Radiative properties of a two-level system in the presence of mirrors

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Using a first-order approximation in time-dependent perturbation theory, we evaluate the probability per unit time of spontaneous emission by a two-level system coupled to a Hermitian massless scalar field in the presence of either one or two infinite perfectly reflecting plates (mirrors). We consider the effects of a finite interaction time between the system and the quantized field. Furthermore, using the image method in imaginary time, we study radiative processes at finite temperature. We show that vacuum and thermal fluctuations give independent contributions to the total transition rate and that both are modified by the presence of the mirrors. In the two-parallel-plate geometry, we find that the position average of the spontaneous emission rate is equal to the free-space rate for resonant cavities, but is suppressed in nonresonant cavities.

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I. INTRODUCTION

In the derivation of Planck's radiation law, Einstein introduced the idea of spontaneous emission, where a quantum system makes a transition to a lower eigenstate without external stimulation [1]. Recently, it was experimentally shown that conducting plates can modify the rate of spontaneous emission of excited atoms [2] as well as the rate of absorption of blackbody radiation [3]. This effect was first discussed by Purcell [4], and there are two different ways to interpret this phenomenon: the first is based on the modification of the density of modes of the electromagnetic field from continuous to discrete modes, by the presence of the cavity [4]. The second is based on the interaction between the atom and its electric image reflected on the plates [5]. In the latter case, the changes on the spontaneous emission rate can be described as a cooperative effect of the emitter and its mirror image. The presence of plates affects both the energies of the atomic levels and the natural lifetime of the states. Using the method of stationary perturbation, the shifts of the atom levels can be calculated. In the same way, the transition probability per unit time of the radiative transition can be determined by using time-dependent perturbation theory. For a recent treatment see, for example, Ref. [6].

Recently, some authors determined the rate of spontaneous emission of excited atoms taking into account the atomic center of mass motion [7]. This can be done by introducing into the total Hamiltonian the contributions of the atomic center of mass momentum and position operators. The resulting total Hamiltonian is a mixed Hamiltonian in the sense that it has terms that are Galilean invariants and others that are Lorentz invariants. However, there is a way to take into account the atom's motion without breaking the Lorentz invariance of the Lagrangian. It can be done in a perturbative scheme by not assuming the rotating-wave approximation, i.e., instead of using the Glauber correlation function [8], one may use the positive Wightman function in the response-function equation. In this way the atom measures the vacuum fluctuations along its world line. This procedure can be implemented for atoms in a generic state of motion, i.e., either inertial or accelerated. In the latter case, the Unruh effect appears [9,10]. But there are many situations where the Lorentz invariance is broken even in the scheme discussed above, as, for example, when plates are introduced in the system. In the absence of plates, the Minkowski vacuum is the state which minimizes the energy subject to the constraint of invariance under the full Poincaré group. In the presence of plates, the ground state is a state which minimizes the energy subject to the constraint of invariance under a subgroup of the Poincaré group. Therefore the positive Wightman function contains information about how the spectral density of the field and the rate of spontaneous emission of an excited atom is modified by the presence of the plates.

In this paper, we continue to investigate the problem of radiative processes of atoms and detectors by evaluating the probability of transition between distinct eigenstates after finite observation time intervals [11–13]. We use

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an oversimplified model in which the atom is represented by a two-level monopole system coupled to a Hermitian massless scalar field [9,10]. This is the simplest model that contains all the ingredients needed to understand the basic features of the radiative processes. We can regard this as a model for the interaction of an atom with the quantized electromagnetic field which should reproduce the essential spectral features. Of course it does not contain any information about polarization features. As noted above, in order to take into account the antiresonant terms in the response function, we use the Wightman function instead of the Glauber correlation function of the field. Thus we are evaluating the probability of transition per unit time from an excited state to a lower-energy state of the atom induced by the vacuum fluctuations of the field on its world line. Furthermore, we determine how the rate of spontaneous emission is modified by the presence of either one or two infinite perfectly reflecting plates (mirrors). This will generalize the results of Davies, Liu, and Ottewill [14]. The radiative processes are also studied in the presence of thermal radiation, and the rates of spontaneous emission and induced absorption are computed for the geometries of one and two infinite mirrors.

We would like to emphasize that the general formalism presented in this paper is exactly the same formalism used in the theory of photodetection. In the Glauber theory of photodetection [8], the antiresonant terms are disregarded, and the basis of photodetection is just photoabsorption processes. However, one can imagine a nonstandard photodetection scheme, as, for example, the one suggested by Mandel [15], where the idea of detectors operating by stimulated emission was proposed. This concept was later developed by Wilkens and Lewenstein [16] and Grochmalicki and Lewenstein [17]. Of course, whether or not to take the antiresonant terms is exactly the ambiguity in the ordering of the operator products in quantum mechanics and quantum field theory. This problem was also discussed by Cahill and Glauber in Ref. [18], where the idea of s ordering was introduced, in which a continuous parameter s interpolates from normal to antinormal ordering in a smooth way. A context in which the antiresonant terms play a crucial role is that of accelerated detectors [9,10]. A uniformly accelerated monopole detector coupled to a scalar field in the Minkowski vacuum has the same response as an inertial detector interacting with a bath of thermal radiation at a temperature of $\beta^{-1} =$ (proper acceleration)$/2\pi$. This behavior is revealed only if one includes the antiresonant terms. For a careful discussion of this effect see, for example, Ref. [19].

