Polarization-sensitive population trapping in an optically pumped laser

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We prove that population trapping depends on the relative polarizations between the pump and generated fields in a double-V laser system. Orthogonal linear field polarizations can lead to total suppression of emission. An interpretation of this alternative mechanism of inversion without amplification is given in the context of a dressed-atom formalism.

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I. INTRODUCTION

Over the last few years the generation of atomic coherence has been attracting much interest because of derivative phenomena such as lasing without population inversion and inversion without lasing [1–3] and, very recently, enhancement of the refractive index in a transparent medium [4]. Although much effort is being devoted to the study of the mechanisms that could lead to the generation of coherent radiation without the necessity of inversion between atomic levels, considerably less attention has been paid to the opposite phenomenon, i.e., the creation of a population inversion between atomic states, *a priori* sufficient for the amplification of coherent radiation, which actually does not give rise to this process [1(a)]. This effect can be related to the accummulation of population in trap states [5], i.e., states created by the interaction between the pump and the bare states involved in this process, whose connection with the lower lasing level is forbidden. Nevertheless, this need not be the only possibility since, e.g., suppression of emission can be achieved *via* quantum interferences [1,2].

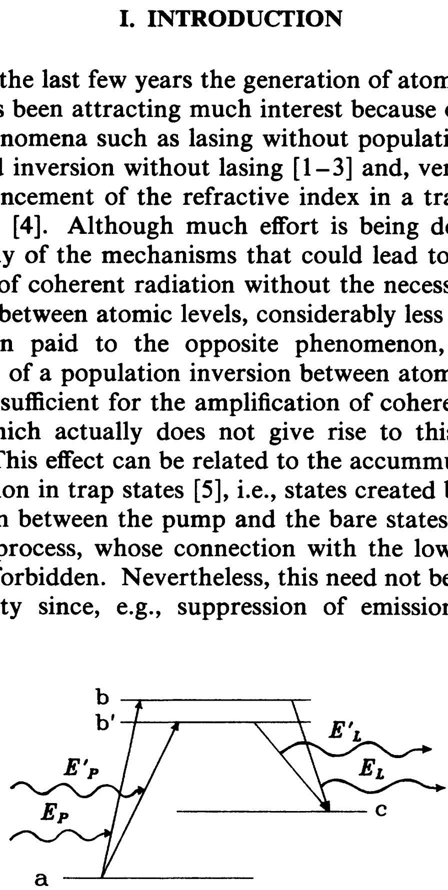

FIG. 1. Schematic of the four-level optically pumped laser. E_P (E_L) and E'_P (E'_L) are the dextro and levo components of the pump (generated) field. The bare states $|a\rangle$ and $|c\rangle$ have the same quantum number $J=0$. The upper bare states $|b\rangle$ and $|b'\rangle$ are degenerated states with $J=1$ and $m_J=+1,-1$, respectively.

In the present Brief Report we discuss a physical situation of inversion without amplification which is determined, among other factors, by the polarizations of the optical fields acting on the atomic (or molecular) system. Specifically, we study the influence of the relative polarization of a driving field and a generated field on the behavior of a double-V system (Fig. 1). This level scheme corresponds to an optically pumped laser with two M-degenerate upper levels (with $J=1$ and $m_J=\pm1$) shared by the pump and lasing transitions [6]. We concentrate on situations in which the pump and generated fields are linearly polarized. Recently we have found strong differences [7] in the behavior of this system depending on the parallel or orthogonal relative polarizations between the two fields: while in the first case amplification is always possible, in the second case amplification can be completely suppressed if the relaxation time of the coherence between the two upper levels is above a certain threshold. This occurs in spite of the existence of population inversion in the lasing transition. As will be shown below, this polarization-sensitive behavior has its origin in (i) the total suppression of the Raman process in the orthogonal case (independent of the relaxations) and (ii) in the different interferences between channels in both cases. In this Brief Report we give a clear explanation of these facts in terms of dressed states.

