Quantum collapse and revival in the motion of a single trapped ion

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Within certain limits, the dynamics of a single trapped ion oscillating about the node of a standing-wave light field is described by the Jaynes-Cummings model, which is routinely used for cavity-QED experiments. We propose a technique to measure quantum collapse and revival in the population inversion of a single trapped ion, and show that this method is uniquely suited for the measurement of final temperatures of the trapped ion as well as for the analysis of nonclassical states of the ion motion, such as Fock and squeezed states. The results are discussed with particular consideration given to the effects of a finite laser bandwidth.

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I. INTRODUCTION

The purpose of this paper is to present a theoretical investigation of collapses and revivals in the population inversion of a single trapped ion moving in a harmonic potential. The motivation for this work is two-fold. First, collapses and revivals are features of the quantized motion of the ion which are directly related to the discreteness of the vibrational states in the trap [1]. Secondly, experimental observation of these collapses and revivals provides a novel means of measuring the statistics of the quantum motion of the ion, and thus of detecting nonclassical states of motion (such as Fock states and squeezed states of motion [2,3]).

The phenomenon of collapses and revivals is well known in quantum optics in the context of the Jaynes-Cummings model (JCM). The JCM describes the coupling of a two-level atom to a single quantized harmonic oscillator. Cavity quantum electrodynamics (CQED) provides an experimental realization of the JCM, both in the optical and the microwave domain [4–7]. With n photons present in the cavity, a two-level atom initially in its ground state undergoes Rabi oscillations with a Rabi frequency proportional to $\sqrt{n}$. Generally, a cavity does not contain a fixed number of photons, but rather is characterized by a density operator with photon number distribution P_n. In this case, Rabi oscillations corresponding to different photon numbers are superimposed and give rise to collapses and revivals in the time evolution of the atomic population inversion [8].

The first observation of quantum collapse and revival phenomena in CQED was reported in an experiment in which single Rydberg atoms from an atomic beam interact with the field of a superconducting microwave cavity [9]. In an experiment with very low atomic-beam fluxes, the cavity contains essentially thermal photons only, since the photons deposited by each atom are lost as thermal equilibrium is restored before the arrival of the next atom. At higher fluxes, however, the system operates as a maser and the mean photon number in the cavity increases. In the first observation of collapse and revival phenomena photons deposited by incoming atoms typically contributed 20% to 60% of the total number of photons in the cavity.

The broad photon distribution characteristic of a thermal state means that collapses and revivals of the atomic inversion tend to be "smeared out" [10] compared to the case, for instance, of an initial coherent state, which yields very prominent collapse and revival features [8]. Hence, for a detailed investigation of collapse and revival phenomena, one would like to be able to investigate a single two-level system at rest interacting with a single cavity mode prepared, for example, in a coherent state. One would also like to be able to perform repeated measurements on a system that can be prepared in an identical way over and over again.

In recent work we have pointed out that the dynamics of a two-level ion in a standing-wave laser field which moves in a harmonic trapping potential leads to an alternative realization of the JCM [2,3,11]. The conditions for such a realization are that the vibrational amplitude of the ion motion is much less than the wavelength of the light (Lamb-Dicke limit), and that the trap frequency ν is much larger than the atomic spontaneous decay rate Γ (strong confinement limit). Given these conditions, we showed that the well-established field of ion trapping can be applied to the investigation of the JCM and of effects taken directly from cavity QED studies. For instance, one should be able to observe quantum collapses and revivals of the population inversion [1] and from this infer statistical properties of the motion of the ion. This would provide a unique way of analyzing nonclassical states of motion in an ion trap [2,3].

The paper is organized as follows. In Sec. II the basic scheme for an observation of the collapse and revival phenomenon with trapped ions is outlined. Section III

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comprises the theoretical description of such an experiment with particular emphasis on the role of laser fluctuations, and in Sec. IV results for various initial nonclassical states of the quantized motion are presented and discussed.