This paper is prepared as follows. In Sec. II the rate of spontaneous emission of the atom in the presence of a perfectly reflecting plate at zero temperature is calculated. In Sec. III we extend the results of the preceding section in order to consider finite-temperature effects. We present in Sec. IV the rate of spontaneous emission and the rate of blackbody radiation absorption in the geometric configuration of two infinite parallel mirrors at finite temperatures. Conclusions are given in Sec. V. In this paper we use units in which $\hbar = c = k_B = 1$.

II. SPONTANEOUS EMISSION IN THE PRESENCE OF ONE MIRROR AT ZERO TEMPERATURE

For simplicity we will use the following model. Our atom will be represented by a pointlike system with an internal structure defining two energy levels ω_g and ω_e ($\omega_e - \omega_g = \omega > 0$), with eigenstates $|g\rangle$ and $|e\rangle$, and with nonzero monopole matrix element between these two states. This is an oversimplification of the usual model used to describe the interaction between an atom and the electromagnetic field. The electromagnetic field is replaced by a Hermitian massless scalar field and the dipole operator of the atom is replaced by a monopole operator. It is clear that the generalization to a multilevel system with a flat ionization continuum is straightforward, but it seems that this is not necessary in order to understand the fundamental features of radiative processes. The coupling between the scalar field $\phi(x)$ and the atom is given by a monopole interaction Hamiltonian

$$H_{\rm int} = c_1 m(\tau)\phi(x(\tau)) \quad , \tag{1}$$

where $m(\tau)$ is the monopole operator of the atom, $\phi(x)$ is the scalar field operator, and c_1 is a small coupling constant between the atom and the field. As we stressed before, this model was also used by Unruh [9] and DeWitt [10], as a detector model of scalar particles. It was shown that a detector moving with a constant acceleration a in the Minkowski vacuum can be excited. The asymptotic probability per unit time of the detector to make a transition to an excited state is the same as obtained for an inertial detector in contact with a bath of thermal radiation at the Davies-Unruh temperature $\beta^{-1} = a/2\pi$ [9,20]. (We use the terms atom and detector interchangeably.)

In order to describe radiative processes of atoms, let us define the Hilbert space of the system (the field plus the atom) as the direct product of the Hilbert space of the field and the Hilbert space of the atom:

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_F \quad . \tag{2}$$

The Hamiltonian of the system can be written as

$$\begin{aligned} H &= H_0 + H_{\rm int} \\ &= H_A + H_F + H_{\rm int} \quad , \end{aligned} \tag{3}$$

where the unperturbed Hamiltonian of the system H_0 is composed of the noninteracting atom Hamiltonian H_A and the free scalar field Hamiltonian H_F. We can define the initial state to be

$$|\tau_0\rangle = |e\rangle \otimes |\Phi_i\rangle = |e\Phi_i\rangle, \tag{4}$$

where $|e\rangle$ is the excited state of the atom and $|\Phi_i\rangle$ is the initial state of the field. In the interaction picture, the evolution of the combined system is governed by the Schrödinger equation

$$i\frac{\partial}{\partial\tau}|\tau\rangle = H_{\rm int}|\tau\rangle \quad , \tag{5}$$

$$|\tau\rangle = U(\tau,\tau_i)|\tau_i\rangle \quad , \tag{6}$$

where the evolution operator $U(\tau,\tau_i)$ obeys

$$U(\tau,\tau_i) = 1 - i\int_{\tau_i}^{\tau} H_{\rm int}(\tau')U(\tau',\tau_i)d\tau' \quad . \qquad (7)$$

In the weak-coupling regime, the evolution operator can be expanded in a power series of the interaction Hamiltonian. To first order, it is given by

$$U(\tau,\tau_i) = 1 - i\int_{\tau_i}^{\tau} d\tau' H_{\rm int}(\tau') \quad . \qquad (8)$$

The amplitude probability of the transition from the initial state $|e\Phi_i\rangle$ at $\tau_i = 0$ into $|g\Phi_f\rangle$ at τ is given by

$$\langle g\Phi_f|U(\tau,0)|e\Phi_i\rangle = -ic_1\int_0^{\tau}\langle g\Phi_f|m(\tau')\phi(x(\tau'))|e\Phi_i\rangle d\tau', \qquad (9)$$

with $|\Phi_f\rangle$ an arbitrary state of the field and $|g\rangle$ is the final state of the atom. This yields the probability of transition

$$P(E,\tau,0) = c_1^2|\langle e|m(0)|g\rangle|^2 F(E,\tau,0) \quad , \qquad (10)$$

where $c_1^2|\langle g|m(0)|e\rangle|^2$ is the selectivity of the atom, and

$$F(E,\tau,0) = \int_0^{\tau} d\tau' \int_0^{\tau} d\tau'' e^{-iE(\tau'-\tau'')} \times\langle\Phi_i|\phi(x(\tau'))\phi(x(\tau''))|\Phi_i\rangle \quad (11)$$

is the response function. Here $E = \pm\omega$, where the signs $(+)$ and $(-)$ represent an excitation process and a decay process, respectively. Let us suppose that the initial state of the field is the vacuum state, $|\Phi_i\rangle = |0\rangle$. Then in Eq. (11) we are using the positive Wightman function associated with the scalar field evaluated on the world line of the atom. The selectivity of the atom will not be discussed here because it appears just as a constant factor in the probability given by Eq. (10) and depends only on the atom's internal structure. We are studying the evolution of the system between two different instants of time where the initial state is known. We suppose that at $\tau = \tau_i = 0$ the atom is in the excited state and the field is in the vacuum state, and we want to know the state of the atom at $\tau = \tau_f$. One way to arrange this is to have a finite interval of interaction between the atom and the quantum field, as in the case of the switched detectors discussed by Grove [21]. Our response function corresponds to switching on the interaction at τ_i and switching it off at τ_f. Note that if $\Delta\tau = \tau_f - \tau_i \to \infty$, then $\frac{d}{d\Delta\tau}F(E,\Delta\tau)$ is the spectral density of the vacuum fluctuations. This is the quantum version of the Wiener-Khinchin theorem, which asserts that the spectral density of a stationary random variable is the Fourier transform of the autocorrelation function.

