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Generation of pure states in a two-photon micromaser: Effects of finite detuning and cavity losses

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In this paper, we study the generation of pure states in a two-photon micromaser with finite detuning of the intermediate atomic level. This study was done via the temporal evolution of the discrete master equation. We show that it is possible to generate the ideal squeezed vacuum for a broad range of detunings in a lossless cavity and away from the trapping condition. We also consider the effect of cavity losses.

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I. INTRODUCTION

Recently, pure states have been found for various micromaser systems, such as the one-photon [1,2] and two-photon [3,4] micromaser as well as in the λ system [5], under idealized conditions such as having a lossless cavity, infinite atomic lifetimes, and a large detuning of the intermediate atomic level, in the case of the two-photon micromaser [3]. Some of these pure states present a strong quadrature noise reduction [2,6]. Nonideal effects have also been studied in the past in a one-photon micromaser [7,8]. Examples of the above-mentioned states are the *tangent* and *cotangent* states for the one-photon micromaser and the *even* and *odd* states for the two-photon micromaser. The even and odd states [3] for the two-photon micromaser were derived under the condition of high detuning of the intermediate atomic level (three-level system) with respect to the midpoint between the upper and lower levels and for special values of the interaction time (trapping condition). In addition, the initial state of the field, say for an even state, has to be a coherent superposition of even number states within the given trap. Alternatively, it could also be an incoherent field with nonzero density matrix elements corresponding only to even indices, again within the trap. These initial conditions are, in general, difficult to realize experimentally, with the exception of the vacuum state, which we use here. In this work, we will study how the even states are modified by the fact that the detuning is finite and also by the effect of cavity losses. We further find in this work that pure states can be generated in the two-photon micromaser without satisfying the trapping conditions. These states turn out to be the perfectly squeezed vacuum [9].

This paper is organized as follows: In Sec. II we obtain the pure states for the field, for an arbitrary finite detuning, in a lossless cavity. In Sec. III we derive the discrete master equation for the reduced field density matrix. This equation enables us to verify dynamically the existence of those pure states. In Sec. IV we include the cavity losses in the temporal evolution and compare them with the previous results. Finally, Sec. V is a summary and conclusion.

II. TRAPPING STATES FOR ARBITRARY DETUNING

We consider the three-level atom, shown in Fig. 1, interacting with a single mode of the electromagnetic field in a lossless microwave cavity. The well-known temporal evolution operator for this system [10] is given by

$$U(\tau)=\begin{bmatrix} 1-a\hat{G}a^{\dagger} & -ia\hat{H}e^{-i\epsilon\phi} & -a\hat{G}a \\ -ie^{i\epsilon\phi}\hat{H}a^{\dagger} & e^{i\epsilon\phi}\hat{L}^{\dagger} & -ie^{i\epsilon\phi}\hat{H}a \\ -a^{\dagger}\hat{G}a^{\dagger} & -ia^{\dagger}\hat{H}e^{-i\epsilon\phi} & 1-a^{\dagger}\hat{G}a \end{bmatrix}, \quad (1)$$

where we have defined

$$\hat{H}=\frac{\sin(\phi\sqrt{2a^{\dagger}a+1+\epsilon^2})}{\sqrt{2a^{\dagger}a+1+\epsilon^2}}, \quad (2)$$

$$\hat{L}=\cos(\phi\sqrt{2a^{\dagger}a+1+\epsilon^2})-i\epsilon\hat{H}, \quad (3)$$

and

$$\hat{G}=\frac{1-e^{-i\epsilon\phi}\hat{L}}{2a^{\dagger}a+1} \quad (4)$$

with $\epsilon=\Delta/2g$ and $\phi=g\tau$, Δ being the detuning (Fig. 1), g the coupling constant, and τ the interaction time between the atoms and the cavity mode.

