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Lasing without inversion: Gain enhancement through spectrally colored population pumping

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We propose the use of spectrally colored rather than white (i.e., incoherent) population pumping in lasing-without-inversion schemes. We show that colored pumping can lead to a significant enhancement in gain by presenting explicit results for the gain in a three-level ladder system. These results are obtained from Monte Carlo simulations of the coupled density-matrix-equations, which, for a colored pump, acquire the structure of Langevin equations.

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Most models [1–7] of lasing without inversion use incoherent pumping [8] to place population in the upper state of the transition in which lasing takes place. One possible way to pump incoherently is to use, as demonstrated in a recent experiment [9], an electric discharge to populate the relevant state. Incoherent pumping can be achieved optically through the use of broad bandwidth (white) light. In many real experimental situations it is convenient to employ a laser to perform the population pumping function. Lasers typically have the property of being spectrally narrow compared to the spectral quantities of importance in an inversionless lasing system. We refer to population-pumping lasers having such a property as colored. The use of a colored population pump introduces another control parameter into the description of the inversionless lasing system, i.e., the frequency at which the colored pump is effective.

In this Rapid Communication, we examine the consequences of replacing the incoherent pumping mechanism with a colored pump. We show that one can realize a significant enhancement in gain. We also discuss the physical mechanism responsible for this gain increase.

We consider the ladder scheme shown in Fig. 1(a). The transition $|1\rangle \leftrightarrow |2\rangle$ is the lasing transition. We assume that a strong coherent pump drives the lower transition $|2\rangle \leftrightarrow |3\rangle$. The middle, thick arrow on the upper transition represents the colored pump of frequency ν. Each of the two upper levels exhibits radiative decay to the most closely adjacent lower-lying level, and as a consequence these levels display the radiative widths of γ_1 and γ_2, respectively. To obtain gain, we consider the interaction of the driven three-level system with a probe field of frequency ω_1 applied on the upper transition. This ladder scheme is relevant, for example, to the energy levels of ^{138}Ba. The Hamiltonian for the system shown in Fig. 1(a) is

$$H = \hbar(\omega_{12}+\omega_{23})|1\rangle\langle 1| + \hbar\omega_{23}|2\rangle\langle 2| - \hbar\{(G_1 e^{-i\omega_1 t} + G_c(t)e^{-i\nu t})|1\rangle\langle 2| + G_2 e^{-i\omega_2 t}|2\rangle\langle 3| + \text{H.c.}\} , \quad (1)$$

where ω_{ij}'s are the frequencies of the atomic transitions and the G's are defined by

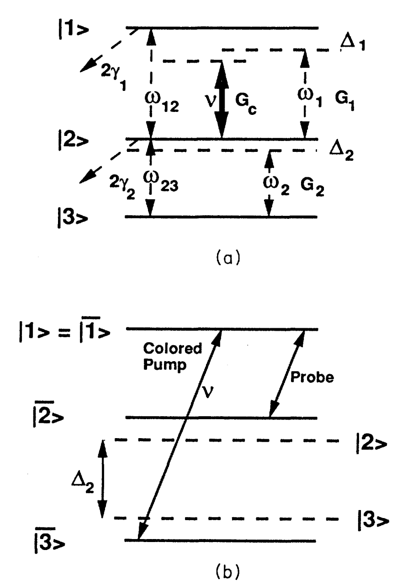

FIG. 1. (a) Energy-level diagram for the ladder system. The $|1\rangle \leftrightarrow |2\rangle$ transition is the lasing transition and the thick arrow on the upper transition is the colored pump of frequency ν. (b) Schematic diagram of the semiclassical dressed states corresponding to the bare states of (a).

$$G_1=\frac{\vec{d}_{12}\cdot\vec{\epsilon}_1}{\hbar},\quad G_2=\frac{\vec{d}_{23}\cdot\vec{\epsilon}_2}{\hbar},\quad G_c=\frac{\vec{d}_{12}\cdot\vec{\epsilon}_c(t)}{\hbar}\ . \tag{2}$$

