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ARTICLES

Feynman's approach to negative probability in quantum mechanics

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Feynman introduces the concept of negative probability in the context of Young's double-slit experiment and in doing so sheds a new light on the problem. However, there are, as Feynman notes, conceptual problems as well as insights associated with this point of view. The micromaser which-path (*Welcher-Weg*) detector eliminates these conceptual difficulties. We also emphasize that the concept of negative probability yields useful insight into the Einstein-Podolsky-Rosen (EPR) problem.

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I. INTRODUCTION

Young's double-slit experiment contains the basic mystery of quantum mechanics. In fact, to quote Feynman [1], it contains the *only* mystery of quantum mechanics. The particle goes through *both* holes. How to think about this? In a recent paper [2], Feynman shows that the concept of negative probability may be useful in this context. Specifically, he considers Young's experiment from the perspective of "two-state" quantum mechanics and develops joint quasiprobability distributions appropriate to the problem. These "Wigner-like" distributions can, of course, be negative; and it is this facet of the problem which is most interesting in the present context.

There are two aspects to the problem of negative probabilities and Young's experiment. First, one must establish the quasiprobability distribution to be used for spin one-half systems. This has been a problem of long-standing interest to us and our approach [3–6] to the subject is presented in Sec. II. Secondly, the application of the spin one-half distribution function to certain correlations involving the two-slit interference pattern is considered and intriguing results are obtained. However, as Feynman notes, we here meet conceptual difficulties. Concerning this second point, we find that our previous studies on micromaser which-path (*Welcher-Weg*) detectors [7] allows us to sharpen Feynman's treatment and eliminate the conceptual difficulties, as is discussed in Sec. III. Finally, we demonstrate how these considerations can shed light on the Einstein-Podolsky-Rosen (EPR) paradox [5,8] in Sec. IV.

II. JOINT QUASIDISTRIBUTIONS FOR SPIN ONE-HALF SYSTEMS

We here are interested in the question: What is the joint probability of finding a spin in, e.g., the $+z$ and $+x$ direction? This is the same sort of question that the Wigner distribution [8] addresses in that it gives us a joint probability for p and q.

To this end Feynman simply writes down the joint probability distributions

$$P_{++}=\tfrac{1}{4}[1+\langle\hat{\sigma}_z\rangle+\langle\hat{\sigma}_x\rangle+\langle\hat{\sigma}_y\rangle]\ , \tag{1a}$$

$$P_{+-}=\tfrac{1}{4}[1+\langle\hat{\sigma}_z\rangle-\langle\hat{\sigma}_x\rangle-\langle\hat{\sigma}_y\rangle]\ , \tag{1b}$$

$$P_{-+}=\tfrac{1}{4}[1+\langle\hat{\sigma}_z\rangle+\langle\hat{\sigma}_x\rangle-\langle\hat{\sigma}_y\rangle]\ , \tag{1c}$$

$$P_{--}=\tfrac{1}{4}[1-\langle\hat{\sigma}_z\rangle-\langle\hat{\sigma}_x\rangle-\langle\hat{\sigma}_y\rangle]\ , \tag{1d}$$

where, for example, P_{++} is the joint probability of finding the system to have its spin along $+z$ and $+x$ simultaneously, and $\hat{\sigma}_x$, $\hat{\sigma}_y$, and $\hat{\sigma}_z$ are the usual Pauli spin operators.

As we noted, Feynman does not motivate or derive (1a)–(1d). In his words he simply "defines" the joint probabilities in this way. But why not some other definition? In fact, there is no unique way to define such probability distributions.