Let us study the response function given by

$$F(E,\tau,0) = \int_0^{\tau} d\tau' \int_0^{\tau} d\tau'' e^{-iE(\tau'-\tau'')} G^+(x(\tau'),x(\tau'')), \qquad (12)$$

where, in the absence of boundaries, $G^+(x,x') = \langle 0|\phi(x)\phi(x')|0\rangle$ is the positive scalar field Wightman function in the Poincaré invariant vacuum state $|0\rangle$ evaluated on the world line of the atom. Note that we do not use the rotating-wave approximation [22–24]. This approximation neglects the antiresonant terms in the response function. That is, in Eq. (11) this approximation would replace $\langle\Phi_i|\phi(x(\tau'))\phi(x(\tau''))|\Phi_i\rangle$ by the Glauber normal-ordered correlation function, $\langle\Phi_i|\phi^-(x(\tau'))\phi^+(x(\tau''))|\Phi_i\rangle$, where ϕ^+ and ϕ^- denote the positive and negative frequency parts of the quantum field. In this case, Eq. (12) would vanish identically when $|\Phi_i\rangle = |0\rangle$, i.e., a purely absorptive detector always gives a zero response function in the vacuum state of the field. If we prepare the atom (detector) in the excited state and the field in the Minkowski vacuum, the Glauber correlation function would appear to predict that the excited state is stable. Of course, this can be avoided by calculating the finite energy width of the excited states and verifying that they are not eigenstates of the full Hamiltonian of the system. The use of the Wightman function avoids the need to do this explicitly in that it is nonzero even though the quantum state is the vacuum. In effect, the Wightman function includes the vacuum fluctuation contributions which are omitted in the Glauber function.

At this point, it is worth commenting upon the relationship between our scalar model and the case of an atom which interacts with the quantized electromagnetic field via an electric dipole coupling. The general forms of Eqs. (10) and (11) will be the same; however, the monopole moment operator m will be replaced by the electric dipole moment operator, and the scalar field correlation function will be replaced by an electric field correlation function. (See, for example, Pike and Sarkar [25].) This latter correlation function may be written as a sum of the empty-space contributions and a portion of which is determined by the presence of the boundaries. The second portion will in general lead to results which depend upon the orientation of the electric dipole with respect to the boundary. For example, in the presence of a conducting plate, we may choose the TE modes to be those which have no component of the electric field normal to the plate. These modes will not contribute when the dipole is oriented normal to the plate, but will contribute otherwise, thus contributing to the orientation dependence. The extension of the results of the current paper to the electromagnetic case is planned to be the topic of a later publication.

Instead of the interaction picture as described above, some authors used the Heisenberg picture with the Dicke spin operators to gain insights into the problem of the radiative processes [26]. Using this picture, and assuming that the spontaneous decay is a very slow process, one can identify the vacuum and the source contributions in the evolution equation of the creation and annihilation field operators of quanta of the field. This source field modifies the atom's characteristics and produces the decay rate. There are many attempts to separate the contribution of each of these mechanisms (vacuum fluctuations and radiation reaction) in the total spontaneous decay rate. An enlightening discussion is given in Ref.

[27]. Both pictures yield identical results and are equivalent approaches for the same quantum mechanical phenomenon. The magnitude of these separate effects can be varied by means of the particular ordering chosen for counting atomic and field operators. Dalibard, Dupont-Roc, and Cohen-Tannoudji proposed that the ambiguity could be removed by requiring the corresponding rates of variation of vacuum and source fields to be Hermitian [28]. We prefer to use a more conservative approach in which both effects are interdependent and we cannot separate their contribution, although through this paper we use the expression "radiative process induced by vacuum fluctuations."

Equation (12) can also tell us the change in the probability of transition if we introduce boundaries into the system. We are interested in measuring the change of the vacuum fluctuations, evaluated on the world line of the atom, due to the presence of perfect mirrors. Let us introduce in the system an infinite perfectly reflecting plate at $z = 0$, and suppose that the atom is at rest at a distance $\eta/2$ from the plate, i.e., its world line is

$$x^\mu(\tau) = (\tau, 0, 0, \eta/2) \quad . \tag{13}$$

The assumption of perfect reflection at the plate is equivalent to supposing that we have a perfect conductor plate in the case of electromagnetic fields. However, real plates are not perfect conductors at arbitrarily high frequencies. Some authors used this argument to explain the unbounded nature of the vacuum expectation value of the renormalized energy momentum tensor of the scalar field near a reflecting plate [29]. In this paper we will be working in the high reflectivity limit. The extension for the case of finite reflectivity is under investigation.

