\rangle$, and the right state, $|R\rangle$, are included, we will call this the *LR* system. Before we formulate a more tractable master equation we will introduce some further approximations. First, we will neglect in the following the nondiagonal matrix element of the boost operator:

$$S_{\mathrm{LR}}(q_l)=\int_{-\infty}^{\infty}dx\, e^{-iq_l x}\psi_L(x)\psi_R(x) \simeq e^{-(a/2\sigma)^2}e^{-[q_l(\sigma/2)]}\to 0\ ,\quad \text{if } a\gg\sigma\ . \tag{17}$$

Here, we used Eq. (C6) from Appendix C. Since ψ_L and ψ_R are very narrow, they hardly overlap. This excludes kicks through the barrier. Second, we will neglect the Doppler shift as well as the recoil energy, since they are small compared to the transition frequency.

Next we define a dimensionless coupling constant g for the interaction between the laser and the longitudinal motion of the atom

$$g=S_{RR}(Q_l)\to g^*=S_{LL}(Q_l)\ . \tag{18}$$

Having restricted the space in which our master equation acts, we can evaluate all integrals in our general master equation. This gives us rates and Lamb-shift terms which govern the irreversible behavior of the atom:

$$\Gamma=\frac{2\pi}{\hbar^2}\sum_{q,\sigma}|E_q^0|^2|S_{RR}(q_l)|^2|\epsilon_{q\sigma}\cdot\mu_{21}|^2\delta(\omega_q-\omega_{21})\ , \tag{19}$$

$$\bar\Gamma=\frac{2\pi}{\hbar^2}\sum_{q,\sigma}|E_q^0|^2S_{LL}(-q_l)S_{RR}(q_l)|\epsilon_{q\sigma}\cdot\mu_{21}|^2\delta(\omega_q-\omega_{21})\ , \tag{20}$$

$$\Delta=\frac{1}{\hbar^2}\sum_{q,\sigma}|E_q^0|^2|S_{RR}(q_l)|^2|\epsilon_{q\sigma}\cdot\mu_{21}|^2\mathrm{P}\frac{1}{\omega_q-\omega_{21}}\ . \tag{21}$$

($\sum_{q,\sigma}\{\mathrm{P}[1/(\omega_q-\omega_{21})]\}$ designates the principal value of the corresponding **q**-space integral.) Note the difference between Γ and $\bar\Gamma$. It will disappear with vanishing photon momentum or with vanishing well separation.

We apply a rotating-frame transformation to remove rapidly oscillating terms and we eliminate the Lamb shift from the equations by choosing the laser frequency as $\omega_q=\omega_{21}-\Delta$. Thus, we finally obtain the master equation

$$\begin{aligned}
\dot\rho_{22}^{RR}&=-\frac{i}{\hbar}T(\rho_{22}^{LR}-\rho_{22}^{RL}) &&+i\Omega\{g\rho_{12}^{RR}-\text{c.c.}\} &&-\Gamma\rho_{22}^{RR}\ ,\\
\dot\rho_{22}^{LL}&=+\frac{i}{\hbar}T(\rho_{22}^{LR}-\rho_{22}^{RL}) &&+i\Omega\{g^*\rho_{12}^{LL}-\text{c.c.}\} &&-\Gamma\rho_{22}^{LL}\ ,\\
\dot\rho_{11}^{RR}&=-\frac{i}{\hbar}T(\rho_{11}^{LR}-\rho_{11}^{RL}) &&-i\Omega\{g\rho_{12}^{RR}-\text{c.c.}\} &&+\Gamma\rho_{22}^{RR}\ ,\\
\dot\rho_{11}^{LL}&=+\frac{i}{\hbar}T(\rho_{11}^{LR}-\rho_{11}^{RL}) &&-i\Omega\{g^*\rho_{12}^{LL}-\text{c.c.}\} &&+\Gamma\rho_{22}^{LL}\ ,\\
\dot\rho_{22}^{LR}&=-\frac{i}{\hbar}T(\rho_{22}^{RR}-\rho_{22}^{LL}) &&+i\Omega g^*\{\rho_{12}^{LR}-\rho_{21}^{LR}\} &&-\Gamma\rho_{22}^{LR}\ ,\\
\dot\rho_{11}^{LR}&=-\frac{i}{\hbar}T(\rho_{11}^{RR}-\rho_{11}^{LL}) &&-i\Omega g\{\rho_{12}^{LR}-\rho_{21}^{LR}\} &&+\bar\Gamma\rho_{22}^{LR}\ ,\\
\dot\rho_{21}^{RR}&=+\frac{i}{\hbar}T(\rho_{21}^{RL}-\rho_{21}^{LR}) &&-i\Omega g\{\rho_{22}^{RR}-\rho_{11}^{RR}\} &&-\frac{\Gamma}{2}\rho_{21}^{RR}\ ,\\
\dot\rho_{21}^{LL}&=-\frac{i}{\hbar}T(\rho_{21}^{RL}-\rho_{21}^{LR}) &&-i\Omega g^*\{\rho_{22}^{LL}-\rho_{11}^{LL}\} &&-\frac{\Gamma}{2}\rho_{21}^{LL}\ ,\\
\dot\rho_{21}^{LR}&=-\frac{i}{\hbar}T(\rho_{21}^{RR}-\rho_{21}^{LL}) &&-i\Omega\{g\rho_{22}^{LR}-g^*\rho_{11}^{LR}\} &&-\frac{\Gamma}{2}\rho_{21}^{LR}\ ,\\
\dot\rho_{21}^{RL}&=+\frac{i}{\hbar}T(\rho_{21}^{RR}-\rho_{21}^{LL}) &&-i\Omega\{g^*\rho_{22}^{RL}-g\rho_{11}^{RL}\} &&-\frac{\Gamma}{2}\rho_{21}^{RL}\ .
\end{aligned} \tag{22}$$

In this master equation the first column on the right-hand side (RHS) corresponds to coherent tunneling. The second column refers to interaction with the laser beam. The last column describes irreversible interaction with the electromagnetic vacuum field. We will solve this system of first-order ordinary differential equations in the next section.

III. SOLVING THE MASTER EQUATION

We will treat the master equation analytically as well as numerically. However, it is not possible to solve the full system of equations analytically. Therefore we take advantage of the very different time scales of the measurement process, i.e., the interaction between the laser, vacuum, and atom, and the coherent tunneling process

$$\frac{T}{\hbar} \ll \Omega, \Gamma, \overline{\Gamma} \; . \tag{23}$$

This enables us to employ an adiabatic elimination scheme. The adiabatic solution can be found on a coarse-grained time scale which averages over fast electronic dynamics. Hence, it is useful to contrast the results from an adiabatic solution to the numerical solution of the full master equation. Finally, we investigate the conditions for the appearance of the quantum Zeno effect using the results from this section.

