Magnetic moment of the negative muon

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The most accurate value of the negative-muon magnetic moment, $\mu_{\mu^-}=3.183\,345(10)\mu_p$, is obtained by combining the negative-muon mass from x-ray transitions in muonic atoms and the reanalyzed negative-muon g factor with the proton magnetic moment in Bohr magnetons. The fractional uncertainty of 3.1×10^{-6} is 15 times more accurate than any previous direct measurement. The obtained ratio of the magnetic moments of the positive muon and the negative muon, $\mu_{\mu^+}/\mu_{\mu^-}=1.000\,000\,2(31)$, provides a test of *CPT* invariance at the level of 3 ppm. The negative-muon g factor g_{μ^-} and the magnetic moment anomaly a_{μ^-} derived for a test of *CPT* theorem are obtained from an approach now made possible using the new value of the negative-muon magnetic moment. To substantially improve the verification of the *CPT* theorem for the muon g factor, a much more precise value for μ_{μ^-}/μ_p is needed to determine the anomalous g factor a_{μ^-} for the new Brookhaven National Laboratory muon g-2 experiment.

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The muon magnetic moment μ_μ, g-factor anomaly a_μ, and mass m_μ are fundamental constants of the muon and hence they must be determined experimentally and theoretically as accurately as possible. The muonium atom (μ^+e^-) [1] consisting of a positive muon and an electron is a simple leptonic hydrogenlike atom which can be studied experimentally and theoretically to a very high precision. The high-precision measurements for the positive-muon magnetic moment μ_{μ^+} and mass m_{μ^+} have been vigorously pursued by means of the microwave spectroscopy for muonium atoms [1] and their values have been determined to an impressive accuracy of sub-ppm level [2]. However, there is no antimuonium atom (μ^-e^+), a bound state of a negative muon and a positron, available for the corresponding measurements at the present time. The negative-muon magnetic moment μ_{μ^-} and mass m_{μ^-} have been measured in more complicated muonic atoms and their values are known to less accuracy than that of the positive muon. A precise determination of the ratio μ_{μ^-}/μ_p of the magnetic moments of the negative muon and the proton which can be compared with the more accurately known μ_{μ^+} can provide an experimental verification of the *CPT* theorem. A precise value for μ_{μ^-}/μ_p is also needed for the determination of the negative-muon magnetic-moment anomaly a_{μ^-} and its g factor g_{μ^-} for a test of *CPT* invariance. The *CPT* theorem states that any quantum-field theory described by a local Lorentz-invariant Hermitian Hamiltonian is invariant under the composite operations of charge conjugation (*C*), parity inversion (*P*), and time reversal (*T*) in any order [3,4]. Currently there is no successful quantum-field theory that can explicitely incorporate *CPT* violations. The proof of the *CPT* theorem, one of the most important principles of quantum-field theory, is based on very general assumptions and is regarded as valid (in theory) in the sense that it is difficult to formulate field theories which are not automatically *CPT* invariant. Quantum electrodynamics theory is one of the most successful quantum-field theories and naturally it allows *CPT* invariance. However, the *CPT* theorem has yet to be further verified experimentally and all available experimental data should be used to check the invariance theorem whenever possible. The theorem implies that a particle and its antiparticle must have the same magnetic moment (with opposite sign), the same mass, and the same mean life. Therefore simple tests of *CPT* invariance can be carried out by comparing the magnetic moment, mass, and lifetime of a particle with that of its antiparticle. High-precision tests of the *CPT* symmetry have been carried out [5] over decades and there have been great interest and important progress in this field recently [6–10].

In this paper we report the most accurate value of the negative-muon magnetic moment in proton magnetic moments μ_{μ^-}/μ_p [11] deduced by combining m_{μ^-} obtained from x-ray transitions in muonic atoms and the reanalyzed negative-muon g factor g_{μ^-} based on the CERN g-2 experiment with the proton magnetic moment in Bohr magnetons μ_p/μ_B. The electric-charge quantization for muons is assumed [10]. Our value is a factor of 15 more accurate than any previous direct measurement. We demonstrate a method of obtaining a_{μ^-} which was determined differently in the previously published paper [12] for a test of *CPT* invariance. We emphasize that the main goal of the muon g-2 experiment at CERN, which has been very successful, was to compare the experimental value for a_μ with the theoretical one, thus verifying the quantum electrodynamics theory. In the CERN experiment, the g-2 precession frequencies for both positive and negative muons were measured. The equality of these frequencies (which are different quantities from muon g-factor anomalies) may serve as a check of CPT invariance. The requirement of knowing μ_{μ^-}/μ_p for a more accurate determination of a_{μ^-} from future muon g-2 experiment to substantially improve the CPT test for the muon magnetic moment is discussed.