If the initial state of the system at $\tau = \tau_i$ is $|e\rangle \otimes |0\rangle$, the probability, to first order, for the system to evolve to $|g\rangle \otimes |\Phi_f\rangle$, where $|\Phi_f\rangle$ is an arbitrary final field configuration, in a finite time interval $\Delta\tau = \tau_f - \tau_i$ is proportional to the response function

$$F(E, \tau_f, \tau_i) = \int_{\tau_i}^{\tau_f} d\tau \int_{\tau_i}^{\tau_f} d\tau' e^{-iE(\tau-\tau')} G^+(x(\tau), x(\tau')). \tag{14}$$

Here

$$G^+(x, x') = G_M(x, x') - G_{\partial\Gamma}(x, x') \tag{15}$$

is the positive Wightman function satisfying Dirichlet boundary conditions on the mirror, and $G_M(x, x')$ is the positive Wightman function of the scalar field in empty Minkowski spacetime. Both are evaluated on the atom's world line. Introducing the variables $\zeta = \tau - \tau'$ and $\lambda = \tau + \tau'$, we have

$$G^+(x, x') = -\frac{1}{4\pi^2}\frac{1}{(\zeta - i\epsilon)^2} + \frac{1}{4\pi^2}\frac{1}{(\zeta - i\epsilon)^2 - \eta^2} \quad , \tag{16}$$

which, by construction, vanishes at the mirror. Substituting Eq. (16) into Eq. (14), we obtain that

$$F(E, \Delta\tau, \eta) = F_M(E, \Delta\tau) + F_{\partial\Gamma}(E, \Delta\tau, \eta) \quad , \tag{17}$$

where

$$F_M(E, \Delta\tau) = -\frac{1}{4\pi^2}\int_{-\Delta\tau}^{\Delta\tau} d\zeta \frac{(\Delta\tau - |\zeta|)e^{-iE\zeta}}{(\zeta - i\epsilon)^2} \tag{18}$$

and

$$F_{\partial\Gamma}(E, \Delta\tau, \eta) = \frac{1}{4\pi^2}\int_{-\Delta\tau}^{\Delta\tau} d\zeta \frac{(\Delta\tau - |\zeta|)e^{-iE\zeta}}{(\zeta - i\epsilon)^2 - \eta^2} \tag{19}$$

are the empty-space contribution and the correction due to the boundary, respectively. Here the infinitesimal parameter ϵ is introduced to correctly specify the singularities of the Wightman function. We define an instantaneous rate as $R = \frac{\partial F(E,\Delta\tau)}{\partial \Delta\tau}$, i.e., the transition probability per unit switching time, normalized by the selectivity of the atom. It is given by

$$R(E, \Delta\tau, \eta) = R_M(E, \Delta\tau) + R_{\partial\Gamma}(E, \Delta\tau, \eta) \quad , \tag{20}$$

where

$$R_M(E, \Delta\tau) = -\frac{1}{4\pi^2}\int_{-\Delta\tau}^{\Delta\tau} d\zeta \frac{e^{-iE\zeta}}{(\zeta - i\epsilon)^2} \tag{21}$$

and

$$R_{\partial\Gamma}(E, \Delta\tau, \eta) = \frac{1}{4\pi^2}\int_{-\Delta\tau}^{\Delta\tau} d\zeta \frac{e^{-iE\zeta}}{(\zeta - i\epsilon)^2 - \eta^2} \quad . \tag{22}$$

Both the integrals in Eqs. (21) and (22) can be evaluated using complex variable methods. Let us first study $R_M(E, \Delta\tau)$, which gives the transition rate in empty Minkowski spacetime. Using the residue theorem and taking the limit $\epsilon \to 0$, we have

$$\int_{-\Delta\tau}^{\Delta\tau} d\zeta \frac{e^{-iE\zeta}}{(\zeta - i\epsilon)^2} = 2\pi E\Theta(-E) - 2\int_{\Delta\tau}^{\infty} d\zeta \frac{\cos E\zeta}{\zeta^2} \quad . \tag{23}$$

Substituting Eq. (23) in Eq. (21), we obtain, after some algebra, the empty-space rate [12]

$$R_M(E, \Delta\tau) = \frac{1}{2\pi}\left\{-E\Theta(-E) + \frac{\cos(E\Delta\tau)}{\pi\Delta\tau} + \frac{|E|}{\pi}\left(-\frac{\pi}{2} + \text{Si}(|E|\Delta\tau)\right)\right\} \quad , \tag{24}$$

where $\text{Si}(z)$ is the sine integral function [30]. Equation (24) has the expected asymptotic behavior when $|E|\Delta\tau \gg 1$: the spontaneous decay rate ($E < 0$) is given by $-E/2\pi$ and spontaneous excitation ($E > 0$) is forbidden. Figure 1 depicts the behavior of the instantaneous rate as a function of $E\Delta\tau$ [Eq. (24)]. In the limit of small $\Delta\tau$, both rates diverge as $1/\Delta\tau$. This divergence may be regarded as the result of a very short switching time, which produces a large perturbation in the system. Alternatively, if we think of this as the result of making

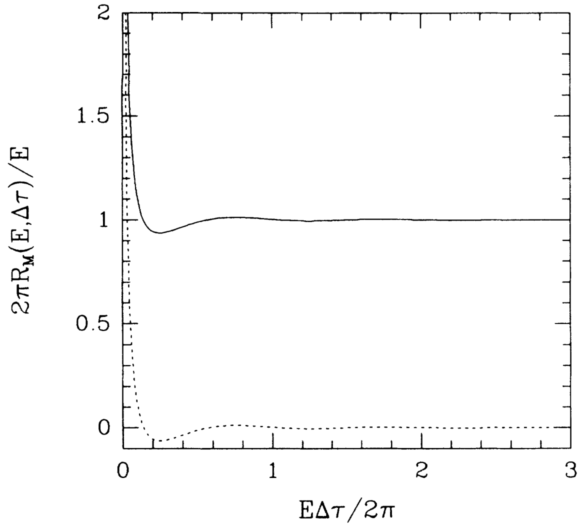

FIG. 1. The rate of spontaneous emission (continuous line) and excitation (dashed line) $R(E,\Delta\tau)$ after a finite observation time. Note the fast convergence to the asymptotic limit. For $E\Delta\tau < 2\pi$ (here E is the absolute value of the energy) the rate of excitation is positive, due to the excitation of the detector by the act of switching.

two measurements on the system separated by a short time interval, we should expect a large disturbance in the system to be introduced. Note that when $E\Delta\tau \lesssim 1$, the excitation probability becomes significant. Of greater physical interest is the case of a switching time for which $E\Delta\tau \gtrsim 1$. In this case, the energy uncertainty $1/\Delta\tau$ introduced by the switching is less than the level separation of our detector. The rate which we have defined is the usual transition probability per unit time when $\Delta\tau$ is long. However, more generally it is the derivative of the transition probability as a function of the switching time.