II. MODEL

We consider a four-level medium interacting with a pump and a generated field of frequencies ω_P and ω_L, respectively (Fig. 1). Since this model deals with molecular transitions between $J_a=0\rightarrow J_b=1\rightarrow J_c=0$ levels, the pump ($\mathcal{E}_P$) and generated ($\mathcal{E}_L$) laser fields are decomposed in terms of their dextro ($\mathbf{E}_P,\mathbf{E}_L$) and levo ($\mathbf{E}'_P,\mathbf{E}'_L$) components, becoming

$$\mathcal{E}_{P(L)}=\mathbf{E}_{P(L)}+\mathbf{E}'_{P(L)} \; ,$$

$$\mathbf{E}_{P(L)}=\tfrac{1}{2}\hat{\mathbf{e}}_{+} A_{P(L)}\exp[-i(\omega_{P(L)}t+\phi_{P(L)})]+\text{c.c.} \; ,$$

$$\mathbf{E}'_{P(L)}=\tfrac{1}{2}\hat{\mathbf{e}}_{-} A'_{P(L)}\exp[-i(\omega_{P(L)}t+\phi'_{P(L)})]+\text{c.c.} \; , \tag{1}$$

where A and A' are real amplitudes, ϕ and ϕ' denote the corresponding phases, and $\hat{\mathbf{e}}_{+(-)}=\mp 1/\sqrt{2}(\hat{x}\pm i\hat{y})$ are the usual circular polarization vectors. The upper molecular sublevels $[b(m_J=+1), b'(m_J=-1)]$ are considered to be degenerate. Within the semiclassical formalism the Hamiltonian of the system is given by

$$H=H_0+H_1 \; , \tag{2a}$$

$$H_0=-\hbar[|a\rangle\omega_a\langle a|+|c\rangle\omega_c\langle c|] \; , \tag{2b}$$

$$H_1=-[\boldsymbol{\mu}_{ab}\cdot\mathbf{E}_p|a\rangle\langle b|+\boldsymbol{\mu}_{ab'}\cdot\mathbf{E}'_p|a\rangle\langle b'| + \boldsymbol{\mu}_{bc}\cdot\mathbf{E}_L|b\rangle\langle c|+\boldsymbol{\mu}_{b'c}\cdot\mathbf{E}'_L|b'\rangle\langle c|]+\text{H.c.} \; , \tag{2c}$$

where the selection rule $\Delta m_J=\pm 1$ has been invoked. The origin of energies has been taken to be that of states $|b\rangle$ and $|b'\rangle$, and $-\hbar\omega_a$ and $-\hbar\omega_c$ are the energies of the bare states $|a\rangle$ and $|c\rangle$ in the absence of interaction.

The Liouville–von Neumann equation for the density matrix is then written

$$\frac{d\rho}{dt}=-\frac{i}{\hbar}[H,\rho]+\Gamma\rho \; , \tag{3}$$

$\Gamma\rho$ describing the relaxation and incoherent pump processes. We assume that the population relaxation for level i $(i=a,b,b',c)$ occurs at a constant rate γ_i and that the coherence between levels i and j $(i,j=a,b,b',c)$ decays with a constant rate γ_{ij}. We also assume that $\gamma_b=\gamma_{b'}$, and that in the absence of fields a thermal distribution with level populations ρ_{11}^0 $(i=a,b,b',c)$ exists.

The stationary solution of Eq. (3) in the case of the optically pumped laser (i.e., when this equation is self-consistently complemented with the equation of propagation of the generated laser field inside the cavity) has been recently derived in Ref. [7] for the case of a polarization-sensitive cavity which imposes the polarization state of the laser light. Fixed linear polarizations for both pump and laser fields, either parallel or orthogonal to each other, have been considered separately. We summarize next the features of these results that are relevant for the present Brief Report.