In considering questions of this type in previous work [3,4], we developed the associated quasiclassical distributions in the z,x plane as

$$P^{(z,x)}(s_z,s_x)\equiv\langle\delta(s_z-\hat{\sigma}_z)\delta(s_x-\hat{\sigma}_x)\rangle\ , \tag{2}$$

where the (z,x) superscript reminds us that we have chosen z,x ordering, s_z and s_x are the "classical" parameters associated with the corresponding Pauli operators $\hat{\sigma}_z$

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and $\hat{\sigma}_x$ and the δ functions appearing in Eq. (1) are defined by their Fourier transforms

$$\delta(s_j-\hat{\sigma}_j)\equiv\frac{1}{2\pi}\int d\xi\, e^{i\xi(s_j-\hat{\sigma}_j)} , \tag{3a}$$

which we may write as

$$\delta(s_j-\hat{\sigma}_j)=\frac{1}{2\pi}\int d\xi[\cos\xi-i\hat{\sigma}_j\sin\xi]e^{i\xi s_j} . \tag{3b}$$

Inserting (3b) into (2) we find

$$\begin{aligned}P^{(z,x)}(s_z,s_x)=\tfrac{1}{4}[&\{\delta(s_z+1)+\delta(s_z-1)\}\{\delta(s_x+1)+\delta(s_x-1)\}-\{\delta(s_z+1)+\delta(s_z-1)\}\{\delta(s_x+1)-\delta(s_x-1)\}\langle\hat{\sigma}_x\rangle\\&-\{\delta(s_z+1)-\delta(s_z-1)\}\{\delta(s_x+1)+\delta(s_x-1)\}\langle\hat{\sigma}_z\rangle\\&+\{\delta(s_z+1)-\delta(s_z-1)\}\{\delta(s_x+1)-\delta(s_x-1)\}\langle\hat{\sigma}_z\hat{\sigma}_x\rangle] ,\end{aligned} \tag{4}$$

noting that $\hat{\sigma}_z\hat{\sigma}_x=i\hat{\sigma}_y$ and rearranging terms, (4) becomes

$$\begin{aligned}P^{(z,x)}(s_z,s_x)=\tfrac{1}{4}\{&\delta(s_z+1)\delta(s_x+1)[1-\langle\hat{\sigma}_z\rangle-\langle\hat{\sigma}_x\rangle+i\langle\hat{\sigma}_y\rangle]+\delta(s_z+1)\delta(s_z-1)[1-\langle\hat{\sigma}_z\rangle+\langle\hat{\sigma}_x\rangle+i\langle\hat{\sigma}_y\rangle]\\&+\delta(s_z-1)\delta(s_x+1)[1+\langle\hat{\sigma}_z\rangle-\langle\hat{\sigma}_x\rangle-i\langle\hat{\sigma}_y\rangle]+\delta(s_z-1)\delta(s_x-1)[1+\langle\hat{\sigma}_z\rangle+\langle\hat{\sigma}_x\rangle+i\langle\hat{\sigma}_y\rangle]\} .\end{aligned} \tag{5}$$

The physical interpretation of the various terms in Eq. (5) is clear. For example, $P^{(z,x)}_{++}$, the joint probability of finding $+z$ and $+x$, is now associated with $s_z=1$ and $s_x=1$; i.e., with the coefficient of the δ function pair $\delta(s_z-1)\delta(s_x-1)$, which is

$$P^{(z,x)}_{++}=1+\langle\sigma_z\rangle+\langle\sigma_x\rangle+i\langle\sigma_y\rangle .$$

But consider the case of x,z ordering defined by

$$P^{(x,z)}=\langle\delta(s_x-\hat{\sigma}_x)\delta(s_z-\hat{\sigma}_z)\rangle . \tag{6}$$

Now everything goes through as before, except that the $\langle\hat{\sigma}_z\hat{\sigma}_x\rangle$ term in Eq. (4) is replaced by $\langle\hat{\sigma}_x\hat{\sigma}_z\rangle$. And since $\langle\hat{\sigma}_x\hat{\sigma}_z\rangle=-\langle\hat{\sigma}_z\hat{\sigma}_x\rangle=-i\langle\sigma_y\rangle$ we have only to replace $\langle\sigma_y\rangle$ by $-\langle\sigma_y\rangle$ in Eq. (5) to obtain $P^{(x,z)}(s_z,s_x)$, that is

