Quantum-nondemolition measurement of photon number using radiation pressure

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We propose a quantum-nondemolition measurement of the amplitude quadrature and the photon-number statistics by using the effect of radiation pressure on a freely suspended mirror. We propose to measure the momentum fluctuations of the mirror which will give us a readout of the amplitude quadrature fluctuations. The scheme we propose is able to avoid the back-action noise leaving the state of the field after the measurement practically undegraded.

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I. INTRODUCTION

There has recently been considerable interest in the mechanical effects of light [1]. It has been proposed that by measuring the deflection of atoms from a standing-wave light field a quantum-nondemolition (QND) measurement may be made on the photon number of the field [2].

The mechanical effects of light on macroscopic objects have also been considered. The theory of radiation-pressure-induced optical bistability has been given by Meystre *et al.* [3]. Experimental demonstrations of this effect have been given in both the optical [4] and microwave regimes [5]. The radiation-pressure coupling of the light field to a freely suspended mirror introduces a nonlinearity which gives rise to the bistability.

In this paper we give a quantum-mechanical analysis of the radiation-pressure coupling of a light field to a freely suspended mirror in the adiabatic limit. We propose to use the momentum fluctuations of the mirror as a meter that provides information on the fluctuations in the amplitude quadrature of the external field and in the photon number. We shall evaluate under what conditions a measurement of the momentum fluctuations of the mirror gives us a QND measurement of the amplitude quadrature statistics of the field. We shall use the criteria introduced by Holland *et al.* [6] to evaluate the quality of the proposed scheme as a QND measurement, taking into account the cavity loss and the back action of the noise on the meter.

The effect of radiation pressure on a freely suspended mirror has also been considered in the context of optical interferometers for gravitational wave detection [7–9]. Whereas in those systems one measures the optical phase shift to determine the position of the mirror, in our application we measure the momentum fluctuations of the mirror to determine the amplitude statistics of the light field.

II. MODEL

Let us consider a single-mode optical cavity at frequency ω_0 with one partially reflecting mirror with transmittivity $\mathcal{T}$ and one totally reflecting mirror at the other end. This second mirror can be considered as a harmonic oscillator with mass m and frequency ω_m. A coherent radiation field transmitted through the first mirror hits the second mirror which starts its oscillation because of the radiation pressure force.

The total Hamiltonian can be written as

$$H=H_1+H_2 \tag{1}$$

with

$$H_1=\hbar\omega_0 a^\dagger a+\frac{\hat{p}^2}{2m}+\tfrac{1}{2}m\omega_m^2\hat{x}^2-\hbar g a^\dagger a\hat{x} \ , \tag{2}$$

$$H_2=\tfrac{1}{2}\int_0^\infty d\omega[(P(\omega)+k(\omega)\hat{x})^2+\omega^2 Q^2(\omega)] +\Gamma a^\dagger+\Gamma^\dagger a \ . \tag{3}$$

The optical mode of the cavity is described by the Bose operators $a, a^\dagger$, while $\hat{x}$ represents the displacement of the oscillating mirror and $\hat{p}$ its momentum. The coupling constant g can be expressed in terms of the cavity length L and frequency ω_0 as $g=\omega_0/L$. The first term of H_2 de-

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scribes the coupling of the harmonic oscillating mirror to the heat bath described by the bath variables P and Q. This coupling to the bath is linear but it is more realistic than the usual rotating-wave approximation [10]. It gives the damping in the momentum of the oscillator and the heat bath is considered in equilibrium at temperature T. The other two terms of H_2 represent the damping of the cavity mode in the usual rotating-wave approximation.

In the input-output formalism [11] the equations of motion for the system variables are written as

$$\begin{aligned} \frac{da}{dt} &= \frac{1}{i\hbar}[a,H_1] - \frac{\gamma_a}{2}a + \sqrt{\gamma_a}a_{\text{in}}(t) , \\ \frac{da^\dagger}{dt} &= \frac{1}{i\hbar}[a^\dagger,H_1] - \frac{\gamma_a}{2}a^\dagger + \sqrt{\gamma_a}a^\dagger_{\text{in}}(t) , \\ \frac{d\hat{x}}{dt} &= \frac{1}{i\hbar}[\hat{x},H_1] , \\ \frac{d\hat{p}}{dt} &= \frac{1}{i\hbar}[\hat{p},H_1] - \frac{\gamma_m}{2m}\hat{p} - \sqrt{\gamma_m}\epsilon_{\text{in}}(t) , \end{aligned} \tag{4}$$

