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X-ray laser with photon energy of about 10 keV

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A possible way of constructing x-ray lasers of photon energy in the keV range is suggested. An arrangement in which x-ray lasing of photon energies 7.18 and 11.7 keV is expected is also given.

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Shortly after the first appearance of optical lasers continuous efforts were made to shorten the wavelength of lasers [1]. The results of the research of three decades indicate that the main problems that hinder the construction of a laser in the real x-ray range, i.e., with a photon energy above 1 keV, are the absence of mirrors of high reflectivity, the relatively large absorption coefficient of the possible lasing media, and the very fast and intense pumping required. In this paper it is suggested that the above difficulties may be overcome by applying the phenomenon of anomalous x-ray absorption (the so-called Borrmann effect) [2] and the fact that radiation is emitted by relativistic electrons between channeling states [3]. Furthermore, devices producing subpicosecond laser pulses of high intensity that have been developed very recently [4] are also proposed to be utilized for the pumping.

If x rays are transmitted in crystals, their absorption can be essentially decreased, propagating in certain directions in which the field strengths have nodal points at crystal planes, i.e., have near zero value around atomic sites [2]. It is just the case of the Borrmann effect, i.e., anomalous x-ray absorption, the use of which was suggested to reduce the pump intensity of a γ-ray laser a few years ago [5]. According to Ref. [5] superradiance at subnanometer wavelengths could be achieved by stimulating resonant, nuclear Mössbauer transitions. In this case the sources, which are atomic nuclei, are located at crystal lattice sites where not only the x-ray absorption but the induced emission have their minima. Thus a more effective application of the Borrmann mode is expected if the sources, unlike in Ref. [5], are located between crystal planes, where the field strengths have maxima. This situation suggests taking free electrons moving in the crystal as sources. If electrons of relativistic energy move in crystals, then they can show the phenomenon of axial and planar channeling, i.e., they can cover large distances without a large-angle collision in the so-called channels determined by the atomic rows or planes; meanwhile their energy has level-like structure [3]. It is expected that if the energy difference between two of the channeling electronic states equals the photon energy selected by the distance of crystal planes in anomalous x-ray absorption, then above a threshold current density of the electrons laser radiation of keV photon energy arises.

In treating relativistic electron channeling, two frames of reference, the so-called laboratory (L) and rest (R) frames, are commonly introduced, which are the frames where the crystal and the electron, respectively, are at rest. For the sake of simplicity we restrict ourselves to the case of axial channeling, where the electron "senses" the averaged potential of an atomic row. In the R system the atoms are spaced in the row at a distance $d_R = d_L/\gamma$, where d_L is their distance in the L system and $\gamma = (1-\beta^2)^{-1/2}$, with $\beta = v_z/c$, and v_z and c are the velocities of the electron and light (Fig. 1). The electron moves in the z direction with $v_z \sim c$. The solution of the energy eigenvalue problem results in wave functions and eigenvalues in the transverse motion characterized by radial and angular quantum numbers n and l. The energy eigenvalues $E_{\pm R}(n,l)$ in the R system implicitly depend on the velocity of the electron as the atomic distance in the row changes according to Lorentz contraction as $d_R = d_L/\gamma$. In a transition between channeling states of energy $E_{\perp R}(n_1,l_1), E_{\perp R}(n_2,l_2)[\Delta E_{\perp R}(n_1,n_2,l_1,l_2) = E_{\perp R}(n_2,l_2) - E_{\perp R}(n_1,l_1)]$ a photon of energy $\hbar\omega_R = \Delta E_{\perp R}$ is emitted. Thus the energy $\hbar\omega_R$ also changes according to the change in the electron velocity. If the angular frequency and wave vector of the emitted radiation ω_R and $\mathbf{K}_R$ are given in the R frame, then their values ω_L and $\mathbf{K}_L$ in the L system can be obtained by

