Uniqueness of the chaotic attractor of a single-mode laser

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Measurements on an optically pumped NH_3 single-mode laser show three different types of chaotic dynamics, Lorentz-type spiral chaos, period-doubling chaos, and type-III intermittency. Analysis of the measurements shows that the peak-intensity return maps of these three types of dynamics have the same shape, indicating that a unique attractor exists for the laser whose topological structure is independent of laser parameters.

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Experimental investigations of the chaotic dynamics of an optically pumped NH_3 single-mode laser have shown different kinds of deterministic chaos, such as period-doubling chaos [1,2], type-III intermittency [3], and Lorenz-type spiral chaos. Because of the small relaxation rates of medium and high gain [5], optically pumped NH_3 far-infrared (FIR) lasers can fulfill the necessary conditions to realize the dynamics of Lorenz equations, one of the protype models for chaotic dynamics [6]. Experimental studies of the properties of the spiral chaos observed in these lasers, such as the characteristic spiral-type pulse train, the statistical distribution of the spiral lengths, the typical cusp form of the peak-intensity return map, etc., have shown all characteristics of the predictions of the Lorenz equations [4,7–9].

Here we report investigations of the three types of dynamics as observed in the NH_3 laser. We find that in spite of different types of dynamics, their peak-intensity return maps are of the same form, indicating an invariance of the attractor on the laser parameters and on the type of dynamics. The experimental results show that the period-doubling chaos and the type-III intermittent chaos have the same underlying attractor as the Lorenz-spiral chaos.

Details of the experimental setup used for the measurements is given in [1]. In short, the laser resonator is a unidirectional ring, which is designed to approach as closely as possible the conditions of the laser Lorenz equations [10–12]. The laser transition is the $aR(7,7)$ rotational transition in the $v_2=1$ vibrational state of $^{14}NH_3$, which is pumped optically with the $P(13)$ line of an N_2O laser *via* the vibrational $aQ(8,7)$ transition. The backward emission at 81 μm wavelength of the laser is used for reasons discussed in [5,13]. The FIR-laser radiation is detected by a Schottky-barrier diode. The signal is stored digitally. Since the cavity tuning is easy to change for the laser, the laser detuning is chosen as one control parameter in the experiment.

The above-mentioned three types of deterministic chaos are observed experimentally for different combinations of the pump intensity and NH_3 gas pressure. The Lorenz-type spiral chaos is observed when the laser resonator is tuned to, or close to, the center of the medium-gain line. The gas pressure for spiral chaos varies between ~4 and 11 Pa, when the pump intensity is changed between 1.5 and 6 W/cm^2. At lower gas pressure, apart from period-doubling chaos, no spiral chaos is found; at a gas pressure higher than 11 Pa, because the "bad-cavity" condition for the chaotic dynamics of th single-mode laser is no longer fulfilled here, the laser emits only stably. In general, chaotic pulses can be emitted from this laser that differ slightly from the pure Lorenz spirals [8,9]. This is due to the fact that the NH_3 laser is not a two-level laser as described by the Lorenz equations but an optically pumped three-level laser with a coherent pump [14–16]. Thus in addition to the phenomena predicted by the Lorenz equations, further effects occur which are due to three-level coherence effects such as ac stark splitting of the gain line. The limits in which the three-level laser behaves like a two-level laser have been investigated in [17]. The most prominent effect of three-level coherence is the appearance of an extra large pulse at the beginning of the pulse spirals [9]. These pulses were found to produce "double maps" as described as follows. In [9] it was found that three-level-coherence effects can be suppressed by pressure broadening; thus the cases corresponding to two-level dynamics are recorded at the highest possible NH_3 pressures.

Figure 1 shows a typical intensity recording of the observed Lorenz-type spiral chaos. Characteristics of the

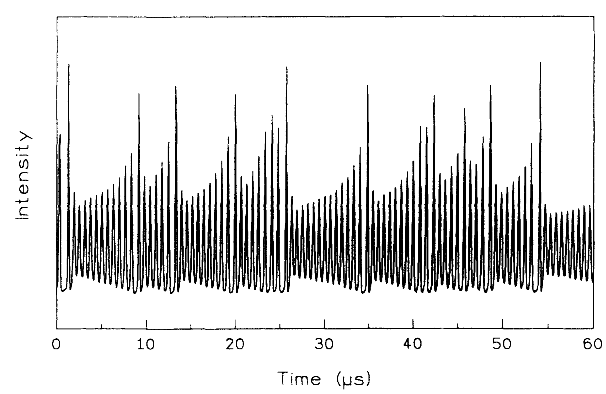

FIG. 1. Typical intensity variation of the Lorenz-type spiral chaos of the laser: pump intensity, 5 W/cm^2; NH_3 gas pressure, 10 Pa.

spiral chaos are the monotonic growth of the intensity pulses in time during individual spirals and the irregular length of these spirals. Experimental measurements of the optical field on the spiral chaos [11,18] have shown that the monotonic growth of the intensity pulses corresponds to the spiral motion of the laser state vector on the two leaves of the Lorenz attractor, and the irregular length of spirals is caused by the irregular jumps between them.

