Detection of quantum noise

I. R. Senitzky
Center for Laser Studies, University of Southern California, Los Angeles, California 90089-1112
(Received 28 July 1993)

Noise that can be attributed to vacuum fluctuations, usually referred to as quantum noise, is examined. It is shown that vacuum fluctuations of a single uncoupled mode do not constitute noise, in the sense of a random process, but the superposition of the fluctuations of a large number of modes does constitute, formally, noise. The effect of vacuum fluctuations of the free-space radiation field on a harmonic oscillator, a nondegenerate parametric amplifier, and a degenerate parametric amplifier, all driven by a prescribed sinusoidal field, is compared with the effect of classical noise. It is found that the coordinates of all systems respond in a formally similar manner to both vacuum fluctuations and classical noise. However, the resonance fluorescence spectrum—the evidence of "detection"—is completely different for the two kinds of noise. The spectrum of the harmonic oscillator does not exhibit noise in response to vacuum fluctuations, but does so in response to classical noise. The spectra of the two types of parametric amplifiers do exhibit noise in response to vacuum fluctuations, but this noise differs from that in the classical case. An explanation for the difference is offered, based on the fact that the quantum fluctuations cannot do work, but can noise-modulate power from an outside source, which, for the parametric amplifiers, is the pump. In the analysis of noise from the degenerate parametric amplifier, it is shown that squeezed noise, viewed as an oscillation of the dispersion with a sufficiently low minimum, is generated in the same manner in the case of classical noise as in the case of quantum noise, and is due to phase conjugation.

PACS number(s): 42.50.Lc

I. INTRODUCTION

The term "quantum noise" has been used widely in recent years to characterize noise due to quantum-mechanical—in the sense of nonclassical—phenomena. In many instances, this term is used interchangeably with quantum fluctuations. An illustration of this use, which can also serve as motivation for the questions to be discussed in the present paper, is the following quotation from a review article on squeezed light by Loudon and Knight [1]: "Optical fields obey the laws of quantum mechanics and have an inherent quantum indeterminacy which cannot be removed no matter how carefully the light source is controlled. A measure of this optical noise for a single-mode field of frequency ω is given by the quantity $\mathcal{E}_0=(\hbar\omega/2\epsilon_0 V)^{1/2}$, where V is the volume over which the field is excited. This noise has the same magnitude for any strength of excitation. Indeed, the same size of field fluctuation is present even in the absence of any field excitation. For this reason, $\mathcal{E}_0$ is often associated with the vacuum fluctuations of the electromagnetic field."

Now, the generally accepted meaning of the word "noise" is a random fluctuation in time of some physical variable, or, in mathematical terms, a random process. If a mode of the electromagnetic field is lossless, the time variation of the field, as will be seen shortly, is sinusoidal, and thus should not be regarded as noise. Quantum fluctuations of a single lossless mode refer to the "indeterminacy" mentioned in the above quotation. This is an indeterminacy in one or both of the dynamical variables of the field when the mode is in a given quantum state. The fluctuation is that in the magnitude of the field variables among members of an ensemble of identically prepared modes at a single time. As will be shown later, a certain randomness in the time variation does exist if the mode is coupled to a loss mechanism, even if the latter is at absolute zero temperature. At this temperature, the random time variation may legitimately be called quantum noise, since classically, no noise is present. There exists, however, a significant difference between this noise and classical noise. As will become apparent, the quantum noise in this case will not be "detected," that is, detected on a measurement instrument that is activated by energy from the mode, even if the mode is excited.

If, instead of a single lossless mode, we consider the field of a large—or infinite—number of modes, such as the free-space radiation field, the situation is different. Here, there does exist a random fluctuation in time. In order to distinguish clearly between classical noise and quantum noise, we will consider the radiation field to be at absolute zero temperature. Then, classically, the radiation field displays no noise. Quantum mechanically, the field consists of the superposition of the zero-point oscillations of all the modes. This superposition is usually referred to as vacuum fluctuations and is, according to the mathematical definition, noise—quantum noise. In contrast to real, or classical noise, vacuum fluctuations cannot be detected directly, since zero-point oscillations can do no work and thus cannot activate a detection mechanism. However, vacuum fluctuations can induce other noise effects, some of which *can* be detected. These effects may also be regarded as quantum noise. It is the purpose of the present article to compare quantum noise to classical noise and to investigate their similarities and differences. The kind of classical noise to which comparison should be made will become apparent in the following analysis.

II. SINGLE MODE

The electric and magnetic fields of a single standing-wave mode of frequency ω can be described by

$$\mathbf{E}(\mathbf{r},t) = -(4\pi\hbar\omega)^{1/2}\mathbf{u}(\mathbf{r})p(t)\ ,$$
$$\mathbf{H}(\mathbf{r},t) = \left[\frac{4\pi c^2\hbar}{\omega}\right]^{1/2}\nabla\times\mathbf{u}(\mathbf{r})q(t)\ , \tag{2.1}$$

where $\mathbf{u}(\mathbf{r})$ describes the spatial dependence of the field and is normalized over a volume V, and q and p are dimensionless quantities. The Hamiltonian for the field in V is given by

$$H = \tfrac{1}{2}\hbar\omega(q^2+p^2)\ , \tag{2.2}$$

with $[q,p]=i$. The dynamical variables q and p can be regarded as the dimensionless coordinate and momentum, respectively, of a harmonic oscillator—the radiation oscillator. The annihilation and creation operators are given, respectively, by

$$a = 2^{-1/2}(q+ip),\quad a^\dagger = 2^{-1/2}(q-ip)\ , \tag{2.3}$$

with $[a,a^\dagger]=1$. The equation of motion in the Heisenberg picture for any variable X, $i\hbar\dot{X}=[X,H]$, leads to the solution

$$q(t) = q(0)\cos\omega t + p(0)\sin\omega t\ ,$$
$$p(t) = -q(0)\sin\omega t + p(0)\cos\omega t\ , \tag{2.4}$$

or

$$a(t) = a(0)e^{-i\omega t},\quad a^\dagger(t) = a^\dagger(0)e^{i\omega t}\ .$$

It is clear that there does not exist any randomness in the time variation of q and p. There does exist a quantum-mechanical randomness in the values of $q(0)$ and $p(0)$ which depends on the state $|\psi\rangle$ of the radiation oscillator. This state describes a conceptual ensemble of oscillators with a spread in the values of $q(0)$ and $p(0)$ determined by the probability densities $|\langle q(0)|\psi\rangle|^2$ and $|\langle p(0)|\psi\rangle|^2$, respectively, but each member of the ensemble oscillates purely sinusoidally. The spread, or uncertainty, in the values of $q(0)$ and $p(0)$ among members of the ensemble, for any state, as is well known, obeys the Heisenberg uncertainty principle $\Delta q\Delta p \geq \frac{1}{2}$, where $(\Delta q)^2 \equiv \langle q^2\rangle - \langle q\rangle^2$, with a similar definition for Δp. If, for a given state, either $[\Delta q(t)]^2 < \frac{1}{2}$ or $[\Delta p(t)]^2 < \frac{1}{2}$ for any t during a cycle of oscillation, the state has been called squeezed. Squeezed states have received much discussion in connection with quantum noise [1,2].