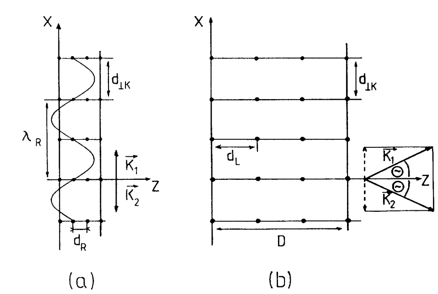

FIG. 1. (a) The notation used in the rest (R) frame, where the electron is at rest. d_R and $d_{\perp k}$ are the distances between adjacent atoms in directions z and x, respectively, λ_R is the wavelength of the radiation determined by the crystal resonator, the vectors $\mathbf{K}_1$ and $\mathbf{K}_2$ denote the wave vectors of those two oppositely running waves in direction x, which form the standing wave in the resonator. $|\mathbf{K}_1 - \mathbf{K}_2| = 2\pi/d_{\perp k}$ ($\lambda_R = 2d_{\perp k}$). (b) The notation in the laboratory (L) system, where the crystal rests and the electron moves in direction z with a relativistic velocity. D is the path length of the electrons in the crystal and Θ is the angle of the wave vectors of the outcoming radiation.

Lorentz transformation. Although the emitted radiation has dipolelike angular distribution in the R frame, the radiation in the L system can be observed in a narrow angle around the forward (z) direction. It was also shown earlier [6] that between states of different eigenvalues in the transverse motion population inversion can be achieved and changed by a slight modification of the angle of incidence of the electron beam.

At certain values of the electron velocity, i.e., the electron energy, half the wavelength of the emitted radiation equals the distance between atomic planes $d_{\perp K} (k=1,2)$ in one of the transverse directions (it is denoted by x, Fig. 1), i.e.,

$$\hbar\omega_R = \pi\hbar c / d_{\perp k} \ . \tag{1}$$

If Eq. (1) holds, then standing waves can be formed in the R frame in direction x. This fact appears as anomalous x-ray transmittion in the L frame at a definite angle Θ and in a state of polarization perpendicular to the plane xz. It is expected that this mode can be easier to amplify by induced emission.

In a crystal resonator it is useful to expand the photon field in terms of standing waves [7], which are linear combinations of plane waves having momentum only of the x component K_{Rx} and $-K_{Rx}$. If we transform the corresponding two four-vectors $(\omega_R/c, -K_{Rx}, 0, 0)$ and $(\omega_R/c, K_{Rx}, 0, 0)$ with $K_{Rx} = \omega_R/c$ into the L system, then we obtain those four-vectors, which determine the energy and momentum of the outcoming radiation in the L frame. These are $(\omega_L/c, K_{Lx}, 0, K_{Lz})$ and $(\omega_L/c, -K_{Lx}, 0, K_{Lz})$ (Fig. 1), where

$$\omega_L = \gamma\omega_R \ , \quad K_{Lx} = K_{Rx}, \quad K_{Lz} = \beta\gamma\omega_R / c \ . \tag{2}$$

The radiation in the L system is emitted at an angle Θ relative to the direction z in the xz plane, and since $K_L = (K_{Lx}^2 + K_{Lz}^2)^{1/2} = \omega_L / c$, we have

$$\sin\Theta = 1/\gamma \ . \tag{3}$$

It is expected that x-ray laser radiation may arise in the xz plane above and below the beam of electrons at angle Θ determined by Eq. (3) and with a state of linear polarization perpendicular to the xz plane.

On the basis of observations [8] the following estimations for the energy and angle of the outcoming radiation and the energy of the channeling electron can be made. From Fig. 2(a) of Ref. [7], $\hbar\omega_R^{31} = 520$ eV and $\hbar\omega_R^{21} = 320$ eV can be determined in the case of electrons of energy 3.81 MeV channeling in Si in the $\langle 110 \rangle$ direction. The superscripts and subscripts 31 and 21 refer to transitions $3p \rightarrow 1s$ and $2p \rightarrow 1s$ throughout.