Here we assume that all the atoms enter the cavity in the same coherent superposition:

$$|\Phi\rangle=\alpha|a\rangle+\gamma|c\rangle. \quad (5)$$

The evolution of a pure state of the atom-field system is

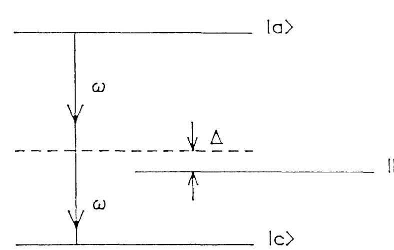

FIG. 1. Energy levels relevant to the two-photon micromaser.

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given by

$$\sum_n S_n|n\rangle(\alpha|a\rangle+\gamma|c\rangle)\rightarrow\sum_n S_n\left[\left\{\alpha\left[1-\frac{n+1}{2n+3}R(n+1)\right]|n\rangle-\gamma\frac{\sqrt{n(n-1)}}{2n-1}R(n-1)|n-2\rangle\right\}|a\rangle\right.$$
$$-ie^{i\epsilon\phi}\left\{\alpha\left[\frac{n+1}{2n+3+\epsilon^2}\right]^{1/2}\sin(\phi\sqrt{2n+3+\epsilon^2})|n+1\rangle\right.$$
$$\left.+\gamma\left[\frac{n}{2n-1+\epsilon^2}\right]^{1/2}\sin(\phi\sqrt{2n-1+\epsilon^2})|n-1\rangle\right\}|b\rangle$$
$$+\left\{-\alpha\frac{\sqrt{(n+1)(n+2)}}{2n+3}R(n+1)|n+2\rangle\right.$$
$$\left.\left.+\gamma\left[1-\frac{n}{2n-1}R(n-1)\right]|n\rangle\right\}|c\rangle\right], \tag{6}$$

where we have defined

$$R(n)=1-e^{-i\epsilon\phi}\left[\cos(\phi\sqrt{2n+1+\epsilon^2})+i\epsilon\frac{\sin(\phi\sqrt{2n+1+\epsilon^2})}{\sqrt{2n+1+\epsilon^2}}\right]. \tag{7}$$

From Eq. (6) we see that the upward trapping conditions are

$$\phi\sqrt{2n_u+3+\epsilon^2}=2p\pi , \tag{8a}$$

where p is an integer, and

$$\epsilon\phi=2r\pi , \tag{8b}$$

where r is an integer, which remove the transition from the $|n_u\rangle$ state to the $|n_u+2\rangle$ and $|n_u+1\rangle$ states.

The following additional condition is necessary in order to remove the two-photon transition from $|n_u-1\rangle$ to $|n_u+1\rangle$:

$$\phi\sqrt{2n_u+1+\epsilon^2}=2q\pi , \tag{8c}$$

where q is an integer. Conditions (8a) and (8c) are inconsistent because there is no ϕ such that will satisfy both conditions.

For $n_u\gg1$ or $\epsilon\gg1$ the conditions (8a) and (8c) are approximately the same. When $\epsilon\gg1$ all the one-photon transitions are removed, and we recover the two-photon case [3]. There is a third case with no trapping condition that will be considered next. Let us assume that the field reached a pure steady state. If one additional atom crosses the cavity, it only modifies the state of the system by a global phase factor and in general yields a different atomic state, that is,

$$\sum_{n=0} S_n|n\rangle(\alpha|a\rangle+\gamma|c\rangle)\rightarrow\sum_{n=0} S_n|n\rangle(\alpha'|a\rangle+\beta'|b\rangle+\gamma'|c\rangle) . \tag{9}$$

Comparing the right-hand side of Eqs. (6) and (9), we obtain

$$\alpha'S_n=\alpha\left\{1-\frac{n+1}{2n+3}R(n+1)\right\}S_n-\gamma\frac{\sqrt{(n+1)(n+2)}}{2n+3}R(n+1)S_{n+2} , \tag{10a}$$

$$\beta'S_n=-ie^{i\epsilon\phi}\frac{\sin(\phi\sqrt{2n+1+\epsilon^2})}{\sqrt{2n+1+\epsilon^2}}\times\{\alpha\sqrt{n}S_{n-1}+\gamma\sqrt{n+1}S_{n+1}\} , \tag{10b}$$

and

$$\gamma'S_n=-\alpha\frac{\sqrt{n(n-1)}}{2n-1}R(n-1)S_{n-2}+\gamma\left\{1-\frac{n}{2n-1}R(n-1)\right\}S_n . \tag{10c}$$