The question now arises, how does noise enter into the behavior of a single mode? The only way noise can be introduced in the time development of the coordinates of a lossless oscillator is by introducing jumps, at successive times, from one member of the ensemble to another. The jumps produce a change of random magnitude in q and p with a spread determined by Δq and Δp. Physically, this means that the experiment that measures the value of either q or p is performed many times in succession, the oscillator being prepared anew in the same state each time. Only when the results of this succession of measurements is considered to be a single measurement will the oscillator coordinates display noise. A squeezed state will then display less noise in one quadrature, properly chosen, than in the other.

III. HARMONIC-OSCILLATOR COUPLED TO RADIATION FIELD

We consider now the effect of the electromagnetic field on a harmonic oscillator, that is, the problem of a harmonic oscillator radiating into free space. The radiation field is represented by a denumerably infinite set of modes, closely spaced in frequency and covering the entire spectrum. The oscillator is coupled to the field through an electric dipole moment proportional to q and sufficiently small so that the relaxation time is much longer than a period. The oscillator will be considered to be driven by a prescribed sinusoidal field near resonance. Here we have, essentially, the description of a harmonic oscillator with dissipation, which has been studied a number of times in the literature (the radiation field being a favorite model for a loss mechanism) [3–10]. We review this problem briefly.

The Hamiltonian for the combined system, which will be referred to as the driven damped oscillator (DDO), is given by

$$H = H_1 + H_2 + H_{12} + H_{13} \tag{3.1}$$

with

$$H_1 = \hbar\omega(a^\dagger a + \tfrac{1}{2})\ ,$$
$$H_2 = \sum_k \hbar\omega_k(a_k^\dagger a_k + \tfrac{1}{2})\ ,$$
$$H_{12} = -\tfrac{1}{2}i\hbar\sum_k \gamma_k(a_k - a_k^\dagger)(a+a^\dagger)\ ,$$
$$H_{13} = -i\hbar(\Omega e^{-i\omega_0 t} - \Omega^* e^{i\omega_0 t})(a+a^\dagger)\ .$$

Here H_2 describes the modes of the radiation field, ω_k is the frequency of the kth mode, H_{12} describes the coupling between the oscillator and the radiation field, and H_{13} describes the effect of the driving field on the oscillator, where the driving-field frequency is ω_0 and the amplitude is proportional to the constant Ω. We introduce, for convenience, the reduced variables A and A_k, defined by

$$a(t) = A(t)e^{-i\omega t},\quad a_k(t) = A_k(t)e^{-i\omega_k t}\ . \tag{3.2}$$

The Heisenberg equations of motion for the oscillator then become

$$i\hbar\dot{A}(t) = [A, H_{12}+H_{13}]\ , \tag{3.3}$$

and the corresponding H.c. equation. With the utilization of certain approximations, discussed in detail elsewhere [10], the equations of motion can be reduced to those in which only the oscillator variables are the unknowns:

$$\dot{A} = -\Omega e^{-i\Delta t} - F - \beta A + i(\beta_2+\beta_3)A \ , \tag{3.4}$$

and the corresponding H.c. equation, with

$$\Delta = \omega_0 - \omega \ ,$$

$$F = \tfrac{1}{2}\sum_k \gamma_k A_k^{(0)} e^{-i(\omega_k-\omega)t} \ ,$$

$$\beta = \tfrac{1}{4}\pi\rho(\omega)\gamma^2(\omega) \ ,$$

$$\beta_2 = \tfrac{1}{4}\mathrm{P}\int_0^\infty d\omega' \rho(\omega')\gamma^2(\omega')\frac{1}{\omega'-\omega} \ ,$$

$$\beta_3 = \tfrac{1}{4}\int_0^\infty d\omega' \rho(\omega')\gamma^2(\omega')\frac{1}{\omega'+\omega} \ ,$$

where $A_k^{(0)}$ refers to the unperturbed (zero order) radiation field, $\gamma^2(\omega')$ is the value of γ_k^2 averaged over modes in a small frequency range about ω', $\rho(\omega')$ is the density of modes at ω', and P indicates the principal value of the integral. The last term in Eq. (3.4) indicates a radiative frequency shift, which we absorb into ω [replacing ω by $\tilde{\omega} = \omega - (\beta_2+\beta_3)$ and dropping the tilde], and obtain

$$\dot{A} = -\Omega e^{-i\Delta t} - \beta A - F \ . \tag{3.5}$$

This is an equation of the Langevin type; the Ω term exhibits the action of the driving field, the β term provides the damping, and F is the fluctuation term. Since the radiation field is at zero temperature, $A_k^{(0)}$ describes the zero-point oscillation of the kth mode and $F(t)$ describes the effect of the vacuum fluctuations on the oscillator. Formally, $F(t)$ has the appearance of noise, and any noise properties of A formally due to $F(t)$ constitute quantum noise. The properties of $F(t)$ are the following [10,11]:

$$\begin{aligned} &F(t)|\rangle = \langle|F^\dagger(t) = 0 \ , \\ &\langle F(t_1)F^\dagger(t_2)\rangle \\ &\quad = 2\beta\delta(t_1-t_2) \\ &\qquad + i\beta_2 \lim_{\epsilon\to 0+}[\delta(t_1-t_2-t)-\delta(t_1-t_2+\epsilon)] \ , \end{aligned} \tag{3.6}$$

where the limit $\epsilon \to 0+$ is to be taken after the time integration of the δ function is carried out. It is obvious that the second term plays a role only if one of the limits of integration with respect to t_1 is t_2, and vice versa. In the following analysis, the limits of integration will be such that the second term vanishes.

The solution of Eq. (3.5) is given by

$$A(t) = A(0)e^{-\beta t} - \frac{\Omega(e^{-i\Delta t} - e^{-\beta t})}{\beta - i\Delta} - \int_0^t dt_1 F(t_1) e^{-\beta(t-t_1)} \ . \tag{3.7}$$