In previous papers we have compared the dynamic details of this spiral chaos with those predicted from the Lorenz equations, and we have found quantitative agreement [7-9]. The so-called Lorenz map [19] was found to be a good indicator of the Lorenz-type chaos. The Lorenz map is constructed by plotting successive local maxima of the Z variable of the real Lorenz equations, one versus the next. The map so obtained has a typical cusp shape which differs from the shape of return maps of other chaotic attractors and is characteristic of the Lorenz dynamics. Numerically, we find that the same cusp form can be constructed from the successive local maxima of the square of variable X or Y of the real or extended (complex) [11,12] Lorenz equations.

We have examined the peak-intensity return map of the observed laser spiral chaos. We find that it has the cusp form of the Lorenz map. In particular, we find that the shape of the map depends sensitively on the dynamics of the laser. Small changes of the dynamics produce a drastic change of the map [9].

Figure 2 shows the peak-intensity return maps of experimentally observed spiral chaos and the Lorenz return map calculated from the extended Lorenz equations [11] for comparison. Figure 2(a) shows the intensity return map of the spiral chaos under conditions in which the pump-coherence effects of the laser have been reduced as much as possible, Fig. 2(b) shows the intensity return map under influence of the pump-coherence effects, and Fig. 2(c) is the intensity return map calculated from the extended Lorenz equations with experimentally accessible parameters. The similarity between the intensity return map of the experiment [Fig. 2(a)] and the Lorenz return map [Fig. 2(c)] is obvious. Figure 2(b) shows that when the pump-coherence effects are strong, the intensity return map splits into two cusps, indicating deviation from the Lorenz dynamics. Experimentally, we find that the stronger the pump-coherent effects, the more pronounced is the splitting of the cusp map.

To illustrate further how sensitive the shape of the map is to deviation from Lorenz dynamics, Fig. 3 shows laser spiral chaos when there is a weak forward emission of the laser. Figure 3(a) shows the intensity in the backward direction as usual; Fig. 3(b) shows the forward emission recorded simultaneously. Becuase of the laser resonator setting and the mode competition with the backward mode, the forward emission is in this case very weak. It exists only at times when the backward emission is zero or near zero. Although the intensity shown in Fig. 3(a) is difficult to distinguish from that of the Lorenz model or the dynamics shown in Fig. 1, Fig. 3(c) shows that the intensity return map is completely destroyed.