A. The analytical solution

We are especially interested in the influences on the center-of-mass motion. Hence, we will trace over the electronic degrees of freedom,

$$\sigma = \mathrm{Tr}_{\mathrm{elec}}\rho = \sum_i \rho_{ii} \; . \tag{24}$$

This gives us the master equation for the center-of-mass motion

$$\begin{aligned}
\dot{\sigma}^{RR} &= -\frac{i}{\hbar}T(\sigma^{LR}-\sigma^{RL}) \, , \\
\dot{\sigma}^{LL} &= \frac{i}{\hbar}T(\sigma^{LR}-\sigma^{RL}) \, , \\
\dot{\sigma}^{LR} &= -\frac{i}{\hbar}T(\sigma^{RR}-\sigma^{LL}) - i(g-g^*)\Omega(\rho_{12}^{LR}-\rho_{21}^{LR}) \\
&\quad + (\overline{\Gamma}-\Gamma)\rho_{22}^{LR} \; .
\end{aligned} \tag{25}$$

Note that the first RHS column describes the undisturbed (i.e., unobserved) coherent tunneling, driven by the nondiagonal element σ^{LR}. We will therefore refer to σ^{LR} as "tunneling coherence." The tunneling coherence is coupled to the laser field as well as to the irreversible decay. To evaluate this dependence we have to resort to the full master equation (22). We will soon see that this coupling leads to a decay of σ^{LR} which again implies stopping the coherent tunneling. This is the quantum Zeno effect. Let us look at the coupling "constants" in the equation for σ^{LR} in greater detail. First, there is a coupling to the laser field,

$$g-g^* = 2i\int_{-\infty}^{\infty} dx \sin(Q_l x)|\psi_R(x)|^2 \; . \tag{26}$$

For a perpendicular incident laser beam, $Q_l=0$ and the coupling $g-g^*$ vanishes. Second, we find the tunneling coherence to be coupled to the irreversible decay of the electronic transition by the difference of the rates Γ and $\overline{\Gamma}$,

$$\begin{aligned}
\overline{\Gamma}-\Gamma = \frac{2\pi}{\hbar^2}\sum_{q,\sigma} |E_q^0|^2 |\boldsymbol{\epsilon}_{q\sigma}\cdot\boldsymbol{\mu}_{21}|^2 \delta(\omega_q-\omega_{21}) \\
\times \int\int dx\, dy \cos[q_l(x-y)]|\psi_R(x)|^2 \\
\times \{|\psi_L(y)|^2 - |\psi_R(y)|^2\} \; .
\end{aligned} \tag{27}$$

This coupling, however, does not vanish, since q_l denotes the longitudinal momentum of the spontaneously emitted photon. Because we sum over all resonant q there is always a component $q_l \neq 0$. The dependence of $g-g^*$ and $\Gamma-\overline{\Gamma}$ on longitudinal momentum transfer indicates that it is the longitudinal recoil which is responsible for the dephasing in the center-of-mass motion which finally leads to the quantum Zeno effect.

In order to solve the set of equations (25) on the relevant intermediate time scale between the fast electronic time scale and the slow time scale of the center-of-mass motion, we combine an adiabatic elimination of the rapid electronic motion with a perturbation treatment for coherent tunneling. We are interested in the solution for the diagonal elements in second order of $T/\hbar$. Therefore we need σ^{LR} in first order of $T/\hbar$ only.

1. *The decay of tunneling coherence*

The tunneling coherence σ^{LR} in first order of $T/\hbar$ can be obtained by iterating the corresponding differential equations from the full master equation (22) to first order in $T/\hbar$. Terms in zeroth order in $T/\hbar$ are replaced by their stationary values for the case of $T\equiv 0$. The stationary solution is obtained from the full master equation (22) with $\dot{\rho}\equiv 0$. This systematic adiabatic elimination procedure renders a closed first-order differential equation for $\sigma^{LR(1)}(t)$:

$$\begin{aligned}
\dot{\sigma}^{LR(1)} &= -\frac{i}{\hbar}T(\sigma^{RR(0)}-\sigma^{LL(0)}) \\
&\quad - (\Gamma_{\mathrm{SE}}^{\mathrm{eff}} + \Gamma_{\mathrm{laser}}^{\mathrm{eff}} - i\Omega^{\mathrm{eff}})\sigma^{LR(1)} \; .
\end{aligned} \tag{28}$$

(The superscript in parentheses indicates the order in $T/\hbar$.)

In Eq. (29) appears the effective decay constants $\Gamma_{\mathrm{SE}}^{\mathrm{eff}}$ and $\Gamma_{\mathrm{laser}}^{\mathrm{eff}}$ and the effective frequency Ω^{eff}. The subscript SE indicates the origin from the spontaneous emission process while the subscript laser designates a decay rate which owes its existence to the perpendicular incident laser photon. The constants are defined as

$$\Gamma_{\mathrm{SE}}^{\mathrm{eff}} = \frac{\dfrac{\Gamma-\overline{\Gamma}}{2}}{1+\dfrac{\Gamma(\Gamma+\overline{\Gamma})}{4\Omega^2(g+g^*)^2}} = \frac{2\Omega^2(g+g^*)^2(\Gamma-\overline{\Gamma})}{4\Omega^2(g+g^*)^2+\Gamma(\Gamma+\overline{\Gamma})} \, , \tag{29}$$

$$\Gamma_{\mathrm{laser}}^{\mathrm{eff}} = \frac{\dfrac{\Gamma+\overline{\Gamma}}{2}\dfrac{|g-g^*|^2}{(g+g^*)^2}}{1+\dfrac{\Gamma(\Gamma+\overline{\Gamma})}{4\Omega^2(g+g^*)^2}} = \frac{2\Omega^2|g-g^*|^2(\Gamma+\overline{\Gamma})}{4\Omega^2(g+g^*)^2+\Gamma(\Gamma+\overline{\Gamma})} \, , \tag{30}$$

$$i\Omega^{\text{eff}}=\frac{\bar{\Gamma}\dfrac{g-g^*}{g+g^*}}{1+\dfrac{\Gamma(\Gamma+\bar{\Gamma})}{4\Omega^2(g+g^*)^2}}=\frac{4\Omega^2(g-g^*)(g+g^*)\bar{\Gamma}}{4\Omega^2(g+g^*)^2+\Gamma(\Gamma+\bar{\Gamma})}\ . \tag{31}$$