$$\begin{aligned}P^{(x,z)}(s_z,s_x)=\tfrac{1}{4}\{&\delta(s_z+1)\delta(s_x+1)[1-\langle\hat{\sigma}_z\rangle-\langle\hat{\sigma}_x\rangle-i\langle\hat{\sigma}_y\rangle]+\delta(s_z+1)\delta(s_x-1)[1-\langle\hat{\sigma}_z\rangle+\langle\hat{\sigma}_x\rangle-i\langle\hat{\sigma}_y\rangle]\\&+\delta(s_z-1)\delta(s_x+1)[1+\langle\hat{\sigma}_z\rangle-\langle\hat{\sigma}_x\rangle+i\langle\hat{\sigma}_y\rangle]+\delta(s_z-1)\delta(s_x-1)[1+\langle\hat{\sigma}_z\rangle+\langle\hat{\sigma}_x\rangle-i\langle\hat{\sigma}_y\rangle]\} .\end{aligned} \tag{7}$$

Thus if we consider the symmetric distribution

$$P(s_z,s_x)=\tfrac{1}{2}[P^{(z,x)}(s_z,s_x)+P^{(x,z)}(s_z,s_x)] \tag{8}$$

we find

$$\begin{aligned}P(s_z,s_x)=\delta(s_x-1)\delta(s_z-1)P_{++}&+\delta(s_x+1)\delta(s_z-1)P_{+-}\\&+\delta(s_x-1)\delta(s_z+1)P_{-+}\\&+\delta(s_x+1)\delta(s_z+1)P_{--} ,\end{aligned} \tag{9}$$

where

$$P_{++}=\tfrac{1}{4}\langle 1+\hat{\sigma}_z+\hat{\sigma}_x\rangle , \tag{10a}$$

$$P_{+-}=\tfrac{1}{4}\langle 1+\hat{\sigma}_z-\hat{\sigma}_x\rangle , \tag{10b}$$

$$P_{-+}=\tfrac{1}{4}\langle 1-\hat{\sigma}_z+\hat{\sigma}_x\rangle , \tag{10c}$$

$$P_{--}=\tfrac{1}{4}\langle 1-\hat{\sigma}_z-\hat{\sigma}_x\rangle . \tag{10d}$$

Finally, following Ref. [4], we give the joint probability distribution along any two directions θ_1, and θ_2 expressed in terms of the operators

$$\hat{\sigma}_j|\pm\theta_j\rangle=\pm|\pm\theta_j\rangle,\quad j=1,2 . \tag{11}$$

We now find

$$P_{++}=\tfrac{1}{4}\langle 1+\hat{\sigma}_1+\hat{\sigma}_2+\cos(\theta_1-\theta_2)\rangle , \tag{12a}$$

$$P_{+-}=\tfrac{1}{4}\langle 1+\hat{\sigma}_1-\hat{\sigma}_2-\cos(\theta_1-\theta_2)\rangle , \tag{12b}$$

$$P_{-+}=\tfrac{1}{4}\langle 1-\hat{\sigma}_1+\hat{\sigma}_2-\cos(\theta_1-\theta_2)\rangle , \tag{12c}$$

$$P_{--}=\tfrac{1}{4}\langle 1-\hat{\sigma}_1-\hat{\sigma}_2+\cos(\theta_1-\theta_2)\rangle . \tag{12d}$$

III. YOUNG'S EXPERIMENT AND THE MICROMASER *WELCHER-WEG* DETECTOR

Next let us recall the micromaser-double-slit setup of Ref. [7], as per Fig. 1. As discussed in Ref. [7] and Fig. 2, the passage through hole 1 (2) is always associated with a photon in cavity 1 (2) which is to say the state $|1,0\rangle$ ($|01\rangle$). Furthermore, the symmetric (antisymmetric) states

$$|s\rangle=\frac{1}{\sqrt{2}}[|10\rangle+|01\rangle] , \tag{13a}$$

$$|\bar{s}\rangle=\frac{1}{\sqrt{2}}[|10\rangle-|01\rangle] , \tag{13b}$$

are associated with symmetric and antisymmetric interference fringe patterns.