The period-doubling route to chaos is observed in the

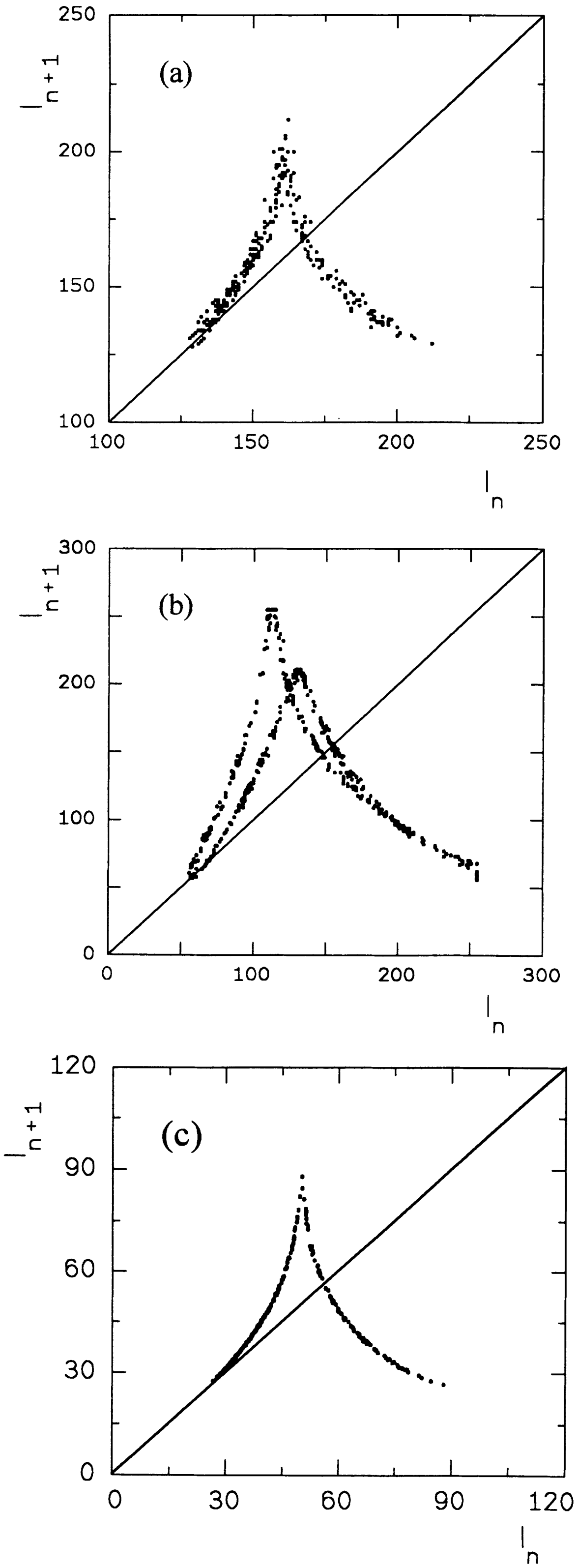

FIG. 2. First-peak-intensity return map constructed from the Lorenz-type spiral chaos of the laser: (a) under the condition that the pump-coherence effects can be neglected, (b) under the influence of the pump-coherence effects of the laser, and (c) peak-intensity return map constructed from the solution of extended Lorenz equations [11] with $\kappa=2$, $b=0.25$, $r=15$, and $\delta=-\Delta=0.02$.

NH_3 laser as the laser detuning is decreased from large detuning to the resonance. Figure 4 shows the typical intensity dynamics of this chaos. The characteristic here is that the chaotic dynamics occurs after a cascade of period-doubling bifurcation. It shows the typical motion on unstable period-n orbits. While the spiral chaos of the laser allows one to view some typical characteristics of the Lorenz dynamics directly from the intensity, the intensity of the period-doubling chaos exhibits no properties obviously related to the Lorenz equations. This type of dynamics has shown all the universal properties such as the existence of different periodic windows in the chaotic range, the period-doubling route to chaos of some of the periodic windows, etc. [20,21]. The period-doubling chaos has also been quantitatively characterized by metric properties including Lyapunov exponents, correlation dimension, and entropy [7]. These metric properties, although found in agreement with the Lorenz equations depend, however, on the laser parameters and do therefore not constitute clear indications of the relation of period-doubling chaos with the extended Lorenz

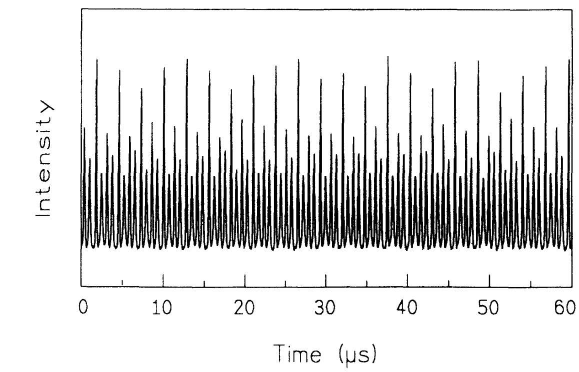

FIG. 4. Typical intensity variation for period-doubling chaos of the laser: pump intensity, 4.6 W/cm^2; NH_3 gas pressure, 7.5 Pa.

model. We find that such an indication is given by the peak-intensity return map.

Figure 5(a) shows the first-peak-intensity return map of the period-doubling chaos. The intensity return map shows the same cusp form as the spiral chaos, although the intensity of this period-doubling chaos does not show any obvious similarities with the Lorenz-spiral chaos. Like the case of spiral chaos, the intensity return map of the period-doubling chaos shown in Fig. 5(a) is observed

