Three-dimensional relativistic model of a bound particle in an intense laser pulse. II

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Using an exactly solvable model of a bound Klein-Gordon particle in an intense laser field, we obtain the above-threshold energy spectra of the ejected electron for intensities in the range $I = 7\times 10^{14}$ W/cm^2 to 10^{20} W/cm^2, and compare them with the predictions of the Schrödinger theory. The present calculations also allow us to test an approximation for relativistic intensities, which should be useful for real systems.

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Current developments [1] in high-power lasers promise to provide laser intensities beyond 10^{18} W/cm^2 for which the oscillation energy of a free electron in the electromagnetic field (the so-called quiver energy) could approach or exceed the rest mass energy mc^2 of the free electron. In this condition (in spite of the fact that the laser photon energy is nonrelativistic) laser-atom interactions need to be analyzed relativistically and nonperturbatively. Investigations of exactly solvable model problems, in this context, are of special interest in view of the possibility of gaining useful qualitative understanding of physical processes such as ionization of atoms or detachment of negative ions or possible photodisintegration of nuclei, in relativistically intense laser fields. Furthermore, they can help test new approximation methods which might be needed to tackle a real system. In a recent paper [2] (to be referred to below as I) we have introduced and solved a model of a charged Klein-Gordon particle bound in a separable potential and simultaneously interacting with a circularly polarized plane-wave electromagnetic field (of arbitrary frequency and intensity). The model studied has one discrete state and the full continuum spectrum and therefore may be applied to quantum systems which exhibit effectively one bound state, e.g., a negative ion or the deuteron nucleus. The purpose of this article is to present quantitative results for the above-threshold energy spectra of the ejected electron in the relativistic region of intensities and compare them with the prediction of the corresponding Schrödinger theory.

The three-dimensional relativistic model system of interest is defined by the Klein-Gordon (KG) equation

$$\{(i\partial_t - \hat{V})^2 - [\hat{\mathbf{p}} - e\mathbf{A}(\mathbf{x},t)]^2 - m^2\}\Psi(t) = 0 \;. \tag{1}$$

In natural units $c = 1$, $\hbar = 1$. The short-range binding potential is chosen in the separable form

$$\hat{V} = V_0|\tilde{\phi}\rangle\langle\tilde{\phi}| \;, \quad \langle\tilde{\phi}|\tilde{\phi}\rangle = 1 \;, \tag{2}$$

with

$$\tilde{\phi}(\mathbf{x}) = N_0\frac{1}{x}e^{-\lambda x} \;, \quad N_0 = \sqrt{\frac{\lambda}{2\pi}} \;, \tag{3}$$

and

$$\mathbf{A}(x) = A_0[\mathbf{e}_1\cos(k\cdot x+\delta) - \mathbf{e}_2\sin(k\cdot x+\delta)] \tag{4}$$

is assumed to be a circularly polarized vector potential with peak amplitude A_0, frequency ω, and unit polarization vectors $\mathbf{e}_1$ and $\mathbf{e}_2$. Note that in the relativistic domain the usual dipole approximation is inapplicable and hence the full wave-vector dependence is retained in the present theory. It is shown in I that in the absence of the field, $\mathbf{A} = \mathbf{0}$, Eq. (1) supports a single bound state with positive binding energy

$$E_0 = \sqrt{m^2 - \frac{3}{16}V_0^2} + \frac{1}{4}V_0 \;, \tag{5}$$

for $V_0 < 0$ and $\frac{3}{16}V_0^2 < m^2$, so that the arbitrary potential parameter V_0 can be chosen to reproduce the actual binding energy of the system of interest. The wave function of the associated bound state is

$$\begin{aligned}\Psi(\mathbf{x},t) &= e^{-iE_0t}\Psi_{E_0}(\mathbf{x})\\ &= \frac{(m^2-E_0^2)^{3/4}}{\sqrt{2\pi E_0}}\exp\left[-\sqrt{m^2-E_0^2}\,|\mathbf{x}|\right]e^{-iE_0t} \;.\end{aligned} \tag{6}$$