For these equations to be consistent it is necessary that

$$\alpha'=\alpha , \quad \beta'=0 , \quad \gamma'=\gamma . \tag{11}$$

With the conditions given above, Eqs. (10) become a single equation

$$S_{n+2}=-\frac{\alpha}{\gamma}\left[\frac{n+1}{n+2}\right]^{1/2}S_n . \tag{12}$$

This is the well-known recursion relation for the even and odd states found in the two-photon micromaser model [3].

It is interesting to set $n=0$ in Eqs. (10). Equations (10a) and (10c) are consistent with Eq. (11); however, from Eq. (10b) we get $S_1=0$. Therefore, in the case of finite detuning the steady state of the field contains only a superposition of even photon-number states since all the odd components vanish. Hence, when we have no trapping condition, the solution for the recursion relation (12) gives us the squeezed vacuum state (see the Appendix). Comparing the recursion relation (12) with Eq. (A3) we find that the squeezing parameter is given by

$$\xi = e^{i\theta_{at}} \ln \left[\left(\frac{1+\left|\frac{\alpha}{\gamma}\right|}{1-\left|\frac{\alpha}{\gamma}\right|} \right)^{1/2} \right] , \tag{13}$$

where θ_{at} is the relative atomic phase between α and γ. For $\theta_{at}=\pi$ we get noise reduction in the a_2 quadrature [9]. The fluctuations of a_2 are given by (A4b),

$$\langle (\Delta a_2)^2 \rangle = \frac{1}{4} \left\{ \frac{1-\left|\frac{\alpha}{\gamma}\right|}{1+\left|\frac{\alpha}{\gamma}\right|} \right\} . \tag{14}$$

From this last equation we see that in the neighborhood of $|\alpha/\gamma|=1$, slightly to the left, the quantum noise can be made nearly zero.

A valid question is, how can we generate one of these states? This question is answered in the following section.

III. GENERATION OF PURE STATES VIA THE TEMPORAL EVOLUTION

In this section we consider the dynamic behavior of the reduced density matrix of the field including arbitrary detuning. We have a special interest in the steady state reached by the cavity field. We calculate the reduced field density matrix by using the time evolution operator given by Eq. (1), after the $(k+1)$th atom has crossed the cavity, that is,

$$\rho^{k+1} = \mathrm{Tr}_{at}[U(\tau)\rho_{at}\rho^k U^\dagger(\tau)] , \tag{15}$$

where ρ_{at} is the initial atomic density matrix and ρ^k is the reduced density matrix of the field after interacting with the kth atom. Using the photon-number basis, after a straightforward calculation, we obtain the following discrete master equation:

$$\begin{aligned}
\rho^{k+1}_{n,m} = & \left\{ |\alpha|^2 \left[1-\frac{n+1}{2n+3}R(n+1) \right] \left[1-\frac{m+1}{2m+3}R^*(m+1) \right] \right. \\
& \left. +|\gamma|^2 \left[1-\frac{n}{2n-1}R(n-1) \right] \left[1-\frac{m}{2m-1}R^*(m-1) \right] \right\} \rho^k_{n,m} \\
& +|\alpha|^2 \frac{\sqrt{n(n-1)m(m-1)}}{(2n-1)(2m-1)} R(n-1)R^*(m-1)\rho^k_{n-2,m-2} \\
& +|\gamma|^2 \frac{\sqrt{(n+1)(n+2)(m+1)(m+2)}}{(2n+3)(2m+3)} R(n+1)R^*(m+1)\rho^k_{n+2,m+2} \\
& -\gamma\alpha^* \left[1-\frac{m+1}{2m+3}R^*(m+1) \right] \frac{\sqrt{(n+1)(n+2)}}{2n+3} R(n+1)\rho^k_{n+2,m} \\
& -\gamma\alpha^* \left[1-\frac{n}{2n-1}R(n-1) \right] \frac{\sqrt{m(m-1)}}{2m-1} R^*(m-1)\rho^k_{n,m-2} \\
& -\alpha\gamma^* \left[1-\frac{n+1}{2n+3}R(n+1) \right] \frac{\sqrt{(m+1)(m+2)}}{2m+3} R^*(m+1)\rho^k_{n,m+2} \\
& -\alpha\gamma^* \left[1-\frac{m}{2m-1}R^*(m-1) \right] \frac{\sqrt{n(n-1)}}{2n-1} R(n-1)\rho^k_{n-2,m} \\
& +\frac{\sin(\phi\sqrt{2n+1+\epsilon^2})\sin(\phi\sqrt{2m+1+\epsilon^2})}{\sqrt{(2n+1+\epsilon^2)(2m+1+\epsilon^2)}} \\
& \times [|\alpha|^2\sqrt{nm}\,\rho^k_{n-1,m-1} + |\gamma|^2\sqrt{(n+1)(m+1)}\rho^k_{n+1,m+1} \\
& \quad +\gamma\alpha^*\sqrt{(n+1)m}\,\rho^k_{n+1,m-1} + \alpha\gamma^*\sqrt{n(m+1)}\rho^k_{n-1,m+1}] .
\end{aligned} \tag{16}$$

