Nonlocal interferometry with high-intensity fields

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Two-photon interferometry can produce violations of Bell's inequality that demonstrate the nonlocal nature of such effects. It is shown here that high-intensity fields containing large numbers of photons can violate Bell's inequality in a similar manner. The macroscopic nature of these fields is evident from the fact that they can produce large bursts of energy in two distant absorbers with nonlocal correlations between the two.

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There has been considerable interest [1] recently in possible violations of Bell's inequality [2] for systems containing N separated particles. One reason for the interest in such systems is the hope that "macroscopic" effects can be obtained in the limit of large N, which may be of relevance to the correspondence principle, for example. However, most [3] of the proposed experiments require that quantum-mechanical measurements be made on the individual particles, such as a measurement of the spin of each of N electrons. Such a situation corresponds to a more complicated quantum system rather than a macroscopic phenomenon approaching the domain of classical physics.

This paper considers nonlocal interference effects that can occur in high-intensity electromagnetic fields containing large numbers of photons [4]. These fields can produce large bursts of energy in two distant absorbers or detectors that are correlated in such a way as to violate Bell's inequality (given the usual assumption that the observed events are a representative sample). No measurements are required on the individual photons and the effects can be viewed as truly macroscopic in nature as a result.

Reid and Munro [5] have recently discussed nonlocal effects based upon photon polarizations that are also macroscopic in the sense that a large number of photons are incident upon each detector. They considered a single-frequency mode of the field, whereas here the quantum state of interest is a coherent superposition of many modes, which gives rise to nonlocal effects involving dynamic variables such as the time of emission of the photons or their localization in space. As a result, the effects considered here are observable using interferometry rather than polarization measurements. In addition, there are nonlocal correlations between the expectation values of the fields themselves and not just their intensities, so that the measurements could, in principle, be made using a voltmeter.

Sanders has shown [6] that a superposition of two macroscopic coherent states can produce nonlocal correlations. However, the correlation coefficient decreases exponentially with increasing photon number and Bell's inequality can only be violated in the limit of weak fields [7]. This is because the measurement process must project the two orthogonal states that are superposed in the initial wave function onto a common final state in order to produce the interference responsible for the nonlocal correlations, which is very difficult when the two states are macroscopically different. This argument suggests that "Schrödinger cats" in general are not likely candidates for violations of Bell's inequality.

The quantum-mechanical state of interest here is somewhat similar to the entangled photon pairs produced by parametric down-conversion and studied in a number of recent experiments [8]. For sufficiently low intensities, the two-photon field can be adequately described by

$$|\psi_p\rangle = c^\dagger|0\rangle \ , \tag{1}$$

where $|0\rangle$ is the vacuum state and the operator $c^\dagger$ creates a pair of entangled photons:

$$c^\dagger = \sum_k f_k a_k^\dagger b_{k_0-k}^\dagger \ . \tag{2}$$

Here, $a_k^\dagger$ creates a photon of momentum $\hbar k$ in beam A, $b_{k_0-k}^\dagger$ creates a photon of momentum $\hbar(k_0-k)$ in beam B, and f_k represents the effects of a filter inserted into the two beams. Phase-matching conditions require that the sum of the momenta of the two down-converted photons be equal to the momentum k_0 of the incident pump photons. For simplicity, it will be assumed that $f_k=f$, a constant, for $k_L \le k \le k_U$ and zero otherwise; normalization then gives $f=1/N_k^{1/2}$, where N_k is the number of states between k_L and k_U. It will also be assumed that the two beams are sufficiently well collimated that the momenta of the photons in beam A can all be taken to be in the same direction, and similarly for beam B.

The high-intensity field of interest is then given by

$$|\Psi\rangle = \gamma \sum_n \frac{(\alpha c^\dagger)^n}{n!}|0\rangle = \gamma e^{\alpha c^\dagger}|0\rangle \ , \tag{3}$$

where α is a complex constant and γ is required for normalization. Unfortunately, this state cannot be produced by parametric down-conversion using a high-intensity pump field and the effects described below must be viewed, at least for the time being, as a gedanken experiment that illustrates the kind of nonlocal effects that can occur at high field intensities. However, some (but not

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all) of the effects described below also occur for a state $|\psi_n\rangle$ containing precisely n pairs of entangled photons, as discussed in the Appendix, and such states can be produced, at least in principle, using a series of down-conversion crystals to double repeatedly the number of photons.