The exact solution of Eq. (1) in the presence of the field (4) is, as may be expected, much more involved. We have given this solution in I and used it to derive the above-threshold energy spectrum of the ejected electron analytically. We shall not repeat the derivation here, but quote the final expressions which are used to carry out the present numerical investigation of the relativistic spectra. These spectra define the probability distributions of the energy of the ejected electron in the continuum states which are given by the relativistic KG Volkov states

$$\begin{aligned}\Psi^{(0)}_{\epsilon,\mathbf{p}}(\mathbf{x},t) = \sum_{n=-\infty}^{+\infty} & e^{in(k\cdot x+\delta)}J_n(\alpha_0^p|\mathbf{p}|\sin\theta_p)\\ &\times e^{in\phi_p}e^{-i\epsilon(\mathbf{p})t+i\mathbf{p}\cdot\mathbf{x}} \;,\end{aligned} \tag{7}$$

where

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$$\epsilon = \epsilon(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m^2 + e^2 A_0^2} \tag{8}$$

and

$$\alpha_0^p = \frac{eA_0}{k \cdot p} \, . \tag{9}$$

This definition, therefore, corresponds to the so-called *intrinsic* spectrum [3], in the presence of the field. The appropriate amplitude is given by the following analytic expressions:

$$A(p) = \sum_{N=-\infty}^{+\infty} e^{iN\delta} A^{(N)}(p) \, , \tag{10}$$

with

$$A^{(N)}(p) = J_{-N}(\alpha_0^p |\mathbf{p}| \sin\theta_p) e^{iN\phi_p} \left[(E_0 + p_0 + Nk_0)\breve{\Phi}_i(\mathbf{p} + N\mathbf{k}) - \frac{V_0}{E_0^{1/2}} \breve{\phi}(\mathbf{p} + N\mathbf{k}) \right]$$
$$+ V_0[2(P_0 + Nk_0) - V_0] J_{-N}(\alpha_0^p |\mathbf{p}| \sin\theta_p) e^{iN\phi_p} \breve{\phi}(\mathbf{p} + N\mathbf{k}) [W_0(p_0 + Nk_0)]^{-1} C^{(N)}(p_0) \tag{11}$$

and

$$C^{(N)}(p_0) = \sum_{M=-\infty}^{+\infty} \int \frac{d^3q}{2\pi^3} \frac{\breve{\phi}(\mathbf{q} + N\mathbf{k}) J_{-M}(\alpha_0^{p_0,\mathbf{q}} |\mathbf{q}| \sin\theta_q) \left[(E + p_0 + Nk_0)\breve{\Phi}_i(\mathbf{q} + N\mathbf{k}) - \frac{V_0}{E_0^{1/2}} \breve{\phi}(\mathbf{q} + N\mathbf{k}) \right]}{\{[p_0 - (M - N)k_0]^2 - [\mathbf{q} - (M - N)\mathbf{k}]^2 - m^2 - e^2 A_0^2 + i\varepsilon\}} \, , \tag{12}$$

where

$$W_n(E) = 1 - V_0[2(E - nk_0) - V_0] \sum_{N=-\infty}^{+\infty} \int_0^\infty \frac{dpp^2}{4\pi^2} \int_0^\pi d\theta_p \sin\theta_p \frac{\breve{\phi}^2(\mathbf{p} - n\mathbf{k}) J_{n-N}^2(\alpha_0^{E,\mathbf{p}} |\mathbf{p}| \sin\theta_p)}{(E - Nk_0)^2 - (\mathbf{p} - N\mathbf{k})^2 - m^2 - e^2 A_0^2 + i\varepsilon} \, . \tag{13}$$

In the present case the Fourier transform of the initial state (6) is

$$\breve{\Phi}_i(\mathbf{p}) = \frac{8\pi\lambda^2 N_0}{\sqrt{E_0}} \frac{1}{[\mathbf{p}^2 + \lambda^2]^2} \tag{14}$$

and that of the potential function (3) is

$$\breve{\phi}(\mathbf{p}) = \frac{4\pi N_0}{\mathbf{p}^2 + \lambda^2} \, . \tag{15}$$