It can be shown that $A(t)$ obeys the correct commutation rule $[A, A^\dagger] = 1$. While the initial commutator $[A(0), A^\dagger(0)]$ is damped by the factor $e^{-2\beta t}$, the quantum noise term yields the commutator $(1-e^{-2\beta t})$. After the transient period, therefore, the quantum-mechanical properties of the oscillator are due entirely to the vacuum fluctuations. The steady state solution, obtained by taking the initial time at $-\infty$, is expressed by

$$A(t) = A_s(t) + A_F(t) \ , \tag{3.8}$$

where $A_s(t)$, which can be regarded as the signal term, is given by

$$A_s(t) = \langle A(t)\rangle = -\frac{\Omega e^{-i\Delta t}}{\beta - i\Delta} \ ,$$

and $A_F(t)$, the quantum noise term, is given by

$$A_F(t) = -\int_{-\infty}^t dt_1 F(t_1) e^{-\beta(t-t_1)} \ .$$

It is interesting to compare the zero-point motion of the free oscillator to that of the damped oscillator. One obtains for the symmetrized correlation function of $q(t)$,

$$\tfrac{1}{2}\langle 0|q(t_1)q(t_2)+q(t_2)q(t_1)|0\rangle = \tfrac{1}{2}\cos[\omega(t-t_1)] \tag{3.9a}$$

in the case of the free oscillator and

$$\tfrac{1}{2}\langle q(t_1)q(t_2)+q(t_2)q(t_1)\rangle_{\Omega=0} = \tfrac{1}{2}e^{-\beta|t_1-t_2|}\cos[\omega(t_1-t_2)] \tag{3.9b}$$

in the case of the damped oscillator, using Eqs. (3.6). Thus the zero-point oscillation of the free oscillator is replaced by quantum noise, that is, by vacuum-fluctuation-driven motion, in the damped oscillator. Since the correlation functions for $p(t)$ are the same as those for $q(t)$, it is clear that the expectation value of the zero-point energy of the DDO is the same as that of the free oscillator. In other words, the energy due to quantum noise in the DDO is the same as that due to zero-point oscillation in the free oscillator. It is also easily seen that $(\Delta q)^2 = (\Delta p)^2 = \frac{1}{2}$ for the steady-state DDO, so that it oscillates as a minimal wave packet. The total steady-state energy E of the DDO is given by

$$E = \tfrac{1}{2}\hbar\omega(q^2+p^2) = \hbar\omega\left[\frac{|\Omega|^2}{\beta^2+\Delta^2} + \frac{1}{2}\right] \ .$$

The Hamiltonian and the equations of motion for the DDO can also be read classically (with the omission of the zero-point energy in the oscillators, an omission that does not affect the equations of motion). Classically, $A_k^{(0)}$ and F vanish, of course. However, in order to investigate the difference between classical and quantum noise, we replace the operator F by the c number F_c, which obeys the relationships

$$\begin{aligned} &\langle F_c\rangle = 0 \ , \\ &\langle F_c(t_1)F_c^*(t_2)\rangle = \langle F_c^*(t_1)F_c(t_2)\rangle = \alpha\delta(t_1-t_2) \ . \end{aligned} \tag{3.10}$$

This is the classical description of Gaussian white noise [12], where α indicates its magnitude. The classical solution looks, formally, exactly like the quantum-mechanical solution [Eqs. (3.7) and (3.8)], except that F is replaced by F_c. It should be noted that quantum-mechanical properties of the oscillator cannot be maintained if the vacuum fluctuations are replaced by classical noise and the radiation field is considered to be classical (acting only as a loss mechanism). This is obvious from Eq. (3.7), which shows that initial quantum-mechanical properties of the oscillator, if any, would be damped, and is an illustration of the general principle that in the interaction of two systems, both must be treated either classically or quantum mechanically in order to avoid unphysical results [13]. One can always, of course, subject a quantum-mechanical system to a *prescribed* c-number perturbation.

We calculate now certain properties of the harmonic oscillator coupled to the classical radiation field and subject to classical noise described by F_c. Using the subscript c to designate classical variables, we obtain for the correlation functions of q and p in the absence of a driving field,

$$\langle q_c(t_1)q_c(t_2)\rangle_{\Omega=0}=\langle p_c(t_1)p_c(t_2)\rangle_{\Omega=0} = \frac{1}{2}\frac{\alpha}{\beta}e^{-\beta|t_1-t_2|}\cos[\omega(t_1-t_2)] \ . \tag{3.11}$$

The zero-point energy, that is, the energy in the absence of a driving field, is given by

$$\tfrac{1}{2}\hbar\omega\langle q_c^2+p_c^2\rangle_{\Omega=0}=\frac{1}{2}\frac{\alpha}{\beta}\hbar\omega \ . \tag{3.12}$$

Generally, in the steady state, we have, for the total energy E_c,

$$E_c=\tfrac{1}{2}\hbar\omega\langle q_c^2+p_c^2\rangle=\hbar\omega\left[\frac{|\Omega|^2}{\beta^2+\Delta^2}+\frac{1}{2}\frac{\alpha}{\beta}\right] , \tag{3.13}$$

and

$$(\Delta q_c)^2=(\Delta p_c)^2=\frac{1}{2}\frac{\alpha}{\beta} \ . \tag{3.14}$$

It is seen that if the magnitude of the classical noise is chosen such that $\alpha=\beta$, this noise produces exactly the same results for the zero-point energy and spread of the "wave packet" in the classical case as the vacuum fluctuations do in the quantum-mechanical case.

Let us assume that the noise exhibited by the coordinates of the oscillator in the steady state is to be detected by an examination of its resonance fluorescence spectrum. This is a reasonable assumption, since spectral analysis of radiation from the oscillator is a simple method of distinguishing between signal energy and noise energy available for detection and, incidentally, corresponds most closely to the process of hearing in the acoustic range, from which the word noise was borrowed [14]. An expression for the resonance fluorescence spectrum that is valid both classically and quantum mechanically has been derived previously [15] and is given, for the range $|\omega'-\omega| \ll \omega$ and for $\omega \gg \beta$, by

$$P(\omega')=\frac{\beta}{\pi}\hbar\omega\int_0^\infty d\tau\langle A^\dagger(t)A(t-\tau)\rangle_{\text{av}}e^{-i(\omega'-\omega)\tau}+\text{c.c.} \ , \tag{3.15}$$

where $P(\omega')$ is the expectation value of the power per unit frequency range about ω' radiated into the field and the notation $\langle\ \rangle_{\text{av}}$ indicates both an ensemble and time average. Considering the classical case first, we obtain, by use of Eq. (3.10),

$$\langle A_c^\dagger(t)A_c(t-\tau)\rangle=\frac{|\Omega|^2e^{i\Delta\tau}}{\beta^2+\Delta^2}+\frac{\alpha}{2\beta}e^{-\beta\tau} , \tag{3.16}$$

which yields

$$P_c(\omega')=2\beta\hbar\omega\frac{|\Omega|^2}{\beta^2+\Delta^2}\delta(\omega'-\omega_0)+\frac{\hbar\omega}{\pi}\frac{\alpha\beta}{\beta^2+(\omega'-\omega)^2} \ . \tag{3.17}$$

The spectrum consists of a δ function at the driving-field frequency and a Lorentzian distribution about the resonant frequency, which indicates a sinusoidal oscillation at ω_0 and noise in the neighborhood of ω. Since the total power radiated is $2\beta E_c$, we can divide the oscillator energy E_c into two parts:

$$E_c=E_c^{(s)}+E_c^{(n)} , \tag{3.18}$$

with the signal energy $E_c^{(s)}$ given by

$$E_c^{(s)}=\frac{\hbar\omega|\Omega|^2}{(\beta^2+\Delta^2)} ,$$

and the noise energy $E_c^{(n)}$, given by

$$E_c^{(n)}=\tfrac{1}{2}\hbar\omega\frac{\alpha}{\beta} ,$$

both of which produce, independently, radiation. Consider now the quantum-mechanical case. From Eq. (3.8), we have

$$\langle A^\dagger(t)A(t-\tau)\rangle =\langle[A_s^\dagger(t)+A_F^\dagger(t)][A_s(t-\tau)+A_F^\dagger(t-\tau)]\rangle , \tag{3.19}$$

and, from Eq. (3.6), we have

$$\langle|A_F^\dagger=A_F|\rangle=0 ,$$

so that

$$\langle A^\dagger(t)A(t-\tau)\rangle=\frac{|\Omega|^2}{\beta^2+\Delta^2}e^{i\Delta\tau} \ . \tag{3.20}$$