Here $d_{\alpha,\beta}$ is the dipole matrix element associated with the $\alpha\beta$ transition and $\epsilon_c(t)$ represents the stochastic amplitude of the colored pump. We assume that the colored-pump field can be described as a complex, Gaussian stochastic process with

$$\langle \epsilon_c(t)\rangle=0,\quad \langle G_c^*(t)G_c(t')\rangle=D\Gamma e^{-\Gamma|t-t'|}\ , \tag{3}$$

which implies that the field has a Lorentzian spectral profile with a full width at half maximum of 2Γ. The limit of incoherent pumping is obtained by letting $\Gamma\to\infty$, whence

$$\langle G_c^*(t)G_c(t')\rangle=2D\delta(t-t')\ . \tag{4}$$

We transform to the appropriate frame so as to eliminate the fast optical frequencies. In this frame, H reduces to

$$\frac{H}{\hbar}=(\Delta_1+\Delta_2)|1\rangle\langle 1|+\Delta_2|2\rangle\langle 2| -[g(t)|1\rangle\langle 2|+G_2|2\rangle\langle 3|+\text{H.c.}]\ , \tag{5}$$

where

$$g(t)=G_1+G_c(t)e^{-i(\nu-\omega_1)t} \tag{6a}$$

and

$$\Delta_1=\omega_{12}-\omega_1,\quad \Delta_2=\omega_{23}-\omega_2\ . \tag{6b}$$

The density-matrix equations for this system can be obtained by using (5) and by adding the dissipative terms corresponding to spontaneous decay of the levels $|1\rangle$ and $|2\rangle$. These are given by

$$\dot\rho_{11}=-2\gamma_1\rho_{11}+ig(t)\rho_{21}-ig^*(t)\rho_{12}-2\Lambda(\rho_{11}-\rho_{22})\ , \tag{7a}$$

$$\dot\rho_{12}=-(\gamma_1+\gamma_2+i\Delta_1)\rho_{12}+ig(t)(\rho_{22}-\rho_{11}) -iG_2^*\rho_{13}-2\Lambda\rho_{12}\ , \tag{7b}$$

$$\dot\rho_{13}=-(\gamma_1+i\Delta_1+i\Delta_2)\rho_{13}+ig(t)\rho_{23}-iG_2\rho_{12}-\Lambda\rho_{13}\ , \tag{7c}$$

$$\dot\rho_{22}=2\gamma_1\rho_{11}-2\gamma_2\rho_{22}-ig(t)\rho_{21}+ig^*(t)\rho_{12} +iG_2\rho_{32}-iG_2^*\rho_{23}+2\Lambda(\rho_{11}-\rho_{22})\ , \tag{7d}$$

$$\dot\rho_{23}=-(\gamma_2+i\Delta_2)\rho_{23}+ig^*(t)\rho_{13}+iG_2(\rho_{33}-\rho_{22})-\Lambda\rho_{23}\ , \tag{7e}$$

$$\dot\rho_3=2\gamma_2\rho_{22}-iG_2\rho_{32}+iG_2^*\rho_{23}\ . \tag{7f}$$

We have included a term (with the coefficient Λ) in these equations to account for an incoherent pump instead of a colored pump. If a colored pump only is used, we take $\Lambda=0$. Conversely, if the colored pump laser is replaced by an incoherent pump, $\Lambda\neq 0$ and $g(t)\to G_1$. Note that the above density-matrix equations are really Langevin equations because of the nature of the colored pump. The stochastic noise appears in a multiplicative form. We thus have a set of eight coupled Langevin equations, which we solve using Monte Carlo simulation methods [10]. The gain G on the upper transition is obtained from

$$G=-\text{Im}\left[\frac{\langle\rho_{12}\rangle\gamma_1}{G_1}\right],\quad I\propto\exp\left\{\frac{4\pi n\omega_1|d_{12}|^2}{\hbar c\gamma_1}Gz\right\}\ . \tag{8}$$

Here $\langle\rho_{12}\rangle$ is the ensemble average of the steady-state value of the density-matrix element ρ_{12} over the fluctuations of the colored pump. Note that the factor

$$\frac{4\pi n\omega_1|d_{12}|^2}{\hbar c\gamma_1}$$

gives the on-resonance absorption coefficient for the transition $|1\rangle\leftrightarrow|2\rangle$ if the atom is initially prepared in the $|2\rangle$ state. Clearly the factor G and even its sign will depend on the frequency ν of the colored pump. To guide us in choosing the most efficacious colored-pump frequency, we examine the eigenstates of the Hamiltonian (5) for $g(t)=0$. These eigenstates, indicated by overhead bars and shown in Fig. 1(b), correspond to the semiclassical dressed states. The $|\bar{2}\rangle$ and $|\bar{3}\rangle$ states are linear combinations of the bare atom $|2\rangle$ and $|3\rangle$ states. We will choose ν so as to make the colored pump resonant with the $|\bar{3}\rangle\leftrightarrow|\bar{1}\rangle$ transition, i.e., we set