In the parallel case habitual conditions for laser emission are found (pump and gain parameters must exceed certain thresholds). In the orthogonal case, however, two important features are found. On the one hand there is an absence of Raman contribution to amplification for any value of the detunings and independently of the value of the relaxation rates. On the other hand, in order to achieve amplification, two additional conditions to that of the pump and gain parameters must be fulfilled:

$$\gamma_{bb'}>\gamma_{bb'}^{\text{thr}}\equiv\gamma_b\frac{N_{ab}+N_{cb}}{N_{ab}-(1+2\gamma_b/\gamma_a)N_{cb}} \; , \tag{4a}$$

$$N_{ab}>(1+2\gamma_b/\gamma_a)N_{cb} \; , \tag{4b}$$

where $N_{ij}\equiv\rho_{ii}^0-\rho_{jj}^0$. Condition (4b) is commonly verified in a Stokes configuration (Fig. 1). Nevertheless, if level a were the ground state, condition (4b) would not be fulfilled, preventing amplification (in fact, in this case one should take into account a more realistic relaxation scheme including internal relaxations). Thus, if the relaxation rate of the coherence $\rho_{bb'}$ is lower than $\gamma_{bb'}^{\text{thr}}$ the laser will remain off, independently of the pump strength and gain parameter. This situation $(\gamma_{bb'}<\gamma_{bb'}^{\text{thr}})$ is of interest since for pump strengths large enough the stationary value of the inversion $D_{bc}\equiv(\rho_{bb}-\rho_{cc})=(\rho_{b'b'}-\rho_{cc})$ turns out to be a positive quantity [7]. Thus it corresponds to the existence of population inversion between the lasing levels, without the counterpart of amplification.

Let us remark that in a Stokes configuration and with γ_a of the same order of magnitude as γ_b, $\gamma_{bb'}^{\text{thr}}$ is slightly larger than γ_b. Then, since

$$\gamma_{bb'}=\tfrac{1}{2}(\gamma_b+\gamma_{b'})+\Gamma_{bb'}^{\text{col}}=\gamma_b+\Gamma_{bb'}^{\text{col}} \tag{5}$$

(where $\Gamma_{bb'}^{\text{col}}$ describes the dephasing collisions) $\gamma_{bb'}<\gamma_{bb'}^{\text{thr}}$ implies that the contribution of dephasing collisions to the relaxation of the coherence $\rho_{bb'}$ must be small enough. This is not a very severe restriction. Now we proceed to interpret these results in terms of the dressed-atom formalism.

III. INTERPRETATION IN TERMS OF DRESSED STATES

In order to obtain the dressed states of our system it is necessary to work in an interaction picture [8]. Using the unitary operator

$$U(t)=\exp\{iH_2t/\hbar\} \; , \qquad H_2=-\hbar[|a\rangle\omega_p\langle a|+|c\rangle\omega_L\langle c|] \; , \tag{6}$$

the Liouville–von Neumann equation becomes

$$\frac{d\bar{\rho}}{dt}=-\frac{i}{\hbar}[\bar{H}_3,\bar{\rho}]+(\overline{\Gamma\rho}) \; , \tag{7}$$

where the overbar denotes this interaction picture, and

$$\bar{H}_3=\hbar\begin{bmatrix} \Delta_P & -\Omega_P e^{-i\phi_P} & -\Omega'_P e^{-i\phi'_P} & 0 \\ -\Omega_P e^{i\phi_P} & 0 & 0 & -\Omega_L e^{i\phi_L} \\ -\Omega'_P e^{-i\phi'_P} & 0 & 0 & -\Omega'_L e^{i\phi'_L} \\ 0 & -\Omega_L e^{-i\phi_L} & -\Omega'_L e^{-i\phi'_L} & \Delta_L \end{bmatrix} \; , \tag{8}$$

where $\Delta_P = \omega_P - \omega_a$ and $\Delta_L = \omega_L - \omega_c$ are the detunings, and

$$\Omega_{P(L)} = \frac{\boldsymbol{\mu}_{ab(bc)} \cdot \hat{\mathbf{e}}_+}{\hbar} A_{P(L)} , \qquad \Omega'_{P(L)} = \frac{\boldsymbol{\mu}_{ab'(b'c)} \cdot \hat{\mathbf{e}}_-}{\hbar} A'_{P(L)} \tag{9}$$

are the Rabi frequencies of the fields, which are taken to be real. Let us point out that $\bar{H}_3$ is time independent if the Rabi frequencies are constant (stationary regime).