The resonance fluorescence spectrum corresponding to this correlation function is given by

$$P(\omega')=2\beta\hbar\omega\frac{|\Omega|^2}{\beta^2+\Delta^2}\delta(\omega'-\omega_0) \ . \tag{3.21}$$

Thus only a pure signal is radiated. It is seen that even though the coordinates of the DDO exhibit noise, this noise is not radiated in the quantum-mechanical case. The reason lies in the fact that zero-point noise, which is due to vacuum fluctuations, can do no work, just like the zero-point oscillations of the undamped oscillator can do no work. In the classical case, power is supplied by the noise source to the oscillator which, in turn, radiates this power. In the quantum-mechanical case, the vacuum fluctuations supply no power. In other words, quantum noise of the DDO cannot be "heard."

IV. NONDEGENERATE PARAMETRIC AMPLIFIER

We consider next a parametric amplifier, which, in idealized form, consists of two coupled oscillators, a signal oscillator and an idler oscillator, driven by a pump at their sum frequency. The precise description of the system is given by

$$H=\hbar\omega_1 a_1^\dagger a_1+\hbar\omega_2 a_2^\dagger a_2+\sum_k \hbar\omega_k a_k^\dagger a_k - i\hbar(\Omega_0 e^{-i\omega_0 t}a_1^\dagger a_2^\dagger-\Omega_0^* e^{i\omega_0 t}a_1 a_2)-i\hbar(\Omega_1 e^{-i\omega_1 t}a_1^\dagger-\Omega_1^* e^{i\omega_1 t}a_1)$$
$$-\tfrac{1}{2}i\hbar\sum_k \gamma_k^{(1)}(a_k a_1^\dagger - a_k^\dagger a_1)-\tfrac{1}{2}i\hbar\sum_k \gamma_k^{(2)}(a_k a_2^\dagger - a_k^\dagger a_2)\ . \quad (4.1)$$

Here a_1 and a_2 are the annihilation operators of the signal and idler oscillators, respectively; ω_0, ω_1, and ω_2 are the respective pump, signal, and idler frequencies; Ω_0 and Ω_1 describe the respective pump and signal inputs; and $\gamma_k^{(1)}$ and $\gamma_k^{(2)}$ indicate the coupling of the signal and idler oscillators, respectively, to the radiation field. The signal input is taken to be at resonance with the signal oscillator, and the pump frequency is given by

$$\omega_0=\omega_1+\omega_2\ .$$

For simplicity, the couplings are all taken to be of the rotating-wave type. In this section, we consider the case $\omega_1\neq\omega_2$, with the $\gamma_k^{(1)}$'s and the $\gamma_k^{(2)}$'s uncorrelated.

Using the reduced variables A_j, defined by

$$a_j(t)=A_j(t)e^{-i\omega_j t},\quad j=1,2\ , \quad (4.2)$$

we obtain, for the equations of motion,

$$\dot{A}_1=-\Omega_0 A_2^\dagger-\Omega_1-\tfrac{1}{2}\sum_k \gamma_k^{(1)}A_k e^{-i(\omega_k-\omega_1)t}\ , \quad (4.3a)$$

$$\dot{A}_2^\dagger=-\Omega_0^* A_1-\tfrac{1}{2}\sum_k \gamma_k^{(2)}A_k^\dagger e^{i(\omega_k-\omega_2)t}\ . \quad (4.3b)$$

With approximations for the last term in both equations similar to those used in obtaining the equations of motion for the DDO, and neglecting radiative frequency shifts, we obtain

$$\dot{A}_1=-\Omega_0 A_2^\dagger-\Omega_1-F_1-\beta A_1\ , \quad (4.4a)$$

$$\dot{A}_2^\dagger=-\Omega_0^* A_1-F_2^\dagger-\beta_2 A_2^\dagger\ , \quad (4.4b)$$

where F_j and β_j, $j=1,2$, are defined similarly to F and β for the DDO,

$$\beta_j=\tfrac{1}{4}\pi\rho(\omega_j)\gamma^{(j)2}(\omega_j)\ ,$$

$$F_j=\tfrac{1}{2}\sum_k \gamma_k^{(j)}A_k^{(0)}e^{-i(\omega_k-\omega_j)t}\ .$$

In order to determine the parameter range appropriate for an amplifier rather than for an oscillator, we examine first the equations for the expectation values $\langle A_1\rangle$ and $\langle A_2^\dagger\rangle$, which are the same as the classical equations for the parametric amplifier. These are

$$\langle \dot{A}_1\rangle=-\Omega_0\langle A_2^\dagger\rangle-\Omega_1-\beta_1\langle A_1\rangle\ , \quad (4.5a)$$

$$\langle \dot{A}_2^\dagger\rangle=-\Omega_0^*\langle A_1\rangle-\beta_2\langle A_2^\dagger\rangle\ . \quad (4.5b)$$

The solution is given by

$$\langle A_1(t)\rangle=K_+^{(1)}e^{-i\alpha_+ t}+K_-^{(1)}e^{-\alpha_- t}-\frac{\beta_2\Omega_1}{\beta_1\beta_2-|\Omega_0|^2}\ , \quad (4.6a)$$

$$\langle A_2^\dagger(t)\rangle=K_+^{(2)}e^{-\alpha_+ t}+K_-^{(2)}e^{-\alpha_- t}+\frac{\Omega_0^*\Omega_1}{\beta_1\beta_2-|\Omega_0|^2}\ , \quad (4.6b)$$

where

$$\alpha_\pm=\beta\pm\Omega$$

with

$$\beta=\tfrac{1}{2}(\beta_1+\beta_2),\quad \Omega=\tfrac{1}{2}[(\beta_1-\beta_2)^2+4|\Omega_0|^2]^{1/2}\ ,$$

and the K's are determined by the initial conditions. In order that a steady state be reached, we must have $\alpha_->0$, which implies

$$\beta_1\beta_2-|\Omega_0|^2>0\ . \quad (4.7)$$

The steady-state solution can be written as

$$\langle A_1\rangle_{\rm SS}=-\frac{\Omega_1}{\beta_1}\mathcal{A}\ , \quad (4.8)$$