The velocity dependence of the energy of a transition in the R frame can be written as

$$\hbar\omega_R = \gamma\hbar\omega_0 \ , \tag{4}$$

where $\hbar\omega_0$ is characteristic of the transition. Thus $\hbar\omega_0^{31} = 61$ eV and $\hbar\omega_0^{21} = 38$ eV for crystalline Si in the $\langle 110 \rangle$ direction. The distance between atomic rows in the transverse direction is $d_{\perp k} = 5.43 \times 2^{-3/2} \times 10^{-8}$ cm $= 1.920 \times 10^{-8}$ cm. It determines the energy $\hbar\omega_R$ of that transition, which matches the condition of resonance in the R frame, through Eq. (1). Having obtained $\hbar\omega_R = 3.23$ keV the quantity γ characteristic of the energy of the electron needed to produce $\hbar\omega_R$ in transitions between channeling states can be computed from Eq. (4). In the above case, $\gamma_{31} = 52.4$ and $\gamma_{12} = 85.4$ are obtained. Applying Eqs. (3) and (2), we obtain for the angles, $\Theta_{31} = 1.09°$ and $\Theta_{21} = 0.67°$, and for the photon energies, $\hbar\omega_L^{31} = 169$ keV and $\hbar\omega_L^{21} = 276$ keV. The kinetic energies of the electrons needed are $E_{\rm kin}^{31} = 26.3$ MeV and $E_{\rm kin}^{21} = 43.1$ MeV, respectively. As the above values are uncomfortable from an experimental point of view, we try to find a more convenient situation.

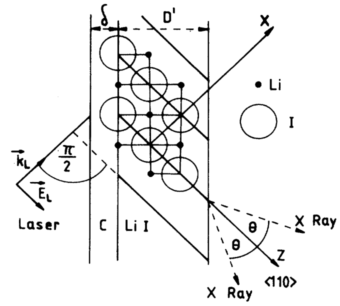

FIG. 2. The scheme of the experimental arrangement proposed. $\mathbf{k}_L$ and $\mathbf{E}_L$ are the wave vector and the electric field strength of the laser radiation, δ and D' are the widths of the graphite layer and the LiI crystal ($D = 2^{1/2} D'$). The other notation is the same as in Fig. 1.

The two-dimensional potential, which describes the interaction between the electron and an atomic row has the form $V = -K/\rho$, where ρ is the radial coordinate in the two-dimensional polar coordinate system. $\hbar\omega_0 \sim K^2$ and K depends on the characteristic of the atomic row, such as d and Z, which are the distance and the proton number of the atoms in the row [9]. If the material, i.e., d and Z, are changed, then the basic energy $\hbar\omega_0$ of a transition also changes as

$$\hbar\omega_0^n = \eta^2 \hbar\omega_0^o \ , \tag{5}$$

with

$$\eta = K^n / K^o = (Z^n / Z^0)^{2/3} d^n / d^o \ , \tag{6}$$

where the superscripts n and o refer to the new and the old material, respectively. With the aid of Eqs. (5) and (6) we can find a more appropriate material for x-ray lasers. The detailed analysis shows that it is advantageous if η is as large as possible, because it makes $\hbar\omega_0$ large and γ (and thus the kinetic energy of the electrons needed) small. Thus LiI seems to be one of the candidate materials. If we take an atomic row in the $\langle 110 \rangle$ direction in Si and convert it to an atomic row of I in LiI also in the $\langle 110 \rangle$ direction, then we get from Eq. (7) $\eta^2 = 4.84$. Using it in Eq. (5) with the $\hbar\omega_0^{\rm Si}$ values of Si, we obtain $\hbar\omega_0^{\rm I,31} = 298$ eV and $\hbar\omega_0^{\rm I,21} = 183$ eV. The new distance between iodine atomic rows ($d_\perp = 4.24 \times 10^{-8}$ cm) determines $\hbar\omega_R = 1.462$ keV by Eq. (1). Thus we obtain from Eqs. (4), (3), and (2) $\gamma_{31} = 4.91$, $\gamma_{21} = 7.99$, $\Theta_{31} = 11.75°$,