We consider the pump field polarized in the $\hat{\mathbf{x}}$ direction, i.e., $\Omega_P = \Omega'_P$ ($\boldsymbol{\mu}_{ab} \cdot \hat{\mathbf{e}}_+ = \boldsymbol{\mu}_{ab'} \cdot \hat{\mathbf{e}}_-$) and $\phi'_P = \phi_P + \pi$. Since we concentrate on the conditions for the emission of $\mathcal{E}_L$, we take $\Omega_L = \Omega'_L = 0$ in Eq. (8) and diagonalize the Hamiltonian in order to obtain the eigenstates of the medium molecules dressed by the pump field. After some algebra one obtains

$$\begin{aligned} \bar{H}_3 |\tilde{b}\rangle &= 0 |\tilde{b}\rangle , \\ \bar{H}_3 |r_\pm\rangle &= \frac{\hbar}{2} (\Delta_2 \pm \sqrt{\Delta_P^2 + 8\Omega_P^2}) |r_\pm\rangle , \\ \bar{H}_3 |c\rangle &= \hbar \Delta_L |c\rangle , \end{aligned} \tag{10}$$

where

$$\begin{aligned} |\tilde{b}\rangle &= \frac{1}{\sqrt{2}} (|b'\rangle + |b\rangle) , \\ |r_\pm\rangle &= N_\pm \left[|a\rangle + \frac{2\Omega_P}{\Delta_P \pm \sqrt{\Delta_P^2 + 8\Omega_P^2}} (|b'\rangle - |b\rangle) \right] , \end{aligned} \tag{11}$$

represent the dressed states, and $N_\pm$ is the normalization factor.

The transition probabilities from these states to the lower lasing level $|c\rangle$ in the presence of a small signal $\mathcal{E}_L$ are, in the Born approximation,

$$\begin{aligned} W(r_\pm \to c) &\propto \frac{2\Omega_P^2}{4\Omega_P^2 + \Delta_P^2 \pm \sqrt{\Delta_P^2 + 8\Omega_P^2}} |\Omega_L e^{i\phi_L} - \Omega'_L e^{i\phi'_L}|^2 , \\ W(\tilde{b} \to c) &\propto |\Omega_L e^{i\phi_L} + \Omega'_L e^{i\phi'_L}|^2 . \end{aligned} \tag{12}$$

In general, these probabilities are not null. Nevertheless, for the case of a linearly polarized generated field $\mathcal{E}_L$ (for which $A_L = A'_L$ and therefore $\Omega_L = \Omega'_L$ if $\boldsymbol{\mu}_{bc} \cdot \hat{\mathbf{e}}_+ = \boldsymbol{\mu}_{b'c} \cdot \hat{\mathbf{e}}$), some of these probabilities can become equal to zero, depending on the value of the relative phase $\phi_L - \phi'_L$. This can be interpreted as the result of the quantum interference between the lasing channels $b \to c$ and $b' \to c$ which give the contributions $\Omega_L \exp(i\phi_L)$ and $\Omega'_L \exp(i\phi'_L)$, respectively, in Eq. (12).

In the case of a $\hat{\mathbf{x}}$-polarized generated field, i.e., $\phi_L = \phi'_L + \pi$ (parallel case), it is observed that $W(\tilde{b} \to c) = 0$ and $W(r_\pm \to c) \neq 0$. Then the states $|r_\pm\rangle$ are connected with $|c\rangle$ via $\mathcal{E}_L$, revealing the existence of Raman processes (since $|a\rangle$ is contained in $|r_\pm\rangle$). In addition, it is easy to show that either $|r_+\rangle$ or $|r_-\rangle$ is more populated than $|c\rangle$.