Equation (3) can be written in the more useful form

$$|\Psi\rangle=\gamma\prod_k e^{\alpha f_k a_k^\dagger b_{k_0-k}^\dagger}|0\rangle \; . \tag{4}$$

The mean number of photon pairs is very high when $|\alpha|\gg 1$. Nevertheless, it can still be assumed that the product $|\alpha f|$ is much less than 1, which allows the use of certain approximations in the power-series expansion of the exponential in Eq. (4). This condition corresponds to an intense field with negligible probability of more than one photon per mode (nondegenerate case) which is well satisfied at the surface of the sun, for example. In the limit of large N_k and $|\alpha f|\ll 1$, it is straightforward to show from Eq. (4) that

$$\langle\Psi|\Psi\rangle=\gamma^2\left[1+\frac{\alpha^*\alpha}{N_k}\right]^{N_k}=\gamma^2 e^{\alpha^*\alpha} \; . \tag{5}$$

This gives $\gamma=\exp(-\alpha^*\alpha/2)$ and demonstrates the convergence of Eqs. (3) and (4) for $|\alpha f|<1$. The mean number of photon pairs in the field can also be shown to be $\alpha^*\alpha$ in the same limit.

Although the form of Eq. (3) is similar to that of the usual single-photon coherent state [9], it does not possess the same properties as a coherent state. In particular,

$$[c,c^\dagger]\neq 1 \; , \tag{6}$$

for which reason

$$c|\Psi\rangle\neq\alpha|\Psi\rangle \; , \tag{7}$$

as would be the case for a coherent state. This state does, however, possess the useful property that

$$a_k|\Psi\rangle=\alpha f_k b_{k_0-k}^\dagger|\Psi\rangle \tag{8}$$

and

$$b_k|\Psi\rangle=\alpha f_k a_{k_0-k}^\dagger|\Psi\rangle \; . \tag{9}$$

The coincidence rate between two detectors in beams A and B is proportional to

$$\langle I_A(x,t_A)I_B(x,t_B)\rangle$$
$$=\langle E_A^-(x,t_A)E_B^-(x,t_B)E_B^+(x,t_B)E_A^+(x,t_A)\rangle \; , \tag{10}$$

where the detectors have been assumed to be equidistant from the source and $E^\pm$ represents the positive and negative frequency components of the electric-field operators. This expression is just the square of the norm of the state $|\phi\rangle$ given by

$$|\phi\rangle=E_A^+(x,t_A)E_B^+(x,t_B)|\Psi\rangle$$
$$=\left[\sum_{k_A}\left[\frac{2\pi\hbar\omega_A}{V}\right]^{1/2}e^{i(k_Ax-\omega_At_A)}a_{k_A}\right]$$
$$\times\left[\sum_{k_B}\left[\frac{2\pi\hbar\omega_B}{V}\right]^{1/2}e^{i(k_Bx-\omega_Bt_B)}b_{k_B}\right]$$
$$\times\gamma\prod_k e^{\alpha f_k a_k^\dagger b_{k_0-k}^\dagger}|0\rangle \; , \tag{11}$$

where V is the volume of the system and ω_A and ω_B are the frequencies corresponding to k_A and k_B. If the filter bandwidth is sufficiently small, the slowly varying factors of ω_A and ω_B can be approximated by $\omega_0/2$ and taken outside the integrals, after which the use of Eq. (9) gives

$$E_A^+(x,t_A)E_B^+(x,t_B)|\Psi\rangle$$
$$=\sum_{k_A}\sum_{k_B}e^{i(k_Ax-\omega_At_A)}e^{i(k_Bx-\omega_Bt_b)}$$
$$\times a_{k_A}(\alpha f)a_{k_0-k_B}^\dagger|\Psi\rangle \; , \tag{12}$$

where an irrelevant constant has been omitted. The commutator $[a_{k_A},a_{k_0-k_B}^\dagger]=\delta_{k_A,k_0-k_B}$ allows this to be rewritten as

$$E_A^+(x,t_A)E_B^+(x,t_B)|\Psi\rangle$$
$$=\alpha f\sum_{k_B}e^{i[(k_0-k_B)x-(\omega_0-\omega_B)t_A]}e^{i(k_Bx-\omega_Bt_B)}|\Psi\rangle$$
$$+\alpha f\sum_{k_A}\sum_{k_B}e^{i(k_Ax-\omega_At_A)}e^{i(k_Bx-\omega_Bt_B)}a_{k_0-k_B}^\dagger a_{k_A}|\Psi\rangle \; . \tag{13}$$