where the amplification factor $\mathcal{A}$ is given by

$$\mathcal{A}=\frac{\beta_1\beta_2}{\beta_1\beta_2-|\Omega_0|^2}\ .$$

The steady-state (operator) solution of Eqs. (4.4) is found to be

$$A_1=-\frac{\beta_2\Omega_1}{\beta^2-\Omega^2}-\int_{-\infty}^t dt_1 e^{-\beta(t-t_1)}\left[F_1(t_1)\left[\cosh[\Omega(t-t_1)]-\frac{1}{2}\frac{\beta_1-\beta_2}{\Omega}\sinh[\Omega(t-t_1)]\right]-\frac{\Omega_0}{\Omega}F_2^\dagger(t_1)\sinh[\Omega(t-t_1)]\right]\ , \quad (4.9a)$$

$$A_2^\dagger=\frac{\Omega_0^*\Omega_1}{\beta^2-\Omega^2}-\int_{-\infty}^t dt_1 e^{-\beta(t-t_1)}\left[F_2^\dagger(t_1)\left[\cosh[\Omega(t-t_1)]-\frac{1}{2}\frac{\beta_2-\beta_1}{\Omega}\sinh[\Omega(t-t_1)]\right]-\frac{\Omega_0^*}{\Omega}F_1(t_1)\sinh[\Omega(t-t_1)]\right]\ . \quad (4.9b)$$

The definition of F_j [Eqs. (4.4)] shows that F_1 describes the action of the vacuum fluctuations on the signal oscillator (oscillator 1) and F_2 describes the action of these fluctuations on the idler oscillator (oscillator 2).

We consider first the effect of the vacuum fluctuations on the dispersion of the coordinates of both oscillators. In the calculation of $\langle q_i^2\rangle$ and $\langle p_i^2\rangle$ the expectation value $\langle F_i(t)F_j^\dagger(t_2)\rangle$, $i\neq j$, given by

$$\langle F_i(t_1)F_j^\dagger(t_2)\rangle=\tfrac{1}{4}\sum_k \gamma_k^{(i)}\gamma_k^{(j)}e^{-i(\omega_j-\omega_i)(t_1-t_2)} ,$$

is encountered. Since the $\gamma_j^{(1)}$'s and $\gamma_k^{(2)}$'s are uncorrelated, this expectation value will be considered negligible. A calculation then yields

$$\begin{aligned}\langle q_i^2\rangle-\langle q_i\rangle^2&=\langle p_i^2\rangle-\langle p_i\rangle^2\\&=\frac{1}{4\beta(\beta^2-\Omega^2)}\{\beta_i[2\beta(\beta+k_{ij}\Omega)\\&\qquad+(k_{ij}^2-1)\Omega^2]\\&\qquad+\beta_j|\Omega_0|^2\} ,\end{aligned} \tag{4.10}$$

where

$$k_{ij}=\frac{1}{2}\frac{\beta_j-\beta_i}{\Omega},\quad i\neq j .$$

[For $|\Omega_0|=0$, Ω is to be interpreted as $\frac{1}{2}(\beta_i-\beta_j)$.] This expression becomes more transparent if we take $\beta_1=\beta_2$ while retaining the separate identity of the two oscillators both with respect to frequency and to coupling to the radiation field. We then have $k_{ij}=0$, $\Omega=|\Omega_0|$, and

$$\langle q_i^2\rangle-\langle q_i\rangle^2=\langle p_i^2\rangle-\langle p_i\rangle^2=\frac{1}{2}\frac{\beta^2}{\beta^2-|\Omega_0|^2} . \tag{4.11}$$

As a check, one can note that for $|\Omega_0|=0$, in which case the two oscillators become uncoupled, both Eqs. (4.10) and (more immediately) Eq. (4.11) indicate that the dispersion in the coordinates reduces to the correct value $\frac{1}{2}$.

As in the case of the harmonic oscillator, we compare this result with that obtained when a classical noise source, instead of the vacuum fluctuations, acts on the parametric amplifier. We replace F_1 and F_2 in Eq. (4.4) by the c numbers $F_1^{(c)}$ and $F_2^{(c)}$, respectively, defined by

$$\begin{aligned}&\langle F_i^{(c)}\rangle=0 ,\\&\langle F_i^{(c)}(t_1)F_j^{(c)*}(t_2)\rangle=\langle F_i^{(c)*}(t_1)F_j^{(c)}(t_2)\rangle\\&\qquad=\delta_{ij}\alpha_i\delta(t_1-t_2) ,\end{aligned} \tag{4.12}$$

$i,j=1,2$, and obtain, in a calculation parallel to that of the quantum-mechanical case,

$$\begin{aligned}\langle q_i^{(c)2}\rangle-\langle q_i^{(c)}\rangle^2&=\langle p_i^{(c)2}\rangle-\langle p_i^{(c)}\rangle^2\\&=\frac{1}{4\beta(\beta^2-\Omega^2)}\{\alpha_i[2\beta(\beta+k_{ij}\Omega)+(k_{ij}^2-1)\Omega^2]+\alpha_j|\Omega_0|^2\},\quad i\neq j ,\end{aligned} \tag{4.13}$$

where the superscript c indicates classical variables. It is seen that if we set $\alpha_i=\beta_i$, the classical noise source produces the same dispersion in the coordinates of each oscillator as that produced by the vacuum fluctuations. Thus, as far as the coordinates are concerned, both noise sources have similar effects. It is also clear that the dispersion (whether quantum mechanical or classical) of the coordinates of *each* oscillator is due to the fluctuations, or noise, acting on *both* oscillators.

We examine next the radiation spectrum of the parametric amplifier. Most of the power radiated can be expected to lie in the neighborhoods of the resonant frequencies of the two oscillators. We can therefore use the resonance fluorescence expression of Eq. (3.15) for each oscillator:

$$\begin{aligned}P_i(\omega')=&\frac{\beta_i}{\pi}\hbar\omega_i\int_0^\infty d\tau\langle A_i^\dagger(t)A_i(t-\tau)\rangle_{\text{av}}e^{-i(\omega'-\omega_i)\tau}\\&+\text{c.c.},\quad i=1,2 .\end{aligned} \tag{4.14}$$

For oscillator 1, we obtain

$$\langle A_1^\dagger(t)A_1(t-\tau)\rangle=\frac{\beta_2^2|\Omega_1|^2}{(\beta^2-\Omega^2)^2}+N_1 , \tag{4.15a}$$

and for oscillator 2, we obtain

$$\langle A_2^\dagger(t)A_2(t-\tau)\rangle=\frac{|\Omega_0\Omega_1|^2}{(\beta^2-\Omega^2)^2}+N_2 , \tag{4.15b}$$

where

$$\begin{aligned}N_i&=\frac{|\Omega_0|^2}{\Omega^2}\int_{-\infty}^t dt_1\int_{-\infty}^{t-\tau}dt_2\langle F_j(t_1)F_j^\dagger(t_2)\rangle e^{-\beta(2t-\tau-t_1-t_2)}\sinh[\Omega(t-t_1)]\sinh[\Omega(t-\tau-t_2)]\\&=\frac{1}{4}\frac{|\Omega_0|^2\beta_j}{\Omega\beta(\beta^2-\Omega^2)}[(\beta+\Omega)e^{-(\beta-\Omega)\tau}-(\beta-\Omega)e^{-(\beta+\Omega)\tau}],\quad i\neq j .\end{aligned} \tag{4.15c}$$