The magnitude of the final momentum $|\mathbf{p}|$ is given in terms of the kinetic energy of the ejected particle in the presence of the field

$$|\mathbf{p}| = \left[\varepsilon_{\text{kin}}^2 + 2\varepsilon_{\text{kin}} \sqrt{m^2 + e^2 A_0^2} \right]^{1/2} \, , \tag{16}$$

where

$$\varepsilon_{\text{kin}} = p_0 - \sqrt{m^2 + e^2 A_0^2} \, . \tag{17}$$

Before presenting the quantitative results we note that a proper comparison of the theoretically calculated *intrinsic* spectra with laboratory results, in general, will require considerations regarding the effective duration of the laser pulse. For very long pulses the ejected electrons in the field can be accelerated by the so-called ponderomotive force arising from the macroscopic field gradient of the laser beam [4]. The associated *extrinsic* spectra would be accordingly shifted greatly to higher energies compared to the *intrinsic* spectra. The latter may be referred directly to the experimental spectra only for pulses which are short enough with respect to the electron escape time through the spatial profile of the laser field (yet long enough compared to the period of the central frequency). For example, if we assume that a typical extension of the laser beam, e.g., the beam waist is about 0.1 mm and if the typical energy associated with the drift motion of the electron in the field is about 10 eV, the observed spectrum may then be treated as the intrinsic spectrum for pulse durations in the range

$$10^{-16} \text{ s} \ll \tau_p \ll 5 \times 10^{-11} \text{ s} \, . \tag{18}$$

This range covers most, if not all, of the super intense lasers currently available. We emphasize that in the case of longer pulses or greater drift velocity of the electron these spectra will be shifted far into greater kinetic energies as a result of the conversion of the large amount of quiver energy into drift energy through the mechanism of ponderomotive acceleration.

In order to be able to compare with the available nonrelativistic results for the same system, we have chosen, for the numerical computations, the nominal binding energy of the model equal to that of the H atom, $\varepsilon_0 = 13.6$ eV and the laser frequency $\hbar\omega = 6.419$ eV (the frequency of the ArF laser).

In Fig. 1 we present the calculated (intrinsic) spectrum of the ejected electron for the intensity $I = 0.7 \times 10^{15}$ W/cm^2. In this case we find the usual "above-threshold-ionization"- (ATI) like peaks obtained earlier from the solution of the corresponding Schrödinger problem [3, 5]. The individual peaks can be characterized by the net number of photons absorbed during the ejection process; in this case the successive peaks are $-N = 3$, 4, 5, and 6. No essential differences are observed in this case whether the problem is solved relativistically or nonrela-

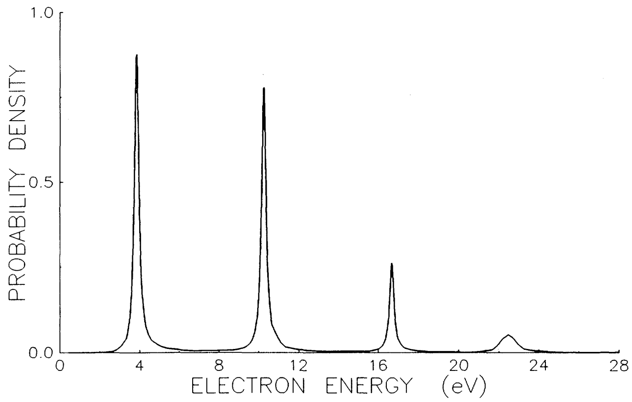

FIG. 1. Above-threshold electron ejection spectrum for a nonrelativistic intensity $I = 0.7 \times 10^{15}$ W/cm^2 and for $\omega = 6.419$ eV (ArF laser). Note the typical individual ATI peaks as in nonrelativistic model [3] with definite number of photon absorption: $-N = 3, 4, 5, 6$.

tivistically [6]. This is easily understood by considering the ratio of the quiver energy to the rest mass energy of the electron, which is given for the present laser frequency, as

$$\frac{E_{\text{quiver}}}{mc^2} \approx 10^{-20} I \,, \tag{19}$$

where I is in W/cm^2. This ratio is a *measure of relativity* of our problem and suggests that significant relativistic corrections would come into play for intensities sufficiently above 10^{18} W/cm^2. Before we present the spectra for such high intensities, we mention parenthetically that the present relativistic model also predicted the other features of electron ejection spectra, e.g., the so-called "peak reversal" for $I = 10^{15}$ W/cm^2 and the "peak suppression" for $I = 1.5 \times 10^{15}$ W/cm^2, found in the nonrelativistic model [3, 5].