The first term in Eq. (13) is just

$$\alpha f e^{i(k_0x-\omega_0t_A)}\sum_{k_B}e^{-i\omega_B(t_B-t_A)}|\Psi\rangle$$
$$=2\pi\rho\alpha f e^{i(k_0x-\omega_0t_A)}\delta_A(t_A-t_B)|\Psi\rangle \; , \tag{14}$$

where ρ is the density of states and δ_A is a strongly peaked function that approaches a δ function for large N_k. The use of Eq. (8) allows the second term in Eq. (13) to be written as

$$\alpha f\sum_{k_A}\sum_{k_B}e^{i(k_Ax-\omega_At_A)}e^{i(k_Bx-\omega_Bt_B)}a_{k_0-k_B}^\dagger a_{k_A}|\Psi\rangle$$
$$=(\alpha f)^2\sum_{k_A}\sum_{k_B}e^{i(k_Ax-\omega_At_A)}e^{i(k_Bx-\omega_Bt_B)}$$
$$\times a_{k_0-k_B}^\dagger b_{k_0-k_A}^\dagger|\Psi\rangle \; . \tag{15}$$

In the limit of $|\alpha f|\ll 1$, the second term is negligible compared to the first due to the factor of $(\alpha f)^2$, as can be verified by explicitly comparing [10] the contributions of the two terms to $\langle\phi|\phi\rangle$. In that case the state of interest is given by

$$E_A^+(x,t_A)E_B^+(x,t_B)|\Psi\rangle = 2\pi\rho\alpha f e^{i(k_0 x-\omega_0 t_A)}\delta_A(t_A-t_B)|\Psi\rangle \ . \quad (16)$$

Equation (16) shows that the probability of detecting two photons in a sufficiently short time interval is dominated by coincident events corresponding to correlated pairs of photons, while the accidental events from unpaired photons are unlikely in comparison.

It is possible to derive an exact solution for $E_A^+E_B^+|\Psi\rangle$ that is valid for any value of $|\alpha f|<1$ by further use of Eqs. (8) and (9) and the commutation relations. The results obtained in that case agree with those obtained here in the limit of $|\alpha f|\ll 1$. Similar calculations can also be done in a more straightforward manner without the use of Eqs. (8) and (9) for the case of a field containing precisely n pairs of entangled photons, as discussed in the Appendix.

It follows immediately from Eq. (16) that

$$E_A^+(x,t+\Delta t)E_B^+(x,t+\Delta t)|\Psi\rangle = e^{-i\omega_0\Delta t}E_A^+(x,t)E_B^+(x,t)|\Psi\rangle \ . \quad (17)$$

It was noted earlier [11] that Eqs. (16) and (17) are logically inconsistent for classical fields, which must satisfy the following inequality:

$$\langle|E_A^*(t)E_B^*(t)E_B(t+\Delta t)E_A(t+\Delta t)|\rangle \le \tfrac{1}{2}\langle E_A^*(t)E_B^*(t+\Delta t)E_B(t+\Delta t)E_A(t)\rangle + \tfrac{1}{2}\langle E_A^*(t+\Delta t)E_B^*(t)E_B(t)E_A(t+\Delta t)\rangle \ . \quad (18)$$

This inequality is violated by these high-intensity fields. McNeil and Gardiner have previously shown that high-intensity squeezed states can violate the Cauchy-Schwarz inequality [12].

It was also noted earlier [13] that fields satisfying Eqs. (16) and (17) can violate Bell's inequality in experiments [8] employing two distant, identical interferometers with one optical path longer than the other, giving a difference Δt in propagation times. The coincidence rate R_c is then proportional to the square of the norm of the state vector

$$|\phi'\rangle = [E_B^+(x,t_B)+e^{i\phi_B}E_B^+(x,t_B+\Delta t)] \times [E_A^+(x,t_A)+e^{i\phi_A}E_A^+(x,t_A+\Delta t)]|\Psi\rangle \ , \quad (19)$$

where ϕ_A and ϕ_B are phase shifts introduced into the longer optical paths. The norm of this state vector can be evaluated using Eqs. (16) and (17) in the same manner as in Ref. [13], resulting in a coincidence rate R_c given by

$$R_c = \eta\cos^2\left[\frac{\phi_A+\phi_B+\omega_0\Delta t}{2}\right] \ , \quad (20)$$

where η is a constant related to the detection efficiency. This nonlocal dependence on the sum of the phases of the two interferometers violates Bell's inequality even though the field intensity may be quite high.