$$\nu\sim\omega_{12}+\Delta_2+\left[\frac{\Delta_2}{2}-\left[\frac{\Delta_2^2}{4}+G_2^2\right]^{1/2}\right]\ , \tag{9}$$

where we assume Δ_2 positive. A similar argument can be developed for negative values of Δ_2. Since it it predominantly ground-state (i.e., $|3\rangle$) -like, one expects the $|\bar{3}\rangle$ level to be more populated than the $|\bar{2}\rangle$ level. Thus, when the colored pump is tuned into resonance with the $|\bar{3}\rangle\leftrightarrow|\bar{1}\rangle$, we expect, on the basis of a Stokes process, to see amplification of a probe field, provided its frequency is close to the transition $|\bar{2}\rangle\to|\bar{1}\rangle$, i.e., if

$$\omega_1\sim\omega_{12}-\left\{\frac{\Delta_2}{2}+\left[\frac{\Delta_2^2}{4}+G_2^2\right]^{1/2}-\Delta_2\right\}\ . \tag{10}$$

In the numerical results shown in Fig. 2, the frequency of the colored pump is chosen so that (9) is satisfied. All rates in this work, i.e., Γ, D, γ_2, Δ_1, Δ_2, G_1, and G_2, are expressed in units of γ_1. We show some typical gain profiles for $\Gamma=1$ and for different values of the intensity of the color pump [as represented by D in Eq. (3)]. Note that the plotted gain G is expressed [see Eq. (8)] in terms of the weak-field resonant absorptivity on the upper transition of the atom prepared in the bare state $|2\rangle$. We find that for $D=60$, the colored-pump gain assumes a *maximum* value of 0.0122.

We wish to compare the colored-pump gain profiles with one obtained with an incoherent pump of appropriate magnitude instead of a colored pump. If Λ [see Eq. (7)] is chosen to satisfy

$$\Lambda \sim \frac{D\Gamma(\Gamma+\gamma_1+\gamma_2)}{(\Gamma+\gamma_1+\gamma_2)^2+\left\{\Delta_2+\left[\frac{\Delta_2}{2}-\left[\frac{\Delta_2^2}{4}+G_2^2\right]^{1/2}\right]\right\}^2}, \tag{11}$$

the excitation rate of the colored and incoherent pumps will be the same. Note that both D and Λ are in units of γ_1.

In all cases, (a)–(c), shown in Fig. 2, we find a decrease in gain if a colored pump is replaced by an incoherent pump with the same excitation rate as determined from Eq. (11). Note that $D=60$ implies $\Lambda=1.1$ and the gain for a Λ of 1.1 is approximately 0.0037, as shown in Fig. 2(d). The use of a colored pump thus provides a threefold enhancement in gain. For an incoherent pump, we find a *maximum* gain of 0.005 for a Λ of 1.7 (result not shown). Thus, the maximum gain that can be obtained with an incoherent pump is smaller by more than a factor of 2 than the maximum gain that can be obtained with a colored pump. Furthermore, the colored pump achieves its higher maximum gain at a lower optimum excitation rate.

In parts (e) and (f) of Fig. 2, we have calculated the gain produced by a larger bandwidth colored pump ($\Gamma=2$). As expected, the gain produced is comparable to that observed in the first three parts of Fig. 2. The colored pump gain should drop to the incoherent pump value when the effective spectral linewidth of the colored pump becomes so large that differential pumping on the pump and probe transitions of Fig. 1(b) becomes impossible.