For the very high intensities in the range 10^{18} W/cm^2–10^{20} W/cm^2 we found that hundreds and even thousands of terms of the summation in (10) contribute to the spectra in an essential way. This is in strong contrast with respect to the calculations for intensities below 10^{16} W/cm^2, where only a few terms of the summation (10) were found to dominate. Nevertheless, a very significant simplification occurs in the domain of relativistic intensities which considerably facilitates the use of the relativistic formula (11). Thus, we observed that with increasing intensity the contribution of the second term in (11) (which is responsible for the very long computational time) becomes a small fraction of the first term. This circumstance may be understood by noting that the ratio of the quiver energy to the binding energy is

$$\frac{E_{\text{quiver}}}{E_0} \approx 3.8 \times 10^{-16} I \,, \tag{20}$$

where the intensity I is given in W/cm^2. Thus, for intensities of order 10^{18} W/cm^2 or more, the binding energy of the electron in the potential is negligibly small compared to its vibrational energy due to the field. The effect of the binding potential, therefore, enters in consideration primarily through the preparation of the initial state for $t = 0$ and not after the field has started to interact with the electron. This would suggest that all additional effects due to small binding potential, during the evolution in the relativistically intense field, would be likely to be small in comparison with the effect of the field itself.

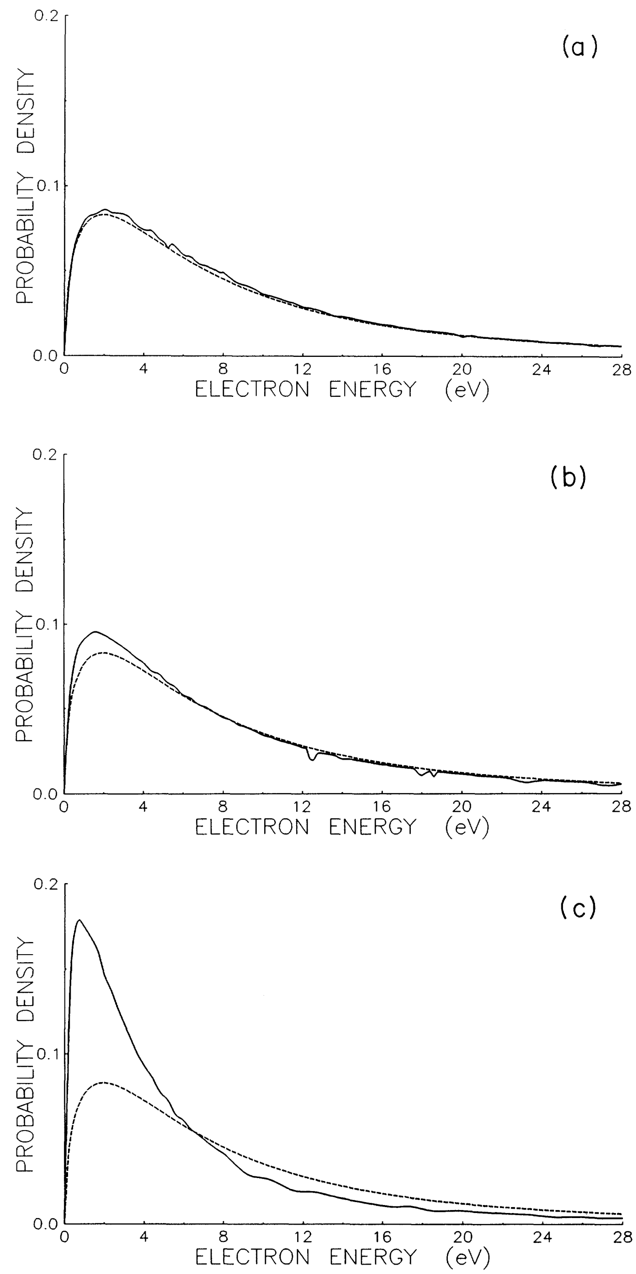

FIG. 2. Above-threshold electron ejection spectra at relativistic intensities: (a) $I = 10^{18}$ W/cm^2, (b) $I = 10^{19}$ W/cm^2, (c) $I = 10^{20}$ W/cm^2. The dashed curve is the prediction of the non-relativistic model for asymptotically large intensities, which is intensity independent. Note the disappearance of prominent individual peaks similar to those in Fig. 1 and bunching of electrons near the threshold, with increasing intensity.

Mathematically, the second term in (11) corresponds exactly to the additional effect of the potential during the interaction with the field, while the first term corresponds to the evolution of the initial wave packet under the influence of the field only. The numerical smallness of the second term compared to the first term at relativistic intensities thus confirms what may be expected intuitively. The approximation of neglecting the contribution of the second term (which also requires long computational time on the computer) has been tested and confirmed by us by calculating upper bounds of the second term (using upper bounds of the Bessel functions appearing in the formula). The results presented in Fig. 2, for (a) $I = 10^{18}$ W/cm^2, (b) $I = 10^{19}$ W/cm^2, and (c) $I = 10^{20}$ W/cm^2, were obtained using the same approximation. The errors at these intensities were found to be within a few percent at the main part of the spectra. This finding from the exactly soluble model strongly suggests that for relativistic intensities electron ejection probabilities for real systems may also be obtained within similar accuracy using the same approximation, namely the binding potential is required for the initial wave packet and the subsequent evolution is governed by the relativistic Volkov propagator [7] only.