The coincidence rate of Eq. (20) corresponds to the same correlations between pairs of photons that have already been observed [8] for the case of weak fields. In addition, Reid and Walls [14] have previously shown that two-photon coincidence events can violate Bell's inequality in high-intensity light beams. The truly macroscopic nature of the fields considered here becomes apparent only when we consider events in which a large number of photons are detected or absorbed at two distant locations.

First consider the probability P_2 that two pairs of photons will be detected, which is proportional to the square of the norm of the state $|\phi_2\rangle$ given by

$$|\phi_2\rangle = E_A^+(x,t_{A1})E_A^+(x,t_{A2})E_B^+(x,t_{B1})E_B^+(x,t_{B2})|\Psi\rangle \ . \quad (21)$$

The same approach that gave the factor of $a_{k_A}a^\dagger_{k_0-k_B}$ in Eq. (12) now gives a factor of

$$\begin{aligned} a_{k_{A1}}a_{k_{A2}}a^\dagger_{k_0-k_{B1}}a^\dagger_{k_0-k_{B2}} &= \delta_{k_{A1},k_0-k_{B1}}\delta_{k_{A2},k_0-k_{B2}}+\delta_{k_{A1},k_0-k_{B2}}\delta_{k_{A2},k_0-k_{B1}} \\ &+\delta_{k_{A1},k_0-k_{B1}}a^\dagger_{k_0-k_{B2}}a_{k_{A2}}+\delta_{k_{A2},k_0-k_{B2}}a^\dagger_{k_0-k_{B1}}a_{k_{A1}}+\delta_{k_{A1},k_0-k_{B2}}a^\dagger_{k_0-k_{B1}}a_{k_{A2}} \\ &+\delta_{k_{A2},k_0-k_{B1}}a^\dagger_{k_0-k_{B2}}a_{k_{A1}}+a^\dagger_{k_0-k_{B1}}a^\dagger_{k_0-k_{B2}}a_{k_{A1}}a_{k_{A2}} \ , \end{aligned} \quad (22)$$

where the right-hand side of the equation was obtained directly from the commutation relations. Equation (8) can once again be used to show that the last five terms in the above equation can be neglected for $|\alpha f|\ll 1$, while the remaining two terms give

$$|\phi_2\rangle = (2\pi\rho)^2(\alpha f)^2 e^{i(k_0x-\omega_0t_{A1})}e^{i(k_0x-\omega_0t_{A2})} \times[\delta_A(t_{A1}-t_{B1})\delta_A(t_{A2}-t_{B2}) + \delta_A(t_{A1}-t_{B2})\delta_A(t_{A2}-t_{B1})]|\Psi\rangle \ . \quad (23)$$

The probability of detecting two pairs of photons in the same output channels of two interferometers can then be shown to be

$$P_2 = 2\eta^2\cos^4\left[\frac{\phi_A+\phi_B+\omega_0\Delta t}{2}\right] = 2P_1^2 \ , \quad (24)$$

where P_1 is the corresponding probability of detecting a single pair of photons in the same time interval. This result can be generalized to

$$P_N = N!\eta^N\cos^{2N}\left[\frac{\phi_A+\phi_B+\omega_0\Delta t}{2}\right] = N!P_1^N \ , \quad (25)$$

where P_N is the probability of detecting exactly N photon pairs.

The factor of $N!$ in Eq. (25) causes P_N to be much larger than the product of the probabilities of N independent events. The origin of the $N!$ factor for the case of $N=2$ is evident from the two δ functions in Eq. (23), which show that photon $A1$ can be coincident with photon $B1$ while $A2$ is coincident with $B2$, or $A1$ can be coincident with $B2$ while $A2$ is coincident with $B1$. Constructive interference between these amplitudes causes each photon to be strongly correlated with all the others, despite the fact that the state of Eq. (3) appears to be a product of uncorrelated pairs. The origin of these correlations is thus similar in some respects to the photon bunching that occurs for stochastic (thermal) light.

The probability that N_1 pairs will be detected in output channel 1 and N_2 in channel 2 of each interferometer can be shown to be $N_1!N_2!\eta^{N_1+N_2}$ for $\Phi=\phi_1+\phi_2+\omega_0\Delta t=0$. This virtually ensures that all photons will be detected in the same output channel for large N and $\Phi=0$, since $N_1!N_2! \ll (N_1+N_2)!$ unless N_1 or N_2 is zero.