$\Theta_{21}=7.19°$, $\hbar\omega_L^{31}=7.18$ keV, and $\hbar\omega_L^{21}=11.7$ keV. The kinetic energy of the electron must be $E_{\rm kin}^{31}=2$ MeV and $E_{\rm kin}^{21}=3.57$ MeV, respectively.

However, because of the relatively large width of the channeling states ($\Gamma\sim 0.15\hbar\omega_L$, which gives $\Gamma_{31}=1.08$ keV and $\Gamma_{21}=1.76$ keV here), the incident electron beam need not be monoenergetic. We can estimate the energy interval of the useful electrons from the expression $\hbar\omega_L=\gamma^2\hbar\omega_0$ and the relations $\Delta\hbar\omega_L=\Gamma=2\gamma\hbar\omega_0\Delta\gamma$ and $\Delta E_{\rm kin}=\Delta\gamma m_0c^2$. We obtain $\Delta\gamma_{31}=0.369$ and $\Delta\gamma_{21}=0.602$, which give $\Delta E_{\rm kin}^{31}=0.185$ MeV and $\Delta E_{\rm kin}^{21}=0.30$ MeV.

Naturally, if the population inversion is achieved, then lasing can start above a threshold in pumping. As the pumping is produced by electrons of relatively definite energy, the system is similar to a semiconductor laser, and so the threshold is expected in the current density of the electrons. In order to estimate the threshold current of such a device, we borrow its method of computation from that of the semiconductor lasers. So the gain constant $G(\lambda)$ can be expressed as [10]

$$G(\lambda)=\frac{\lambda^2\xi J}{8\pi^2 eD}g(\nu)\ , \tag{7}$$

where λ is the wavelength of the emitted radiation, $\xi=(N_2-N_1)/N_2$ gives the magnitude of the population inversion, N_2 and N_1 are the number of electrons in states 2 and 1 ($E_2>E_1$), J is the current density, D is the thickness of the crystal, and e is the elementary charge. The quantity $g(\nu)$, which is the line-shape function, is estimated at $\nu=\nu_0$ as $g(\nu_0)=2T_2$, where T_2 is the lifetime of state 2. T_2 can be expressed here by the measured width $\Gamma_2=2\gamma^2\hbar c/1_c$ of the channeling state, where l_c is the so-called coherence length ($l_c\sim 0.5$ μm [3]). It is supposed that $D>1_c$ in the following. [In obtaining Eq. (7), the internal quantum efficiency η and the index of refraction n of the original expression [10] were taken for unity.]

The condition of gain is

$$G(\lambda)>\mu_B(\lambda)\ , \tag{8}$$

where $\mu_B(\lambda)$ is the absorption coefficient of the x ray of wavelength λ in the Borrmann mode. μ_B is about a magnitude less than the normal absorption coefficient μ_n [2]. From Eqs. (7) and (8) we obtain for the threshold current density

$$J_{\rm th}\geq 4\pi^2 ec\lambda^{-2}\mu_B(\lambda)D/1_c\xi\ . \tag{9}$$

Equation (9) has the form $J_{\rm th}\geq 1.89\times 10^9\lambda^{-2}\mu_B(\lambda)D/l_c\xi$, where μ_B, D, l_c, and λ have to be substituted in units of cm^{-1}, cm, cm and 10^{-8} cm, and $J_{\rm th}$ is obtained in units of A/cm^2. The normal absorption coefficients of crystalline LiI for the two photon energies of lasing obtained above are approximated by the absorption coefficients of I only, as the contribution of Li is negligible. Thus $\mu_n^{31}=1.38\times 10^4$ cm^{-1} and $\mu_n^{21}=3.46\times 10^3$ cm^{-1}. Using the approximation $\mu_B=0.1\mu_n$, we obtain $\mu_B^{31}/\lambda_{31}^2=461$ and $\mu_B^{21}/\lambda_{21}^2=308$ in the units mentioned above. Thus we get $J_{\rm th}^{31}\geq 8.7\times 10^{11}D/l_c\xi$ and $J_{\rm th}^{21}\geq 5.8\times 10^{11}D/l_c\xi$.

