BRIEF REPORTS

Brief Reports are accounts of completed research which do not warrant regular articles or the priority handling given to Rapid Communications; however, the same standards of scientific quality apply. (Addenda are included in Brief Reports.) A Brief Report may be no longer than 4 printed pages and must be accompanied by an abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.

Electric charges of positive and negative muons

X. Fei
Department of Physics, Yale University, New Haven, Connecticut 06511
(Received 5 May 1993)

Electric charges carried by positive and negative muons in units of an electric charge, $e_{\mu^+}/e=1\pm1\times10^{-7}$ and $e_{\mu^-}/e=-1\pm2\times10^{-5}$, are derived from the theoretical and experimental values of the ground-state hyperfine-structure intervals $\Delta\nu$ in muonium and muonic helium atoms as tests of electric-charge quantization for muons. These results also provide a test of *CPT* symmetry for positive and negative muon charges, with an accuracy of 2 parts in 10^5. Further improvements can be made from more precise theoretical and experimental values for $\Delta\nu$ in muonium and muonic helium atoms with the approach described in this paper.

PACS number(s): 36.10.Dr, 11.30.Er, 14.60.Ef, 31.30.Gs

There have been important developments recently in high-precision tests of charge quantization for electrons and positrons [1,2], protons and antiprotons [1], and neutrality of matter [3,4]. As the famous Millikan oil drop experiment [5] first demonstrated in high precision, the electric charge on natural materials carries an integral multiple of a fundamental unit of electric charge e with the magnitude equal to an electron charge. The charge quantization in units of e for elementary particles is a fundamental principle that is subject to tests. There are fundamentally important cosmological, metrological, and theoretical reasons to determine the electric charges carried by elementary particles for tests of electric-charge quantization [1]. Theoretical understanding for the charge quantization has been reviewed [6]. In this paper we shall determine muon charges by combining precision measurements of the ground-state hyperfine-structure intervals in muonium and muonic helium atoms $\Delta\nu_{\text{expt}}$ with their theoretical values $\Delta\nu_{\text{theor}}$. The equality of particle and antiparticle charges (opposite in sign) [7] required by *CPT* symmetry is as fundamental as the charge quantization and thus should also be tested. There are only two tests of *CPT* invariance for particle and antiparticle charges, with precisions of 4×10^{-8} [1] and then 10^{-18} [2] for electron and positron charges, and 2×10^{-5} for proton and antiproton charges, that are of comparable or higher precision than that for the muons reported here. Since the equality of the charges on the electron and the proton has been established to better than 10^{-21} [9], their charges are denoted as e (with $e>0$ for protons) in this paper.

The muonium atom [8] is a simple leptonic hydrogen-like atom consisting of a positive muon and an electron that can be studied experimentally and theoretically to a very high precision. The hyperfine structure in spectra arises from the interaction of nuclear magnetic moments and the magnetic field due to the electron cloud. Generally, the leading term of the hyperfine energy in the s states is given by the Fermi formula

$$E_{\text{hfs}}=\frac{16\pi}{3}\mu_e\frac{\mu_n}{I}\frac{Z^3}{\pi a_0^3n^3}(\mathbf{I}\cdot\mathbf{J})\ , \tag{1}$$

where μ_e and μ_n are the electron and nuclear magnetic moments, respectively, a_0 is the Bohr radius, $\mathbf{I}$ is the nuclear spin, $\mathbf{J}$ is the electron angular momentum, Z is the nuclear charge number, and n is the principal quantum number. Equation (1) is the most elegant form as the leading term of the hyperfine energy that involves the muon charge and its mass. We identify the leading term of the muon electric charge dependence of the hyperfine structure for the muonium in the ground state to be $\mu_n Z^3$. The Z^3 term comes from the wave function of the atom. We substitute $\mu_n=\mu_{\mu^+}$ and $Z=e_{\mu^+}/e$ in the Fermi formula and include other small corrections. Then the formula for the muonium hyperfine-structure interval in the ground state may be expressed as

$$\Delta\nu=\mu_{\mu^+}e_{\mu^+}^3F(1+C)\left[1+\frac{m_e}{m_{\mu^+}}\right]^{-3}\ , \tag{2}$$

where F is a coefficient independent of the muon charges from the Fermi formula, C is of order 1%, representing the recoil corrections, radiative-recoil corrections, and other quantum-electrodynamics contributions as discussed in Ref. [10], and $(1+m_e/m_{\mu^+})^{-3}$ is the reduced-mass correction for the electron mass m_e and the muon mass m_{μ^+}. The mass ratio of the electron and the muon is given by