In Fig. 2 the dashed curve corresponds to the result of the Schrödinger theory for asymptotically large intensities. It is found that in this limit the Schrödinger theory leads to the following analytical spectrum independent of the intensity [8]:

$$w(\varepsilon_{\mathrm{kin}}) = \frac{32\lambda^5 m\sqrt{2m\varepsilon_{\mathrm{kin}}}}{\pi(2m\varepsilon_{\mathrm{kin}} + \lambda^2)^4}\,. \tag{21}$$

It will be seen that at $I = 10^{18}$ W/cm^2 (curve a) the departure of the relativistic distribution is marginal compared to the nonrelativistic limit. Significant departure from the nonrelativistic limit appears at $I = 10^{19}$ W/cm^2 (curve b) and it becomes rather large at $I = 10^{20}$ W/cm^2 (curve c). In the last case, it is seen from (19) that the quiver energy essentially equals the rest mass energy of the electron. It is seen clearly from the curves in Fig. 2 that in the relativistic intensity domain the above-threshold energy spectra lose all distinctions of individual peaks found at lower intensities (cf. Fig. 1) and they become continuous humps. This is predicted both by the nonrelativistic theory and by the relativistic theory. In the relativistic theory, however, the emitted electrons tend to bunch together more strongly near the threshold. Note that the latter is determined by the threshold of ejection inside the field and is therefore greatly shifted from the unperturbed threshold by the mean quiver energy. We may point out, finally, that the drift energy of the electron, on ejection into the field, remains quite small in these cases and hence our estimate (18) of the pulse durations relevant for the intrinsic spectra remains self-consistent.

In conclusion, we have investigated above-threshold electron ejection spectra using an exactly soluble relativistic model of a bound KG particle interacting with an intense circularly polarized laser field. The results obtained for intensities below 10^{16} W/cm^2 show the same features of individual ATI peaks as obtained earlier from the corresponding nonrelativistic theory. For intensities in the range between 10^{18} and 10^{20} W/cm^2 the individual peaks disappear completely and the spectra become broad humps with the maximum near the threshold energy. The nonrelativistic theory predicts an intensity independent distribution for asymptotically large intensities while the relativistic theory predicts distributions with increasingly more pronounced maximum, which also moves closer to the threshold, with increasing intensity. Finally, it is found that in the relativistic domain of intensity the binding potential essentially determines the initial wave packet and the subsequent evolution is dominated by the relativistic Volkov propagator only.

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