II. COLLAPSE AND REVIVAL WITH A SINGLE TRAPPED ION

The interaction of a harmonic oscillator coupled to a two-level system is generally investigated in quantum optics by studying the Jaynes-Cummings Hamiltonian in the rotating wave approximation ($\hbar = 1$),

$$H = \omega_f a^\dagger a + \tfrac{1}{2}\omega_0 \sigma_z + g(\sigma_+ a + a^\dagger \sigma_-), \tag{1}$$

where $a^\dagger$ and a are creation and annihilation operators for the harmonic oscillator oscillating at frequency ω_f, and $\sigma_{\pm,z}$ are the Pauli spin matrices describing a two-level system with transition frequency ω_0. The last term in (1) describes the coupling of the two-level system to the harmonic oscillator with a coupling strength g.

In Ref. [2] it was shown that the master equation for a single two-level ion trapped in a harmonic potential and located close to the node of a standing-wave light field [12] is given by

$$\frac{d\rho}{dt} = -i\left[\nu a^\dagger a + \frac{\Delta}{2}\sigma_z + \frac{\Omega}{2}\eta(\sigma_+ a + a^\dagger \sigma_-), \rho\right] + \frac{\Gamma}{2}(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-). \tag{2}$$

Here ν is the trap frequency, Δ is the detuning of the two-level transition from the laser frequency, Ω is the laser Rabi frequency, and Γ denotes the decay rate of the excited atomic level; $\eta = 2\pi a_0/\lambda$ is the Lamb-Dicke parameter with a_0 the amplitude of the ground state of the trap and λ the optical wavelength. The master equation (2) was derived with the assumptions

$$\nu, |\Delta| \gg \eta\Omega, \Gamma, |\Delta - \nu| \ , \tag{3a}$$

$$\eta \ll 1. \tag{3b}$$

Moreover, for interaction times τ that are short compared to the spontaneous lifetime,

$$\tau \ll \Gamma^{-1}, \tag{4}$$

the dissipation part of master equation (2) is negligible. In this case, with the associations

$$\begin{aligned} \Delta &\leftrightarrow \omega_0, \\ \nu &\leftrightarrow \omega_f, \\ g &\leftrightarrow \eta\Omega/2, \end{aligned} \tag{5}$$

we have a direct correspondence with the Jaynes-Cummings Hamiltonian (1), and the level scheme of the coupled ion-trap system reproduces the JCM in which the harmonic oscillator describes the motion of the ion in the trap [2]. Thus, localizing a laser-cooled and laser-trapped two-level ion close to the node of a standing-wave field should allow one to perform experiments analogous to those in cavity QED. The main advantages in the trapped ion case, as compared to conventional cavity QED experiments, are (i) the harmonic oscillator is not damped, and (ii) the coupling is proportional to the laser amplitude, i.e., it can be made very strong simply by increasing laser power. Such straightforward control of the coupling strength is usually not possible in cavity QED experiments.

For an observation of the collapse and revival phenomenon we propose to employ a single ion trapped in a harmonic trap (e.g., a Paul trap) and an internal level scheme as indicated in Fig. 1(a). The two-level transition $|g\rangle \leftrightarrow |r\rangle$ used to observe the collapse and revival is a forbidden transition (e.g., an electric quadrupole transition [12]), for which the spontaneous decay time (Γ_r^{-1}) is long compared to the observation time τ and the trap period ν^{-1} [see conditions (3a) and (4)]. It is assumed that this transition is driven by a very strong standing-wave field in such a way that the effective coupling constant $\eta\Omega_r/2$ leads to several Rabi oscillations during the interaction time, but it still satisfies condition (3a). The third level $|e\rangle$ is used for optical cooling of the ion (initial preparation), and for a measurement of the population inversion (on the $|g\rangle \leftrightarrow |r\rangle$ transition) as explained below.

In a first step, one laser cools the ion into the Lamb-Dicke limit [see condition (3b)] [13,14] using the strong $|g\rangle \leftrightarrow |e\rangle$ dipole transition. One then prepares the ion in a given state of motion, which serves as the initial harmonic oscillator state for interaction with the two-level system ($|g\rangle \leftrightarrow |r\rangle$). In the usual experimental situation where one has $\Gamma_e > \nu$ (weak confinement condition for