$$\frac{m_e}{m_\mu}=\left[\frac{\mu_\mu}{\mu_p}\right]\left[\frac{g_p}{g_\mu}\right]\left[\frac{m_e}{m_p}\right]\left[\frac{e}{e_\mu}\right]\approx\frac{1}{207}\left[\frac{e}{e_\mu}\right], \quad (3)$$

where the magnetic-moment ratio of the muon and the proton μ_μ/μ_p, the proton g factor g_p, the muon g factor g_μ, and the mass ratio of the electron and the proton m_e/m_p have been measured. The reduced-mass correction $(1+m_e/m_\mu)^{-3}$ only contributes about 10^{-3} to the leading term in $\Delta\nu$; thus it is negligible for our estimates of e_μ/e. For the same reason, all the combinations of the muon charge and mass in C are also negligible. Since $\mu_\mu=(g_\mu\hbar/4c)(e_\mu/m_\mu)$ is determined directly [11] and other m_μ terms are negligible, the only relevant term for the muon charge that is independent of its mass is proportional to the cube of the muon charge.

The present experimental value for $\Delta\nu$ from a Los Alamos Meson Physics Facility (LAMPF) experiment on muonium is [11]

$$\Delta_{\nu_{\rm expt}}=4\,463\,302.88(16)\text{ kHz}\quad(0.036\text{ ppm}) \quad (4)$$

and the value for μ_{μ^+}/μ_p measured directly from the same experiment is [11]

$$\mu_{\mu^+}/\mu_p=3.183\,346\,1(11)\quad(0.36\text{ ppm})\,. \quad (5)$$

Calculation of the hyperfine structure of the muonium requires knowledge of the magnetic moment of the positive muon μ_{μ^+}, which is obtained by combining Eq. (5) and the muon spin rotation measurement carried out in liquid bromine [12]. The theoretical value for $\Delta\nu$ may be expressed as [10]

$$\Delta\nu_{\rm theor}=4\,463\,303.11(1.33)\,(0.40)\,(1.0)\text{ kHz}\quad(0.4\text{ ppm})\,. \quad (6)$$

The principal uncertainty in $\Delta\nu_{\rm theor}$ arises from the uncertainty in μ_{μ^+}/μ_p. The second uncertainty is from that in α based on the quantized Hall-effect experiment, and the third is an estimate of uncalculated QED terms. The experimental and theoretical values for $\Delta\nu$ agree well:

$$\Delta\nu_{\rm expt}-\Delta\nu_{\rm theor}=-0.23(1.7)\text{ kHz} \quad (7)$$

or

$$\frac{\Delta\nu_{\rm expt}-\Delta\nu_{\rm theor}}{\Delta\nu_{\rm theor}}=(-0.05\pm0.4)\text{ ppm}\,. \quad (8)$$

We should note that the electric charge e has been used instead of the muon charge in the theoretical calculations, while the experimental value for μ_{μ^+} was used directly. We assume the theoretical value for $\Delta\nu$ is correct and thus the equality of the experimental and theoretical values for $\Delta\nu$ gives a limit of the equality of the positive muon charge and the electric charge. From Eq. (2), we have

$$\left[\frac{e_{\mu^+}}{e}\right]^3-1=\frac{\Delta\nu_{\rm expt}-\Delta\nu_{\rm theor}}{\Delta\nu_{\rm theor}} \quad (9)$$

or

$$\frac{e_{\mu^+}}{e}=1+(\tfrac{1}{3})\frac{\Delta\nu_{\rm expt}-\Delta\nu_{\rm theor}}{\Delta\nu_{\rm theor}}\,. \quad (10)$$

Substituting Eq. (8) into the above equation yields

$$\frac{e_{\mu^+}}{e}=1+(-0.2\pm1.3)\times10^{-7}\,. \quad (11)$$

Neutral muonic helium is a system consisting of three particles: a negative muon (μ^-), an electron, and a $^4He^{2+}$ nucleus (α particle). Since the spin of the nucleus is equal to zero, the hyperfine splitting is defined only by the interaction of the electron and the negative muon. In the ground state, the subsystem $(^4He^{2+}\mu^-)^+$ is a factor of about 400 smaller than a hydrogen atom in radius and can be regarded as a pseudonucleus with an effective charge $q=2e+e_{\mu^-}$ and with spin and magnetic moment equal to those of the muon. In the pseudonucleus, the muon orbits the $^4He^{2+}$ nucleus in a $Z=2$ hydrogenic $1s$ state. Because of the similarity of the muonium atom and the muonic helium atom, the leading term of the hyperfine interaction can be expressed by the Fermi formula. Analogous to Eq. (2), the hyperfine splitting in the ground state of muonic helium is given by

$$\Delta\nu(\mu^-)=\mu_{\mu^-}(2e+e_{\mu^-})^3F'(1+C')\,, \quad (12)$$

where F' and C' (the reduced-mass correction is included here) are constants for muonic helium atoms. Therefore we have

$$\frac{e_{\mu^-}}{e}=-1+(\tfrac{1}{3})\frac{\Delta\nu_{\rm expt}-\Delta\nu_{\rm theor}}{\Delta\nu_{\rm theor}}\,. \quad (13)$$