The muon g-factor anomaly $a=(g-2)/2$ can be expressed as [12]

$$a_\mu = \frac{R}{\dfrac{\mu_\mu}{\mu_p} - R} , \quad (1)$$

where $R = f_a/f_p$, measured to 10 ppm from the CERN g-2 experiment, is the ratio of the muon anomalous precession frequency and the proton NMR frequency. The quantity μ_{μ^-}/μ_p should be used to obtain the negative-muon g-factor anomaly a_{μ^-}. From a direct measurement of the hyperfine-structure interval and the Zeeman effect in the muonic helium atom, we have [15]

$$\frac{\mu_{\mu^-}}{\mu_p} = 3.183\,28(15) \quad (47 \text{ ppm}) . \quad (2)$$

We note that the published value for a_{μ^-} was obtained by using μ_{μ^+}/μ_p, which is valid for the determination of a_μ for a comparison with the QED theory [13], but is not consistent for a comparison with a_{μ^+} for a test of the CPT symmetry. At the time of the publication [12], the quantity μ_{μ^-}/μ_p was not known precisely. Historically μ_{μ^+} was used for determining the g_{μ^-} since the value $R(\mu^-) = f_a(\mu^-)/f_p$ was measured [14]. The equality of the precession frequencies $f_a(\mu^-) = f_a(\mu^+)$ could be regarded as an indication of the CPT invariance. We derive a_{μ^-} and g_{μ^-} in a more consistent way as follows. The R value of μ^- from the CERN g-2 experiment $R(\mu^-) = 3.707\,256(37)\times 10^{-3}$ together with μ_{μ^-}/μ_p in Eq. (2) yield

$$a_{\mu^-} = 1.165\,96(6)\times 10^{-3} \quad (48 \text{ ppm}) . \quad (3)$$

Thus we obtain

$$\frac{g_{\mu^-}}{2} = 1.001\,165\,96(6) \quad (60 \text{ ppb}) , \quad (4)$$

which is sufficiently accurate to deduce a more accurate value for μ_{μ^-}/μ_p.

The negative-muon magnetic moment (μ_{μ^-}), the proton magnetic moment (μ_p), and the Bohr magneton (μ_B) are given by

$$\mu_{\mu^-} = g_{\mu^-} \frac{e\hbar}{4m_{\mu^-}c} , \quad (5)$$

$$\mu_p = g_p \frac{e\hbar}{4m_p c} , \quad (6)$$

and

$$\mu_B = \frac{e\hbar}{2m_e c} , \quad (7)$$

respectively. From Eqs. (5)–(7), the magnetic-moment ratio of the negative muon and the proton is given by

$$\frac{\mu_{\mu^-}}{\mu_p} = \left[\frac{g_{\mu^-}}{2}\right]\left[\frac{m_e}{m_{\mu^-}}\right]\left[\frac{\mu_B}{\mu_p}\right] . \quad (8)$$

The precise value of the negative-muon mass comes from the muonic atom x-ray studies [16–18]. The wavelengths of the $3d_{5/2}$-$2p_{3/2}$ muonic x-ray transitions in ^{24}Mg and ^{28}Si have been measured with the bent-crystal spectrometer at the Swiss Institute for Nuclear Research (SIN) muon channel and the most accurate value for the negative-muon mass is given as [18]

$$\frac{m_{\mu^-}}{m_e} = 206.768\,30(64) , \quad (9)$$

which has a fractional uncertainty of 3.1 ppm. The proton magnetic moment in Bohr magnetons is [19]

$$\frac{\mu_p}{\mu_B} = 1.521\,032\,202(15)\times 10^{-3} \quad (0.010 \text{ ppm}) . \quad (10)$$

Combining Eqs. (4), (9), and (10) into Eq. (8) yields the most accurate value,

$$\frac{\mu_{\mu^-}}{\mu_p} = 3.183\,345(10) , \quad (11)$$

which agrees with the direct measurement of Eq. (2) but with an accuracy improved by a factor of more than 15. This is the deduced value with higher accuracy limited by our knowledge of m_{μ^-}. Our value for the negative-muon magnetic moment is in good agreement with the much better known positive-muon magnetic moment [1]

$$\frac{\mu_{\mu^+}}{\mu_p} = 3.183\,345\,47(47) \quad (0.15 \text{ ppm}) . \quad (12)$$

Combining Eqs. (11) and (12) yields

$$\frac{\mu_{\mu^+}}{\mu_{\mu^-}} = 1.000\,000\,2(31) , \quad (13)$$

which provides a CPT test for the muon magnetic moment at 3-ppm level.

We may now use our value of μ_{μ^-}/μ_p to determine a_{μ^-} according to Eq. (1), which gives

$$a_{\mu^-} = 1.165\,937(12)\times 10^{-3} \quad (10 \text{ ppm}) . \quad (14)$$

This value is the same as the previously published value [12] since the main error in determining a_{μ^-} is now from R. Taking the value $R = 3.707\,173(36)\times 10^{-3}$ for the positive muon [12] and μ_{μ^+}/μ_p from Eq. (12) leads to the following result for the g-factor anomaly:

$$a_{\mu^+} = 1.165\,910(11)\times 10^{-3} \quad (10 \text{ ppm}) . \quad (15)$$

Equations (14) and (15) imply that

$$\frac{a_{\mu^+}}{a_{\mu^-}} = 1-(2.3\pm 1.4)\times 10^{-5} \quad (16)$$

and

$$\frac{g_{\mu^+}}{g_{\mu^-}} = 1-(2.7\pm1.6)\times10^{-8} \quad (17)$$

with minor changes from the published values [12] due to the use of the new value for μ_{μ^+}/μ_p.

In conclusion, we obtained an accurate value for the negative-muon magnetic moment and determined the g-factor anomaly for the negative muon to serve as a test of *CPT* invariance. A new muon g-2 experiment [20] at Brookhaven National Laboratory aims to measure a_μ to 0.35 ppm, a factor of 20 improvement over the previous CERN measurement, and thus a test of *CPT* symmetry for a_{μ^+} and a_{μ^-} could also be improved. Further knowledge of a_{μ^-} requires even better known μ_{μ^-}/μ_p at a level of 0.1 ppm. Therefore the more accurate measurements of the magnetic moment of the negative muon are very desirable.

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