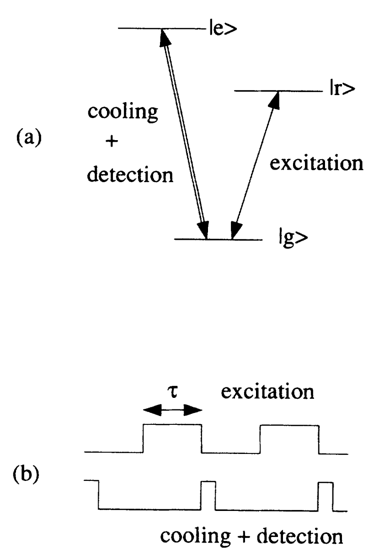

FIG. 1. (a) Three-level ion with a strong transition $|g\rangle \leftrightarrow |e\rangle$ for laser cooling and weak transition $|g\rangle \leftrightarrow |r\rangle$ which serves as the effective two-level system for the observation of collapse and revival phenomena. (b) The lasers on the strong and weak transition are alternately switched on and off. After a certain interaction time τ on the weak transition, interaction on the strong transition serves to measure the ground state population.

strong transition), a single trapped ion can be cooled to the Doppler limit, in which the steady state of motion in the harmonic trap is a thermal state of mean quantum number $< n > \approx \Gamma_e/2\nu$. With conditions for strong confinement on the weak transition, i.e., $\Gamma_r < \nu$, only the ground state is occupied [15], and various nonclassical states of trap motion such as, e.g., Fock states and coherent squeezed states, can be produced with special standing- and traveling-wave arrangements [2,3]. In what follows, we will concentrate on situations in which the initial state of the harmonic oscillator is one of the following: thermal, Fock, coherent, or squeezed state. Obviously, the present approach is valid for any initial state.

Collapse and revival in the three-level atom shown in Fig. 1(a) can then be observed by exciting the narrow transition $|g\rangle \leftrightarrow |r\rangle$ in the following way [cf. Fig. 1(b)]: (i) after the ion is laser cooled and an initial state distribution $\rho(t_i)$ is prepared, the cooling radiation is turned off, (ii) then laser radiation on the narrow (two-level) transition is applied for a certain time τ with the ion placed at the node of the standing-wave laser field, and (iii) at the end of this time interval τ the laser exciting the strong transition is turned on again; the absence or presence of fluorescence from the strong transition will indicate whether or not the ion has made a transition (quantum jump) to the excited metastable level $|r\rangle$ [16–18]. We repeat the same sequence in order to determine the probability $P_r(\tau)$ that after the interaction time τ the ion is in the state $|r\rangle$ [i.e., to determine the population inversion $\langle\sigma_z(\tau)\rangle = 1-2P_r(\tau)$]. Repeating this measurement for different interaction times τ, we build up the population inversion as a function of time τ. Depending on the initial state distribution, this population inversion exhibits different quantum collapses and revivals which provide a signature of the statistical distribution of the initially prepared state $\rho(t_i)$.

In Sec. III we give a quantitative analysis of the collapse and revival phenomena in an ion trap. We will neglect spontaneous transitions on the $|r\rangle \to |g\rangle$ transition, $\Gamma_r = 0$ (assuming that $|r\rangle$ is a metastable state). Our calculation includes the effect of laser fluctuations. Since the coupling constant is proportional to the laser amplitude we note that a slow drift of the laser amplitude by 1% will allow the observation of at least 100 Rabi oscillations, which is sufficient to observe the collapses and revivals in any case. In contrast, laser phase fluctuations producing a laser bandwidth comparable to $\eta\Omega$ are found to spoil the collapses and revivals.

III. THEORETICAL DESCRIPTION

In this section we consider the evolution of a single two-level ion with levels $|g\rangle$ and $|r\rangle$ trapped in a harmonic potential and interacting with a standing-wave light field. The Hamiltonian describing this situation is

$$H = \frac{1}{2}\omega_0\sigma_z + \nu a^\dagger a + \frac{\Omega}{2}\sin[\eta(a+a^\dagger)+\varphi]\left[\sigma_+ e^{-i\omega_L t - i\phi(t)} + \text{H.c.}\right], \tag{6}$$

where Ω is the Rabi frequency for the atom-laser interaction and φ denotes the center of the trap with respect to the standing wave [12]. The stochastic phase $\phi(t)$ describes phase fluctuations arising from a finite laser bandwidth which may influence the collapse and revival phenomenon. Transforming to an interaction picture defined by the unitary operator