Now, it is clear that there is an immediate correspondence between the states of the *Welcher-Weg* detectors and a spin one-half system, as is summarized in Table I.

Having set the stage, introducing the micromaser-which-path-detector description of the double-slit experiment, we next use Eqs. (10a)–(10d) to reconsider

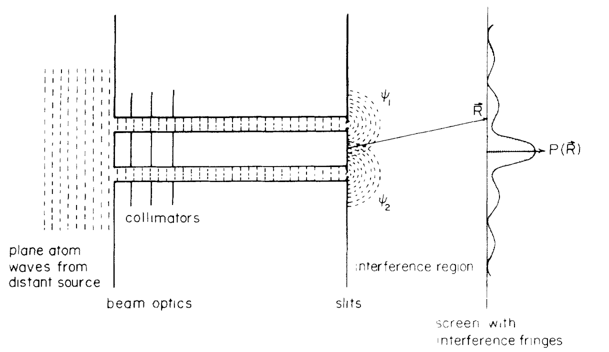

FIG. 1. Two-slit experiment with atoms. A set of wider slits collimates two atom beams which illuminate the narrow slits where the interferences pattern originates.

Feynman's question. To quote Feynman: "What is the joint probability of finding the particle to go through hole 1 and be 180° out of phase with hole 2 (whatever that could mean)?"

First we note that the question, posed in terms of the micromaser *Welcher-Weg* approach, is more clearly defined. The joint probability of "going through one hole and being out of phase with the other hole" takes on a precise and clear meaning when expressed in terms of the detector states of Table I. Thus, P_{+-} governs the "probability" that the atom goes through hole 1 and that the micromasers are in the state $|\bar{s}\rangle$.

We proceed to calculate P_{+-}, P_{++}, etc. by writing the state of the micromaser atom system which, in the region following the slits, is given by

$$\psi=\psi_1(\xi)|10\rangle+\psi_2(\xi)|01\rangle \ , \tag{14}$$

where

$$\psi_1(\xi)=Ne^{-\frac{(\xi-d)^2}{4w^2}}e^{-i\frac{(\xi-d)^2}{w^2}} \tag{15}$$

in which N is a normalization factor, d is the slit spacing

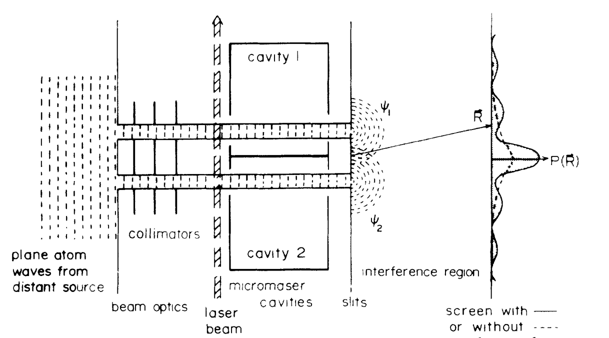

FIG. 2. Two-slit interference pattern at a screen associated with the state of a micromaser *Welcher-Weg* detector setup. The setup of Fig. 1 is supplemented by two high-quality micromaser cavities and a laser beam to provide which-path information. This is accomplished by arranging things such that every time an atom passes through the pair of micromaser cavities it leaves a photon in one or the other cavity, thus providing which-path information, see Ref. [7].

TABLE I. *Welcher-Weg* detector in photon and spin one-half notation.

Photons	Spin one-half
$\|10\rangle$	$\|\uparrow\rangle$
$\|01\rangle$	$\|\downarrow\rangle$
$\|s\rangle$	$\|+x\rangle$
$\|\bar{s}\rangle$	$\|-x\rangle$

while w governs the width of the initial Gaussian wave packet, and, for convenience, we have taken the screen to be at $z=2\pi\omega^2/\lambda$. The wave function $\psi_2(\xi)$ is obtained by replacing $d\rightarrow -d$ in Eq. (15).