where $\gamma_a = c\mathcal{T}/2L$ is the cavity damping constant with c the speed of light; γ_m represents the damping of the oscillating mirror in the usual Markoffian approximation [1]; and $a_{\text{in}}(t)$ and $a^\dagger_{\text{in}}(t)$ are the Bose operators describing the input field near the frequency of the cavity mode, i.e., $a^\dagger_{\text{in}}(t)a_{\text{in}}(t)$ is the number of photons per second at frequency ω_0 hitting the partially reflecting mirror. The quantity $\epsilon_{\text{in}}(t)$ is determined in terms of the heat-bath operators at initial time; thus it depends on the state of the bath at this initial time [10]. It can be shown that in the limit of high temperature (i.e., $k_B T \gg \hbar\omega_m$) the statistics of the "noise" variable $\epsilon_{\text{in}}(t)$ is that of a white noise [10]

$$\langle \epsilon_{\text{in}}(t) \rangle = 0, \quad \langle \epsilon_{\text{in}}(t)\epsilon_{\text{in}}(t') \rangle = k_B T \delta(t-t') , \tag{5}$$

where k_B is the Boltzmann constant.

Equations (4) are interpreted as quantum-Langevin equations in the Stratonovich sense. The steady-state solutions can be easily derived; however, it turns out that, due to the presence of the oscillating mirror, the mode inside the cavity is detuned. We can get rid of this detuning by introducing *ab initio* a cavity detuning δ which can be adjusted either by varying the length of the cavity or by varying the frequency of the external field. Thus the frequency ω_0 in Eq. (2) is modified to $\omega_0+\delta$ and the equations of motion (4) become

$$\begin{aligned} \frac{da}{dt} &= -i(\omega_0+\delta-g\hat{x})a - \frac{\gamma_a}{2}a + \sqrt{\gamma_a}a_{\text{in}}(t) , \\ \frac{d\hat{x}}{dt} &= \hat{p}/m , \\ \frac{d\hat{p}}{dt} &= -m\omega_m^2\hat{x} + \hbar g a^\dagger a - \frac{\gamma_m}{2m}\hat{p} - \sqrt{\gamma_m}\epsilon_{\text{in}}(t) . \end{aligned} \tag{6}$$

Within the usual semiclassical approximation we get the steady-state equations, obtained by putting $d\langle a \rangle/dt = d\langle a^\dagger \rangle/dt = d\langle \hat{x} \rangle/dt = d\langle \hat{p} \rangle/dt = 0$,

$$\begin{aligned} & i\alpha_s(\omega_0+\delta-gx_s) + \frac{\gamma_a}{2}\alpha_s = \sqrt{\gamma_a}\alpha_{\text{in}} , \\ & p_s = 0 , \\ & -m\omega_m^2 x_s + \hbar g \alpha_s^* \alpha_s - \frac{\gamma_m}{2m}p_s = 0 , \end{aligned} \tag{7}$$

where $\alpha_s = \langle a \rangle_s$, $x_s = \langle \hat{x} \rangle_s$, $p_s = \langle \hat{p} \rangle_s$ are the steady-state mean values and $\alpha_{\text{in}} = \langle a_{\text{in}} \rangle$. By choosing the detuning δ so that

$$\delta = -(\omega_0 - gx_s) = -\omega_0 \left[1 - \frac{x_s}{L} \right] , \tag{8}$$

the equations simplify and we get the solutions

$$\begin{aligned} x_s &= \frac{8\hbar\omega_0}{mc\omega_m^2 \mathcal{T}}|\alpha_{\text{in}}|^2 , \\ p_s &= 0 , \\ \alpha_s^*\alpha_s &= \frac{8L}{c\mathcal{T}}|\alpha_{\text{in}}|^2 , \end{aligned} \tag{9}$$

where we used the definitions given above for g and γ_a; x_s is the steady-state value of the mean displacement of the oscillating mirror, $|\alpha_s|^2$ represents the steady-state mean value of the photon number inside the cavity, while $|\alpha_{\text{in}}|^2$ is the mean number of photons hitting the first cavity mirror in 1 s at the frequency of the cavity.