$$\mathcal{U}_t = \exp(-i\nu a^\dagger a t)\exp\{-i[\omega_L t + \phi(t)]\sigma_z/2\}, \tag{7}$$

and neglecting the rapidly oscillating terms in the limit

$$\nu, |\omega_0 - [\omega_L + \dot\phi(t)]| \gg \eta\Omega, |\omega_0 - [\omega_L + \dot\phi(t)] - \nu|, \tag{8}$$

one obtains [2]

$$H_I \approx \frac{1}{2}[\Delta - \nu - \dot\phi]\sigma_z + \eta\frac{\Omega}{2}(\sigma_+ a + a^\dagger\sigma_-), \tag{9}$$

where we have defined the detuning $\Delta = \omega_0 - \omega_L$. From this Hamiltonian, it can be easily shown that the subspaces $\mathcal{H}_n = \{|n,g\rangle, |n-1,r\rangle\}$ $(n = 1,2,\ldots)$ remain invariant under the interaction. Defining matrix elements ρ^n_{ij} according to

$$w^n = \langle n-1,r|\rho|n-1,r\rangle - \langle n,g|\rho|n,g\rangle\ , \tag{10a}$$

$$\rho^n_{gr} = \langle n,g|\rho|n-1,r\rangle\ , \tag{10b}$$

$$\rho^n_{rg} = \langle n-1,r|\rho|n,g\rangle\ , \tag{10c}$$

we can write the equations of motion as derived from $\dot\rho = -i[H_I,\rho]$ in the form

$$\begin{pmatrix} \dot\rho^n_{rg} \\ \dot\rho^n_{gr} \\ \dot w^n \end{pmatrix} = \begin{bmatrix} -i(\Delta-\nu)+i\dot\phi & 0 & i\eta\Omega\sqrt{n}/2 \\ 0 & i(\Delta-\nu)-i\dot\phi & -i\eta\Omega\sqrt{n}/2 \\ i\eta\Omega\sqrt{n} & -i\eta\Omega\sqrt{n} & 0 \end{bmatrix} \begin{pmatrix} \rho^n_{rg} \\ \rho^n_{gr} \\ w^n \end{pmatrix}. \tag{11}$$

In writing this equation we have assumed spontaneous decay on the weak transition to be negligible.

Assuming that the laser exhibits a Lorentzian profile according to the phase diffusion model with bandwidth Γ' [half-width at half maximum (HWHM)], and using standard methods [19], one can find the evolution equations for w^n, ρ^n_{eg}, and ρ^n_{ge} averaged over frequency fluctuations. The result is

$$\frac{d}{dt}\begin{pmatrix} \langle\rho^n_{rg}\rangle \\ \langle\rho^n_{gr}\rangle \\ \langle w^n\rangle \end{pmatrix} = \begin{bmatrix} z^* & 0 & \frac{i}{2}\Omega_n \\ 0 & z & -\frac{i}{2}\Omega_n \\ i\Omega_n & i\Omega_n & 0 \end{bmatrix} \begin{pmatrix} \langle\rho^n_{rg}\rangle \\ \langle\rho^n_{gr}\rangle \\ \langle w^n\rangle \end{pmatrix} \equiv A_n \begin{pmatrix} \langle\rho^n_{rg}\rangle \\ \langle\rho^n_{gr}\rangle \\ \langle w^n\rangle \end{pmatrix}, \tag{12}$$

with $z = i[\Delta - \nu] + \Gamma'$ and $\Omega_n = \eta\Omega\sqrt{n}$. Here the notation $\langle \rho_{ij}^n \rangle$ denotes the average over the bandwidth of the exciting laser.

Using the definition (10a), we see that the population inversion averaged over laser phase fluctuations $\langle \sigma_z \rangle = \langle \, \mathrm{Tr}[(|r\rangle\langle r| - |g\rangle\langle g|)\rho] \rangle$ can be written as

$$\langle \sigma_z(\tau) \rangle = \sum_{n=1}^{\infty} \langle w_n(\tau) \rangle - P_g^0 \; , \tag{13}$$

where P_g^0 denotes the initial probability for being in the ground state $|0, g\rangle$. Starting with the two-level ion initially in the ground state $|g\rangle$, we find for the inversion

$$\langle \sigma_z(\tau) \rangle = -\left(P_0 + \sum_{n=1}^{\infty} P_n \left[\exp(A_n \tau) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right]_3 \right) \; , \tag{14}$$

where P_n with $n = 0, 1, 2, \ldots$ denotes the initial occupation number of the harmonic oscillator state $|n\rangle$, and the subindex 3 indicates the third element of the resulting vector.