These extremely high current densities can be reached with the aid of pulsed lasers of extremely short (about 20 fs [11]) pulse length, as initially the electrons produced by multiterawatt lasers of femtosecond pulse length [4] are also bunched in a pulse of a length that is of the same order of magnitude as that of the laser pulse; meanwhile, their kinetic energy can increase to the required MeV range.

Now we deal with one possible arrangement based on the above general thoughts (Fig. 2). The laser pulse of femtosecond duration travels nearly perpendicular to direction z, with a state of linear polarization parallel to the plane of incidence. The solid material is built up from two layers. The first one is a conductor, e.g., graphite, in a very thin layer, the thickness of which is determined by its skin depth. In case of graphite and a laser of photon energy $\hbar\omega=1.16$ eV, the skin depth δ has a value of about 0.04–0.12 μm. That part of the laser pulse that is absorbed in the first layer produces a beam of fast electrons, which mainly move parallel to the electric field strength $\mathbf{E}_L$ [12], i.e., parallel to z. The population inversion is expected to change with small variations of the angle of incidence of the laser beam. The second layer is the LiI crystal resonator, which is oriented so that the iodine atomic rows, along which the channeling occurs, are parallel to the axis z. The width D' of LiI is determined by the condition $\mu_B^{-1}\gtrsim D'>\mu_n^{-1}$ ($D=2^{1/2}D'$). Shortly after the absorption of energy W from the laser pulse, electrons of high energy with an extremely large current density fall on the crystal. As they reach the crystal in a very short time the pulse cannot spread, and if the pulse of electrons is short enough, then x-ray lasing happens before the destruction of the resonator.

Using an absorbing layer (in our case graphite) of a width of about the skin depth, the absorption of the main part of the laser pulse can be achieved in the layer. The resonator has a secondary heating due to the electrons; thus the destruction of the resonator happens on a time scale determined by the electron-phonon energy transfer (relaxation) time, which was found to be several hundred femtoseconds [13]. But in consequence of the use of very short laser pulses (of length about 20 fs) the electron pulse length, which approximately equals the laser pulse length, is much less than the electron-phonon relaxation time. So the crystal resonator can be considered to be in an undisturbed state during the pumping electron pulse.

The energy W, which must be absorbed from the pulse, can be approximated with the aid of the following formulas: $Q_{\rm th}=A\tau J_{\rm th}$, $N_{e\rm th}=Q_{\rm th}/e$, and $W>N_{e\rm th}E_{\rm kin}$, where $Q_{\rm th}$ and $N_{e\rm th}$ are the charge and number of bunched electrons produced by the pulse, A is the focal-spot area, and τ is the pulse length of the laser. Taking $A=4$ μm^2 and $\tau=20$ fs, $N_{e\rm th}^{31}=4.35\times 10^9D/l_c\xi$, $N_{e\rm th}^{21}=2.9\times 10^9D/l_c\xi$, and $W_{31}>1.39\times 10^{-3}D/l_c\xi$, $W_{21}>1.66\times 10^{-3}D/l_c\xi$ (in joule units) are obtained for the $3p\rightarrow 1s$ and $2p\rightarrow 1s$ transitions in the case of axial channeling in crystalline LiI. Pulses that can produce these values, the energy of which is larger than W_{31} and W_{21}, seem to be available now [4] or will be available in the near future, thus making promising the construction of x-ray lasers of photon energies about 10 keV.

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