We now consider the evolution equations of small fluctuations with respect to the steady-state values given in Eq. (9). To this end we define

$$\begin{aligned} \delta a &= a - \alpha_s , \\ \delta x &= \hat{x} - x_s , \\ \delta p &= \hat{p} - p_s , \end{aligned} \tag{10}$$

and the fluctuations with respect to the mean value of the input field

$$\delta a_{\text{in}} = a_{\text{in}} - \alpha_{\text{in}} . \tag{11}$$

To the lowest order in the fluctuations we get

$$\frac{d}{dt}\xi = \underline{A}\xi + \eta . \tag{12}$$

The vectors ξ and η are defined by

$$\begin{aligned} \xi^T &= (\delta a, \delta a^\dagger, \delta x, \delta p) , \\ \eta^T &= (\sqrt{\gamma_a}\delta a_{\text{in}}, \sqrt{\gamma_a}\delta a^\dagger_{\text{in}}, 0, -\sqrt{\gamma_m}\epsilon_{\text{in}}) , \end{aligned}$$

where for simplicity we showed the transpose of the column vectors ξ and η. The matrix $\underline{A}$ is given by

$$\underline{A} = \begin{bmatrix} -\gamma_a/2 & 0 & ig\alpha_s & 0 \\ 0 & -\gamma_a/2 & -ig\alpha_s^* & 0 \\ 0 & 0 & 0 & 1/m \\ \hbar g\alpha_s^* & \hbar g\alpha_s & -m\omega_m^2 & -\gamma_m/2m \end{bmatrix} . \tag{13}$$

By using the Fourier transform $f(t) = (2\pi)^{-1/2}\int d\omega \tilde{f}(\omega)\exp(i\omega t)$, we get

$$\tilde{\xi}(\omega)=\underline{B}^{-1}\tilde{\eta}(\omega) , \tag{14}$$

with $\underline{B}=i\omega\underline{I}-\underline{A}$ where $\underline{I}$ is the identity matrix. After some algebra the matrix $\underline{B}^{-1}$ is given by the expression

$$\underline{B}^{-1}(\omega)=\frac{1}{\Delta}\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{14} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} , \tag{15}$$

where

$$\Delta=\left[\frac{\gamma_a}{2}+i\omega\right]^2\left[\omega_m^2-\omega^2+i\frac{\gamma_m}{2m}\omega\right] \tag{16}$$

is the determinant of $\underline{B}$. The matrix elements b_{ij} are given by

$$\begin{aligned} b_{11}&=\left[\frac{\gamma_a}{2}+i\omega\right]\left[\omega_m^2-\omega^2+i\omega\frac{\gamma_m}{2m}\right] +\frac{i|\alpha_s|^2g^2\hbar}{m}=b_{22}^*(-\omega) , \\ b_{12}&=i\frac{\alpha_s^2g^2\hbar}{m}=b_{21}^* , \\ b_{13}&=i\alpha_s g\left[\frac{\gamma_a}{2}+i\omega\right]\left[\frac{\gamma_m}{2m}+i\omega\right]=b_{23}^*(-\omega) , \\ b_{14}&=i\frac{\alpha_s g}{m}\left[\frac{\gamma_a}{2}+i\omega\right]=\frac{b_{41}^*(-\omega)}{\hbar\omega m}=b_{24}^*(-\omega) \\ &=\frac{b_{42}(\omega)}{\hbar\omega m}=i\frac{b_{31}^*(-\omega)}{\hbar}=i\frac{b_{32}(\omega)}{\hbar} , \\ b_{33}&=\left[\frac{\gamma_a}{2}+i\omega\right]^2\left[\frac{\gamma_m}{2m}+i\omega\right] , \\ b_{34}&=\frac{1}{m}\left[\frac{\gamma_a}{2}+i\omega\right]^2=\frac{1}{m^2\omega_m^2}b_{43} , \\ b_{44}&=i\omega\left[\frac{\gamma_a}{2}+i\omega\right]^2 . \end{aligned} \tag{17}$$