The power spectrum of the radiation is given by

$$P(\omega')=P_1(\omega')+P_2(\omega') , \tag{4.16a}$$

where

$$P_1(\omega')=2\beta_1\hbar\omega_1\frac{\beta_2^2|\Omega_1|^2}{(\beta^2-\Omega^2)^2}\delta(\omega'-\omega_1)+P_1^{(n)}(\omega') , \tag{4.16b}$$

$$P_2(\omega')=2\beta_2\hbar\omega_2\frac{|\Omega_0\Omega_1|^2}{(\beta^2-\Omega^2)^2}\delta(\omega'-\omega_2)+P_2^{(n)}(\omega') , \tag{4.16c}$$

with

$$P_i^{(n)}(\omega')=\frac{\beta_1\beta_2|\Omega_0|^2}{2\pi\beta\Omega}\hbar\omega_i\left\{\frac{1}{(\beta-\Omega)^2+(\omega'-\omega_i)^2}-\frac{1}{(\beta+\Omega)^2+(\omega'-\omega_i)^2}\right\} . \tag{4.16d}$$

The first terms in Eqs. (4.16b) and (4.16c) are obviously due to the signal input. The first term in P_1 represents the amplified signal, the power amplification factor being $\beta_1^2\beta_2^2(\beta_1\beta_2-|\Omega_0|^2)^{-2}$. The first term in P_2 is the result of the mixing of signal and pump. $P_1^{(n)}$ and $P_2^{(n)}$ describe noise with a spectrum centered about the respective frequencies of the two oscillators. In contrast to the case of the DDO, the vacuum fluctuations in this case *do* generate noise power. However, this is accomplished in an indirect manner. Equation (4.15c) shows that the noise output of oscillator 1 is due to the vacuum fluctuations acting on oscillator 2 only, and vice versa. This should be contrasted with the dispersion in the coordinates, where the vacuum fluctuations acting on both oscillators contribute to the dispersion of the coordinates of each.

In order for an oscillator to radiate noise power it must be supplied with noise power. The question therefore arises, how do vacuum fluctuations, which cannot do any work, account for noise power being supplied to each oscillator in the parametric amplifier? An insight into this matter can be gained by examining the expression for $\langle (d/dt)(A_1^\dagger A_1)\rangle$, the power (in units of $h\omega_1$) absorbed by oscillator 1. (A similar consideration applies to oscillator 2.) From Eq. (4.4a), we have

$$\frac{d}{dt}(A_1^\dagger A_1)=-\Omega_0^* A_2 A_1-\Omega_1^* A_1-F_1^\dagger A_1-\beta_1 A_1^\dagger A_1+\text{H.c.} \tag{4.17}$$

The formal interpretation of the operators on the right-hand side is as follows: the first term corresponds to power supplied by the pump through mixing with oscillator 2; the second term corresponds to power supplied by the signal input; the third term corresponds to power supplied by the vacuum fluctuations; the fourth term corresponds to power lost by radiation. When the expectation value of both sides is taken, the third term drops out, which is consistent with the fact that vacuum fluctuations do no work. The second term supplies only monochromatic power. The presence of A_2 in the first term indicates that the pump power contains modulation by the coordinates of oscillator 2, which, as was shown previously, exhibit quantum noise. The noise power supplied to oscillator 1, therefore, comes from the pump. It is easily checked that for $\Omega_1=0$, we have

$$\hbar\omega_1\langle -2\Omega_0^* A_2 A_1\rangle=\int_0^\infty d\omega' P_1^{(n)}(\omega')=\frac{\beta_1\beta_2|\Omega_0|^2\hbar\omega_1}{\beta(\beta^2-\Omega^2)} , \tag{4.18}$$

that is, the noise power supplied by the pump is equal to the noise power radiated. Thus, the vacuum fluctuations, by modulation, produce noise power in oscillator 1 through the quantum noise of the coordinates of oscillator 2 without doing any work.

We consider now the effect of a classical noise source—in place of the vacuum fluctuations—on the radiation spectrum of the parametric amplifier. Use of Eqs. (4.12) and (4.14) yields the same monochromatic part of the spectrum as that in the quantum-mechanical case, but the noise spectrum is now given by

$$P_i^{(n)(c)}(\omega')=\frac{\beta_i}{4\pi\beta}\hbar\omega_i\left\{\alpha_i\left[\frac{(2\beta-\Omega+k_{ij}\Omega)(1+k_{ij})}{(\beta-\Omega)^2+(\omega'-\omega_i)^2}+\frac{(2\beta+\Omega+k_{ij}\Omega)(1-k_{ij})}{(\beta+\Omega)+(\omega'-\omega_i)^2}\right]+\alpha_j\frac{|\Omega_0|^2}{\Omega}\left[\frac{1}{(\beta-\Omega)^2+(\omega'-\omega_i)^2}-\frac{1}{(\beta+\Omega)^2+(\omega'-\omega_i)^2}\right]\right\},\quad i\neq j . \tag{4.19}$$

One can see here both the direct and indirect effects of the classical noise source. The α_i term indicates the direct effect, due to work done by the classical noise source, and the α_j term indicates the indirect effect, due to noise modulation of the pump output that requires no work on the part of the noise source. For $\alpha_j=2\beta_j$, $\alpha_i=0$, the spectrum is the same as that in the quantum-mechanical case.

V. DEGENERATE PARAMETRIC AMPLIFIER

A degenerate parametric amplifier (DPA) is one in which the same oscillator serves as both the signal and idler oscillator. This system, which may be considered a subharmonic amplifier—since the signal frequency is half the pump frequency—has been of wide interest in recent years as a source of squeezed light. Its Hamiltonian, in notation similar to that of the nondegenerate parametric amplifier, is given by

$$H=\hbar\omega a^\dagger a+\sum_k \hbar\omega_k a_k^\dagger a_k - i\hbar(\Omega_0 e^{-i\omega_0 t}a^{\dagger 2}-\Omega_0^* e^{i\omega_0 t}a^2)$$
$$-i\hbar(\Omega_1 e^{-i\omega t}a^\dagger-\Omega_1^* e^{i\omega t}a)$$
$$-\tfrac{1}{2}i\hbar\sum_k \gamma_k(a_k a^\dagger - a_k^\dagger a)\ , \tag{5.1}$$

with $\omega_0=2\omega$. The equations of motion for the reduced variables are

$$\dot A=-2\Omega_0 A^\dagger-\Omega_1-F-\beta A\ ,$$
$$\dot A^\dagger=-2\Omega_0^* A-\Omega_1^*-F^\dagger-\beta A^\dagger\ . \tag{5.2}$$