The constants $\Gamma_{\text{SE}}^{\text{eff}}$, $\Gamma_{\text{laser}}^{\text{eff}}$, and Ω^{eff} are dependent on the Rabi frequency Ω of the laser field, on the angle of incidence of the laser (via g), and on the wavelength of the spontaneously emitted photon (via the boost operator matrix element in Γ and $\bar{\Gamma}$). We will investigate this dependence later. Integrating Eq. (28) yields

$$\sigma^{LR(1)}(t)=\sigma_{\text{stat}}^{LR(1)}+[\sigma^{LR(1)}(0)-\sigma_{\text{stat}}^{LR(1)}]\times\exp[-(\Gamma_{\text{SE}}^{\text{eff}}+\Gamma_{\text{laser}}^{\text{eff}}+i\Omega^{\text{eff}})t]\ . \tag{32}$$

Equation (32) describes the rapid decay of the tunneling coherence with the time constant $(\Gamma_{\text{SE}}^{\text{eff}}+\Gamma_{\text{laser}}^{\text{eff}})^{-1}$ due to a laser pulse. Here, $\sigma^{LR(1)}(0)$ is the initial value of tunneling coherence immediately before the laser pulse while $\sigma_{\text{stat}}^{LR(1)}$ is the stationary value of tunneling coherence after the decay. Physically, the decay can be understood in terms of a dephasing of the coherent tunneling motion by the longitudinal recoil of the laser photons as well as the spontaneously emitted photons.

The stationary tunneling coherence after the laser pulse, i.e., the measurement, is given by

$$\sigma_{\text{stat}}^{LR(1)}=-\frac{iT}{\hbar(\Gamma_{\text{SE}}^{\text{eff}}+\Gamma_{\text{laser}}^{\text{eff}}-i\Omega^{\text{eff}})}(\sigma^{RR(0)}-\sigma^{LL(0)})\ . \tag{33}$$

This stationary rest coherence is of the order $T/\hbar\Gamma$ which is very small. However, this result shows that the nondiagonal element of a measured entity will not vanish completely as it is suggested by the von Neumann approach to measurement theory. This would be true only if the response of the apparatus, i.e., Γ^{-1}, were infinitely fast which is an unphysical assumption.

2. *Bringing the tunneling to a halt*

We obtain the adiabatic behavior of the center of mass if we introduce the stationary value of $\sigma^{LR(1)}$ into the first two equations of Eqs. (25). This gives us rate equations instead of the master equation (25):

$$\dot{\sigma}^{RR}(t)=-\gamma\sigma^{RR}(t)+\gamma\sigma^{LL}(t)\ ,$$
$$\dot{\sigma}^{LL}(t)=\gamma\sigma^{RR}(t)-\gamma\sigma^{LL}(t)\ . \tag{34}$$

Here, we have defined the rate

$$\gamma=\frac{2}{\hbar^2}\frac{T^2(\Gamma_{\text{SE}}^{\text{eff}}+\Gamma_{\text{laser}}^{\text{eff}})}{(\Gamma_{\text{SE}}^{\text{eff}}+\Gamma_{\text{laser}}^{\text{eff}})^2+(\Omega_{\text{laser}}^{\text{eff}})^2}\ . \tag{35}$$

This rate is of the order $T^2/\hbar^2\Gamma$ which is very small compared to $T/\hbar$. If we integrate Eqs. (34) we finally obtain the center-of-mass motion during the laser pulse,

$$\sigma^{RR}(t)=\tfrac{1}{2}(1+e^{-2\gamma t})\sigma^{RR}(0)+\tfrac{1}{2}(1-e^{-2\gamma t})\sigma^{LL}(0)\ , \tag{36}$$
$$\sigma^{LL}(t)=\tfrac{1}{2}(1-e^{-2\gamma t})\sigma^{RR}(0)+\tfrac{1}{2}(1+e^{-2\gamma t})\sigma^{LL}(0)\ .$$

Since $\gamma\ll T/\hbar$ this amounts to

$$\sigma^{RR}(t)\simeq\sigma^{RR}(0),\quad \sigma^{LL}(t)\simeq\sigma^{LL}(0)\ . \tag{37}$$

This means that the evolution during the laser pulse will stop. Thus, we have found an example of a quantum Zeno effect using a real measurement process. However, Eq. (37) is only valid to the order of $T^2/\hbar^2\Gamma$ since the tunneling coherence will not vanish completely. We see that a complete halt is only possible in the limit of an infinitely fast response of the apparatus. This is the same unphysical assumption on which the von Neumann projection hypothesis is based. A physical measurement process instead will not reduce the system in an eigenstate of the investigated observable but will leave the system in a superposition of eigenstates that is heavily concentrated in one of these states. However, since there are contributions of the other states to the superposition after the measurement a physical measurement will sometimes yield a "wrong" answer. The fact that tunneling coherence does not vanish completely is an example for an "imprecise measurement" as introduced by Caves [9].

B. Numerical solution

In this section we discuss the results of the numerical solution of the full master equation. The main advantage of the numerical treatment is the possibility of studying the behavior for long times as well as for short times beyond the perturbative adiabatic approximation. In order not to confuse the issues we will eliminate all laser-related oscillations from our considerations by choosing $g=1$ (perpendicular incidence of the laser beam).

The basic parameters of our problem are Γ, $\bar{\Gamma}$, Ω, and T. Without any measurement performed, $\sigma^{RR}\equiv P_R$ harmonically oscillates. The tunneling coherence is essentially the time derivative of P_R.