We are interested in studying the two-time correlation functions of fluctuations with respect to the steady state which are given, in terms of Fourier transforms, by

$$\langle\tilde{\xi}(\omega)\tilde{\xi}^T(\omega')\rangle=\underline{B}^{-1}(\omega)\langle\tilde{\eta}(\omega)\tilde{\eta}^T(\omega')\rangle[\underline{B}^{-1}(\omega')]^T . \tag{18}$$

The correlations of the noise terms are given by [11]

$$\begin{aligned} &\langle\delta\tilde{a}_{\rm in}(\omega)\delta\tilde{a}_{\rm in}(\omega')\rangle=\langle\delta a_{\rm in}^\dagger(\omega)\delta a_{\rm in}^\dagger(\omega')\rangle=0 , \\ &\langle\delta\tilde{a}_{\rm in}(\omega)\delta\tilde{a}_{\rm in}^\dagger(\omega')\rangle=(1+\bar{n})\delta(\omega+\omega') , \\ &\langle\delta\tilde{a}_{\rm in}^\dagger(\omega)\delta\tilde{a}_{\rm in}(\omega')\rangle=\bar{n}\delta(\omega+\omega') , \\ &\langle\tilde{\epsilon}_{\rm in}(\omega)\tilde{\epsilon}_{\rm in}(\omega')\rangle=k_BT\delta(\omega+\omega') , \end{aligned} \tag{19}$$

where $\bar{n}=\exp[(\hbar\omega_0/k_BT)-1]^{-1}$ is vanishingly small at optical frequencies. Because of the δ function we can rewrite Eq. (18) in the following way:

$$\langle\tilde{\xi}(\omega)\tilde{\xi}^T(\omega')\rangle=\delta(\omega+\omega')\underline{B}^{-1}(\omega)M[\underline{B}^{-1}(-\omega)]^T , \tag{20}$$

with

$$\underline{M}=\begin{bmatrix} 0 & (1+\bar{n})\gamma_a & 0 & 0 \\ \bar{n}\gamma_a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_m k_B T \end{bmatrix} . \tag{21}$$

We are now able to calculate the various correlation functions of interest. The correlation function of fluctuations of the momentum of the oscillating mirror, after lengthy but straightforward algebra, is given by

$$\langle\delta\tilde{p}(\omega)\delta\tilde{p}(\omega')\rangle =\delta(\omega+\omega')\frac{\dfrac{8|\alpha_{\rm in}|^2g^2\hbar^2(\bar{n}+\frac{1}{2})}{\omega^2+\gamma_a^2/4}+\gamma_m k_B T}{[(\omega_m^2-\omega^2)^2+\omega^2\gamma_m^2/4m^2]}\omega^2 . \tag{22}$$

III. QND CRITERIA

Let us try to answer the following question: how good is this system as a measurement device for the amplitude quadrature of the input mode at frequency ω_0? In order that this system be a good measurement device, a criterion introduced by Holland *et al.* [6] should be fulfilled: the quality of the measurement is determined by the level of correlation between the probe field and the signal field, i.e.,

$$C^2_{X^{\rm in}Y^{\rm out}}=\frac{|\langle X^{\rm in}Y^{\rm out}\rangle-\langle X^{\rm in}\rangle\langle Y^{\rm out}\rangle|^2}{V_{X^{\rm in}}V_{Y^{\rm out}}} , \tag{23}$$

where $X^{\rm in}$ is the input signal incident on the apparatus and $Y^{\rm out}$ is the output probe measured by a detector. For a perfect QND measurement device the correlation coefficient $C^2_{X^{\rm in}Y^{\rm out}}$ is unity. In our case $X^{\rm in}$ is the amplitude quadrature of the input signal $X^{\rm in}=(a_{\rm in}+a_{\rm in}^\dagger)$, and the probe is the momentum of the oscillating mirror, so that $Y^{\rm out}=\epsilon_{\rm in}+(\sqrt{\gamma_m}/m)\tilde{p}$. $V_{X^{\rm in}}$ and $V_{Y^{\rm out}}$ are the variances of the two fields defined as $V_Z=\langle Z^2\rangle-\langle Z\rangle^2$. To the lowest order in the fluctuations we have

$$[\langle X^{\rm in}(t)Y^{\rm out}(t')\rangle-\langle X^{\rm in}(t)\rangle\langle Y^{\rm out}(t')\rangle]_{\rm sym} \simeq\frac{\sqrt{\gamma_m}}{m}[\langle\delta a_{\rm in}(t)\delta p(t')\rangle+\langle\delta a_{\rm in}^\dagger(t)\delta p(t')\rangle] , \tag{24}$$