Using the state of the system, as given by Eq. (14) together with the definitions [(10a)–(10d)] we find

$$P_{++}(\xi)=\tfrac{1}{2}[|\psi_1(\xi)|^2+\tfrac{1}{2}\{\psi_1^*(\xi)\psi_2(\xi)+\text{c.c.}\}] \ , \tag{16a}$$

$$P_{+-}(\xi)=\tfrac{1}{2}[|\psi_1(\xi)|^2-\tfrac{1}{2}\{\psi_1^*(\xi)\psi_2(\xi)+\text{c.c.}\}] \ , \tag{16b}$$

$$P_{-+}(\xi)=\tfrac{1}{2}[|\psi_2(\xi)|^2+\tfrac{1}{2}\{\psi_1^*(\xi)\psi_2(\xi)+\text{c.c.}\}] \ , \tag{16c}$$

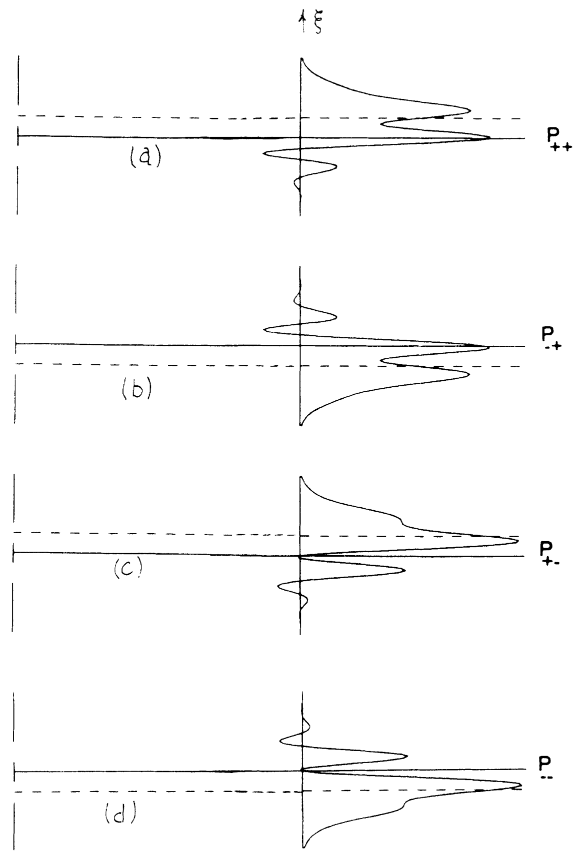

FIG. 3. (a) Joint probability P_{++} for a particle going through hole 1 and the *Welcher-Weg* detector in a symmetric state $|s\rangle$ given by Eq. (17a); ξ is displacement on screen. (b) Joint probability P_{-+} for a particle going through hole 2 and the *Welcher-Weg* detector in symmetric state $|s\rangle$ as given by Eq. (17c). (c) Joint probability P_{+-} for a particle going through hole 1 and the *Welcher-Weg* detector in the antisymmetric state $|\bar{s}\rangle$ as per Eq. (17b). (d) Joint probability P_{--} for passage through hole 2 and the *Welcher-Weg* detector in $|\bar{s}\rangle$ as in Eq. (17d).

$$P_{--}(\xi)=\tfrac{1}{2}[|\psi_2(\xi)|^2-\tfrac{1}{2}\{\psi_1^*(\xi)\psi_2(\xi)+\text{c.c.}\}]\ , \quad (16d)$$

and inserting the expressions for ψ_1 and ψ_2 into 16(a)–16(d) we find

$$P_{++}(\xi)=\tfrac{1}{2}N^2\left[e^{-\frac{(\xi-d)^2}{2w^2}}+e^{-\frac{(\xi^2+d^2)}{2w^2}}\cos\frac{4\xi d}{w^2}\right]\ , \quad (17a)$$

$$P_{+-}(\xi)=\tfrac{1}{2}N^2\left[e^{-\frac{(\xi-d)^2}{2w^2}}-e^{-\frac{(\xi^2+d^2)}{2w^2}}\cos\frac{4\xi d}{w^2}\right]\ , \quad (17b)$$