The population inversion (14) is a superposition of (weakly damped) Rabi oscillations with frequency Ω_n, weighted with the initial state distribution P_n. This gives rise to collapses and revivals in $\sigma_z(\tau)$ as a function of time τ. We note that only the diagonal elements (and no coherences) of the initial density matrix enter into Eq. (14). The inversion of (13), and the reconstruction of the distribution P_n from the time evolution of $\sigma_z(\tau)$ has been discussed by Fleischhauer and Schleich [20].

IV. DISCUSSION

In the present section we give a discussion and numerical results for collapse and revivals of the population inversion of a trapped ion given the following initial population distributions of the harmonic oscillator: (i) coherent state distribution of mean excitation number $\langle n \rangle$,

$$P_n^{\mathrm{coh}} = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}, \tag{15}$$

(ii) thermal distribution of mean excitation number $\langle n \rangle$,

$$P_n^{\mathrm{th}} = \frac{\langle n \rangle^n}{(\langle n \rangle + 1)^n} \; , \tag{16}$$

(iii) squeezed vacuum distribution, with squeezing parameter r,

$$P_{2n}^{\mathrm{sq}} = \frac{(2n)!}{(2^n n!)^2}, \frac{(\tanh r)^{2n}}{\cosh r} \; , \tag{17}$$

and (iv) Fock state ($|n_0\rangle$) distribution,

$$P_n^{\mathrm{Fock}} = \delta_{n,n_0} . \tag{18}$$

Figure 2(a) shows the population inversion according to Eq. (14) for an initial coherent state of the trapped ion with a mean excitation of $\langle n \rangle = 4$. The Rabi frequency was chosen as $\Omega = 2\pi \times 10^5 \ \mathrm{s}^{-1}$. Collapse appears after about 0.75 ms and the revivals occur for $\tau = 2.5$ ms. The solid, dashed-dotted, and dashed lines indicate the inversion as expected for a laser bandwidth of $\Gamma' = 0$, $2\pi \times 100$, $2\pi \times 1000 \ \mathrm{s}^{-1}$ and it is clearly seen that for a bandwidth comparable to the inverse of the interaction time the collapse and revival phenomenon can no longer be observed. Preparation of a coherent state was discussed in Ref. [3].

Standard laser cooling theories, for example, for Doppler cooling of the ion to the Lamb-Dicke limit, predict a thermal (Bose-Einstein) distribution for the final state [13]. Figure 2(b) shows the expected population inversion for a thermal state with mean excitation $\langle n \rangle = 10$, as expected for a typical Doppler cooling experiment with trap frequency $\nu = \Gamma/5$. The other parameters are the same as in Fig. 2(a). Again, collapse and revival may be observed, however, with a rather irregular time evolution [10]. Again collapse and revival can no longer be observed when the laser bandwidth is comparable to the inverse of the interaction time.