where we used the subscript sym for the symmetrized expression

$$V_{X^{\rm in}}(t,t')=\langle\delta a_{\rm in}(t)\delta a_{\rm in}^\dagger(t')\rangle+\langle\delta a_{\rm in}^\dagger(t)\delta a_{\rm in}(t')\rangle , \tag{25}$$

$$V_{Y^{\rm out}}(t,t')=\langle \epsilon_{\rm in}(t)\epsilon_{\rm in}(t')\rangle+\frac{\gamma_m}{m^2}\langle \delta p(t)\delta p(t')\rangle_{\rm sym}$$
$$+\frac{\sqrt{\gamma_m}}{m}[\langle \delta p(t)\epsilon_{\rm in}(t')\rangle+\langle \epsilon_{\rm in}(t)\delta p(t')\rangle] . \quad (26)$$

In terms of the Fourier transforms, by using Eqs. (14)–(17) and the independence of the input fields $\langle \delta a_{\rm in}(\omega)\epsilon_{\rm in}(\omega')\rangle=\langle \delta a_{\rm in}(\omega')\rangle\langle \epsilon_{\rm in}(\omega')\rangle$, after some algebra we get the symmetrized expression

$$\langle X^{\rm in}(\omega)Y^{\rm out}(\omega')\rangle \simeq \delta(\omega+\omega')\frac{-2i\frac{\sqrt{\gamma_m}}{m}\hbar\omega g[\alpha_{\rm in}+\bar n(\alpha_{\rm in}+\alpha_{\rm in}^*)]}{(\gamma_a/2-i\omega)(\omega_m^2-\omega^2-i\gamma_m\omega/2m)} , \quad (27)$$

$$V_{X^{\rm in}}(\omega,\omega')=(1+2\bar n)\delta(\omega+\omega') . \quad (28)$$

In order to calculate $V_{Y^{\rm out}}(\omega,\omega')$ we need the Fourier transform of the correlations $\langle \delta p(t)\epsilon_{\rm in}(t')\rangle$ and $\langle \epsilon_{\rm in}(t)\delta p(t')\rangle$, while the Fourier transform of the other terms in Eq. (25) have been obtained in Eqs. (19) and (22). Finally we have

$$V_{Y^{\rm out}}(\omega,\omega')=V_{Y^{\rm out}}(\omega)\delta(\omega+\omega') ,$$
$$V_{Y^{\rm out}}(\omega)=\frac{\frac{4\gamma_m}{m^2}|\alpha_{\rm in}|^2g^2\hbar^2(1+2\bar n)\frac{\omega^2}{\omega^2+\gamma_a^2/4}+k_BT\left[(\omega_m^2-\omega^2)^2+\frac{1}{4}\left[\frac{\gamma_m\omega}{m}\right]^2\right]}{(\omega_m^2-\omega^2)^2+\frac{1}{4}\left[\frac{\gamma_m\omega}{m}\right]^2} . \quad (29)$$

In Fig. 1 we show the variance of the output field normalized to its maximum value attained at $\omega=\omega_m$ for various temperatures. We are now able to give the explicit form of Eq. (23)

$$C^2_{X^{\rm in}Y^{\rm out}}(\omega)=\left[1+\frac{k_BTm^2\left[\omega^2+\frac{\gamma_a^2}{4}\right]\left[(\omega_m^2-\omega^2)^2+\frac{1}{4}\frac{\gamma_m^2\omega^2}{m^2}\right]}{4\gamma_m\omega^2|\alpha_{\rm in}|^2g^2\hbar^2(1+2\bar n)}\right]^{-1} , \quad (30)$$

where for simplicity we choose the phase of the external field to get $\alpha_{\rm in}$ real. In Fig. 2 the behavior of $C^2(\omega)\equiv C^2_{X^{\rm in}Y^{\rm out}}(\omega)$ near the resonance value is shown for a particular set of experimental parameters and various equilibrium temperatures. The maximum is obtained for $\omega=\pm\omega_m$; then we have

$$(C^2_{X^{\rm in}Y^{\rm out}})_{\rm max}=(1+\beta)^{-1} , \quad (31)$$

with

$$\beta=\tfrac{1}{16}k_BT\gamma_m\frac{\left[\omega_m^2+\frac{\gamma_a^2}{4}\right]}{|\alpha_{\rm in}|^2g^2\hbar^2(1+2\bar n)} . \quad (32)$$