The last four terms account for the one-photon transitions and they depend on a factor $1/(2n+1+\epsilon^2)<1$. These terms are very small in the regime $|\epsilon| \gg 1$.

In the following, we present and discuss some numerical calculations of the temporal evolution of this discrete master equation.

Figure 2(a) shows the quadrature fluctuations $\langle (\Delta a_2)^2 \rangle$ for three times, as a function of ϵ, the detuning parameter, in logarithmic scale. The initial condition is the vacuum state, $\alpha/\gamma = -\sqrt{0.7}$ and the reduced interaction time is $\phi = 4\pi\epsilon/23$, which corresponds to $n_u = 10$ for high detuning [3]. We observe that for $\epsilon > 10^3$ the value of $\langle (\Delta a_2)^2 \rangle$ is in agreement with the value corresponding to the trapped even states. In the region $\epsilon < 10$ the fluctuations are constant and smaller than in the previous region. The value $\langle(\Delta a_2)^2\rangle=0.022$ is in agreement with the corresponding number obtained from Eq. (14) for $|\alpha/\gamma|=\sqrt{0.7}$. In Fig. 2(b) we show the corresponding entropy for the same cases as in Fig. 2(a). Here we observe that the entropy vanishes in the two ϵ regions discussed above. Hence, in those regions we have pure states. Figure 2(c) shows the ratio (versus detuning)

$$r_1=\frac{\sum\limits_{n=1,3,5,\ldots}\rho_{n,n}}{\sum\limits_{n=0,2,4,\ldots}\rho_{n,n}}\,, \tag{17}$$

which gives us an idea about the behavior of the one-photon transitions and how these are removed in time. We see that in the ϵ parameter ranges specified above only the even photon-number components survive. Hence, in the steady state, the one-photon transitions have been totally suppressed. Figure 2(d) shows the ratio (versus detuning)

$$r_2=\frac{\sum\limits_{n=11,12,13,\ldots}\rho_{n,n}}{\sum\limits_{n=0,1,2,\ldots,10}\rho_{n,n}}\,, \tag{18}$$

which gives us information about the probability leakage from the even trap corresponding to $n_u=10$. Here we see that $n_u=10$ is upward trapping state for $\epsilon>10^3$. On the other hand, for $\epsilon<10$, this ratio is a constant different from zero. This number agrees with the r_2 value as obtained from the ideal squeezed vacuum [9].

Figure 3(a) shows the quadrature fluctuations as function of l, which is number of atoms crossing the cavity. Here the interaction time is $\phi=4\pi/23$, $|\alpha/\gamma|=\sqrt{0.7}$, $\epsilon=1$, and the initial condition corresponds to a coherent state with $\langle n\rangle=16$. We observe that in steady state, the quadrature fluctuations reduce to the same value as obtained with the squeezed vacuum. Figure 3(b) shows the entropy for the previous case. It goes to zero when approaching steady state. Figure 3(c) shows the temporal evolution of the ratio r_1, which also vanishes in steady state; thus only the coherent superposition of even photon-number states survive. The same is true with any other initial condition, therefore, we have numerical evidence to state that the two-photon micromaser with arbitrary detuning generates a perfectly squeezed vacuum, with the complex squeezing parameter given by Eq. (13).