In Figs. 3(a)–3(d) the course of P_R and $\text{Im}(\sigma^{LR})$ is

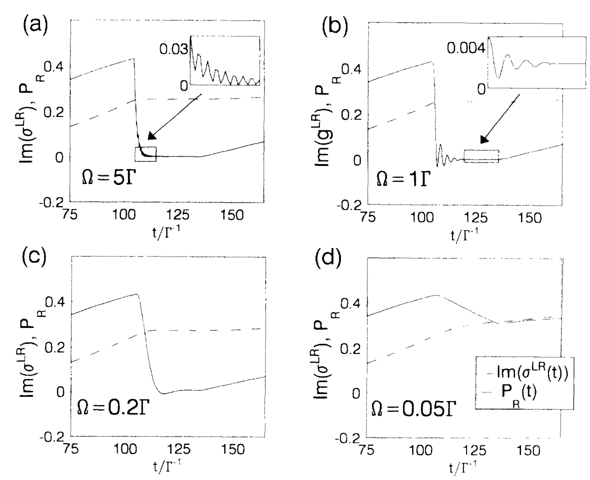

FIG. 3. (a)–(d) The decay of tunneling coherence [$\text{Im}(\sigma^{LR})$: solid line] and the stagnation of the population in the right well (P_R: dashed line) during a laser pulse for four different Rabi frequencies Ω. Between $t=105\Gamma^{-1}$ and $t=135\Gamma^{-1}$ a laser pulse of Rabi frequency Ω is applied. The insets in (a) and (b) magnify the oscillations in the coherence decay. The parameters were chosen as $\bar{\Gamma}=-0.7\Gamma$, $T=5\times10^{-3}\Gamma$, $\Omega=5\Gamma$ (a), 1Γ (b), 0.2Γ (c), and 0.05Γ (d).

plotted during a laser pulse using four different Rabi frequencies Ω. Clearly, the tunneling coherence $\mathrm{Im}(\sigma^{LR})$ decays for Rabi frequencies $\Omega \geq 0.1\Gamma$. Due to this decay the evolution of the population P_R stagnates during the pulse. After the pulse it only evolves slowly according to a t^2 law. This is consistent with our analytical treatment in Sec. III A. However, here we find additional oscillations in $\mathrm{Im}(\sigma^{LR})$ on the electronic time scale. The decay rate as well as the oscillation frequency decreases with decreasing Rabi frequency. In Fig. 3(d) the Rabi frequency is so small that there is no longer a decay of the coherence. Accordingly, the evolution of P_R is only slightly affected. This is an incomplete measurement. It is straightforward to show that in this case on average far less than one laser photon is scattered. Thus, no position measurement within the double-well potential is performed. Consequently, there is no quantum Zeno effect. In Sec. III A we inferred that there has to remain a stationary coherence of the order $T/\hbar\Gamma$. The inset in Fig. 3(b) confirms this result, showing that there is indeed a rest coherence after the rapid decay of σ^{LR}.

In order to investigate the oscillations in σ^{LR} during the decay Fig. 4 shows the Fourier transforms of the coherence decay for five different Rabi frequencies. From this, one finds $\omega \simeq 2\Omega$ for frequencies smaller than approximately 5Γ. Thus, the oscillations in tunneling coherence follow the Rabi oscillations of the electronic system as long as the oscillations do not become too fast.

In Fig. 5 we compare our analytical result for the decay rate $\Gamma_{\mathrm{SE}}^{\mathrm{eff}}$ with the ratio $R=-\mathrm{Im}(\dot{\sigma}^{LR})/\mathrm{Im}(\sigma^{LR})$. The rate $\Gamma_{\mathrm{SE}}^{\mathrm{eff}}$ obviously averages over the fast oscillations in the decay.

Finally, we study the quantum Zeno effect due to a pulse sequence. Figure 6(a) displays the undisturbed behavior of the system. In Figs. 6(b) and 6(c) we apply four, respectively 17, pulses to the double-well potential. Since tunneling coherence decays in each single pulse the evolution is strongly inhibited: In Fig. 6(b) only 30% and

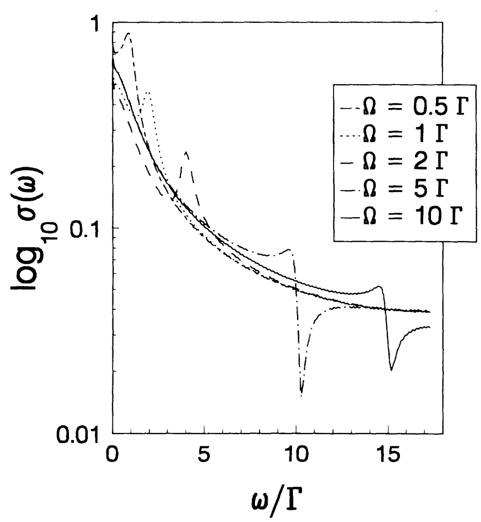

FIG. 4. The Fourier transforms of $\mathrm{Im}(\sigma^{LR})$ for five different Rabi frequencies during the laser pulse in logarithmic units. For $\Omega \leq 5\Gamma$ approximately holds $\omega = 2\Omega$. At higher frequencies the center-of-mass coherence cannot follow the fast electronic oscillations. The parameters are chosen as in Fig. 3.

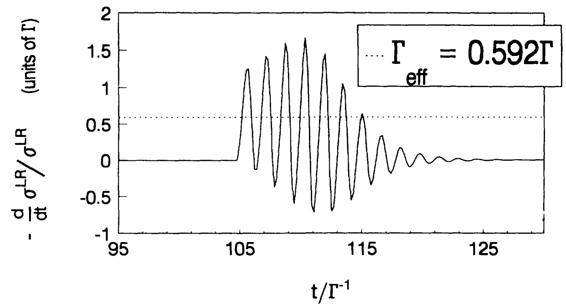

FIG. 5. Comparison between the analytical result, predicting an average exponential decay with decay constant Γ^{eff} and the numerical calculated decay of $\mathrm{Im}(\sigma^{LR})$. As expected, Γ^{eff} reflects the averaged behavior of $\mathrm{Im}(\sigma^{LR})$ during the pulse. The parameters are $\Omega = 2\Gamma$, $\Gamma = -0.2\Gamma$.

in 6(c) only 6% of the population can cross the barrier instead of the full 100% as in Fig. 6(a).