P_N initially decreases for increasing values of N but then increases rapidly for larger values of N, since the factor of $N!$ will eventually become comparable to $1/P_1^N$. This suggests that the most likely event to occur is one in which a large number of photons are simultaneously detected in the same channel of both interferometers. It must be kept in mind, however, that the expressions for the coincidence rates given by Eqs. (10) and (21), although widely used [15], are valid only to lowest order in perturbation theory and neglect the depletion of the initial state, which can produce probabilities greater than unity if applied to sufficiently long time intervals [16]. Equation (25) diverges for the same reason in the limit of large N. As a result, Eq. (25) is valid only for sufficiently short time intervals and for values of $N \le N_{\max}$, with $N_{\max}$ chosen in such a way that all of the included P_N are sufficiently small. A more general treatment of the correlated photon-counting statistics may require the use of generating functions [17] and is beyond the intended scope of this paper.

Events corresponding to arbitrarily large values of N can still be observed experimentally, at least in principle, under conditions for which Eq. (25) remains valid. A correspondingly short time interval must be chosen for which all the relevant probabilities are sufficiently small, after which the observation can be repeated until such an event occurs. No single-photon detectors are required to observe such events, which correspond to correlated bursts of energy that can be deposited in any absorbing material, such as a bolometer.

It may be worth noting that the direct use of perturbation theory rather than Eqs. (10) and (21) would give an additional factor of $1/N!$ associated with the power-series expansion of

$$\exp(-i\int H'(t')dt'/h) \ .$$

It is not difficult to show, however, that this $1/N!$ is canceled by the $N!$ different orders in which the operators

$$E^+(r_1), E^+(r_2), \ldots, E^+(r_N)$$

can occur, leaving only the $N!$ associated with the pairing of the photon momenta, as discussed above. It is for this reason that $1/N!$ does not appear in Eqs. (10) or (21) or their higher-order generalizations [15]. It is also for this reason that the factor-of-2 enhancement in ordinary photon bunching is not canceled out by the $1/2!$ in the corresponding power-series expansion.

Equations (24) and (25) violate Bell's inequality [18] provided that it is assumed that the observed events form a representative sample. This assumption was required in earlier experiments due to the limited detection efficiencies of the available detectors, whereas here the use of perturbation theory requires that the events of interest occur with relatively small probabilities. In either case, a Bell inequality can still be derived provided that the subset of observed events is assumed to be characterized by a set of hidden variables that ensures that they will be detected with a probability of unity regardless of the settings of the adjustable experimental parameters. To be more specific, it suffices here to assume that the total number of photons and their potential arrival times at a set of perfect detectors are unaffected by the settings of the phase shifters. These two conditions would automatically be satisfied as a result of energy conservation and the known speed of light, respectively, but are assumptions nevertheless within the context of hidden-variable theories in general. It should also be noted that the response time of the detectors has been taken to be negligibly small and that noncoincident events are to be rejected, as in the case of two-photon interferometry [11,13], which is necessary to eliminate those events in which one photon of a pair has taken the longer path through an interferometer while the other has taken the shorter path.

The coincidence rates of Eqs. (24) and (25) correspond to a nonlocal correlation between the intensities of the two fields in paths A and B. The same intensity correlations can also be obtained from a state $|\psi_n\rangle$ containing a definite number of pairs of entangled photons, as shown in the Appendix. These intensity correlations are shown in the Appendix to be due to a coherent superposition of the probability amplitudes for the different momenta $\hbar k$ in an expansion of the wave function of an entangled pair, as in Eq. (2); there are a large number of initial momentum states that can all contribute to the same final state if both photons of a pair are annihilated at the same time.

The state $|\Psi\rangle$ also gives rise to nonlocal correlations between the fields themselves, as can be seen by taking the inner product of Eq. (21) with the initial state and using Eq. (23), which gives

$$\begin{aligned}\langle\Psi|E_A^+(x,t_{A1})E_A^+(x,t_{A2})E_B^+(x,t_{B1})E_B^+(x,t_{B2})|\Psi\rangle \\ = c(2\pi\rho)^2(\alpha f)^2 e^{i(k_0x-\omega_0 t_{A1})}e^{i(k_0x-\omega_0 t_{A2})} \\ \times[\delta_A(t_{A1}-t_{B1})\delta_A(t_{A2}-t_{B2}) \\ +\delta_A(t_{A1}-t_{B2})\delta_A(t_{A2}-t_{B1})] \ , \qquad (26)\end{aligned}$$

where c is an irrelevant constant. On the other hand, the expectation value of the fields in path A by themselves, obtained by tracing over the field in path B, is zero:

$$\langle E_A^+(x,t_{A1})\rangle = \langle E_A^+(x,t_{A1})E_A^+(x,t_{A2})\rangle = 0 \ . \qquad (27)$$

This nonlocal correlation of the fields themselves is dependent upon the coherence between the probability amplitudes corresponding to different numbers of pairs in the field and is a feature of the state $|\Psi\rangle$ but not $|\psi_n\rangle$.