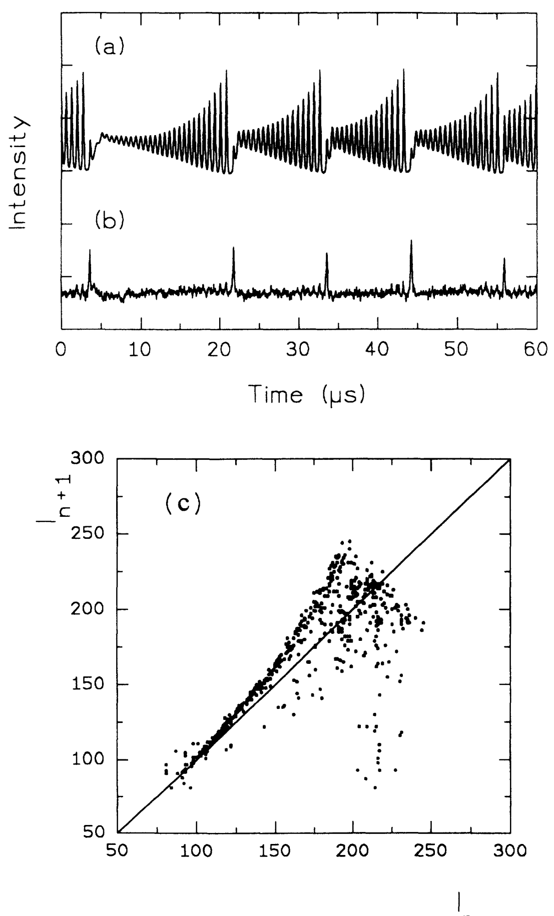

FIG. 3. Influence of weak counterpropagating emission on the Lorenz-type dynamics of the laser spiral chaos: (a) typical intensity of the laser spiral chaos in the presence of the forward emission, (b) intensity variation of the forward emission, and (c) first-peak-intensity return map constructed from the laser spiral chaos shown in (a).

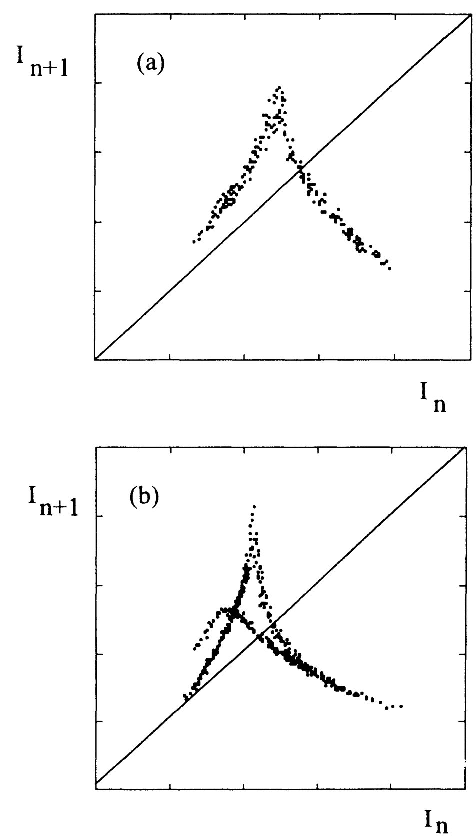

FIG. 5. First-peak-intensity return map constructed from period-doubling chaos of the laser: (a) under the condition that the pump-coherence effects can be neglected, and (b) under the influence of the pump-coherence effects of the laser.

only under conditions in which the pump-coherence effects of the laser are suppressed, so that the dynamics of the laser is the one of the extended (complex) Lorenz equations. More generally, the intensity return map of this period-doubling chaos splits into two cusps, and the stronger the pump-coherence effects, the more pronounced is this splitting, as shown in Fig. 5(b), for example. Thus we find the same properties of the peak-intensity return map of the period-doubling chaos as in the case of the spiral chaos.

Type-III intermittent dynamics of the laser is observed in the gas-pressure range 4–6 Pa with the pump intensity greater than 3.5 W/cm^2. In this gas-pressure range the spiral chaos is also observed when the pump intensity is reduced to 1.5 W/cm^2. The typical measured intensity of this chaotic dynamics is shown in Fig. 6. The characteristic of type-III intermittency is that its intensity peaks fall into two components. During one "laminar" phase, one component grows, while the other one decreases; two successive laminar phases are separated by "chaotic bursts."