C. The dependence of the effective decay rate

Because we have an analytical expression for the effective decay constant $\Gamma^{\mathrm{eff}} = \Gamma_{\mathrm{SE}}^{\mathrm{eff}} + \Gamma_{\mathrm{laser}}^{\mathrm{eff}}$ it is possible to investigate the dependence on the wavelength, the direction of incidence, and the intensity of the probing laser field. First we consider the case of a perpendicular in-

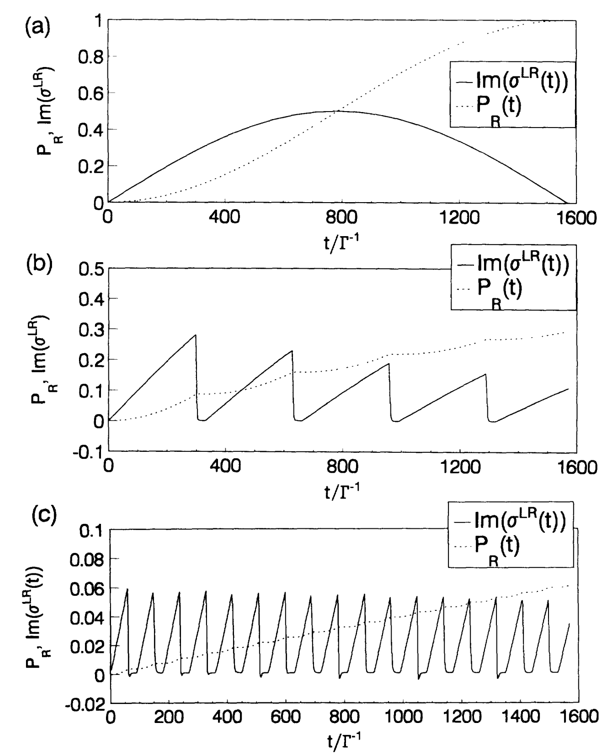

FIG. 6. The quantum Zeno effect due to a pulse sequence. (a) displays the free, undisturbed coherent tunneling which transfers the complete population from left to right. In (b) four laser pulses are applied: Each collapses the coherence, so that the evolution of P_R is strongly inhibited. Within the same time as in (a), only 30% of the population reaches the right well. In (c) 17 pulses are applied: here, only 6% of the population tunnels to the right well. The parameters are $\Gamma = -0.7\Gamma$, $T = 2 \times 10^{-3}\Gamma$, $\Omega = 1\Gamma$. The pulse period is $\tau_P = 30\Gamma^{-1}$.

cident laser beam $(g=1)$. Then we obtain for the effective decay constant $\Gamma^{\rm eff}=\Gamma_{\rm SE}^{\rm eff}$

$$\Gamma_{\rm SE}^{\rm eff}=\frac{\dfrac{\Gamma-\bar{\Gamma}}{2}}{1+\dfrac{\Gamma(\Gamma+\bar{\Gamma})}{16\Omega^2}} \ . \tag{38}$$

The wavelength dependence in Eq. (38) is introduced by the rates Γ and $\bar{\Gamma}$ [Eqs. (19) and (20)] which again contain the boost operator matrix elements [Eq. (7)]. In $S_{MM'}(q_l)$ we need the wave functions ψ_L and ψ_R. These we can construct either numerically (using a model potential) or analytically (using Gaussians as approximations for the wave function, cf. Appendix C).

Introducing Eqs. (19) and (20) into our expression for the effective decay constant [Eq. (38)] yields the sought after wavelength dependence of $\Gamma_{\rm SE}^{\rm eff}$. This is represented in Fig. 7. We see that $\Gamma_{\rm SE}^{\rm eff}/\Gamma_{\rm free}$ is small for small photon momentum and increases with q, reaching a maximum value around $0.7q_0$. This maximum corresponds to a constructively interfering diffraction pattern of the emitted radiation. With increasing momentum destructive interference makes the pattern more difficult to resolve, so $\Gamma_{\rm SE}^{\rm eff}/\Gamma_{\rm free}$ decreases again. At higher photon momentum the pattern becomes resolvable again and approaches a second maximum. On the far right side of the q axis the validity of our assumption that $1>>2(q_l\sigma/2)^2$ eventually worsens.

The dependence of the effective decay constant on the wavelength of the photon illustrates in a more quantitative way what is intuitively expected: The vanishing of the quantum Zeno effect if the laser wavelength is large compared to the well separation. This is due to the fact that our meter readings, i.e., the diffraction pattern of the electromagnetic field, are no longer resolvable. Using a more idealized measurement concept, Peres [10] pointed out that a higher resolution of the measurement (i.e., a smaller laser wavelength) will increasingly effectively lock the evolution of the observed quantum system (i.e., the atomic center-of-mass motion). Thus, it is indeed necessary to use a coarser resolution of the meter to avoid the Zeno effect. Our result here confirms Peres' results using

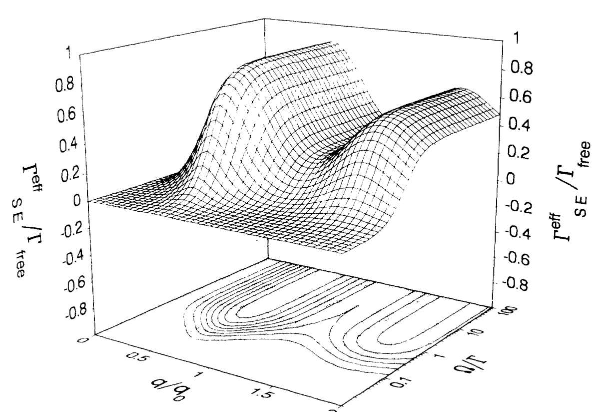

FIG. 7. The effective decay rate $\Gamma_{\rm eff}$ (in units of $\Gamma_{\rm free}$) versus the momentum transfer q and versus the Rabi frequency of the laser field. Here, $\sigma\to 0$ was assumed (very deep potential) and $(g-g^*)=0$ (perpendicular incident laser beam).

a realistic measurement process.

We may differentiate between the regime high above saturation and the regime far below saturation of the electronic transition. Figure 7 shows that the effective decay constant saturates with increasing Rabi frequency around $\Omega\simeq 1\Gamma$ and decreases to almost zero around $\Omega\simeq 0.1\Gamma$. From Eq. (38) we compute the value of $\Gamma_{\rm SE}^{\rm eff}$ below and above saturation:

$$\Gamma_{\rm SE}^{\rm eff}=\frac{8\Omega^2}{\Gamma}\frac{\Gamma-\bar{\Gamma}}{\Gamma+\bar{\Gamma}}, \quad \frac{\Gamma}{4}>>\Omega \tag{39}$$

$$\Gamma_{\rm SE}^{\rm eff}=\frac{\Gamma-\bar{\Gamma}}{2}, \quad \frac{\Gamma}{4}<<\Omega \ . \tag{40}$$

This kind of intensity dependence $\Gamma_{\rm SE}^{\rm eff}$ shares with $\Gamma_{\rm laser}^{\rm eff}$ and $\Omega^{\rm eff}$ because all of them share the same Ω-dependent denominator. For low Rabi frequencies the decay constants increase proportional to Ω^2/Γ, and for high Rabi frequencies the constants approach a saturation value. The latter is given by the decay rates of the electronic system, Γ and $\bar{\Gamma}$, thereby the response time $1/\Gamma^{\rm eff}$ of a physical measurement is principally limited.