Let us concentrate on the much more interesting case of orthogonal polarizations ($\phi_L = \phi'_L$, $\mathcal{E}_L$ is $\hat{\mathbf{y}}$ polarized). In this case $W(r_\pm \to c) = 0$ and $W(\tilde{b} \to c) \propto \Omega_L^2$. Thus, Raman processes are forbidden since $|\tilde{b}\rangle$ does not project onto the $|a\rangle$ subspace. As is evident from Eq. (12) this result holds for any set of relaxation parameter values. On the other hand, the stationary population of the unique connected state $|\tilde{b}\rangle$ is [7]

$$\rho_{\tilde{b}\tilde{b}} = \rho_{bb}^0 + \frac{\gamma_a (\gamma_{bb'} - \gamma_b) N_{ab} \Omega_P^2}{A \Omega_P^2 + B + 4\gamma_a \gamma_b \gamma_{bb'}^2 \Delta_P^2 / (\Omega_P^2 + 2\gamma_{bb'} \gamma_{ab})} , \tag{13}$$

where $A = \gamma_a \gamma_b + (\gamma_a + 2\gamma_b) \gamma_{bb'}$, and $B = 2\gamma_a \gamma_b \gamma_{ab} \gamma_{bb'}$.

As is obvious, the largest population in level $\tilde{b}$ is obtained in the limit $\Omega_P^2 \to \infty$. In this limit the population inversion $D_{\tilde{b}c}^\infty$ between the only two states involved in the lasing process ($|\tilde{b}\rangle, |c\rangle$) is

$$D_{\tilde{b}c}^\infty = \frac{\gamma_a N_{ab} - (\gamma_a + 2\gamma_b) N_{cb}}{\gamma_a \gamma_b + (\gamma_a + 2\gamma_b) \gamma_{bb'}} (\gamma_{bb'} - \gamma_{bb'}^{\text{thr}}) . \tag{14}$$

It is observed that $D_{\tilde{b}c}^\infty$ will be positive if and only if $\gamma_{bb'} > \gamma_{bb'}^{\text{thr}}$ [the numerator in Eq. (14) was already assumed to be positive]. In the opposite case ($\gamma_{bb'} < \gamma_{bb'}^{\text{thr}}$) $D_{\tilde{b}c}$ will be negative for any value of the pump Ω_P^2, in spite of the positive values of D_{bc} and $D_{b'c}$. This situation is accompanied by the accumulation of population in the other dressed states $|r_\pm\rangle$, which, for this orthogonal case, are trap states.

IV. CONCLUSION

We have shown that amplification in an optically pumped laser based on a Λ-type three-level scheme involving an upper degenerate $m_J = \pm 1$ level is strongly dependent on the relative polarizations of the linearly polarized pump and generated fields. The viewpoint of the interactions in the frame of dressed states has allowed us to clearly understand the origin of this behavior in terms of population trapping in some particular dressed states. The sensitivity to the field polarizations lies in the fact that the polarization of the generated field selects which of the dressed states produced by the pump field are connected with the lower lasing level. While in the parallel case no special features are found, it is not so for the orthogonal case. In this last scheme there is an absence of Raman contribution to amplification, which can be understood through the destructive interference of the lasing channels $b \to c$ and $b' \to c$, which disconnects the lower lasing state $|c\rangle$ from the dressed states with projections on $|a\rangle$. Population inversion without amplification occurs for a relaxation rate of the coherence between the two M-degenerate upper levels below a certain threshold $\gamma_{bb'}^{\text{thr}}$. This threshold determines the possibility of population inversion between the dressed states which interact. Inversion between bare states does not guarantee inversion between dressed states, and this last situation is at the origin of amplification inhibition in this orthogonal configuration.

It is worth noting that the conditions found here for inversion without amplification are different from those described in previous works in which the influence of field polarization was not explicitly taken into account [1–3]. These works correspond to some fixed polarization state in which all the channels are open. In view of the present analysis it turns out that in the particular case of the double-Λ scheme this population state was in most cases the parallel-configuration one.

Our results can be used to study lasing action in optically pumped lasers with a polarization-anisotropic cavity, in which field polarization (either parallel or orthogonal to that of the pump beam) can be fixed by some element such as, for instance, a Brewster-angle plate.

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