We see that as long as $\beta\ll1$ it is possible to have

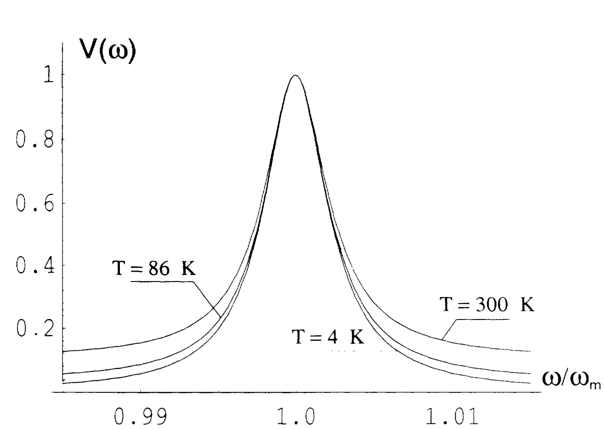

FIG. 1. Spectrum $V(\omega)=V_{Y^{\rm out}}(\omega)/V_{Y^{\rm out}}(\omega_m)$ of output probe fluctuations measured by a detector, normalized at its maximum value reached for $\omega=\omega_m$, is shown vs the dimensionless angular frequency ω/ω_m at various temperatures.

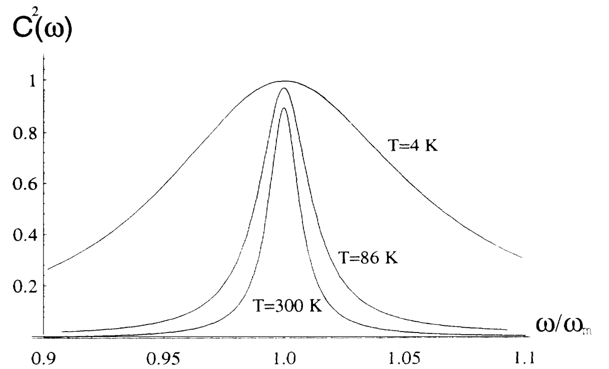

FIG. 2. Correlation coefficient $C^2(\omega)=C^2_{X^{\rm in}Y^{\rm out}}(\omega)=C^2_{X^{\rm out}Y^{\rm out}}(\omega)$ is shown vs ω/ω_m for various temperatures.

$(C_{X^{\rm in}Y^{\rm out}})_{\max}\simeq 1$. For $\omega_m \ll \omega_a$ the relevant quantity is given by

$$r=\sqrt{k_B T \gamma_m}\,\gamma_a/(|\alpha_{\rm in}|g\hbar)=\sqrt{k_B T\gamma_m}\,\frac{c\mathcal{T}}{2\hbar\omega_0|\alpha_{\rm in}|}\ , \tag{33}$$

so that $\beta \cong \frac{1}{64}r^2$. Let us now consider how much the proposed scheme degrades the signal field. Holland *et al.* [6] show that the quantity of interest for such a purpose is

$$C^2_{X^{\rm in}X^{\rm out}}=\frac{|\langle X^{\rm in}X^{\rm out}\rangle-\langle X^{\rm in}\rangle\langle X^{\rm out}\rangle|^2}{V_{X^{\rm in}}V_{X^{\rm out}}}\ . \tag{34}$$

When $C^2_{X^{\rm in}X^{\rm out}}=1$ the scheme is completely able to isolate the noise introduced by the measurement process. Being $X^{\rm out}=\sqrt{\gamma_a}(a+a^\dagger)-X^{\rm in}$ we now consider the symmetrized expression

$$[\langle X^{\rm in}(t)X^{\rm out}(t')\rangle-\langle X^{\rm in}(t)\rangle\langle X^{\rm out}(t')\rangle]_{\rm sym}=[\sqrt{\gamma_a}\langle X^{\rm in}(t)(a(t')+a^\dagger(t'))\rangle]_{\rm sym}-V_{X^{\rm in}}(t,t')\ . \tag{35}$$