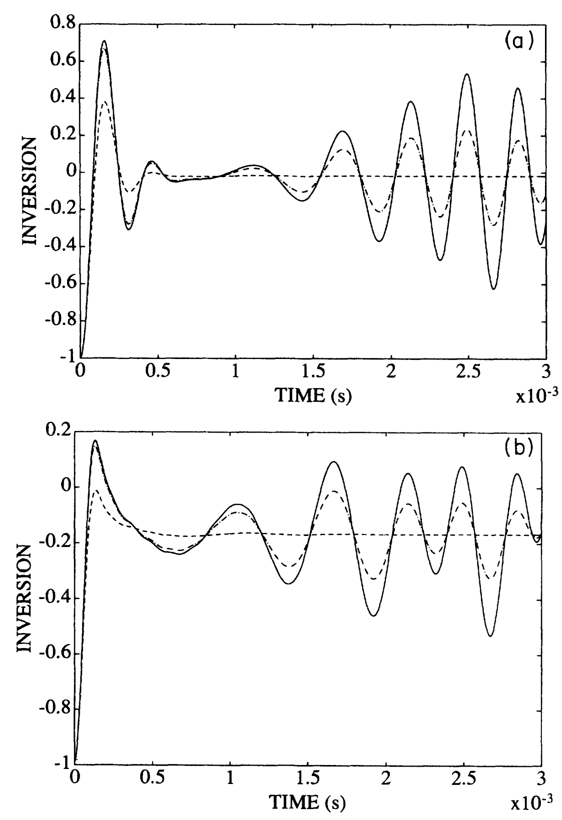

FIG. 2. (a) Population inversion as a function of time for an initial coherent state distribution with $\langle n \rangle = 4$ and for varying laser bandwidth $\Gamma' = 2\pi \times 0$, 100, 1000 s^{-1} (solid, dashed, dotted line, respectively). (b) Population of state $|r\rangle$ as a function of time for an initial thermal state distribution with $\langle n \rangle = 5$ and for varying laser bandwidth; parameters are as in (a). In (a) and (b) the Rabi frequency is $\Omega = 2\pi \times 10^5$.

Figure 3 shows the population inversion as a function of time for an initial squeezed state. The upper (solid) curve shows a result for a large squeezing parameter ($r = 1.5$) which is similar to that found for a thermal distribution [21]. The lower set of curves (dashed, dotted, dashed-dotted, and solid curves) show results for decreasing squeezing parameter ($r = 0.8, 0.55, 0.35, 0.2$). We find a strong dependence on the degree of squeezing which would allow one to derive the squeezing parameter r from a measurement. The calculations shown in Fig. 3 were obtained for a laser bandwidth $\Gamma' = 2\pi 100\ \mathrm{s}^{-1}$; a similar behavior as in Fig. 2 is observed by varying the laser bandwidth.

In Fig. 4 we plot our results for a Fock $|n_0 = 2\rangle$. The different curves indicate the behavior for the laser bandwidths considered in Fig. 2. The solid line shows the sinusoidal behavior as is expected for a Fock state, and a measurement of its period allows the determination of the quantum number n_0. As expected, for larger laser bandwidth, the inversion damps out as a function of time.

As can be seen from Figs. 2, 3, and 4, the appearance of collapse and revival depends sensitively on the initial state distribution. These examples demonstrate that the proposed measurement of collapse and revival phenomena is a new tool to analyze the state distribution in an ion trap. This also requires knowledge of the laser bandwidth. We emphasize, however, that even in the case of a large laser bandwidth (which would not allow the observation of collapse and revival) the temperature of a trapped ion (in a thermal state) can be determined. This can be easily seen from Eq. (14) in the limit $t \to \infty$ (i.e., $\tau \ll 1/\Gamma'$). In this case the inversion in Eq. (14) is given simply by the population of the harmonic oscillator state $|0\rangle$, i.e., $\langle\sigma_z(\tau \gg \Gamma'^{-1})\rangle \longrightarrow P_0$ and for a thermal distribution we obtain

$$\langle n\rangle \equiv \overline{n} = \frac{1}{P_0} - 1\,. \tag{19}$$

Thus a measurement of P_0 yields immediately the temperature of the single trapped ion. This is indicated in Fig. 5 which shows the expected result for the inversion as a function of time for different mean values of the initial thermal distribution. Of course, in an experiment it suffices to take measurements for a single interaction time τ only, and therefore such a measurement would be very simple to perform. Hence this is an alternative to analyzing atomic excitation spectra which usually require a longer measurement time and could perturb the state distribution of the ion.

So far, only frequency fluctuations have been taken into account. Amplitude fluctuations, of course, would also impede the appearance of the collapse and revival effect. However, this is relieved due to the fact that the interaction [cf. Eq. (2)] on the sideband involves the small Lamb-Dicke parameter and hence amplitude fluctuations are accordingly reduced. From our numerical calculations we estimate that amplitude fluctuations of about 1% do not significantly alter the results obtained.