IV. THE EFFECT OF CAVITY LOSSES

In this section we study the effects of the cavity losses on the pure states, through the dynamic behavior of the discrete master equation for the field. Here we consider atoms being injected at a constant rate $R<\tau^{-1}$. We also assume that the cavity damping time $t_{\rm cav}$ is much larger than $t_{\rm at}=R^{-1}$ and τ. With these assumptions, the field

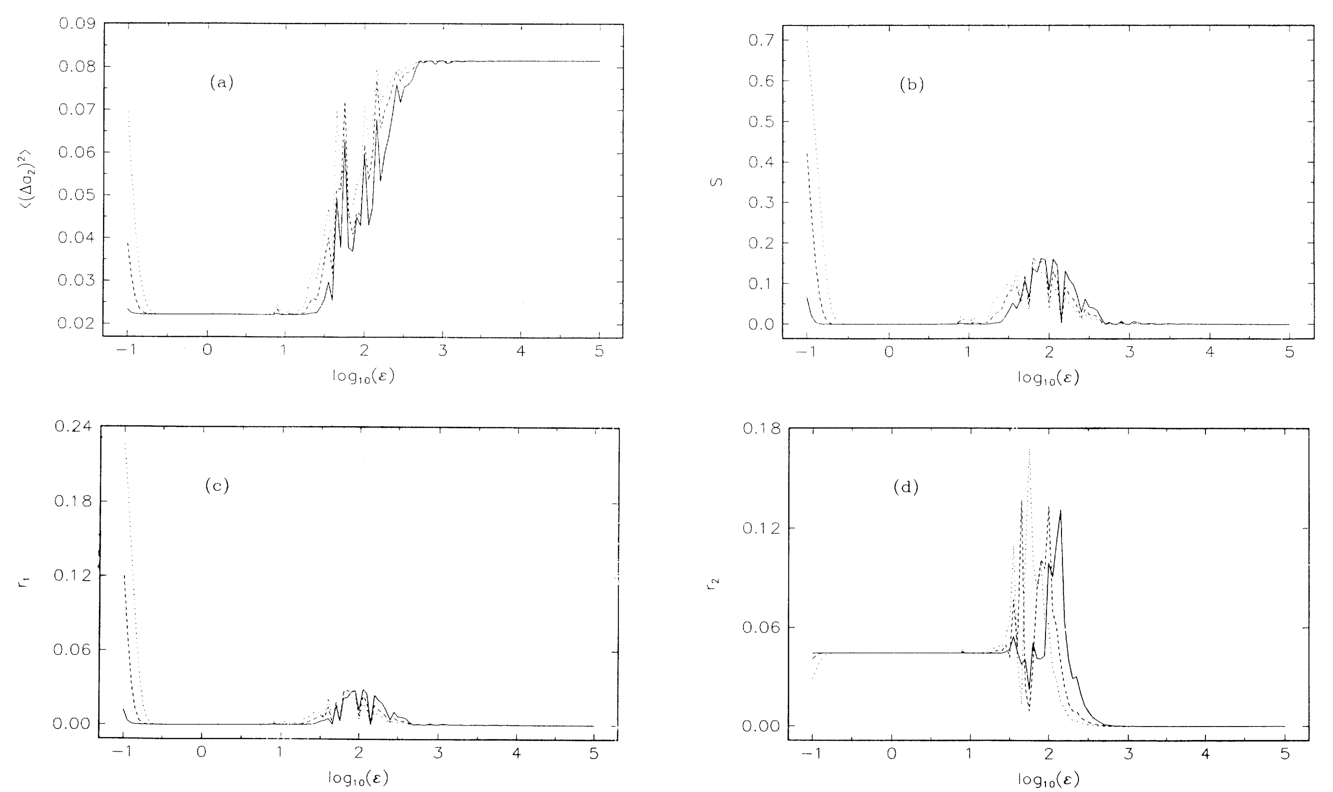

FIG. 2. (a) $\langle(\Delta a_2)^2\rangle$, (b) entropy S, (c) r_1, and (d) r_2 as a function of $\log_{10}(\epsilon)$ for various numbers of atoms crossing the cavity. 3000 atoms (dotted line), 5000 atoms (dashed line), and 10 000 atoms (solid line). Here $\alpha/\gamma=-\sqrt{0.7}$, $\phi=4\pi\epsilon/23$, and the initial condition of the field is $|0\rangle$.

density matrix after the $(k+1)$th atom has gone through the cavity is given by [8]

$$\rho^{k+1}=e^{R^{-1}L}M\rho^{k}, \tag{19}$$

where L is the well-known cavity loss operator and M the one-atom gain operator, that is, it represents the gain for the field due to the effect of one-atom crossing the cavity. Consistent with the assumptions stated above, we neglect the losses while the atom interacts with the field. We define the parameter $N_{ex}=t_{cav}/t_{at}$ which represents the number of atoms crossing the cavity during the time t_{cav}.

In the following numerical calculations we will neglect the thermal photons (low-temperature regime). In Fig. 4(a) we show the steady state of the quadrature fluctuations as a function of N_{ex} in logarithmic scale for $\epsilon=10^4$ and 1. Here $\alpha/\gamma=-\sqrt{0.7}$, $\phi=4\pi\epsilon/23$, and we took the initial condition to be $|0\rangle$. We observe that, for $N_{ex}\geq 10^7$ and $\epsilon=10^4$, $\langle(\Delta a_2)^2\rangle$ settles at about the same value as the one obtained with the trapped even state; however, for $N_{ex}\geq 10^5$ and $\epsilon=1$, this fluctuation stabilizes around the value for the squeezed vacuum. Figure 4(b) shows the entropy as a function of N_{ex} with the same parameters as in Fig. 4(a). We obtain quasipure states in the same N_{ex}