In order to calculate the correction in the rate due to the mirror, $R_{\partial\Gamma}(E,\Delta\tau,\eta)$, let us divide the integration range into $[-\Delta\tau,0]$ and $[0,\Delta\tau]$. Changing the integration variable in the first interval, we get

$$R_{\partial\Gamma}(E,\Delta\tau,\eta)=\frac{1}{2\pi^2}\mathrm{Re}\int_0^{\Delta\tau}d\zeta\frac{e^{-iE\zeta}}{(\zeta-i\epsilon)^2-\eta^2} = \frac{1}{2\pi^2}\mathrm{Re}\int_0^{\Delta\tau}d\zeta f(E,\eta,\zeta,\epsilon) \quad . \tag{25}$$

The function f is analytic except at the points $\{\eta+i\epsilon,-\eta+i\epsilon\}$. For $0\le\Delta\tau\le\eta$, the limit $\epsilon\to0$ can be taken directly. In this interval, $R_{\partial\Gamma}(E,\Delta\tau,\eta)$ is continuous with respect to $\Delta\tau$ and is given by

$$R_{\partial\Gamma}(E,\Delta\tau,\eta)=\frac{1}{2\pi^2}\int_0^{\Delta\tau}d\zeta\frac{\cos E\zeta}{(\zeta^2-\eta^2)} \quad . \tag{26}$$

Note that $R_{\partial\Gamma}(E,\Delta\tau,\eta)\neq0$ for $\Delta\tau<\eta$. This appears to be an acausal effect, as an emitted photon does not have time to travel to the mirror and be reflected back to the atom. However, it can be interpreted as due to the fact that the presence of the plate modifies the quantized field to which the atom is coupled. The atom is not really interacting directly with the plate so much as it is interacting with the modified vacuum fluctuations in the atom's vicinity. Note that in this case it is better to regard the atom as coupled to these vacuum fluctuations than as interacting directly with its image.

The contribution of the plate to the rate of spontaneous radiation for $\Delta\tau>\eta$ can also be computed and straightforward calculations yield

$$R_{\partial\Gamma}(E,\Delta\tau,\eta)=\frac{1}{2\pi}\Theta(-E)\frac{\sin\eta E}{\eta}+\frac{1}{2\pi^2}\int_{\Delta\tau}^{\infty}d\zeta\frac{\cos E\zeta}{\zeta^2-\eta^2}. \tag{27}$$

The function $R_{\partial\Gamma}(E,\Delta\tau,\eta)$ is plotted in Fig. 2 for a fixed value of η. Note that it has a light-cone singularity when $\Delta\tau=\eta$. This response arises if a signal emitted by the detector when it is switched on is reflected by the mirror and returns just as the detector is being switched off. The singularity is presumably an artifact of our approximation of sudden switching, and $R_{\partial\Gamma}$ would be finite for a smoothly switched detector. The total asymptotic rate of spontaneous radiation is then obtained by substituting Eqs. (24) and (27) in Eq. (20) and taking the limit $\Delta\tau\to\infty$, resulting in

$$\lim_{\Delta\tau\to\infty}R(E,\Delta\tau,\eta)=\frac{-E}{2\pi}\Theta(-E)\left(1-\frac{\sin\eta E}{\eta E}\right) \quad . \tag{28}$$

This agrees with the result of Davies *et al.* [14]. In Fig. 3 we plot the asymptotic rate as a function of $E\eta$. As we impose that the field vanishes at the mirror (Dirichlet boundary conditions), the rate vanishes at $\eta=0$. As usual, there are regions where the rate of transition is suppressed and regions where it is enhanced.

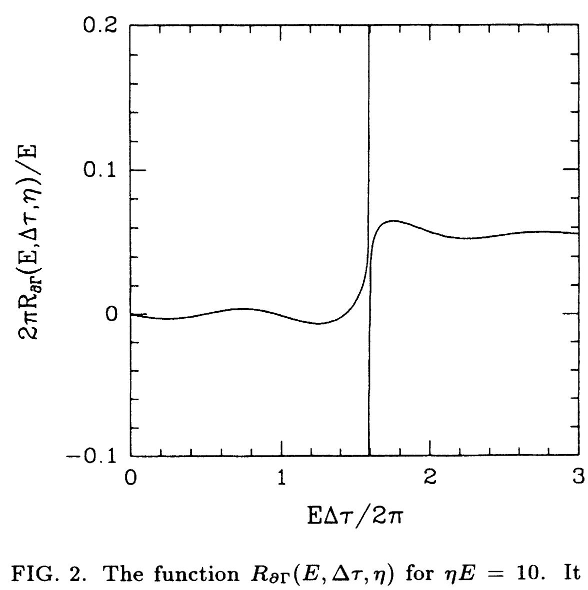

FIG. 2. The function $R_{\partial\Gamma}(E,\Delta\tau,\eta)$ for $\eta E=10$. It becomes singular as the observation time interval $\Delta\tau$ approaches η, the round-trip light travel time to the mirror.