After some algebra, by using Eqs. (14)–(17) and correlations given in Eq. (19), always considering a real input field for the sake of simplicity, we get the symmetrized Fourier transforms

$$[\langle X^{\rm in}(\omega)X^{\rm out}(\omega')\rangle-\langle X^{\rm in}(\omega)\rangle\langle X^{\rm out}(\omega')\rangle]_{\rm sym}=(1+2\bar n)\frac{\gamma_a/2-i\omega}{\gamma_a/2+i\omega}\delta(\omega+\omega') \tag{36}$$

and

$$V_{X^{\rm out}}(\omega,\omega')=(1+2\bar n)\delta(\omega+\omega')\ . \tag{37}$$

Then, the criterion in Eq. (34) becomes

$$C^2_{X^{\rm in}X^{\rm out}}(\omega)=1\ , \tag{38}$$

where we used Eq. (28). Then, the scheme is completely able to isolate the noise introduced by the measurement.

However, in order to have a good QND measurement scheme the state of the field, after the measurement, should remain unaffected by the measurement. Holland *et al.* [6] introduced a third criterion to indicate how good the scheme is as a state-preparation device. It turns out that the quality of state preparation for a QND scheme can be evaluated by considering the quantity

$$C^2_{X^{\rm out}Y^{\rm out}}=\frac{|\langle X^{\rm out}Y^{\rm out}\rangle-\langle X^{\rm out}\rangle\langle Y^{\rm out}\rangle|^2}{V_{X^{\rm out}}V_{Y^{\rm out}}}\ . \tag{39}$$

A perfect state-preparation device would have $C^2_{X^{\rm out}Y^{\rm out}}=1$, which gives the variance in the signal output, given a measured value for the probe field, as

$$V(X^{\rm out}|Y^{\rm out})=V_{X^{\rm out}}(1-C^2_{X^{\rm out}Y^{\rm out}})=0\ . \tag{40}$$

In the present situation we can evaluate

$$\langle[X^{\rm out}(t)Y^{\rm out}(t')\rangle-\langle X^{\rm out}(t)\rangle\langle Y^{\rm out}(t')\rangle]_{\rm sym}=\sqrt{\gamma_a}\langle(\delta a(t)+\delta a^\dagger(t))\epsilon_{\rm in}(t')\rangle+\frac{\sqrt{\gamma_a\gamma_m}}{m}\langle(\delta a(t)+\delta a^\dagger(t))\delta p(t')\rangle_{\rm sym}-\frac{\sqrt{\gamma_m}}{m}\langle(\delta a_{\rm in}(t)+\delta a^\dagger_{\rm in}(t))\delta p(t')\rangle\ . \tag{41}$$

After lengthy but straightforward algebra we get the symmetrized Fourier transform

$$\langle X^{\rm out}(\omega)Y^{\rm out}(\omega')\rangle_{\rm sym}=\frac{-2i\dfrac{\sqrt{\gamma_m}}{m}\hbar\omega g[\alpha_{\rm in}+\bar n(\alpha_{\rm in}+\alpha^*_{\rm in})]}{(\gamma_a/2+i\omega)(\omega_m^2-\omega^2-i\gamma_m\omega/2m)}\delta(\omega+\omega')\ . \tag{42}$$

Thus, finally for a real input field we get

$$C^2_{X^{\rm out}Y^{\rm out}}(\omega)=\left[1+\frac{k_BTm^2\left[\omega^2+\dfrac{\gamma_a^2}{4}\right]\left[(\omega_m^2-\omega^2)^2+\dfrac{1}{4}\dfrac{\gamma_m^2\omega^2}{m^2}\right]}{4\gamma_m\omega^2|\alpha_{\rm in}|^2g^2\hbar^2(1+2\bar n)}\right]^{-1}\ , \tag{43}$$

where we used Eqs. (29) and (37). We thus obtain

$$C^2_{X^{\rm out}Y^{\rm out}}(\omega)=C^2_{X^{\rm in}Y^{\rm out}}(\omega)\ . \tag{44}$$

In the best situation considered above, the value of $C^2_{X^{\rm out}Y^{\rm out}}$ can be $\simeq 1$; in such a case the minimum value of $V(X^{\rm out}|Y^{\rm out})\simeq 0$. It means that the scheme is a very good QND measurement device because the state after the measurement is not degraded.