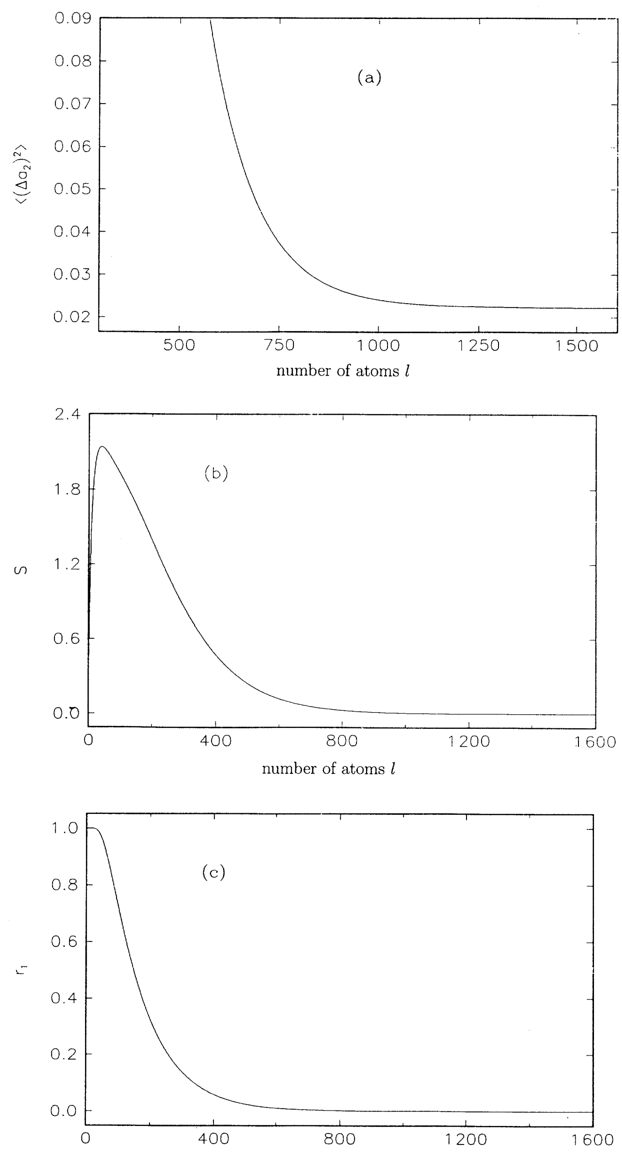

FIG. 3. (a) $\langle(\Delta a_2)^2\rangle$, (b) entropy S, and (c) r_1 versus the number of atoms l crossing the cavity. Here $\alpha/\gamma=-\sqrt{0.7}$, $\phi=4\pi/23$, $\epsilon=1$, and the initial condition is a coherent state with $\langle n\rangle=16$.

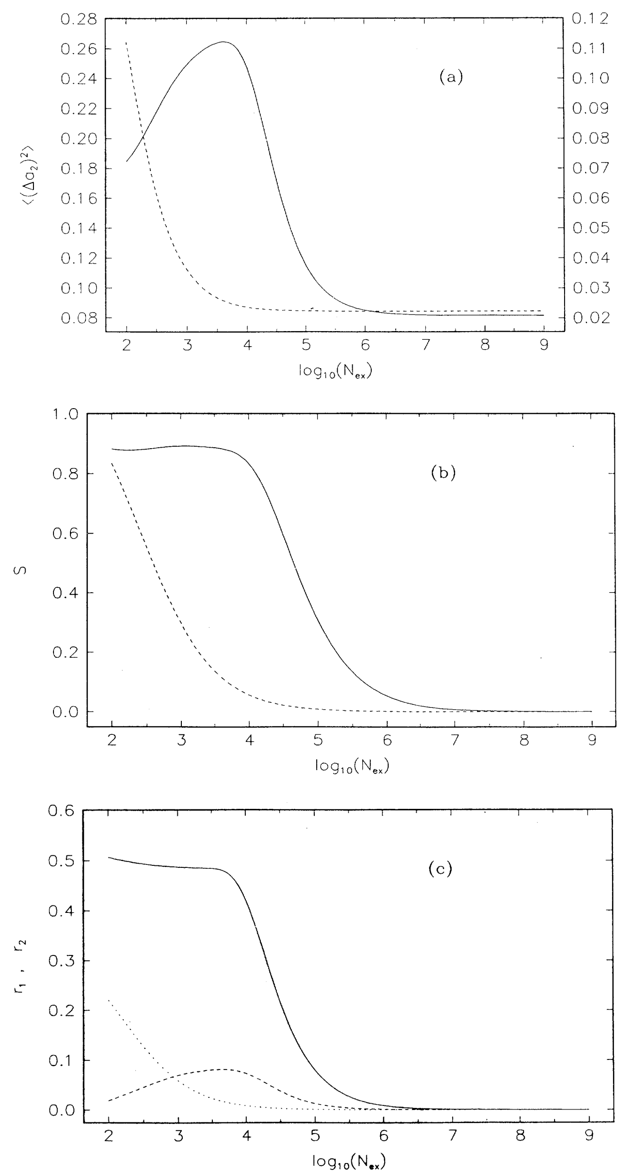

FIG. 4. (a) $\langle(\Delta a_2)^2\rangle$ versus $\log_{10}(N_{ex})$ for $\epsilon=1$ (dashed line, right scale) and $\epsilon=10^4$ (solid line, left scale). (b) Entropy (S) versus $\log_{10}(N_{ex})$ for $\epsilon=1$ (dashed line) and $\epsilon=10^4$ (solid line). (c) r_1 versus $\log_{10}(N_{ex})$ for $\epsilon=1$ (dotted line), $\epsilon=10^4$ (solid line), and r_2 versus $\log_{10}(N_{ex})$ (dashed line) for $\epsilon=10^4$. Here the initial condition for the field is $|0\rangle$, $\phi=4\pi\epsilon/23$, and $\alpha/\gamma=-\sqrt{0.7}$.

range as in the previous cases. Figure 4(c) shows the steady state of the r_1 and r_2 ratios as a function of N_{ex} for $\epsilon=10^4$ and the r_1 ratio for $\epsilon=1$. We see that for $\epsilon=10^4$ and $N_{ex}\geq 10^7$ the trap in $|10\rangle$ does not leak, and only even number states are important. Also for $\epsilon=1$ and $N_{ex}>10^5$ only even number states are important.