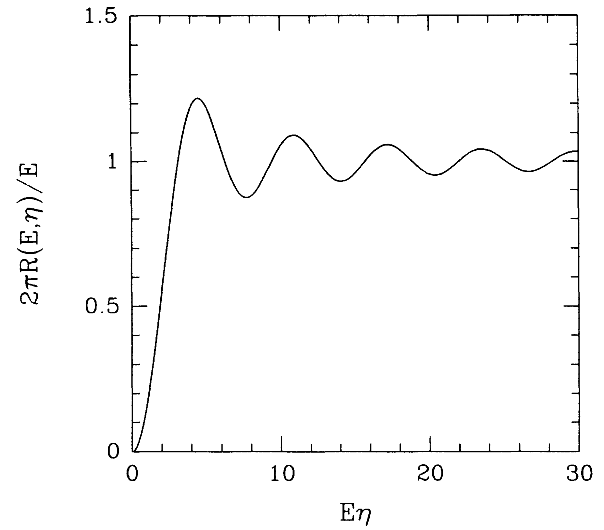

FIG. 3. The asymptotic rate of spontaneous emission in the presence of a perfectly conducting plate as a function of the atom distance to the mirror. It vanishes as $E\eta \to 0$ by construction, and for $E\eta \to \infty$ we recover the free atom result. Here E is the absolute value of the transition energy.

III. SPONTANEOUS AND INDUCED EMISSION AND BLACKBODY RADIATION ABSORPTION IN THE PRESENCE OF ONE MIRROR AT FINITE TEMPERATURE

In this section we generalize the above result for the case of finite temperature, i.e., we will consider the scalar field and the mirror as a system in thermodynamic equilibrium with a thermal reservoir at temperature β^{-1}. Using the image construction in imaginary time, we obtain the temperature-dependent Green's function evaluated on the world line of the atom [31]:

$$G_\beta(x,x') = -\frac{1}{4\pi^2}\sum_{k=-\infty}^{\infty}\frac{1}{(\zeta+i\beta k-i\epsilon)^2} + \frac{1}{4\pi^2}\sum_{k=-\infty}^{\infty}\frac{1}{(\zeta+i\beta k-i\epsilon)^2-\eta^2} \quad . \tag{29}$$

Substituting Eq. (29) in Eq. (14) we obtain

$$F_\beta(E,\Delta\tau,\eta) = F(E,\Delta\tau) + F_{\rm I}(E,\Delta\tau,\beta) + F_{\rm II}(E,\Delta\tau,\eta,\beta) \quad , \tag{30}$$

where $F(E,\Delta\tau)$ is the zero-temperature result given by Eq. (17), and

$$F_{\rm I}(E,\Delta\tau,\beta) = -\frac{1}{4\pi^2}\sum_{k=-\infty}^{\infty}{}'\int_{-\Delta\tau}^{\Delta\tau} d\zeta\frac{(\Delta\tau-|\zeta|)e^{-iE\zeta}}{(\zeta+i\beta k)^2} \quad , \tag{31}$$

$$F_{\rm II}(E,\Delta\tau,\eta,\beta) = \frac{1}{4\pi^2}\sum_{k=-\infty}^{\infty}{}'\int_{-\Delta\tau}^{\Delta\tau} d\zeta\frac{(\Delta\tau-|\zeta|)e^{-iE\zeta}}{(\zeta+i\beta k)^2-\eta^2} \tag{32}$$

are the temperature-dependent empty-space and the boundary terms, respectively. The prime sign indicates that the term $k=0$ is excluded. The instantaneous rate of radiation is now given by

$$R_\beta(E,\Delta\tau,\eta) = R(E,\Delta\tau) + R_{\rm I}(E,\Delta\tau,\beta) + R_{\rm II}(E,\Delta\tau,\eta,\beta) \quad , \tag{33}$$

where

$$R_{\rm I}(E,\Delta\tau,\beta) = -\frac{1}{4\pi^2}\sum_{k=-\infty}^{\infty}{}'\int_{-\Delta\tau}^{\Delta\tau} d\zeta\frac{e^{-iE\zeta}}{(\zeta+i\beta k)^2} \quad , \tag{34}$$

$$R_{\rm II}(E,\Delta\tau,\eta,\beta) = \frac{1}{4\pi^2}\sum_{k=-\infty}^{\infty}{}'\int_{-\Delta\tau}^{\Delta\tau} d\zeta\frac{e^{-iE\zeta}}{(\zeta+i\beta k)^2-\eta^2} \quad , \tag{35}$$

and $R(E,\Delta\tau)$ is given by Eq. (20). Equations (34) and (35) give the finite-temperature correction for the total transition rate.

Using the residue theorem the asymptotic rate can be obtained to be

$$\lim_{\Delta\tau\to\infty} R_\beta(E,\Delta\tau,\eta) = \frac{|E|}{2\pi}\left[\Theta(-E)\left(1+\frac{1}{e^{\beta|E|}-1}\right) + \Theta(E)\frac{1}{e^{\beta E}-1}\right] \times\left(1-\frac{\sin E\eta}{E\eta}\right) \quad . \tag{36}$$

Davies *et al.* [14] calculate the response of an accelerated detector near a reflecting plate. However, unlike the case of an accelerated detector in empty space, the result is not the same as that for an inertial detector in a thermal bath. For $E < 0$, Eq. (36) gives the asymptotic rate of decay induced by both vacuum and thermal fluctuations. Note that the total rate is just the product of the contributions of these two processes, indicating that they occur independently, and are affected in the same way by the presence of the mirror. For $E > 0$, we have the rate of induced absorption of blackbody radiation.