Finally, it should be noted that for an experiment with a single trapped ion the localization at the node of the standing wave is only required to suppress unwanted transitions $|g, n\rangle \to |e, n\rangle$ which may appear at the exact resonance frequency. We remember that placing an

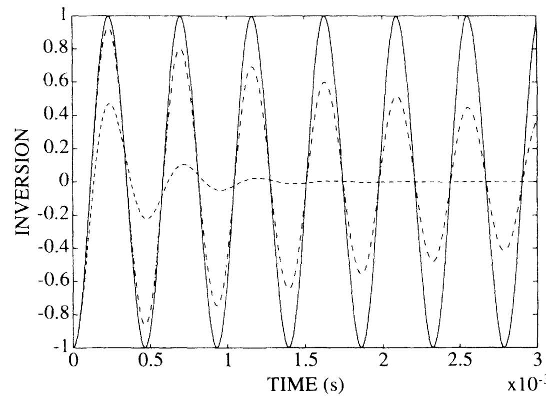

FIG. 4. Population inversion as a function of time for an initial Fock state $n = 4$ and laser bandwidth $\Gamma' = 2\pi \times 0, 100, 1000\ \mathrm{s}^{-1}$ (solid, dashed, dotted, respectively). The Rabi frequency is $\Omega = 2\pi \times 10^5$.

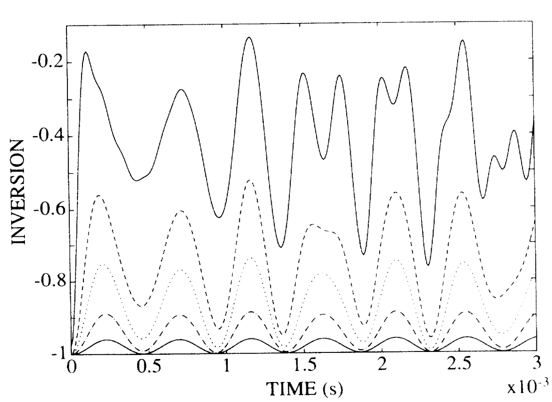

FIG. 3. Population inversion as a function of time for an initial squeezed state distribution with a laser bandwidth $\Gamma' = 2\pi 100\ \mathrm{s}^{-1}$. The solid, dashed, dotted, dashed-dotted, and solid curves show results for decreasing squeezing parameter $r = 1.5, 0.8, 0.55, 0.35$, and 0.2. The Rabi frequency is $\Omega = 2\pi \times 10^5$.

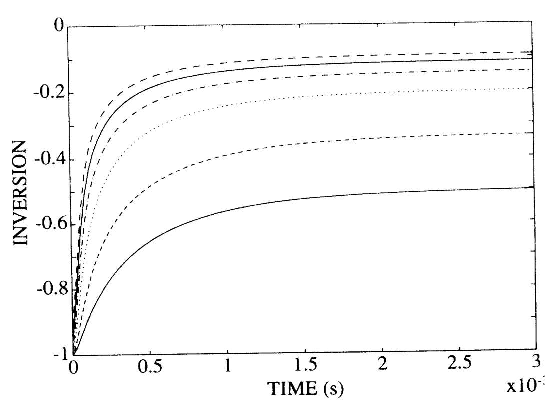

FIG. 5. Population inversion as a function of time for an initial thermal state with $\langle n\rangle = 1, 2, 4, 6, 8, 10$ and laser bandwidth $\Gamma' = 2\pi \times 10\,000\ \mathrm{s}^{-1}$ (from bottom to top). The Rabi frequency is $\Omega = 2\pi \times 10^5$.

ion at the node of the standing-wave field allows interaction only with the motional sidebands corresponding to transitions $|g,n\rangle \rightarrow |e,n\pm 1\rangle$. For a highly forbidden transition, such as, e.g., that used in the quantum jump experiments [18], a traveling wave detuned to the lower sideband (i.e., $\omega_L = \omega_0 - \nu$) would be sufficient for the observation of these phenomena. Thus an experimental realization of the proposed technique seems to be readily feasible with single trapped Ba^+, Ca^+, Sr^+, Hg^+, In^+, and Yb^+ ions.

V. CONCLUSION

The analogy of a single trapped ion at the node of a standing-wave light field with the JCM is used to propose an experiment for the observation of collapse and revival phenomena. Such a technique provides a novel tool to investigate and analyze nonclassical state distributions of a single trapped ion and is suited to the measurement of residual temperatures in a trap. It is expected that this experimental technique will allow the observation of Fock and squeezed states of ion motion in a trap.

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