For notational simplicity, we set $|\Omega_0|=\Omega$. The condition for a stable steady-state solution is then given by

$$4\Omega^2<\beta^2\ , \tag{5.3}$$

with the solution being

$$A=\frac{-\beta\Omega_1+2\Omega_0\Omega_1^*}{\beta^2-4\Omega^2}$$
$$-\int_{-\infty}^t dt_1 F(t_1)e^{-\beta(t-t_1)}\cosh[2\Omega(t-t_1)]$$
$$+\frac{\Omega_0}{\Omega}\int_{-\infty}^t dt_1 F^\dagger(t_1)e^{-\beta(t-t_1)}$$
$$\times\sinh[2\Omega(t-t_1)]\ , \tag{5.4}$$

together with the Hermitian conjugate. Comparison of the solution with that for the nondegenerate parametric amplifier shows that it can be obtained from the latter by setting $\beta_1=\beta_2=\beta$, $F_1=F_2=\frac{1}{2}F$, making the transformation $\Omega_0\to 2\Omega_0$, and letting $A=A_1+A_2$. It should be noted, for later discussion of squeezing, that the response of the DPA (to the signal and vacuum fluctuations) consists of a superposition of two sets of components, one with the phases of Ω_1 and F_1 and the other with corresponding conjugate phases, that is, with those of Ω_1^* and $F_1^\dagger$.

In the present instance, we examine the radiation spectrum first and the coordinates later, since behavior of the coordinates will be related to the discussion of squeezing. The spectrum of the radiation into free space by the DPA is given by

$$P(\omega')=2\beta\hbar\omega\frac{|2\Omega_0\Omega_1^*-\beta\Omega_1|^2}{(\beta^2-4\Omega^2)^2}\delta(\omega'-\omega)$$
$$+\frac{\beta\Omega}{\pi}\hbar\omega\left[\frac{1}{(\beta-2\Omega)^2+(\omega'-\omega)^2}-\frac{1}{(\beta+2\Omega)^2+(\omega'-\omega)^2}\right]\ . \tag{5.5}$$

It is seen that the noise spectrum consists of the difference of two Lorentzians centered about the resonant frequency and is qualitatively similar to the noise spectrum of either one of the oscillators in the nondegenerate parametric amplifier.

For classical noise acting on the DPA, we use, as previously, the quantum-mechanical solution of the equations of motion with F replaced by F_c. The monochromatic part of the spectrum (the signal power) is unchanged, while the noise spectrum becomes

$$P^{(n)(c)}(\omega')=\frac{\alpha\beta}{2\pi}\hbar\omega\left[\frac{1}{(\beta-2\Omega)^2+(\omega'-\omega)^2}+\frac{1}{(\beta+2\Omega)^2+(\omega'-\omega)^2}\right]\ . \tag{5.6}$$

This expression can be rewritten as

$$P^{(n)(c)}(\omega')=P_\alpha(\omega')+P_\Omega(\omega')\ , \tag{5.7}$$

where

$$P_\alpha(\omega')=\frac{\alpha}{2\pi}\hbar\omega\left[\frac{\beta-2\Omega}{(\beta-2\Omega)^2+(\omega'-\omega)^2}+\frac{\beta+2\Omega}{(\beta+2\Omega)^2+(\omega'-\omega)^2}\right]$$

and

$$P_\Omega(\omega')=\frac{\alpha\Omega}{\pi}\hbar\omega\left[\frac{1}{(\beta-2\Omega)^2+(\omega'-\omega)^2}-\frac{2}{(\beta+2\Omega)^2+(\omega'-\omega)^2}\right]\ .$$

$P_\Omega(\omega')$ becomes identical to the noise power in the quantum mechanical case for $\alpha=\beta$. Now, the total power contained in $P_\alpha(\omega')$ is given by

$$\int_0^\infty d\omega' P_\alpha(\omega')=\alpha\hbar\omega\ . \tag{5.8}$$

From the equation

$$\frac{d}{dt}(A^\dagger A)=-2\Omega_0^* A^2-\Omega_1^* A-F_c A-\beta A^\dagger A+\text{H.c.}\ , \tag{5.9}$$

we see that the power input from the classical noise source is given by

$$\hbar\omega\langle -F_c A\rangle+\text{H.c.}$$
$$=\hbar\omega\alpha\int_{-\infty}^t dt_1\langle F_c(t)F_c^*(t_1)\rangle$$
$$\times\cosh[2\Omega(t-t_1)]+\text{H.c.}\ ,$$
$$=\alpha\hbar\omega\ , \tag{5.10}$$

where Eq. (3.10) has been utilized. Thus $P_\alpha(\omega')$ can be viewed as the reradiation of all the power coming from the noise source. Since $P_\Omega(\omega')$ is additional noise power, it follows that $P_\Omega(\omega')$ is due to modulation of the pump input by the noise source without any absorption of power from the noise source. This is just the "effortless" effect that the vacuum fluctuations have in the production of noise power radiated by the DPA.

We consider next the dispersion of q and p. Equation (5.4) yields

$$q(t)=\langle q(t)\rangle-2^{-1/2}\int_{-\infty}^{t}dt_1\{[F(t_1)e^{-i\omega t}+F^{\dagger}(t_1)e^{i\omega t}]\cosh[2\Omega(t-t_1)]-[F^{\dagger}(t_1)e^{-i(\omega t+\theta)}+F(t_1)e^{i(\omega t+\theta)}]\sinh[2\Omega(t-t_1)]\}\ , \tag{5.11}$$

where θ is defined by $\Omega_0=\Omega e^{-i\theta}$ and $\langle q(t)\rangle$ is given by

$$\langle q(t)\rangle=\frac{2^{-1/2}(2\Omega_0\Omega_1^*-\beta\Omega_1)}{\beta^2-4\Omega^2}e^{-i\omega t}+\text{c.c.} \tag{5.12}$$

In calculating $\langle q^2(t)\rangle$ we encounter terms which vary as $e^{\pm 2i\omega t}$. The reason double-frequency terms do not occur in the nondegenerate case is that the coefficients of such terms contain the vanishing factor $\langle F_i(t_1)\,F_j^{\dagger}(t_2)\rangle$, $i\neq j$, while, in the present case, the coefficients of the double-frequency terms contain the nonvanishing factor $\langle F(t_1)\,F^{\dagger}(t_2)\rangle$. A calculation yields

$$\langle q^2(t)\rangle-\langle q(t)\rangle^2=\frac{1}{2}\frac{\beta^2}{\beta^2-4\Omega^2}\left[1-\frac{2\Omega}{\beta}\cos(2\omega t+\theta)\right]\ , \tag{5.13}$$

$$\langle p^2(t)\rangle-\langle p(t)\rangle^2=\frac{1}{2}\frac{\beta^2}{\beta^2-4\Omega^2}\left[1+\frac{2\Omega}{\beta}\cos(2\omega t+\theta)\right]\ . \tag{5.14}$$

It is easily seen that the dispersions of q and p reach minima of $\frac{1}{2}(1+2\Omega/\beta)^{-1}$ with a frequency of 2ω. The DPA is therefore in a squeezed state.