The correspondence principle is sometimes illustrated by saying that we must approach the classical limit "when the beam consists of many, many photons" [19]. The effects derived above show that that is not the case and that Bell's inequality can be violated by macroscopic events that do not require the use of single-photon detectors.

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APPENDIX

The properties of a coherentlike state $|\Psi\rangle$ containing an indefinite number of entangled pairs of photons were derived in the text using Eqs. (8) and (9). It will be shown here that some of the same properties can be obtained from a state $|\psi_n\rangle$ containing a definite number n of pairs of photons:

$$|\psi_n\rangle=\epsilon(c^\dagger)^n|0\rangle\ , \tag{A1}$$

where ϵ is a normalization constant. These calculations can be done without the use of Eqs. (8) and (9) and provide some additional insight into the origin of the effects discussed in the text.

The probability P_1 of detecting a single pair of coincident photons is calculated in a manner very similar to that for the familiar case of $n=1$ and need not be considered in detail. The first nontrivial results are obtained for the probability P_2 of detecting two coincident pairs of photons. As in the text, P_2 is proportional to the norm of the state vector

$$|\phi\rangle=E_A^+(x,t_{A1})E_A^+(x,t_{A2})E_B^+(x,t_{B1})E_B^+(x,t_{B2})|\psi_n\rangle\ . \tag{A2}$$

Equation (A2) corresponds to the situation in which no beam splitters are present, but the results can be readily generalized to the case of an interferometer, as was done in the text. All nonessential constants and the many factors of $\exp(ikx-\omega t)$ will be dropped in what follows in order to make the notation more compact; the effects of the $\exp(ikx-\omega t)$ factors are essentially the same as in the text and can be reinserted at the end of the calculation.

Inserting Eq. (2) into Eq. (A1) and multiplying out all the terms gives

$$|\psi_n\rangle=\epsilon\sum_{k_1}\sum_{k_2}\cdots\sum_{k_n}\prod_{i=1}^{n}(f_{k_i}a_{k_i}^\dagger b_{k_0-k_i}^\dagger)|0\rangle\ . \tag{A3}$$

The state $|\phi\rangle$ is then

$$|\phi\rangle=\frac{\epsilon}{V^2}\sum_{p,p',q,q'}a_pa_{p'}b_qb_{q'}\sum_{k_1,k_2,\ldots,k_n}\prod_{i=1}^{n}(f_{k_i}a_{k_i}^\dagger b_{k_0-k_i}^\dagger)|0\rangle\ , \tag{A4}$$

where the factor of $1/V^2$ comes from the electric-field operators. The photon momenta associated with the field operators in path A have been denoted by $\hbar p$ and $\hbar p'$, while the momenta in path B have been denoted by $\hbar q$ and $\hbar q'$.

It will be useful to separate $|\phi\rangle$ into three components depending on whether or not $q=k_0-p$, etc.:

$$\begin{aligned}|\phi\rangle=&\left\{\sum_p a_pb_{k_0-p}\sum_{p'}a_{p'}b_{k_0-p'}+\sum_p a_pb_{k_0-p'}\sum_{p'}a_{p'}b_{k_0-p}+\sum_p a_pb_{k_0-p}\sum_{p'}\sum_{q'}{}'a_{p'}b_{q'}+\cdots+\sum_p\sum_q{}'\sum_{p'}\sum_{q'}{}'a_pb_qa_{p'}b_{q'}\right\}\\ &\times\frac{\epsilon}{V^2}\sum_{k_1,k_2,\ldots,k_n}\prod_{i=1}^{n}(f_{k_i}a_{k_i}^\dagger b_{k_0-k_i}^\dagger)|0\rangle\\ &\equiv\{|\phi_2\rangle+|\phi_1\rangle+|\phi_0\rangle\}\frac{\epsilon}{V^2}\sum_{k_1,k_2,\ldots,k_n}\prod_{i=1}^{n}(f_{k_i}a_{k_i}^\dagger b_{k_0-k_i}^\dagger)|0\rangle\ .\end{aligned} \tag{A5}$$