IV. SPONTANEOUS AND INDUCED EMISSION AND BLACKBODY RADIATION ABSORPTION IN THE PRESENCE OF TWO MIRRORS AT FINITE TEMPERATURE

Now let us confine the atom between two infinite parallel plates at $z=0$ and $z=\ell$ (again the position of the atom is $x=y=0$, $z=\eta/2$). Using the image method (assuming perfectly reflecting plates), the positive Wightman function evaluated on the world line of the atom is given by

$$G^{+}(x,x') = -\frac{1}{4\pi^2}\sum_{k=-\infty}^{\infty}\frac{1}{(\zeta - i\epsilon)^2 - (2k\ell)^2} + \frac{1}{4\pi^2}\sum_{k=-\infty}^{\infty}\frac{1}{(\zeta - i\epsilon)^2 - (\eta - 2k\ell)^2} \quad . \tag{37}$$

Substituting Eq. (37) in Eq. (14), and using the same techniques as before, the rate of spontaneous emission can be exactly evaluated. The asymptotic rate [14] is given by

$$\lim_{\Delta\tau\to\infty} R(E,\Delta\tau,\eta,l) = \frac{-E\Theta(-E)}{2\pi}\sum_{k=-\infty}^{\infty} \times \left[\frac{\sin 2kE\ell}{2kE\ell} - \frac{\sin E(\eta - 2k\ell)}{E(\eta - 2k\ell)}\right] \quad , \tag{38}$$

and is depicted in Fig. 4 as a function of the plate separation for several atom positions. The rate is a discontinuous function of $E\ell$, which is due to the discreteness of the wave number in the direction normal to the mirrors. For cavities where $|E|\ell < \pi$ the spontaneous decay rate is zero. The behavior for the two mirror case is characteristic of the radiative process inside cavities in general [32]. It reflects the discontinuous spectral density for the Casimir energy for this geometric configuration [33]. In some experimental arrangements the atomic beam samples all the space between the plates, and the measured rate of transition is then an average over all atom positions. Taking the average in Eq. (38) over $0 \le \frac{1}{2}\eta \le \ell$, the resulting series can be exactly evaluated using the identity

$$\sum_{k=-\infty}^{\infty}\int_0^a f(x+ka)dx = \int_{-\infty}^{\infty} f(x)dx \quad . \tag{39}$$

After a straightforward calculation, we obtain

$$\lim_{\Delta\tau\to\infty}\langle R(E,\ell,\Delta\tau)\rangle = \frac{|E|}{2\pi}\left(1 + \frac{y}{n+y}\right) \quad , \tag{40}$$

where $E\ell = \pi(n+y)$, $n = 0,1,2,...,$ $0 < y < 1$, and $\langle\,\rangle$ stands for an average over the atom's position. The mean asymptotic rate behaves as depicted in Fig. 5 as a function of $E\ell$. Whenever $E\ell/\pi$ is an integer, which we can label as a resonant cavity, the mean rate has a

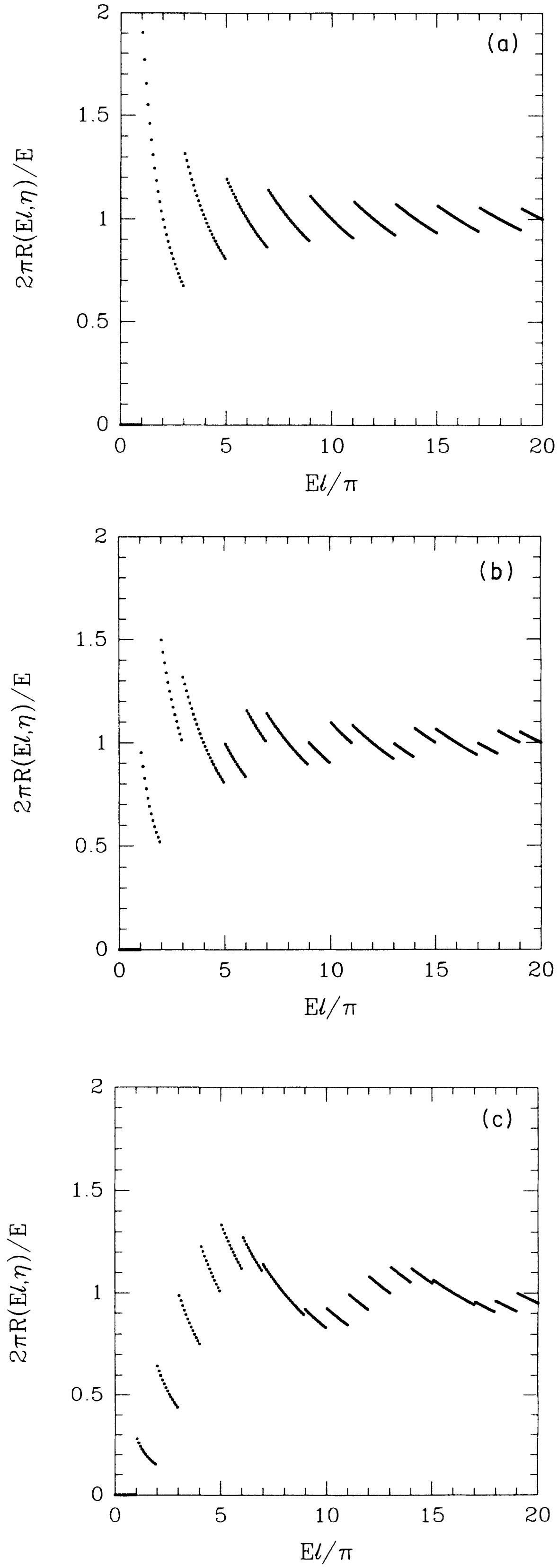

FIG. 4. The asymptotic rate of spontaneous emission in the presence of two parallel plates as a function of the cavity size for (a) $\eta/\ell = 1$ (the center of the cavity); (b) $\eta/\ell = 1/2$, and (c) $\eta/\ell = 1/4$.

discontinuous jump and reaches a value equal to the rate of transition in the free space. In nonresonant cavities, the mean rate is inhibited and consequently the mean lifetime of the excited state is enhanced.