It should be noted that the oscillation of dispersion is not a quantum-mechanical phenomenon, but one that applies to all dispersion, classical as well as quantum mechanical [16]. Thus, if we consider the noise to come from F_c rather than F, use of Eq. (5.4) with F replaced by F_c yields

$$\langle q_c^2(t)\rangle-\langle q_c(t)\rangle^2=\frac{1}{2}\frac{\alpha\beta}{\beta^2-4\Omega^2}\left[1-\frac{2\Omega}{\beta}\cos(2\omega t+\theta)\right]\ , \tag{5.15}$$

$$\langle p_c^2(t)\rangle-\langle p_c(t)\rangle^2=\frac{1}{2}\frac{\alpha\beta}{\beta^2-4\Omega^2}\left[1+\frac{2\Omega}{\beta}\cos(2\omega t+\theta)\right]\ . \tag{5.16}$$

It is seen that *all* noise processed by the DPA, whether classical or quantum mechanical, produces a dispersion which oscillates in magnitude with a frequency of 2ω and phase determined by that of the pump. The reason is the fact that the response of the DPA to incoming noise contains the superposition of a component with the phase of the noise and a component with the conjugate phase [17].

VI. SUMMARY AND CONCLUSIONS

As mentioned in the Introduction, vacuum fluctuations, as well as their effects, are usually referred to in quantum optics as quantum noise. It was seen that, for a single mode, vacuum fluctuations are not noise in the sense of a random process, but for the superposition of the field of a large number of modes and, in particular, for the free-space field, vacuum fluctuations can indeed by regarded as noise. Can this noise be observed, or detected, as is classical noise? In order to answer this question, the effect of vacuum fluctuations on three types of systems, a harmonic oscillator, a nondegenerate parametric amplifier, and a degenerate parametric amplifier, was examined and compared with the effect of classical noise. It was shown that the response of the coordinates to vacuum fluctuation is formally similar to their response to classical noise. In other words, the quantum noise of the coordinates is formally similar to their classical noise. It turns out, however, that the noise part of the radiation from the systems into free space, that is, the noise part of their resonance fluorescence spectrum (which, in the present article, is considered to be the evidence of detecting the noise), is completely different for the quantum noise than for the classical noise. In the case of the harmonic oscillator, the quantum noise present in the coordinates is absent in the radiation spectrum. When driven by a sinusoidal signal, and under the influence of vacuum fluctuations, the spectrum of the radiation is monochromatic. On the other hand, when the vacuum fluctuations are replaced by classical white noise, the radiation spectrum exhibits both the monochromatic part due to the signal and a Lorentzian distribution due to the noise. In the case of the nondegenerate parametric amplifier, the two types of spectra are likewise different. Here there does exist spectral noise due to vacuum fluctuations. Derivation of the result shows that noise radiated by oscillator 1 is due to vacuum fluctuations acting on oscillator 2, and vice versa. The vacuum fluctuations act only indirectly, noise-modulating the pump power in each of the oscillators through the coordinates of the other. With a classical noise source, the noise radiated by each oscillator consists not only of noise similar to that produced indirectly by the vacuum fluctuations but also of additional noise produced by a direct effect on each oscillator. In the case of a degenerate parametric amplifier, the spectra are qualitatively similar to those of either one of the oscillators in the nondegenerate parametric amplifier. The similarity disappears, however, when the dispersion in coordinates is examined. The difference here is not between quantum noise and classical noise but between the degenerate and nondegenerate parametric amplifiers. In the degenerate system, the dispersion of the coordinates, whether classical or quantum mechanical oscillates at the pump frequency. For quantum noise, the minimum is sufficiently low to produce squeezing. For classical noise, the minimum can be made arbitrarily low by a proper choice of parameters. In other words, the degenerate system is described by a squeezed state, quantum mechanically, and by a squeezed probability density, classically. The squeezing is due to the fact that the response of the coordinates contains the superposition of a term with the phase of the incoming noise and a term with the conjugate phase.

It is natural to ask why noise in the coordinates produced by vacuum fluctuations is not radiated in the same manner as formally similar noise produced by classical sources. The answer lies in the fact that zero-point oscillations can do no work. Vacuum fluctuations, which are zero-point oscillations of the vacuum, induce zero-point oscillation in systems coupled to the vacuum. While an uncoupled harmonic oscillator exhibits sinusoidal zero-point oscillation, it is damped out by the vacuum acting as a loss mechanism and replaced by noisy zero-point oscillation induced by the vacuum fluctuations. Thus, although the coordinates of the driven damped harmonic oscillator exhibit noise, this is zero-point noise, and cannot be radiated. The same can be said about the other systems considered. Generally, one can say that the component of the quantum noise in the coordinates that is not radiated is zero-point noise. The component that is radiated is due to modulation by the vacuum fluctuations of power from an outside source, modulation which does not require work. This modulating effect is displayed in the two types of parametric amplifiers, where the outside source is the pump.

[1] R. Loudon and P. L. Knight, J. Mod. Opt. **34**, 709 (1987).

[2] J. Opt. Soc. Am. B, Special Issue on squeezed states of the electromagnetic field **4** (1987); J. Mod. Opt., Special Issue on squeezed light **34** (1987); see Ref. [1] for numerous additional references.

[3] I. R. Senitzky, Phys. Rev. **119**, 670 (1960); **124**, 642 (1961).

[4] W. H. Louisell and L. R. Walker, Phys. Rev. **137**, B204 (1965).

[5] M. Lax, J. Phys. Chem. Solid **25**, 487 (1964); Phys. Rev. **145**, 110 (1966).

[6] W. Weidlich and F. Haake, Z. Phys. **185**, 30 (1965).

[7] W. H. Louisell and J. H. Marburger, J. Quantum Electron. **QE-3**, 348 (1967).

[8] W. H. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1973).

[9] H. Grabert *et al.*, Phys. Rep. **168**, 115 (1988), where additional references can be found.

[10] I. R. Senitzky, Phys. Rev. A **6**, 1175 (1972); **40**, 4435 (1989).

[11] I. R. Senitzky, Phys. Rev. Lett. **31**, 955 (1973), Eq. (9).

[12] R. L. Stratonovich, *Topics in the Theory of Random Noise* (Gordon and Breach, New York, 1963).

[13] I. R. Senitzky, Phys. Rev. Lett. **20**, 1062 (1968); in *Progress in Optics XVI,* edited by Emil Wolf (North-Holland, Amsterdam, 1978), Chap. VII.

[14] There exist other detection methods, such as mixing with a local oscillator output (heterodyning or homodyning), which will not be discussed in the present article.

[15] I. R. Senitzky, Phys. Rev. Lett. **40**, 1334 (1978).

[16] Squeezing of classical noise has been discussed by D. Rugar and P. Grutter, Phys. Rev. Lett. **67**, 699 (1991); and by Frank Di Filipo *et al.*, *ibid.* **68**, 2859 (1992).

[17] In the generation of squeezed light, the oscillator of the DPA is taken to be a mode of the electromagnetic field, so that what appears as squeezing of coordinate noise in the present model becomes squeezing of electromagnetic-field noise, which yields squeezed light.