IV. CONCLUSIONS

The maximum correlation between the momentum fluctuations of the mirror and the amplitude quadrature fluctuations occurs at $\omega=\pm\omega_m$. Thus one should observe the amplitude of the variance of momentum fluctuations at this maximum ($\omega=\pm\omega_m$) as this will give the best measure of the amplitude quadrature of the input mode.

The width of the variance of momentum fluctuations is determined by the damping constant of the oscillating mirror.

For $\omega=\pm\omega_m$ we get

$$V_{Y^{\rm out}}(\omega_m;|\alpha_{\rm in}|^2)=k_BT\left[1+\frac{64}{k_BT}\left[\frac{g\hbar}{\gamma_a}\right]^2\frac{|\alpha_{\rm in}|^2}{\gamma_m}\right]. \qquad (45)$$

Thus, the number of photons impinging the cavity at frequency ω_0 is

$$|\alpha_{\rm in}|^2=\frac{1}{\Lambda}[V_{Y^{\rm out}}(\omega_m;|\alpha_{\rm in}|^2)-V_{Y^{\rm out}}(\omega_m;0)] \qquad (46)$$

with

$$\Lambda=\frac{64}{\gamma_m}\left[\frac{g\hbar}{\gamma_a}\right]^2 \qquad (47)$$

fixed by the experimental setup.

Let us now consider a possible experimental situation specified by the following set of parameters which could be adjusted by an experimentalist: $L=10^{-1}$ m, $\omega_0=10^{18}$ s^{-1}, $\mathcal{T}=2\times10^{-2}$, $\gamma_m/m=10^{-1}$ s^{-1}, $m=10^{-7}$ kg, and $T=4$ K. Then $\beta\sim1.39\times10^{14}|\alpha_{\rm in}|^{-2}$. For an input power $P=\hbar\omega_0|\alpha_{\rm in}|^2\sim0.105$ W we have $|\alpha_{\rm in}|^2\simeq10^{15}$ s^{-1} then $\beta\simeq0.139$ and $(C^2_{X^{\rm in}Y^{\rm out}})_{\rm max}\simeq0.877$. For higher input power $(C^2_{X^{\rm in}Y^{\rm out}})_{\rm max}$ rises and can approach 1. At lower input power with $|\alpha_{\rm in}|^2\sim10^8$ s^{-1}, we can still have a good criterion for smaller values of the cavity damping constant. This could be obtained by using the mirrors recently considered by Rempe *et al.* [12] with a transmittivity $\mathcal{T}=1.6\times10^{-6}$. In this case $\gamma_a=4.8\times10^3$ and we could obtain $(C^2_{X^{\rm in}Y^{\rm out}})_{\rm max}\simeq0.99$ with an input power $P\simeq1.05\times10^{-8}$ W. At lower frequencies with $\omega_0=3\times10^{15}$ s^{-1} with the same mirrors we still could have a good criterion $(C^2_{X^{\rm in}Y^{\rm out}})_{\rm max}\simeq0.97$ with an input power $P\simeq0.1\times10^{-3}$ W and $\gamma_m/m=10^{-1}$ s^{-1} with $m=10^{-5}$ kg. In Fig. 2 we show $C^2(\omega)\equiv C^2_{X^{\rm in}Y^{\rm out}}(\omega)=C^2_{X^{\rm out}Y^{\rm out}}(\omega)$ for these values of the parameters and various temperatures. We conclude that a measurement of the quadrature phase of the external signal, by measuring the momentum fluctuations of a freely suspended mirror induced by radiation-pressure fluctuations, should be feasible. Our conclusion is supported by the values of the above criteria that for $\omega=\pm\omega_m$ are $C^2_{X^{\rm out}Y^{\rm out}}=C^2_{X^{\rm in}Y^{\rm out}}\simeq1$ for the values of the various parameters considered, while always $C^2_{X^{\rm in}X^{\rm out}}=1$ holds.

Therefore, we can consider the scheme as a good QND measurement device, giving a measurement of the number of photons for a sufficiently high input power and in a good range of temperatures, without feeding any spurious noise into the system and leaving the state practically unchanged after the measurement. This scheme could be useful in a tap extracting information from a transmission line by means of a nondestructive measurement of the number of photons.

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