Again, if we suppose that the field and the plates are in thermal equilibrium with a reservoir at temperature β^{-1}, we obtain

$$R_\beta(E,\Delta\tau,\eta,\ell) = \sum_{k=-\infty}^{\infty} [R_{\rm II}(E,\Delta\tau,\eta-2k\ell,\beta) - R_{\rm II}(E,\Delta\tau,2k\ell,\beta)] \quad , \quad (41)$$

where $R_{\rm II}$ is given by Eq. (35). The asymptotic rate is given by

$$\lim_{\Delta\tau\to\infty} R_\beta(E,\Delta\tau,\eta,\ell) = \frac{|E|}{2\pi}\left[\Theta(-E)\left(1+\frac{1}{e^{\beta|E|}-1}\right)+\Theta(E)\frac{1}{e^{\beta E}-1}\right]\sum_{k=-\infty}^{\infty}\left[\frac{\sin 2Ek\ell}{2Ek\ell}-\frac{\sin E(\eta-2k\ell)}{E(\eta-2k\ell)}\right] \quad . \quad (42)$$

Equation (42) is in turn a generalization of the results given in Eqs. (28), (36), and (38), which can be recovered by taking the appropriate limits. Note that the effects of the presence of the boundaries and of the finite temperature are multiplicative. In the low-temperature limit, we obtain the vacuum result, Eq. (38), multiplied by a correction factor which approaches unity as $T=\beta^{-1}\to 0$. In the high-temperature limit, the dominant feature of Eq. (42) is the first, β-dependent, factor. It is multiplied by the discontinuous ℓ-dependent factor, however, in the limit $T\to\infty$ ($\beta\to 0$), the peak of the thermal spectrum is at values of $E \gg \ell^{-1}$, where this latter factor is very close to unity.

V. CONCLUSIONS

In this paper we discussed the radiation process in atomic systems, using first-order time-dependent perturbation theory. Using a simple model of an atom consisting of a two-level system interacting with a Hermitian scalar field, we obtained the rate of spontaneous emission after a finite observation or switching time $\Delta\tau$.

We also evaluate the asymptotic rate of transition in the presence of one and two infinite perfectly conducting plates at zero and finite temperature. We obtained that, in the limit of large $\Delta\tau$, the vacuum and thermal fluctuations act independently, leading to a total rate that is just the product of the contributions of the two processes. It is also shown how the presence of mirrors affects the rate of absorption of quanta of the thermal field. In the geometry of two parallel plates we found that the spontaneous emission rate displays discontinuous jumps. This behavior can be interpreted in either of two ways: The first is that the phase space available for emission of quanta is modified by the presence of the boundaries. Emission is suppressed when there are fewer modes into which the photon may be emitted. The second viewpoint regards the effect of the mirrors as modifying the vacuum fluctuations which are necessary for spontaneous emission. The points where the graph in Fig. 5 dips below unity are situations where the fluctuations measured by the atom in its world line are suppressed relative to empty space. The mean transition rate, averaged over all positions for the atom, reaches a maximum value in resonant cavities which is equal to the free-space rate. For nonresonant cavities the mean transition rate is always inhibited.

The simple model used in this paper has then all the fundamental radiative properties of real systems. In this way, it is valuable to use it in order to explore other problems concerning the spontaneous radiation phenomenon. A natural extension is to generalize the results presented in this paper to the case of moving mirrors [34]. We can also replace the monopole system by a harmonic oscillator [35]. In this case the nonlocalized harmonic oscillator's wave function introduces new interesting problems. The above topics, as well as the electromagnetic case, are under investigation.

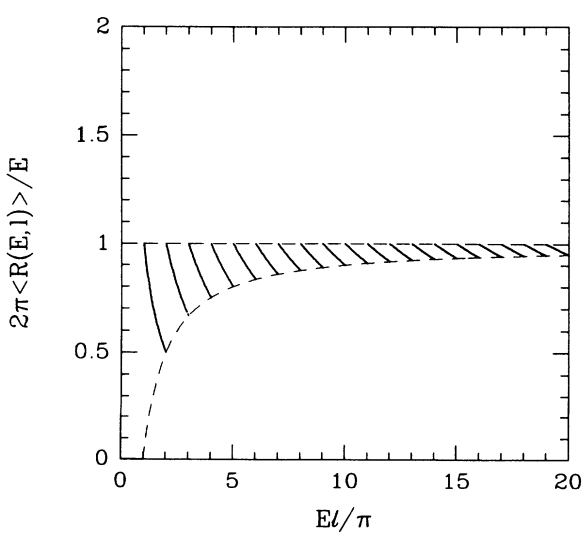

FIG. 5. The mean asymptotic rate of spontaneous emission as a function of the size of the cavity. Note that in resonant cavities the mean rate is the same as the rate at the free space.

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