The increase in photon momentum in $\Gamma_{\rm SE}^{\rm eff}/\Gamma_{\rm free}$ for small momenta can be studied using the analytical solution. We obtain in the case high above saturation:

$$\Gamma_{\rm SE}^{\rm eff}=\Gamma_{\rm free}\sin^2[\tfrac{1}{2}q_{21}(x_L-x_R)] \times\left[1-\frac{\Gamma_{\rm free}^2\cos^2[\tfrac{1}{2}q_{21}(x_L-x_R)]}{16\Omega^2}\right], \quad \frac{\Gamma}{4}<<\Omega \tag{41}$$

In the case far below saturation we derive similarly

$$\Gamma_{\rm SE}^{\rm eff}=\frac{8\Omega^2}{\Gamma}\tan^2[\tfrac{1}{2}q_{21}(x_L-x_R)] \propto\Gamma_{\rm free}\tan^2[\tfrac{1}{2}q_{21}(x_L-x_R)], \quad \frac{\Gamma}{4}>>\Omega\propto\Gamma \ . \tag{42}$$

In both cases $\Gamma_{\rm SE}^{\rm eff}$ increases quadratically with q for small photon momenta. For the sake of compatibility with our numerical study we neglected so far the directional dependence in the effective decay constants by setting $g\equiv 1$. Now we want to investigate the dependence of the decay constants $\Gamma_{\rm SE}^{\rm eff}$, $\Gamma_{\rm laser}^{\rm eff}$, and $\Omega^{\rm eff}$, on the angle of incidence of the laser beam.

Figure 8(a) shows $\Gamma_{\rm SE}^{\rm eff}$ versus the laser photon momentum $\mathbf{Q}$ for $\Omega=1\Gamma$. $\Gamma_{\rm SE}^{\rm eff}$ displays concentric fringes which are interrupted at $Q_l=1/2q_0$, $3/2q_0$. On the plane given by $Q_l=0$ we find, of course, the oscillations in $\Gamma_{\rm SE}^{\rm eff}$ from Fig. 7. The fringes emerge from the isotropic spontaneous emission that depends only on the absolute value of $\mathbf{Q}$ (i.e., the transition frequency). At $Q_l=1/2q_0$ and $Q_l=3/2q_0$ $\Gamma_{\rm SE}^{\rm eff}$ vanishes because then $(g+g^*)\simeq 0$ for narrow wave functions. From the full master equation (22) it can be seen that for $(g+g^*)=0$ the electronically upper level of the tunneling coherence, ρ_{22}^{LR}, is not populated during the pulse: If one studies the dynamics which is imposed on the ρ_{ij}^{LR} by the laser field alone the corresponding nonvanishing eigenfrequencies are found to be $\pm\Omega(g+g^*)$. Thus, the laser field cannot populate ρ_{22}^{LR} if g is imaginary. Without having ρ_{22}^{LR} populated the cou-

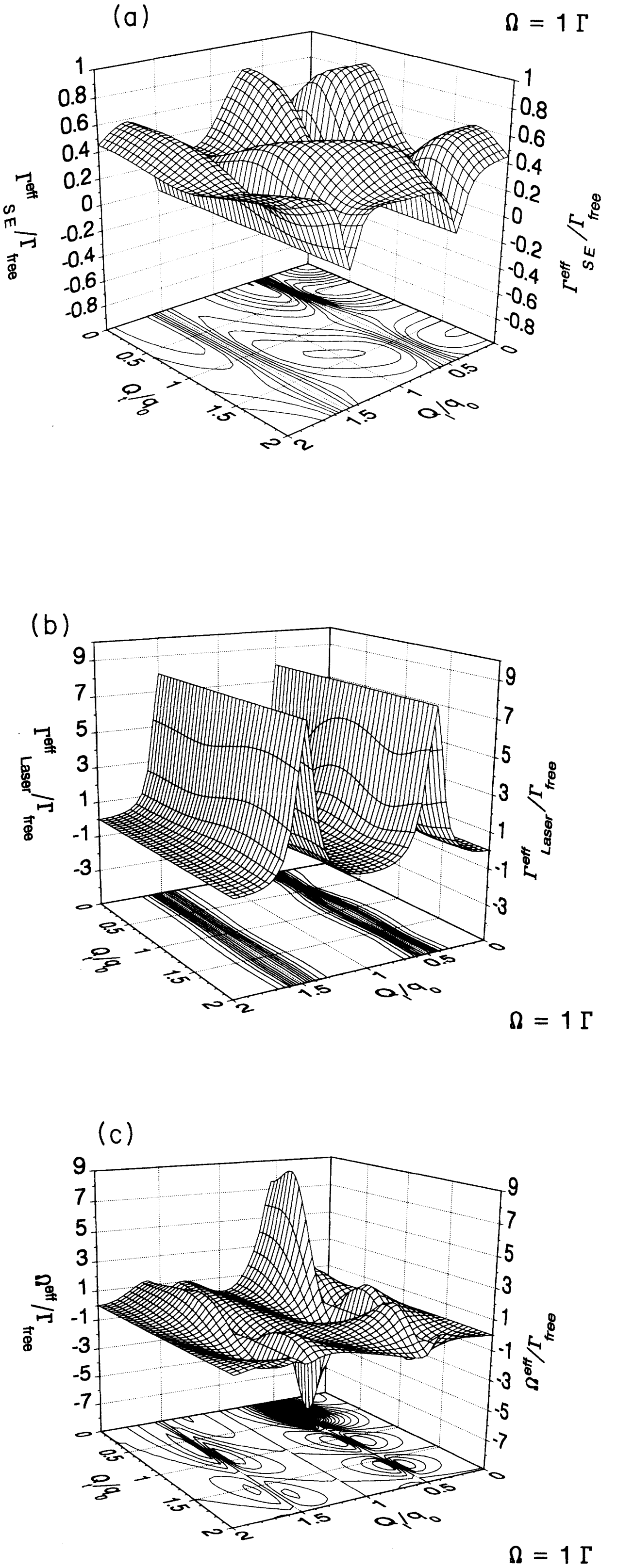

FIG. 8. (a) Γ_{SE}^{eff} versus the momentum of the laser photon $\mathbf{Q}$. The parameters were chosen as $\Omega = 1\Gamma$ and $\sigma \rightarrow 0$. (b) Γ_{laser}^{eff} versus the momentum of the laser photon $\mathbf{Q}$. The parameters were chosen as $\Omega = 1\Gamma$ and $\sigma \rightarrow 0$. (c) Ω^{eff} versus the momentum of the laser photon $\mathbf{Q}$. The parameters were chosen as $\Omega = 1\Gamma$ and $\sigma \rightarrow 0$.

pling of σ^{LR} to the spontaneous emission in Eqs. (25) and consequently Γ_{SE}^{eff} vanishes.