$$P_{-+}(\xi)=\tfrac{1}{2}N^2\left[e^{-\frac{(\xi+d)^2}{2w^2}}+e^{-\frac{(\xi^2+d^2)}{2w^2}}\cos\frac{4\xi d}{w^2}\right]\ , \quad (17c)$$

$$P_{--}(\xi)=\tfrac{1}{2}N^2\left[e^{-\frac{(\xi+d)^2}{2w^2}}-e^{-\frac{(\xi^2+d^2)}{2w^2}}\cos\frac{4\xi d}{w^2}\right]\ . \quad (17d)$$

Equations (17a)–(17d) are the main results of this paper, and are plotted in Fig. 3.

We note that while each of these probability distributions may be negative, the physically meaningful constructive and destructive interference patterns

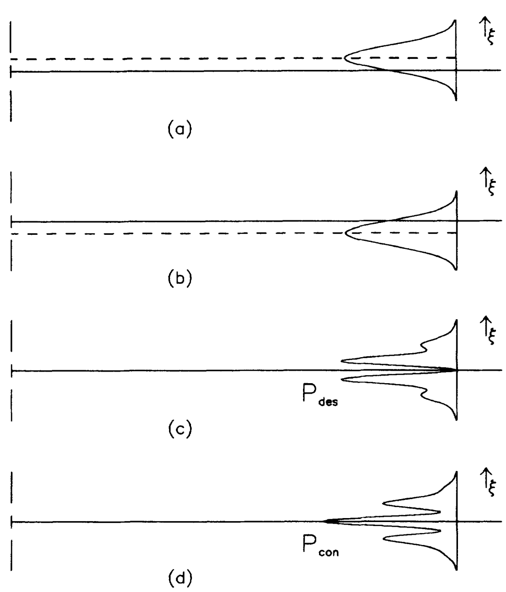

FIG. 4. (a) Atoms pass through hole 1, leaving photons in cavity 1, so that state of the *Welcher-Weg* detector is $|1,0\rangle$. The pattern on the screen is the probability $P_{++}+P_{+-}$. (b) Atoms pass through hole 2, leaving photon in cavity 2, so that the state of the *Welcher-Weg* detector is $|0,1\rangle$. The pattern on the screen is the probability $P_{-+}+P_{--}$. (c) $P(\xi)_{\text{destructive}}=P_{+-}+P_{--}$ destructive interference is found by asking for total probability given the *Welcher-Weg* state $|\bar{s}\rangle$. (d) $P(\xi)_{\text{constructive}}=P_{++}+P_{-+}$ constructive interference is found by asking for total probability given the *Welcher-Weg* detectors in $|s\rangle$.

$$P_{\text{constructive}}(\xi)=P_{++}(\xi)+P_{-+}(\xi)\ , \quad (18a)$$

$$P_{\text{destructive}}(\xi)=P_{+-}(\xi)+P_{--}(\xi)\ , \quad (18b)$$

are, however, everywhere positive as seen in Fig. 4.

IV. NEGATIVE PROBABILITIES AND THE EPR PARADOX

Thus far we have seen that the combinations $P_{++}+P_{-+}$ and $P_{+-}+P_{--}$ give us new insight into Young's double-slit problem. In this section we ask: What do we learn if we consider combinations like $P_{+-}+P_{-+}$? As we shall see, the answer is: a new way to look at the EPR paradox.

To this end let us recall the essence of the EPR dilemma. Following Fry [9] we begin with a spin singlet formed from, for example, the nuclear spin singlet contained within the Hg_2 molecule. Now if we "split" the molecule into its individual Hg atom constituents, the spin state remains a singlet; that is, the nuclear spins are described by

$$|\psi\rangle=\frac{1}{\sqrt{2}}[|\uparrow_1,\downarrow_2\rangle-|\downarrow_1,\uparrow_2\rangle] \quad (19)$$

even though they are propagating to opposite ends of the "universe." The essence of the EPR problem is as follows.