The summations with primes omit the terms with $q=k_0-p$, etc., which are explicitly contained in the first two terms. The state $|\phi_2\rangle$, consisting of the first two terms of Eq. (A5), corresponds to the situation in which two entangled pairs of photons were completely annihilated. The state $|\phi_0\rangle$, the fourth term of Eq. (A5), corresponds to a situation in which no pairs were completely annihilated but four pairs had one of their photons annihilated. Finally, $|\phi_1\rangle$, the third term of Eq. (A5), corresponds to completely annihilating one pair of photons while annihilating only one photon from each of two other pairs; there are three other terms of this type corresponding to other pairings of p and p' with q and q' which are not written out in Eq. (A5) since they will all be found to be negligible.

These three states are orthogonal to each other and they contribute independently to the total coincidence rate as a result. First, consider the contribution to P_2 from $\langle\phi_0|\phi_0\rangle$, which corresponds to the purely accidental or background coincidence rate. The orthogonality of the various final states will be found to dominate the effects obtained and the inner product must be written out explicitly:

$$\langle \phi_0|\phi_0\rangle = \frac{\epsilon^2}{V^4}\langle 0| \sum_{\hat{k}_1,\hat{k}_2,\ldots,\hat{k}_n} \prod_{j=1}^{n} (f^*_{\hat{k}_j} a_{\hat{k}_j} b_{k_0-\hat{k}_j}) \sum_{\hat{p},\hat{q},\hat{p}',\hat{q}'}{}' a^\dagger_{\hat{p}} a^\dagger_{\hat{p}'} b^\dagger_{\hat{q}} b^\dagger_{\hat{q}'} \sum_{p,p',q,q'}{}' a_p a_{p'} b_q b_{q'} \times \sum_{k_1,k_2,\ldots,k_n} \prod_{i=1}^{n} (f_{k_i} a^\dagger_{k_i} b^\dagger_{k_0-k_i})|0\rangle \; . \tag{A6}$$

The carets over the $\hat{p}$, $\hat{q}$, and $\hat{k}$ terms in the bra vector are a convenient way to signify that their values may in general be different from p, q, and k; this is not meant to imply that these quantities are operators.

The operator a_p will give zero unless $p=k_i$ for some value of i; this can happen for n different values of i and contributes a factor of n. The same argument applies for p', q, and q', giving an overall factor of n^4 in the limit of large n and eliminating the sum over p, p', q, and q'.

Since no pair of photons was totally annihilated in the state $|\phi_0\rangle$, the inner product is zero unless $\hat{k}_j=k_i$ for some pairing of the i and j. There are $n!$ different ways in which the k_i and $\hat{k}_j$ can be matched with each other, but this factor also appears in the normalization constant ϵ^2 and cancels out. This leaves only the sum over each k_i and gives

$$\langle \phi_0|\phi_0\rangle = \frac{n^4}{V^4} f_k^{2n} \sum_{k_1,k_2,\ldots,k_n} (1) = \frac{n^4}{V^4} f_k^{2n} N_k^n \tag{A7}$$

in the nondegenerate limit.

The factor of unity that appears in the above summations should contain a rapidly varying phase from the $\exp(ikx-\omega t)$ terms that were not included in the equations. This will greatly reduce the right-hand side of Eq. (A7) but still provides an upper bound of

$$\langle \phi_0|\phi_0\rangle \le \left[\frac{n}{V}\right]^4 , \tag{A8}$$

where $f_k=1/N_k^{1/2}$ has been used.

The contribution from $\langle \phi_2|\phi_2\rangle$ corresponds to the complete annihilation of two entangled pairs of photons and is given by

$$\langle \phi_2|\phi_2\rangle = 2\frac{\epsilon^2}{V^4}\langle 0| \sum_{\hat{k}_1,\hat{k}_2,\ldots,\hat{k}_n} \prod_{j=1}^{n} (f^*_{\hat{k}_j} a_{\hat{k}_j} b_{k_0-\hat{k}_j}) \sum_{\hat{p},\hat{p}'} a^\dagger_{\hat{p}} a^\dagger_{\hat{p}'} b^\dagger_{k_0-\hat{p}} b^\dagger_{k_0-\hat{p}'} \times \sum_{p,p'} a_p a_{p'} b_{k_0-p} b_{k_0-p'} \sum_{k_1,k_2,\ldots,k_n} \prod_{i=1}^{n} (f_{k_i} a^\dagger_{k_i} b^\dagger_{k_0-k_i})|0\rangle \; . \tag{A9}$$

Here, a_p and $a_{p'}$ can each act on n different creation operators but b_q and $b_{q'}$ must then act on the corresponding terms, giving only a total of n^2 possible combinations rather than the n^4 obtained from Eq. (A6). The factor of 2 comes from the fact that there are two terms in Eq. (A5) in which q and q' are interchanged and this corresponds to the factor of 2! in the text.