In Fig. 8(b) Γ_{laser}^{eff} versus $\mathbf{Q}$ is plotted. Here we find almost no dependence on Q_t or $|\mathbf{Q}|$. Instead we find a dependence on the longitudinal component of the laser photon. This indicates the origin of Γ_{laser}^{eff} from the effects of the longitudinal recoil of the laser field. At $Q_l = 0$, $1q_0$, $2q_0$, Γ_{laser}^{eff} goes to zero. At the corresponding wavelengths both wells are exposed to the same laser amplitude, so that no dephasing due to the laser field occurs.

The imaginary part of the decay constants, Ω^{eff}, is pictured in Fig. 8(c). The corresponding oscillations emerge obviously from the spontaneous emission as well as from the laser photon, since Fig. 8(c) displays a combination of the features of Figs. 8(a) and 8(b).

IV. CONCLUSION

In this work we used the quantum Zeno effect as paradigm of quantum-mechanical measurement theory. We showed that it is possible to derive the "collapse of the wave function" in all details from the irreversible dynamics of a real measurement process. Moreover, employing the Zeno effect as an indicator for the quality of a measurement process we were able to perform a more detailed analysis than is possible using the standard approach to measurement theory. From this it becomes clear once again that the von Neumann collapse hypothesis only models the measurement in an unphysical parameter limit, i.e., the infinitely fast response of the apparatus. In dealing with a real process we showed that neither the nondiagonal elements decay completely nor, consequently, does the Zeno effect perfectly stop the evolution. A physical measurement is bound to be an imprecise measurement in the sense of Caves [9]: Sometimes the fluorescence will indicate the atom to be in one well although the measurement process reduced the system in a superposition which is strongly concentrated in the other well.

In particular, we investigated the dependence on wavelength and intensity of the laser beam. It was shown that a minimum Rabi frequency is necessary to ensure the decay of tunneling coherence. This minimum intensity corresponds to the scattering of one photon. As already pointed out by Frerichs and Schenzle [5] the reliability of the photon's occurrence determines the quality of the measurement. Therefore, the variance rather than the mean square of the photon number is important.

One of the main results of this paper is the wavelength dependence of the decay constant. For long wavelength we found $\Gamma_{SE}^{eff}/\Gamma_{free}$ to be proportional to λ^{-2}. The interpretation of this result is straightforward: A process that does not extract information from the object can hardly be regarded as a "measurement." For too long a wavelength the scattering pattern of the spontaneously emitted photons cannot serve as a record of a result, because it is not possible to infer from it what the position of the scattering atom is. Therefore, no measurement is performed and no quantum Zeno effect can be observed. In our model of a real measurement process the wavelength dependence of the quantum Zeno effect is the concrete manifestation of the requirement of "macroscopic distinguished meter states" in standard measurement theory. However, although the measurement theory due to von Neumann is clearly too idealized, one of its strange implications, namely the quantum Zeno effect, can be confirmed using the quantum mechanics of dissipative systems. The way we treated a "measurement-induced effect" in this paper actually suggests that a measurement is only a particular form of irreversible dynamics. In particular, effects like the "collapse of the wave function" need not be traced back to some metaphysical coupling between the observer's mind and the object of his experiment. We feel that it is not necessary to consider quantum mechanics incomplete [11,12]. Rather, we think that the measurement-induced effect can be seen to emerge from the coupling between a object with only a few degrees of freedom to a system with very many degrees of freedom. This, of course, means that measurement theory should be viewed as inherently connected to quantum statistics.

APPENDIX A: DETAILS OF THE DERIVATION OF THE MASTER EQUATION

Following the standard derivation of a master equation for the interaction of a small system with a $T=0$ reservoir (e.g. in [13]) we obtain an integrodifferential equation of the form:

$$\frac{\partial}{\partial t}\tilde{\rho}_A(t)=-\frac{1}{\hbar^2}\int_0^t dt'\sum_{q,\sigma}|E_q^0|^2[e^{-i\omega_q t'}\{\cdots\}+e^{i\omega_q t'}\{\cdots\}]. \tag{A1}$$

In order to transform this to a differential equation we employ the Markov approximation: The state of the reservoir at time t only depends on the reservoir's state immediately before, so that all correlations within the bath decay very fast. In Eq. (A1) this means that the integrand of the integral in Eq. (A1) vanishes rapidly within a correlation time τ_c. However, the terms in parentheses in Eq. (A1) that are still dependent on q due to the inclusion of the photon recoil. Introducing the definition of the boost matrix elements in Eq. (A1) one finds that interference of the reservoir modes within the double-well will limit the spectral bandwidth of the reservoir. Therefore the correlation time τ_c of the bath is of the order a/c. Physically this means that the reservoir of the electromagnetic field modes needs a time τ_c to reach equilibrium again after an interaction. This is reasonable since τ_c is just the time a perturbation in the electromagnetic field on the one side of the potential needs to be sensed on the other side. However, because a has to be chosen rather small to enable tunneling, τ_c is almost zero on an atomic time scale. Thus we can replace $\rho_A(t-t')$ by $\rho_A(t)$ and shift the upper limit of the time integration to infinity. Only then we can execute the time integration which eventually gives us rates and Lamb-shift terms.