The present experimental value from a LAMPF experiment on muonic helium atom is [13]

$$\Delta\nu_{\rm expt}=4465.004(29)\text{ MHz}\quad(6.5\text{ ppm})\,. \quad (14)$$

The best known theoretical value for $\Delta\nu$ is [14]

$$\Delta\nu_{\rm theor}=4465.0(3)\text{ MHz}\quad(67\text{ ppm})\,. \quad (15)$$

Substituting Eqs. (14) and (15) into Eq. (13), we obtain

$$\frac{e_{\mu^-}}{e}=-1\pm2\times10^{-5}\,. \quad (16)$$

Combining Eqs. (11) and (16) yields

$$(e_{\mu^+}+e_{\mu^-})=0\pm2\times10^{-5}e\,, \quad (17)$$

which provides a test of *CPT* symmetry for the positive and negative muon charges. Another relevant test of CPT invariance with muons is discussed in [15].

Our results represent tests of charge quantization for the positive and negative muons and a test of *CPT* symmetry for muon charges. The most recent theoretical value for muonium hyperfine splitting (including an as yet unpublished term by Kinoshita [16,17]) has an uncertainty of 0.3 ppm (limited by our knowledge of the muon magnetic moment), which implies that $e_{\mu^+}/e=1\pm1\times10^{-7}$. Further improvements can be made with more precise theoretical and experimental values for $\Delta\nu$ in muonium and muonic helium atoms. We could expect to know e_{μ^+} to about 10^{-8} from an ongoing muonium experiment [18] that aims to measure the muon magnetic moment with higher precision at LAMPF.

I thank V. W. Hughes for introducing me the muonium hyperfine-structure problem and J. Wang and G. Wu for helpful conversations.

[1] R. J. Hughes and B. I. Deutch, Phys. Rev. Lett. **69**, 578 (1992).

[2] B. Müller and M. Thoma, Phys. Rev. Lett. **69**, 3432 (1992).

[3] V. W. Hughes, L. J. Fraser, and E. R. Carlson, Z. Phys. D **10**, 145 (1988).

[4] A. G. U. Perera, S. R. Betarbet, Byungsung O, and D. D. Coon, Phys. Rev. Lett. **70**, 1053 (1993).

[5] R. A. Millikan, Phys. Rev. **7**, 355 (1916).

[6] R. Foot, H. Lew, and R. R. Volkas, J. Phys. G **19**, 361 (1993).

[7] T. D. Lee, *Particle Physics and Introduction to Field Theory* (Harwood, Chur, Switzerland, 1981), p. 330.

[8] V. W. Hughes and G. zu Putlitz, in *Quantum Electrodynamics,* edited by T. Kinoshita (World Scientific, Singapore, 1990), p. 822.

[9] M. Marinelli and G. Morpurgo, Phys. Lett. **137B**, 439 (1984).

[10] J. R. Sapirstein and D. R. Yennie, in *Quantum Electrodynamics* (Ref. [8]), p. 560.

[11] F. G. Mariam *et al.*, Phys. Rev. Lett. **49**, 993 (1982).

[12] E. Klempt *et al.*, Phys. Rev. D **25**, 652 (1982).

[13] C. J. Gardner *et al.*, Phys. Rev. Lett. **48**, 1168 (1982).

[14] K.-N. Huang and V. W. Hughes, Phys. Rev. A **26**, 2330 (1982).

[15] X. Fei, following paper, Phys. Rev. A **49**, 1470 (1994).

[16] T. Kinoshita (unpublished).

[17] T. Kinoshita (private communication); X. Fei and V. W. Hughes, Brookhaven National Laboratory Muon g-2 Technical Note No. 131, 1992 (unpublished).

[18] V. W. Hughes, G. zu Putlitz, P. Souder, M. Boshier, D. Ciskowski, S. Dhawan, X. Fei, M. Janousch, W. Liu, W. Schwarz, K. Jungmann, B. Matthias, R. Holmes, J. McCracken, X. Wang, C. Pillai, O. van Dyck, and K. Woodle (unpublished).