In a previous paper the good agreement of this observed dynamics with the universal properties of type-III intermittency was proved [3]. To show the relation of this type-III intermittency with the Lorenz-spiral chaos and the period-doubling chaos of the laser, the first-peak-intensity return map of the dynamics is shown in Fig. 7. Figure 7(a) gives the general shape of the first-peak-intensity return map. It has the same cusp form as the spiral chaos and as the period-doubling chaos of the laser. The intensity return map in Fig. 7(a) has two cusps, indicating the influence of pump-coherence effects. Like the other two types of chaos, where the influence of the pump-coherence effects is reduced as much as possible, this deviation from the Lorenz dynamics can be removed. Figure 7(b) shows such an intensity return map. To obtain this map, the pump intensity has been reduced and the gas pressure has been increased to the limit of disappareance of the dynamics. Even the variation of the intensity return map of the dynamics under the influence of the pump-coherence effects has the same behavior as those of the spiral chaos and the period-doubling chaos. The clear cusp form of the intermittency indicates that the intensity during the chaotic bursts between two laminar phases is not irregular but deterministic. This experimental result suggests strongly that the intermittent dynamics of the laser is intrinsic dynamics of the Lorenz system.

We mentioned that the above shapes of the intensity return maps of these three types of dynamics do not depend on the operating laser parameters. Whenever these three dynamics types are observed, their intensity return map has the cusp shape. To explain, we note the relationship between the intensity return map and the structure of the corresponding chaotic attractor. In [22] it is shown that this return map is in practice the Poincaré map on a specific Poincaré section. Furthermore, the one-dimensional structure of this return map is the result of the Hausdorf dimension of the attractor being slightly greater than 2. Since the Poincaré map is uniquely fixed by the dynamic flow in the phase space, and its form reflects the structural characteristics of the attractor, the equal shape of the intensity return maps implies that their attractors are of the same topological structure in the phase space.

Based on the above, it can be said that the three dynamics types originate from the same attractor. From

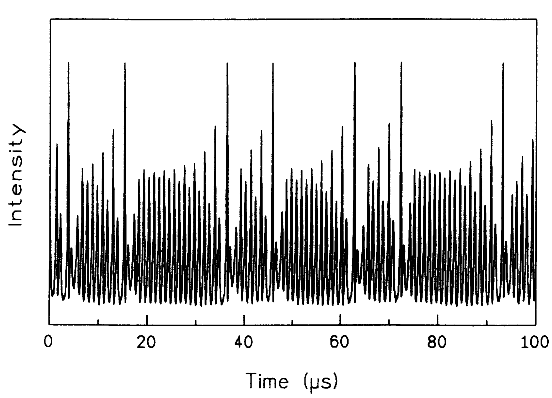

FIG. 6. Typical intensity variation for type-III intermittency of the laser: pump intensity, 7 W/cm^2; NH_3 gas pressure, 4 Pa.

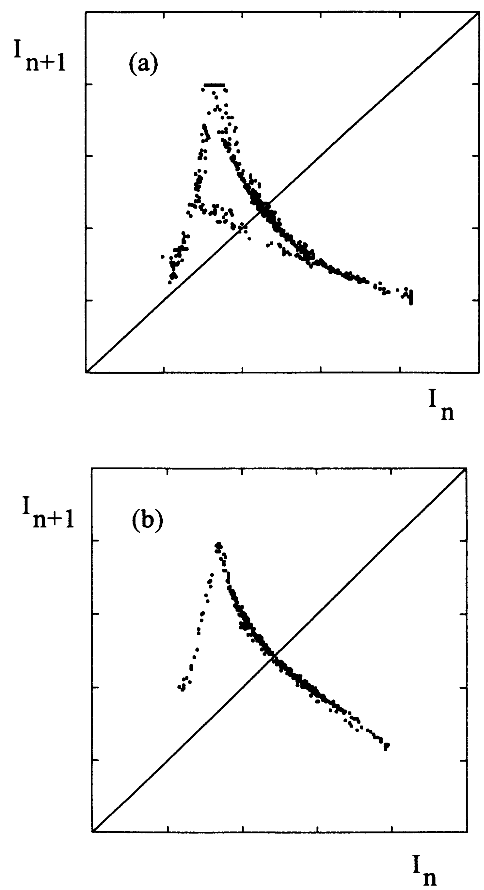

FIG. 7. First-peak-intensity return map constructed from the type-III intermittency of the laser: (a) under the influence of the pump-coherence effects of the laser and (b) under the condition that the pump-coherence effects can be neglected.

this experimental result we see that the topology of the attractor is not only completely parameter invariant, but also independent of dynamics type.

In summary, we find that despite the difference in dynamics of the three types of chaos observed on the single-mode NH_3 FIR laser, their intensity return maps show the same shape as the one of the Lorenz attractor, indicating that they are all produced by the same "laser attractor," which is that of the complex Lorenz equations. Thus, we conclude that inspection of the peak-intensity return map can provide results like those of "topological analysis" of measurements on chaotic systems [23].

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