APPENDIX B: RESTRICTION OF THE TRANSVERSE SPACE

In order to remove the transverse degrees of freedom from the general master equation (22) we trace over the transverse space. In the irreversible part of (22) we find only diagonal elements in the transverse states. Since the summation over the transverse states may be arbitrarily shifted we can replace all diagonal matrix elements by a unique average value

$$\begin{aligned}\langle\rho_A\rangle_t&\equiv\sum_{\mathbf{k}_t}\langle\mathbf{k}_t|\rho_A|\mathbf{k}_t\rangle\\&=\sum_{\mathbf{k}_t}\langle\mathbf{k}_t\pm\mathbf{Q}_t|\rho_A|\mathbf{k}_t\pm\mathbf{Q}_t\rangle\ .\end{aligned} \tag{B1}$$

However, performing the trace over the reversible part of (22) turns out to be somewhat more complicated because all matrix elements are nondiagonal in the transverse states. Tracing over these elements we obtain

$$\begin{aligned}\sum_{\mathbf{k}_t}\langle\mathbf{k}_t\pm\mathbf{Q}_t|\rho_A|\mathbf{k}_t\rangle&=\sum_{\mathbf{k}_t}\langle\mathbf{k}_t|\rho_A|\mathbf{k}_t\mp\mathbf{Q}_t\rangle\\&=\sum_{\mathbf{k}_t}\langle\mathbf{k}_t|\rho_A|\mathbf{k}_t\rangle\mp\mathbf{Q}_t\frac{\partial}{\partial\mathbf{k}_t'}\langle\mathbf{k}_t|\rho_A|\mathbf{k}_t'\rangle|_{\mathbf{k}_t'=\mathbf{k}_t}\pm\cdots\\&\simeq\sum_{\mathbf{k}_t}\langle\mathbf{k}_t|\rho_A|\mathbf{k}_t\rangle\equiv\langle\rho_A\rangle_t\ .\end{aligned} \tag{B2}$$

The last step is only permissible if $Q_t \ll k_t$. This is essentially a semiclassical condition. It can be justified if one compares the natural linewidth Γ with the Doppler shift ω_D. For an interaction between an atom and a resonant laser beam $\Gamma\geq\omega_D$ must be approximately valid. From this we can estimate

$$\frac{k_t}{Q_t}\simeq\frac{\Gamma}{\omega_{21}}\frac{Mc^2}{\hbar\omega_{21}}=\text{const}\times\omega_{21}\gg 1\ . \tag{B3}$$

The last relation can be fulfilled for wavelengths smaller than 0.1 μm even for small atomic numbers. Using this we can replace every matrix element of the transverse states by a unique average value.

APPENDIX C: RESTRICTION OF THE LONGITUDINAL SPACE

We want to derive a more quantitative condition for the validity of a description that takes only two longitudinal states, namely $|L\rangle$ and $|R\rangle$, into account. Let us consider a kicked wave function $|\psi'\rangle$. Before the kick, the center-of-mass motion is assumed to be in the state $|L\rangle$. Using the boost operator from Eq. (6) we can write for the kicked state

$$|\psi'\rangle = S(q_l)|L\rangle \ . \tag{C1}$$

The norm of the kicked state $|\psi'\rangle$ is then

$$1 = \sum_{M'} |S_{M'L}(q_l)|^2 \ . \tag{C2}$$

If we want to limit our considerations to only two states of longitudinal motion ($|L\rangle, |R\rangle$) it must be approximately valid that

$$1 \simeq |S_{LL}(q_l)|^2 + |S_{LR}(q_l)|^2 \ . \tag{C3}$$

We can investigate this analytically if we use approximate solutions for the symmetric ground state, respectively, the antisymmetric first excited state. For deep wells these are approximately parabolic potentials, so we can set

$$\psi_{s,a} = (4\pi\sigma)^{-1/4} \left[\exp\left[-\frac{1}{2}\frac{(x-x_L)^2}{\sigma^2} \right] \pm \exp\left[-\frac{1}{2}\frac{(x-x_R)^2}{\sigma^2} \right] \right] . \tag{C4}$$

Here, two Gaussian functions centered in the left, respectively, right well with width $\sigma/2$ are superposed to give ψ_a and ψ_s. Introducing this in Eq. (7) yields

$$S_{LL}(q_l) = e^{[(1/2)q_l\sigma]^2} \left[\cos\left[\frac{q_l a}{2} \right] - i \sin\left[\frac{q_l a}{2} \right] \right] , \tag{C5}$$

$$S_{LR}(q_l) = e^{[(1/2)q_l\sigma]^2} e^{(-a/2\sigma)^2} \ . \tag{C6}$$

Because $S_{LR}(q_l)$ is very small the condition (C3) may be expressed as

$$1 \simeq e^{-2[(1/2)q_l\sigma]^2} \simeq 1 - 2\left[\frac{q_l\sigma}{2} \right]^2 . \tag{C7}$$

Thus, a criterion for the validity of our two-state approximation is

$$1 \gg 2\left[\frac{q_l\sigma}{2} \right]^2 . \tag{C8}$$

If we want to do better we could use the four lowest potential states instead of just two. An approximation analogous to the one in (C8) then leads to the criterion [8]:

$$1 \gg 4\left[\frac{\sigma q_l}{2} \right]^4 . \tag{C9}$$

[1] Albeit the notion of the quantum Zeno effect was first introduced by B. Misra and E. C. G. Sudarshan, J. Math. Phys. **18**, 756 (1977). The physics of the Zeno effect was investigated even earlier: W. Yourgrau, in *Problems in the Philosophy of Science,* edited by I. Lakatos and A. Musgrave (North-Holland, Amsterdam, 1968), p. 191; H. Ekstein and A. Seigert, Ann. Phys. (N.Y.) **68**, 509 (1971); A. Degasperis, L. Fonda, and G. C. Ghiradi, Nuovo Cimento **21A**, 471 (1974).

[2] J. von Neumann, *Mathematische Grundlagen der Quantentheorie* (Springer, Berlin, 1931).

[3] R. J. Cook, Phys. Scr. T **21**, 49 (1988).

[4] W. M. Itano, D. J. Heinzen, J. J. Bollinger, and D. J. Wineland, Phys. Rev. A **41**, 2295 (1990).

[5] V. Frerichs and A. Schenzle, Phys. Rev. A **44**, 1962 (1991).

[6] E. Block and P. R. Berman, Phys. Rev. A **44**, 1466 (1991).

[7] H. Fearn and W. Lamb, Phys. Rev. A **46**, 1199 (1992).

[8] T. Altenmüller, Diploma Thesis, Technische Universität München, München, 1992 (unpublished).

[9] C. M. Caves, Phys. Rev. D **33**, 1643 (1986).

[10] A. Peres, Phys. Rev. D **39**, 2943 (1989).

[11] J. Bell, Phys. World **3**, 33 (1990).

[12] G. C. Ghiradi, A. Rimini, and T. Weber, Phys. Rev. D **34**, 470 (1986).

[13] C. Cohen-Tannoudji, in *Frontiers in Laser Spectroscopy,* edited by S. Haroche and S. Liberman (North-Holland, Amsterdam, 1977), p. 4.