(1) Pick an arbitrary direction, which we can take to be the z axis, and pass one Hg atom (say atom 1) through a Stern-Gerlach apparatus (SGA) oriented along the z axis. The particle will now be deflected in either the + or $-z$ direction, say $+z$.

(2) Knowing that the spin of nucleus 1 is up, we now know the spin of particle 2 is down. But if we then pass atom 2 through a SGA oriented along the x axis we will find that particle 2 has a definite spin along the x direction (either $+x$ or $-x$).

(3) Therefore, as the argument goes, we know both the z and x components of spin 2–in violation of complementarity.

How and what should we think about this? One appealing approach is to think carefully through the measurement sequence and its mathematical analog. Suppose we ask for the simultaneous passage of both atoms through two SGA's. The first SGA is along the z direction and the SGA for the second atom is oriented at an angle θ to the z axis. Then the eigenstates are $|\pm z\rangle_1$ for spin 1 and

$$|\pm\theta\rangle_2=e^{-\frac{i}{2}\sigma_y\theta}|\pm z\rangle_2 \quad (20)$$

or, upon expanding the exponent in Eq. (20), we have

$$|+\theta\rangle_2=\cos\frac{\theta}{2}|\uparrow\rangle_2+\sin\frac{\theta}{2}|\downarrow\rangle_2\ , \quad (21a)$$

$$|-\theta\rangle_2=-\sin\frac{\theta}{2}|\uparrow\rangle_2+\cos\frac{\theta}{2}|\downarrow\rangle_2\ . \quad (21b)$$

Now the probability of finding particle 1 deflected in the $+z$ direction and the second particle deflected in the $|-\theta\rangle$ direction is

$$P^{(2)}(+z_1,-\theta_2)=|\langle\uparrow_1,\theta_2|\psi\rangle|^2 \quad (22)$$

and from the specification of $|\psi\rangle$ as the spin singlet (19) and using (21b) we find

$$P^{(2)}(+z_1,-\theta_2)=\tfrac{1}{4}(1-\cos\theta)\ . \quad (23)$$

Finally, we note that by symmetry we would expect

$$P^{(2)}(-z_1,+\theta_2)=P^{(2)}(+z_1,-\theta_2) \quad (24)$$

as can be verified by direct calculation.

Next, following Refs. [4] and [5], we return to the single particle case as given by Eq. (12). In particular, we consider the case where $\sigma_1=\sigma_z$, i.e., $\theta_1=0$ and $\sigma_2=\sigma_\theta$, i.e., $\theta_2=\theta$. Then the analog of the two particle joint probability $P^{(2)}(+z_1,-\theta_2)$ is given by Eq. (12c), that is

$$P^{(2)}(+z_1,-\theta_2)\rightarrow P_{+-}=\tfrac{1}{4}\langle 1+\hat{\sigma}_z-\hat{\sigma}_\theta-\cos\theta\rangle\ , \quad (25a)$$

and likewise

$$P^{(2)}(-z_1,+\theta_2)\rightarrow P_{-+}=\tfrac{1}{4}\langle 1-\hat{\sigma}_z+\hat{\sigma}_\theta-\cos\theta\rangle\ . \quad (25b)$$

Physically, the joint count distribution Eq. (23) must be symmetric (depend only on θ and not, e.g., the choice of the z axis) because the singlet state (19) is symmetric. However, P_{+-} and P_{-+} are dependent on $\langle\sigma_z\rangle$, and therefore on the choice of the z axis. Hence we see that the single particle joint probabilities P_{+-} and P_{-+} are very different from the two particle distributions which they are supposed to mimic. However, the symmetrical version,

$$\tfrac{1}{2}(P_{+-}+P_{-+})=\tfrac{1}{4}(1-\cos\theta)\ , \quad (26)$$

is the correct two particle result (23).

V. SUMMARY

This paper is summarized as follows.

(1) The spin one-half quantities P_{++}, etc. are naturally described by a Wigner-like distribution.