If k_p and $k_{p'}$ represent the momenta of the two pairs that were completely annihilated, Eq. (A9) can be written as

$$\langle \phi_2|\phi_2\rangle = 2\frac{n^2}{V^4} f_k^{2n} \sum_{\hat{k}_p} \sum_{\hat{k}_{p'}} \sum_{k_1,k_2,\ldots,k_n} (1) = 2\frac{n^2}{V^4} f_k^{2n} N_k^{n+2} \; . \tag{A10}$$

Here, $\hat{k}_p$ need not equal k_p, and $\hat{k}_{p'}$ need not equal $k_{p'}$, since those pairs were annihilated and no longer appear in the final state, and Eq. (A10) contains two more summations than does Eq. (A7). The physical meaning of this is that there are a large number of different initial momentum states that can all lead to the same final state and the coherent superposition of their probability amplitudes increases the coincident counting rate by a factor of N_k^2 compared to the accidental rate. Equation (A10) can be rewritten as

$$\langle \phi_2|\phi_2\rangle = 2\left[\frac{n}{V}\right]^2 \rho^2 , \tag{A11}$$

since $\rho=N_k/V$. When the factors of $\exp(ikx-\omega t)$ are included, it is found that they give rise to a series of δ_A functions in the measurement times, as in the text, and that these phase factors all cancel out for coincident measurement times, leaving only Eq. (A11) in that case.

The ratio of the accidental to coincident detection events is

$$\frac{\langle \phi_0|\phi_0\rangle}{\langle \phi_2|\phi_2\rangle} = \frac{1}{2}\left[\frac{n}{V}\right]^2 \rho^{-2} = \frac{1}{2}\left[\frac{n}{N_k}\right]^2 . \tag{A12}$$

In the nondegenerate limit $n/N_k \ll 1$ and Eq. (A12) explicitly shows that the accidental rate is negligible in that case. This is equivalent to the approximation $\alpha f \ll 1$ used in the text, since $\alpha=\langle n\rangle^{1/2}$ for the coherentlike state $|\Psi\rangle$. The remaining contribution from $\langle \phi_1|\phi_1\rangle$ can be shown to be negligible in a similar way.

Thus it can be seen that the nonlocal correlations in the intensities are the result of a coherent superposition of momentum components inherent in each pair of entangled photons, all of which lead to the same final state, and this is not dependent on coherence between terms with different values of n. The latter is, however, responsible for the correlations between the fields themselves as seen in Eq. (26), which is satisfied by the state $|\Psi\rangle$ of Eq. (2) but not $|\psi_n\rangle$.

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[10] For $t_A = t_B$ the first term in Eq. (13) contributes $\langle\phi_1|\phi_1\rangle = \alpha^2 N_k$, which follows from the left-hand side of Eq. (14) and $f = 1/N_k^{1/2}$. From Eq. (15), the contribution of the second term, $\langle\phi_2|\phi_2\rangle$, is proportional to the matrix elements of $a_{ka'}b_{kb'}a_{ka}^\dagger b_{kb}^\dagger$ in state $|\Psi\rangle$, which are nonzero only if $k_b = k_0 - k_a$ and $k_b' = k_0 - k_a'$ or if $k_a' = k_a$ and $k_b' = k_b$. This leaves a sum over two independent momenta and gives $\langle\phi_2|\phi_2\rangle \leq 2(\alpha f)^4 N_k^2$. The ratio of the contributions of the two terms is $\langle\phi_2|\phi_2\rangle / \langle\phi_1|\phi_1\rangle \leq 2(\alpha f)^2 << 1$. The cross term $\langle\phi_1|\phi_2\rangle$ cna also be shown to be negligible.

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[18] Equations (24) and (25) violate the original form of Bell's theorem (Ref. [2]), which shows that any local hidden-variable theory must have a linear cusp in P_N at $\Phi = 0$. The fact that the probability may not be unity allows a potential "loophole" that is equivalent to that of the limited detection efficiencies in earlier experiments.

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