We conclude that the use of the colored pump leads to

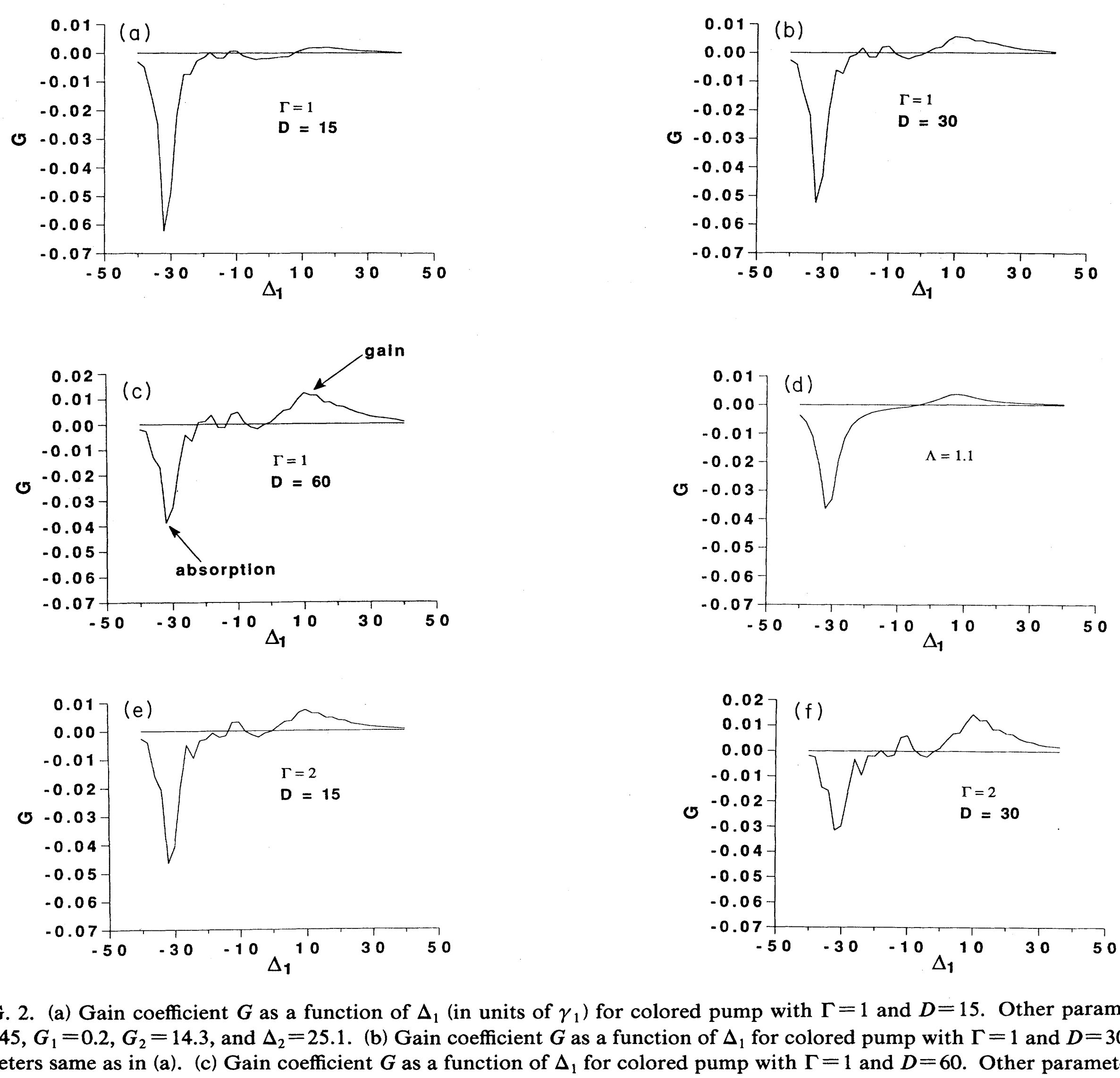

FIG. 2. (a) Gain coefficient G as a function of Δ_1 (in units of γ_1) for colored pump with $\Gamma=1$ and $D=15$. Other parameters are $\gamma_2=5.45$, $G_1=0.2$, $G_2=14.3$, and $\Delta_2=25.1$. (b) Gain coefficient G as a function of Δ_1 for colored pump with $\Gamma=1$ and $D=30$. Other parameters same as in (a). (c) Gain coefficient G as a function of Δ_1 for colored pump with $\Gamma=1$ and $D=60$. Other parameters same as in (a). (d) Gain coefficient G as a function of Δ_1 for incoherent pump with $\Lambda=1.1$ [compare with (c)]. Other parameters same as in (a). (e) Gain coefficient G as a function of Δ_1 for colored pump with $\Gamma=2$ and $D=15$. Other parameters same as in (a). (f) Gain coefficient G as a function of Δ_1 for colored pump with $\Gamma=2$ and $D=30$. Other parameters same as in (a).

substantially larger gain than can be obtained from an incoherent pump under similar conditions. This result is especially important in the context of experimental realizations of inversionless lasing, since available gain levels are quite meager. Our Fig. 1(b) also gives the physical mechanism for the gain involved—it arises from stimulated Raman processes between atomic levels dressed by the strong coherent driving field that interacts with the lower atomic transition. The efficacy of the colored pump in producing gain can be seen to follow from its ability to preferentially transfer population among the dressed levels. The incoherent pump causes population transfer strictly in proportion to transition matrix elements between the levels and steady-state populations that reside in them. One may reasonably expect that many, perhaps all, of the inversionless lasing schemes that have been devised will exhibit substantially higher potential when they are reexamined in the context of a colored population pump.

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