(2) The connection with a double-slit experiment is convincingly made by a micromaser *Welcher-Weg* setup.

(3) The joint distribution $P_{++}(\xi)$ for Young's experiment can be negative but the physical observable $P_{++}(\xi)+P_{-+}(\xi)$ is everywhere positive.

(4) The average $\frac{1}{2}[P_{+-}+P_{-+}]$ corresponds exactly to the EPR result and provides insight into that problem.

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[1] R. Feynman, R. Leighton, and M. Sands, *The Feynman Lectures on Physics III* (Addison Wesley, Reading, MA, 1965).

[2] R. Feynman, in *Negative Probabilities in Quantum Mechanics,* edited by B. Hiley and F. Peat (Routledge, London, 1987).

[3] M. Scully, Phys. Rev. D **28**, 2477 (1983).

[4] L. Cohen and M. Scully, Found. Phys. **16**, 295 (1986).

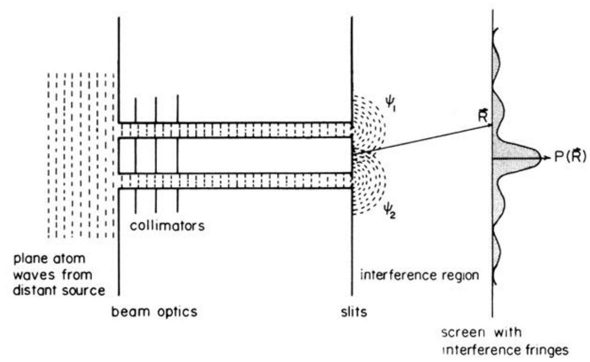
[5] M. Scully and L. Cohen, *New Techniques and Ideas in Measurement Theory,* edited by D. Greenberger (Annals of the New York Academy of Science, 1987).

[6] C. Chandler, L. Cohen, C. Lu, M. Scully, and K. Wodkiewicz, Found. Phys. **22**, 867 (1992).

[7] M. Scully, B.-G. Englert, and H. Walther, Nature **351**, 111 (1991). For an analysis of the *Welcher-Weg* information in a Mach-Zehender interferometer using Pauli matrices see T. Hellmuth, H. Walther, A. Zajonc, and W. Schleich, Phys. Rev. A **35**, 2532 (1987); and for other approaches to the EPR paradox using negative-valued Wigner functions see G. Agarwal, Phys. Rev. A **24**, 2889 (1981); G. Agarwal, D. Home, and W. Schleich, Phys. Lett. A **170**, 359 (1993); M. Scully and K. Wodkiewicz, Found. Phys. (to be published).

[8] For a review of the Wigner distribution see H. Hillery, R. O'Connell, M. Scully, and E. Wigner, Phys. Rep. **106**, 121 (1984).

[9] E. Fry, in *Foundations of Quantum Mechanics,* edited by T. Black, M. Nieto, H. Pilloff, and R. Sinclair (World Scientific, Singapore, 1992).



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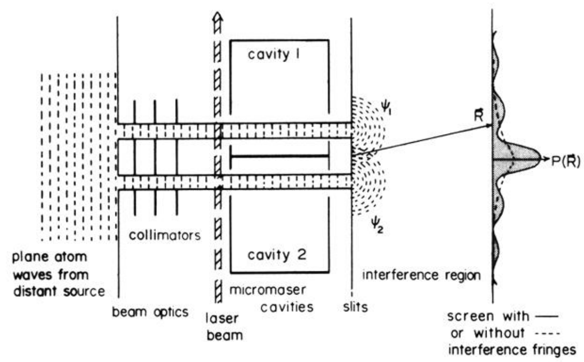

FIG. 2. Two-slit interference pattern at a screen associated with the state of a micromaser *Welcher-Weg* detector setup. The setup of Fig. 1 is supplemented by two high-quality micromaser cavities and a laser beam to provide which-path information. This is accomplished by arranging things such that every time an atom passes through the pair of micromaser cavities it leaves a photon in one or the other cavity, thus providing which-path information, see Ref. [7].