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Hyperfine populations prior to muon capture

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It is shown that the $1S$ level hyperfine populations prior to muon capture will be statistical when either target or beam is unpolarized independent of the atomic level at which the hyperfine interaction becomes appreciable. This assertion holds in the absence of magnetic transitions during the cascade and is true because of minimal polarization after atomic capture and selective feeding during the cascade.

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The muon capture rate, by a nucleus with nonzero spin, depends on the populations of the $1S$ atomic-level hyperfine states f, f_z. The isotropic rate depends only on the total populations of the two hyperfine levels $f^{\pm} = i \pm \frac{1}{2}$, where i is the total angular momentum of the nucleus ($i \neq 0$). In this Rapid Communication, it is shown that these populations are statistical, independent of the atomic level at which the hyperfine interaction becomes appreciable, provided at least one of the target nucleus or the muon beam is unpolarized. This assertion holds in the absence of magnetic transitions during the atomic cascade.

Mukhopadhyay [1] has shown that the populations are statistical when the hyperfine interaction acts only at the $1S$ state and at least one of either the target nucleus or beam is unpolarized. Measurement of residual muon polarizations in light nuclei [2] shows that the hyperfine interaction must be active at atomic states higher than $n = 1$. The question arises as to whether the populations remain statistical when the hyperfine interaction acts before the muon reaches the $1S$ state.

This question is of current interest due to the experimental proposal [3] to measure the statistical capture rate by ^{3}He to a precision of 1% at the Paul Scherrer Institute. A deviation of 2.5% from statisticity in the hyperfine populations would change the measured rate by 1% [4], and so it is important to establish that the populations are indeed statistical.

The population of the atomic states will be described by statistical tensors as were used by Nagamine and Yamazaki [5] and Kuno, Nagamine, and Yamazaki [6] in their study of polarized muonic atoms. The statistical tensor $B_k(j)$, $k = 0, 1 \ldots 2j$ is proportional to the rank k polarization of the level $|\, j\,\rangle$ and is defined below. With this definition, $B_0(j)$ equals the population of the level $|\, j\,\rangle$:

$$B_k(j) = \sqrt{2j+1} \sum_m (-1)^{j-m} P_m \langle k0 \,|\, jjm\, -m \rangle. \tag{1}$$

The population of the state $|\, j, m\,\rangle$ has been written P_m. The conventions for the angular momentum algebra follow Brink and Satchler [7] throughout this Rapid Communication. At atomic capture the atomic orbitals are filled without prejudice to m_l. This corresponds to the direction of motion of the muons being completely random at the moment of their capture. If their motion were not random this would invalidate the final conclusion regarding the populations of the $1S$ levels. Appendix A details an estimate of the angular correlation c between the beam direction and the direction of the muons at capture. It was found that $c \sim 5 \times 10^{-5}$ for the case of hydrogen and is smaller for elements of higher Z. It is therefore a good approximation to fill the atomic orbitals without prejudice to m_l. The smallness of c could be experimentally confirmed by the absence of a $P_2(\cos\theta)$ modulation in the intensity of muonic x rays produced in the cascade.

After some fast internal Auger transitions the spin-orbit interaction splits terms according to j and the statistical tensors for the level n, l, j are

$$B_k(n,l,j) = \left[\frac{(2j+1)^3}{(2l+1)(2s+1)}\right]^{\frac{1}{2}} \times \sum_{k_1,k_2} B_{k_1}(l) B_{k_2}(s) \times \langle k0 \,|\, k_1 k_2 00 \rangle \begin{Bmatrix} l & s & j \\ l & s & j \\ k_1 & k_2 & k \end{Bmatrix}. \tag{2}$$

Only $B_0(l)$ is nonzero, and using the triangular selection rule imposed by the Clebsch-Gordan coefficient we see that only $B_0(n,l,j)$ and $B_1(n,l,j)$ are nonzero. This "minimal polarization" feature is a direct result of the isotropy of the muons just before atomic capture and the fact that muons are spin-$\frac{1}{2}$ particles. $B_1(n,l,j)$ is proportional to P_μ, the polarization of the muon before atomic capture.

There follows an electromagnetic cascade whereby the muonic atom deexcites from $n \approx 14$ to the $1S$ level. The cascade is dominated by charge $E1$ transitions and, in the following, the approximation that other transitions are absent is made. Transitions other than charge $E1$ occur at worst at the $(Z\alpha)^4/72$ level (see Appendix B), and so this approximation is very good.

For electric transitions of multipolarity L, the new statistical tensors after the $n, l, j \to n', l', j'$ transition are

$$B_k(n',l',j') = (2l+1)(2j'+1)\,W(jj'll';L\tfrac{1}{2})^2 \times u_k(jLj')\,B_k(n,l,j), \qquad (3)$$

$$u_k(jLj') = (-1)^{k+L-j-j'}[(2j+1)(2j'+1)]^{\frac{1}{2}} \times W(jjj'j';kL). \qquad (4)$$

Thus, the statistical tensor of rank k is fed only by the rank k statistical tensors for higher levels. This feature may be termed "selective feeding" since the tensor B_k feeds other tensors according to the selection rule $\Delta k = 0$. It follows that B_0 and B_1 will be the only nonzero statistical tensors during the cascade.