V. CONCLUSION

In this paper we have studied the generation of pure states in a cascade three-level system with finite detuning by means of the temporal behavior. We also considered the cavity losses taking realistic values [11–15] of the physical parameters ϵ and N_{ex}.

As a first conclusion, we can say that for α/γ fixed and a reduced interaction time ϕ corresponding to an upward trapping condition for the high detuning [3], the steady state for the field will have a greater quadrature noise reduction for smaller ϵ values. Furthermore, this steady state is insensitive to initial conditions. The experimental range for this parameter ϵ is between -10^3 and 10^4 for actual micromasers with Rydberg atoms [11–15]. Therefore, one could, in principle, generate an ideal squeezed vacuum state, for an arbitrary initial condition, provided that the detuning is small enough so as to have both two- and one-photon transitions. Finally, in the case of lossy cavities we have concluded that within an experimentally reachable range of N_{ex} values and realistics values of ϵ we get quasipure states for the electromagnetic field, with properties not very different from the even or ideal squeezed vacuum states. We emphasize that the large-detuning case is much more sensitive to the cavity losses than the finite-detuning case. This can be seen, for example, from Fig. 4(b), where the entropy vanishes for a N_{ex} value at least two orders of magnitude smaller for the small detuning case ($\epsilon=1$, ideal squeezing) as compared to the large detuning case ($\epsilon=10^4$, even states). This is another advantage for the ideal squeezed vacuum state over the even states.

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APPENDIX

In this appendix we calculate the recursion relation for the expansion coefficients in terms of number states.

A squeezed state [9] can be written as

$$|\bar{\alpha},\xi\rangle=\sum_{n=0}^{\infty}[n!\cosh(s)]^{-1/2}[\tfrac{1}{2}e^{i\theta}\tanh(s)]^{n/2}\times e^{-(1/2)[|\bar{\alpha}|^2+\bar{\alpha}^{*2}e^{i\theta}\tanh(s)]}\times H_n\left[\frac{\bar{\alpha}+\bar{\alpha}^{*}e^{i\theta}\tanh(s)}{\sqrt{2e^{i\theta}\tanh(s)}}\right]|n\rangle\ , \tag{A1}$$

where H_n is the Hermite polynomial of nth degree, $\xi=se^{i\theta}$, $\theta/2$ is a geometrical phase, and $\bar{\alpha}$ is the displacement of the error ellipse with respect to the origin [9]. Now, if we set $\bar{\alpha}=0$ in Eq. (A1), then the probability amplitude for the n-photon number state is given by

$$S_n=\begin{cases}[\cosh(s)]^{-1/2}[e^{i\theta}\tanh(s)]^{n/2}\dfrac{\sqrt{n!}}{2^{n/2}\left[\dfrac{n}{2}\right]!} & \text{for } n \text{ even}\\[2ex] 0 \text{ for } n \text{ odd}\ . \end{cases} \tag{A2}$$

It is simple to verify that the S_n satisfy the recursion relation

$$S_{n+2}=e^{i\theta}\tanh(s)\left[\frac{n+1}{n+2}\right]^{1/2}S_n\ . \tag{A3}$$

From Ref. [9], for an ideal squeezed vacuum state, the quadrature uncertainties are given by

$$\langle(\Delta a_1)^2\rangle=\frac{1}{4}\left\{e^{-2s}\cos^2\left[\frac{\theta}{2}\right]+e^{2s}\sin^2\left[\frac{\theta}{2}\right]\right\}, \tag{A4a}$$

$$\langle(\Delta a_2)^2\rangle=\frac{1}{4}\left\{e^{-2s}\sin^2\left[\frac{\theta}{2}\right]+e^{2s}\cos^2\left[\frac{\theta}{2}\right]\right\}. \tag{A4b}$$

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