At the level where the hyperfine splitting becomes larger than the natural width, the statistical tensors for the states $|\,n,l,j,f\,\rangle$ are

$$B_k(n,l,j,f) = \left[\frac{(2f+1)^3}{(2i+1)(2j+1)}\right]^{\frac{1}{2}} \times \sum_{k_1,k_2} B_{k_1}(i) B_{k_2}(n,l,j) \times \langle k0\,|\,k_1k_2 00\,\rangle \begin{Bmatrix} i & j & f \\ i & j & f \\ k_1 & k_2 & k \end{Bmatrix}, \qquad (5)$$

where $B_{k_1}(i)$ is the statistical tensor for the nucleus. By observing the selection rule for k, k_1, and k_2, the total populations for the hyperfine states must have the following form:

$$B_0(n,l,j,f) = \alpha + \beta P_\mu P_i, \qquad (6)$$

where α and β are constants peculiar to the level $|\,n,l,j,f\,\rangle$ and P_i is the vector polarization of the nucleus. Higher rank polarizations of the nucleus cannot contribute to $B_0(n,l,j,f)$, since the rank of the total angular momentum polarization is no higher than 1.

Allowing the hyperfine levels to decay only via charge electric transitions, we have for the transition $n,l,j,f \rightarrow n',l',j',f'$,

$$B_k(n',l',j',f') = (2l+1)(2j'+1)\,W(jj'll';L\tfrac{1}{2})^2 \times (2j+1)(2f'+1) W(jj'll';L\tfrac{1}{2})^2 \times u_k(fLf')\,B_k(n,l,j,f), \qquad (7)$$

which has the same "selective feeding" property as Eq. (3). Using this fact and Eq. (6) the $1S$ hyperfine populations can be parametrized by

$$B_0(1,0,\tfrac{1}{2},f) = \gamma + \delta P_\mu P_i. \qquad (8)$$

The parameter γ may be found either by demanding that the populations be statistical when $P_\mu = P_i = 0$ or by directly calculating it using Eqs. (2), (3), (5), and (7):

$$\gamma = \frac{2f+1}{2(2i+1)}. \qquad (9)$$

Thus, if either $P_\mu = 0$ or $P_i = 0$, the total populations of the $1S$ hyperfine levels are statistical in the limit that no magnetic transitions occur during the cascade. This is due to minimal polarization after atomic capture and selective feeding during the cascade.

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APPENDIX A: ANGULAR CORRELATION OF THE MUON AT CAPTURE

The angular correlation c between the direction of the muon at kinetic energy $T < T_0$ and the direction of the beam is given in Mann and Rose [8] by

$$c = \left(\frac{T}{T_0}\right)^p, \qquad (A1)$$

where T_0 is the energy below which the number of ionizing collisions becomes small compared to the number of elastic scattering collisions with nuclei. The exponent p is half the muon-nucleus total mass divided by their reduced mass, $p = (M + m_\mu)^2/(2m_\mu M)$.

The number of ionizing collisions becomes small compared to the number of elastic scattering collisions when the speed of the muon becomes less than the speed of the electrons orbiting the nucleus. For hydrogen, this implies $T_0 = 3$ keV.

The pertinent value of T is that energy where muons begin to be captured via an Auger process. Haff and Tombrello [9] have calculated the atomic capture rates of muons by hydrogen, helium, and lithium, and their Fig. 9 indicates $T < 500$ eV for hydrogen.

The exponent p for hydrogen is 5.5, and so Eq. (A1) implies $c < 5 \times 10^{-5}$. For elements other than hydrogen one expects c to be yet smaller since the exponent p is roughly proportional to Z and T/T_0 is roughly independent of Z, both T and T_0 growing as Z.

APPENDIX B: NEGLECT OF MAGNETIC TRANSITIONS

The theory of the cascade used in this Rapid Communication assumes that only charge electric multipoles contribute to the decay amplitude. In this appendix, the quality of this approximation is addressed by calculating the probability that transitions due to other multipoles occur.

In general, spin electric, charge magnetic, and spin magnetic multipoles also contribute to the decay amplitude. The largest multipole is $L = 1$, because the wavelengths of the emitted photons are large compared to the spatial dimensions of the muonic atom ($kr \sim Z\alpha$). Therefore one need only consider the $E1'$, $M1$, and $M1'$ multipoles where $E1'$, $M1$, and $M1'$ stand for spin electric dipole, charge magnetic dipole, and spin magnetic dipole, respectively.

The $M1 + M1'$ amplitude obeys the selection rules

$\Delta n = \Delta l = 0$, and so only causes transitions between levels split by fine structure. The small photon energy in these transitions makes their likelihood very small, and the largest ratio of $M1 + M1'$ to $E1$ transition probability is when the muonic atom is in the $2p$ state. The ratio of the $2p_{3/2} \to 2p_{1/2}$ to $2p_{3/2} \to 1s_{1/2}$ transition probability was calculated using the results given in Brink and Satchler [7], p. 95 for the reduced matrix elements of $M1 + M1'$, and was found to be $(Z\alpha)^8/6912$.

The $E1'$ multipole accompanies $E1$ in every transition for which electric dipole radiation is allowed. Its amplitude is a factor ω/m_μ smaller than the $E1$ amplitude where ω is the energy of the photon emitted and m_μ is the mass of the muon. However, the $E1$-$E1'$ cross term cancels to zero in every case, and so the contribution of $E1'$ to the decay rate is suppressed by $(\omega/m_\mu)^2$. The $E1'^2$ transition probability is largest for the $\infty p \to 1s$ transition since then ω takes its maximum value. The ratio of the $E1'^2$ to $E1^2$ transition probability is $(Z\alpha)^4/72$ in this case. In fact, most atoms decay via the $2p$ state in which case the above ratio is $(Z\alpha)^4/128$.

From the above arguments, it can be concluded that all multipoles except for charge electric may be neglected in the calculation of the cascade at better than the $(Z\alpha)^4/72$ level. For hydrogen this is 4×10^{-11}.

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